15-Phys-202

SUMMER 2003

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Exam 1 Name ____________________

Formulae and constants:

$$x(t) = x_m \cos(\omega t + \phi)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$U(t) = \frac{1}{2} k x^2$$

$$K(t) = \frac{1}{2} m v^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1| |q_2|}{r^2}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$$

$$\Phi = \frac{q_{tot}}{\varepsilon_0}$$

1. An oscillating block-spring system has a mechanical energy of 0.50 J, an amplitude of 10.0 cm, and a maximum speed 1.00 m/s. Find

   (a) the spring constant

   (b) the mass of the block

   (c) the angular frequency of oscillations

   Solution

   (a)

$$E = \frac{1}{2}kx_m^2 \Rightarrow k = \frac{2E}{x_m^2} = \frac{2 \times 0.50}{(0.1)^2} = 100 \text{ N/m}$$

   (b)

$$E = \frac{1}{2}mv_m^2 \Rightarrow m = \frac{2E}{v_m^2} = \frac{2 \times 0.50}{(1.00)^2} = 1.00 \text{ kg}$$

   (c)

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{1.00}} = 10.0 \text{ rad/s}$$

2. Two fixed particles of charges $q_1 = 1\ \mu$C and $q_2 = -4\ \mu$C are 10 cm apart. How far from each should the third charge be located so that no net electrostatic force acts on it?

   Solution

   The third charge q_3 should be to the left of q_1 along the line connecting q_1 and q_2, assuming that q_2 is to the right of q_1. The force on q_3 will be zero when its distance x from q_1 is such that

$$\frac{q_1\,|q_3|}{x^2} = \frac{|q_2|\,|q_3|}{(x+d)^2}$$

   and

$$\begin{aligned}\frac{(x+d)^2}{x^2} &= \frac{|q_2|}{q_1} = 4\\ \frac{(x+d)}{x} &= 2\\ x &= d = 10\text{ cm}\end{aligned}$$

3. A thin glass rod is bent into a semicircle of radius r. A charge $+q$ is uniformly distributed along the upper half and a charge $-q$ is uniformly distributed along the lower half, as shown in the Figure. Find the magnitude and the direction of the electric field $\overrightarrow{E}$ at P , the center of the semicircle.

Solution

By symmetry, the x-components of the electric field of the two quarter-circles cancel out and the y-components add up. Therefore,

$$\overrightarrow{E} = E_y \hat{\mathbf{j}}$$

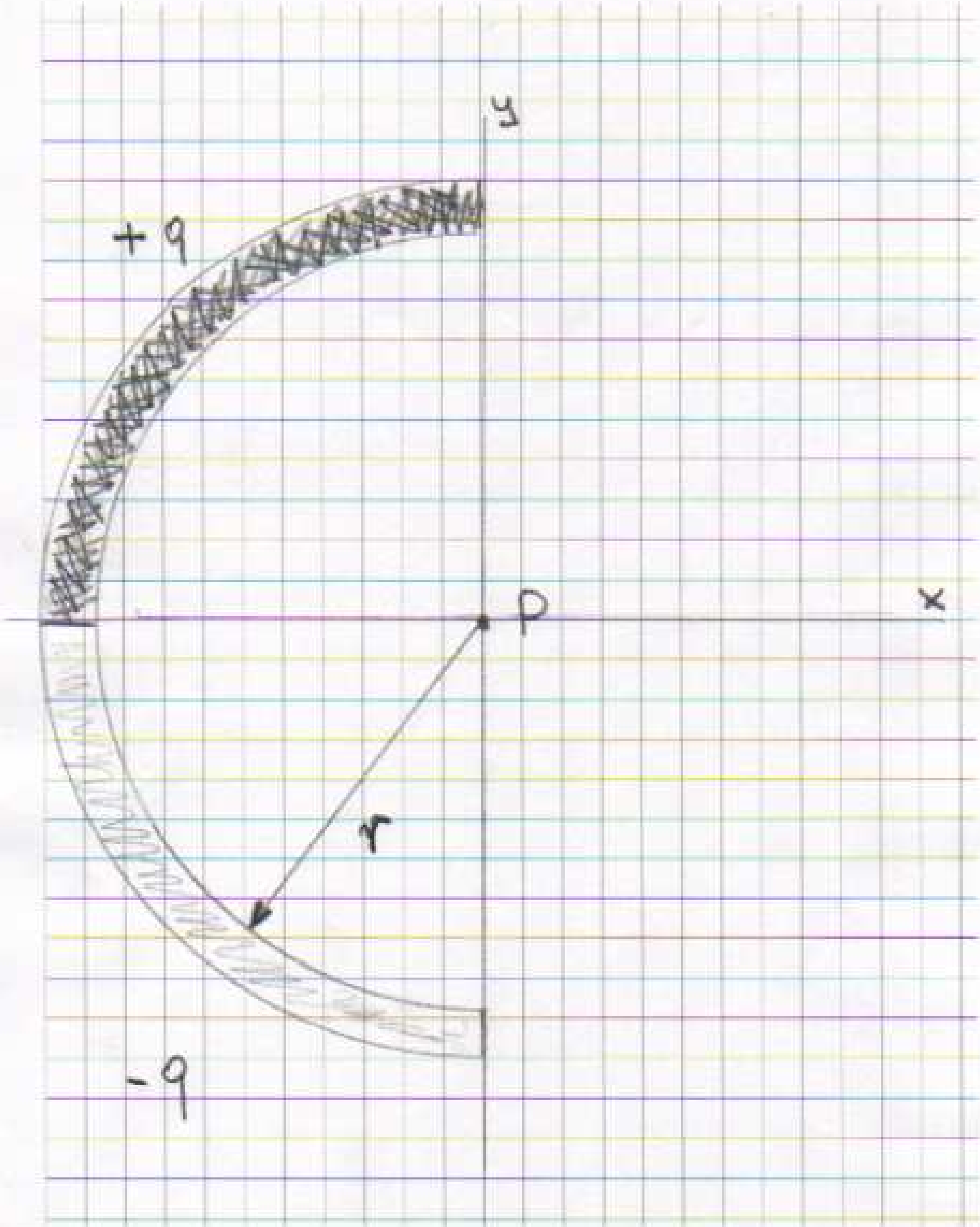
$$E_y = -2 \int_0^{\pi/2} \frac{(\lambda r d\theta) \cos\theta}{4\pi\varepsilon_0 r^2}$$

where

$$\lambda = \frac{q}{(\pi/2)\, r}$$

so that

$$E_y = -\frac{q}{\pi^2 \varepsilon_0 r^2} \int_0^{\pi/2} \cos\theta d\theta = -\frac{q}{\pi^2 \varepsilon_0 r^2} \left[\sin\theta\right]_0^{\pi/2} = -\frac{q}{\pi^2 \varepsilon_0 r^2}$$


y
+q
x
P
r
-q

4. A point charge of 8.85 μC is at the center of a cubical Gaussian surface 1 m on edge.

(a) What is the net electric flux through the surface?

(b) What is the electric flux through each of the cube's faces?

(c) What would be the net flux and flux through each face if each edge of the cube doubled?

Solution

$$\Phi = \frac{q}{\varepsilon_0} = \frac{8.85 \times 10^{-6}}{8.85 \times 10^{-12}} = 10^6 \ \mathrm{N \cdot m^2/C}$$

$$\Phi_1 = \frac{\Phi}{6} = 1.66 \times 10^5 \ \mathrm{N \cdot m^2/C}$$

The answer does not change if the edge length changes.

A cubical Gaussian surface with edge length a is immersed in the uniform electric field $\overrightarrow{E}$ which is perpendicular to a cube's face.

(a) What is the net flux through the cube's surface?

(b) What would be the answer if the angle between the electric field and the normal to the face were $0 < \theta < \pi/2$?

Solution

The flux is zero in both cases.