15-Phys-202

S 2002

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Quiz 4 Name _____

1. Four charges

$$\begin{aligned} q_A &= 12 \text{ nC} \\ q_B &= 6 \text{ nC} \\ q_C &= 4 \text{ nC} \\ q_D &= -8 \text{ nC} \end{aligned}$$

are located in the corners of a square ABCD, whose side is $a = 1$ m. What is the potential energy of this charge configuration?

Hint: The electric potential energy of the pair of point charges is $U = kq_1q_2/r_{12}$, where $k = 1/4\pi\varepsilon_0 = 8.99 \times 10^9$ N·m^2/C^2.

Solution

Noting that $\sqrt{2}a$ is the lengths of AC and BD,

$$\begin{aligned} U &= \frac{k}{a}\left(q_Aq_B + q_Bq_C + q_Cq_D + q_Dq_A + \frac{q_Aq_C + q_Bq_D}{\sqrt{2}}\right) \\ &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)}{(1 \text{ m})}\left(12 \cdot 6 + 6 \cdot 4 + 4 \cdot (-8) + (-8) \cdot 12 + \frac{12 \cdot 4 + 6 \cdot (-8)}{\sqrt{2}}\right)\left(10^{-9}C\right)^2 \\ &= -2.9 \times 10^{-7} \text{ J} \end{aligned}$$

2. Consider two widely separated conducting spheres, 1 and 2, the second having one third the diameter of the first. The larger sphere has initially a charge of $q = 10$ μC and the smaller one is initially uncharged. Now you connect the spheres with a long thing wire.

   (a) How are the final potentials V_1 and V_2 of the spheres related?
   (b) What are the final charges of the spheres?

   Hint: The potential at the surface of a sphere of radius R is $V = q/(4\pi\varepsilon_0 R)$.

   Solution

   (a) The potentials will be the same, $V_1 = V_2$.
   (b) The potential of a sphere is $V = q/4\pi\varepsilon_0 R$ and from $V_1 = V_2$ it follows that $q_1/R_1 = q_2/R_2$ and

   $$q_1/q_2 = R_1/R_2 = 3$$

   but

   $$q_1 + q_2 = q = 10\mu C$$

   so that the charges are $q_1 = 7.5$ μC and $q_2 = 2.5$ μC.