15-Phys-202

S 2002

Prof. R.A. Serota
Quiz 3 Name _____

1. A thin glass rod is bent into a one-quarter circle of radius r, as shown in the Figure. A charge $+q$ is uniformly distributed along the rod.

   (a) What is the charge density of the rod?

$$\lambda = \frac{q}{(\pi r/2)} = \frac{2q}{\pi r}$$

   (b) Evaluate the electric field component E_x at the origin (center of the curvature).

$$\begin{aligned} E_x &= \int dE_x = \int dE\cos\theta = \int_0^{\pi/2} \frac{\lambda\,(rd\theta)}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\int_0^{\pi/2}\cos\theta d\theta \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin\theta\right]_0^{\pi/2} = \frac{\lambda}{4\pi\varepsilon_0 r} = \frac{q}{2\pi^2\varepsilon_0 r^2} \end{aligned}$$

   (c) Evaluate the electric field component E_y at the origin. Compare with E_x and explain.

$$\begin{aligned} E_y &= \int dE_y = -\int dE\sin\theta = -\int_0^{\pi/2} \frac{\lambda\,(rd\theta)}{4\pi\varepsilon_0 r^2}\sin\theta = -\frac{\lambda}{4\pi\varepsilon_0 r}\int_0^{\pi/2}\sin\theta d\theta \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}\left[\cos\theta\right]_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 r} = -\frac{q}{2\pi^2\varepsilon_0 r^2} \end{aligned}$$

   No calculation necessary since, by symmetry, $|E_y| = |E_x|$

   (d) Find the magnitude and the direction of the electric field $\overrightarrow{E}$ at the origin.

$$\overrightarrow{E} = \frac{q}{2\pi^2\varepsilon_0 r^2}\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)$$

$$E = \sqrt{E_x^2 + E_y^2} = \frac{q}{\sqrt{2}\pi^2\varepsilon_0 r^2}$$

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = -\frac{\pi}{4} = -45^\circ$$