15-Phys-203

Summer 2003

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Exam 1

Name ______________________________

Formulae and constants:

$$\triangle E_{int} = Q - W$$

$$W = \int_{V_i}^{V_f} p dV$$

$$pV = nRT$$

$$R = 8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

$$Q = nC\Delta T$$

$$E_{int} = nC_V T$$

$$C_V = \frac{f}{2} R$$

$$C_p = C_V + R$$

$$pV^{\gamma} = const, \; \gamma = \frac{C_p}{C_V} \text{ (adiabatic process)}$$

$$\Delta S = \int_i^f \frac{dQ}{T}$$

1. Gas within a closed chamber undergoes the cycle shown in the p-V diagram.

   (a) What is the net change of internal energy in the cycle?

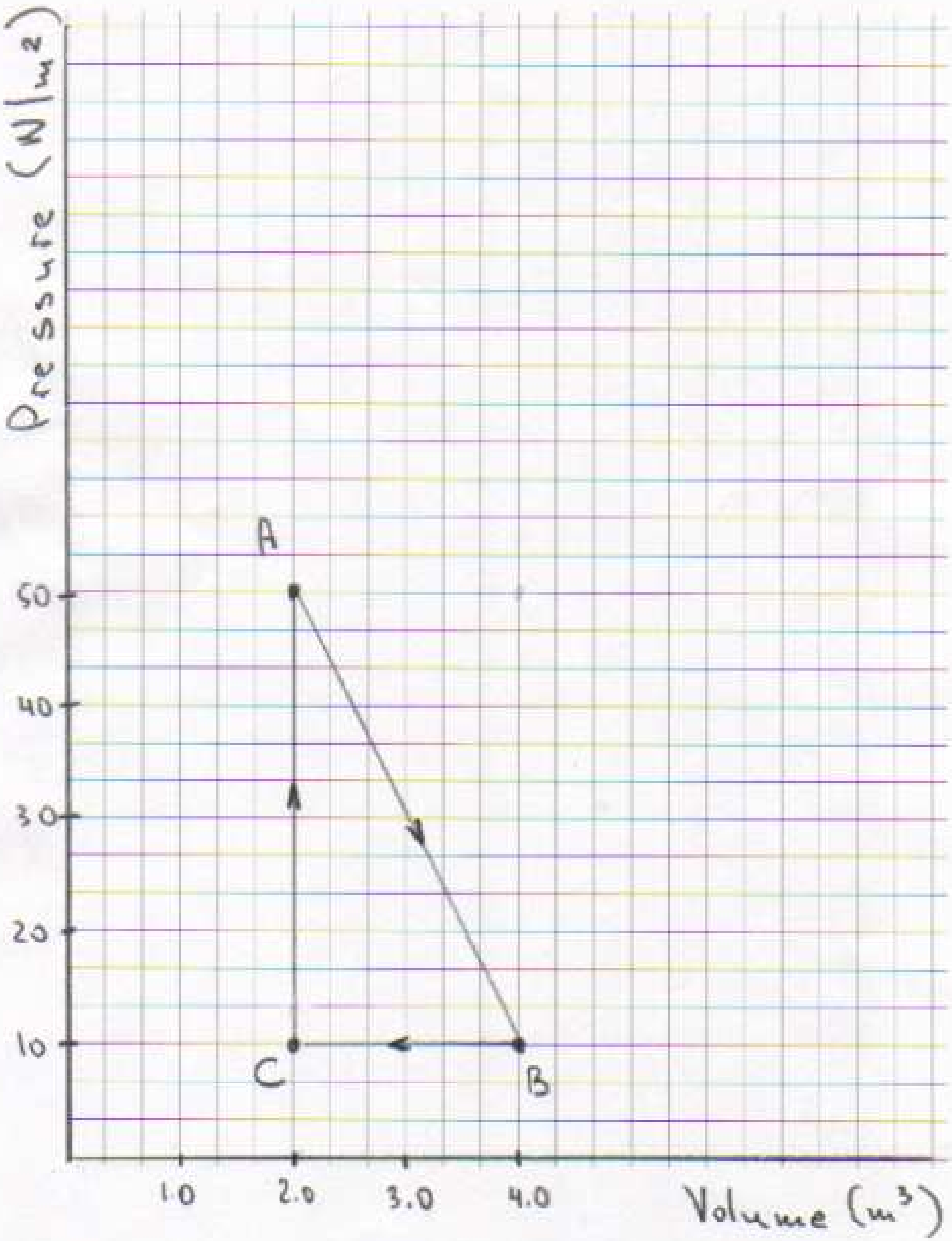
   (b) What is the net change of entropy?

   (c) Is the net energy added or extracted from the system in the form of heat? - Calculate the heat.

   Solution

$$\Delta E_{int} = 0,\ \Delta S = 0$$

$$Q = W = (50 - 10)\,(4 - 2)/2 = 40 \text{ J}$$


Pressure (N/m²)
50
40
30
20
10
A
B
C
1.0
2.0
3.0
4.0
Volume (m³)

3. One mole of oxygen O_2 is heated at constant pressure starting at 273 K (0°C). How much energy must be added to the gas as heat to double its volume? *Hint*: $f = 5$.

Solution

$$Q = nC_p \left(T_f - T_i\right)$$

but for an isobaric process

$$\frac{T_f}{T_i} = \frac{V_f}{V_i}$$

so that

$$\begin{aligned} Q &= nC_p \left(T_f - T_i\right) = nC_p T_i \left(\frac{V_f}{V_i} - 1\right) \\ &= nC_p T_i = \frac{7}{2} R T_i = \frac{7}{2} \left(8.31\right) \left(273\right) \approx 8 \text{ kJ} \end{aligned}$$

since $V_f / V_i = 2$.

4. An ideal diatomic gas, whose molecules are rotating but not oscillating ($f = 5$), is taken through the following cycle:

$$(p_1,V_1,T_1) \overset{\text{isothermal}}{\longrightarrow} (p_2,V_2,T_1) \overset{\text{isochoric}}{\longrightarrow} (p_3,V_2,T_3) \overset{\text{adiabatic}}{\longrightarrow} (p_1,V_1,T_1)$$

where

$$V_2 = 3V_1$$

(a) Sketch the p-V diagram.

(b) Determine p_2, p_3, and T_3 in terms of p_1 and T_1.

Solution

$$p_1V_1 = p_2V_2 \Rightarrow p_2 = p_1\frac{V_1}{V_2} = \frac{p_1}{3}$$

$$p_3V_2^\gamma = p_1V_1^\gamma \Rightarrow p_3 = p_1\left(\frac{V_1}{V_2}\right)^\gamma = \frac{p_1}{3^{7/5}}$$

$$T_1V_1^{\gamma-1} = T_3V_2^{\gamma-1} \Rightarrow T_3 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{T_1}{3^{2/5}}$$

or, equivalently,

$$\begin{aligned}\frac{p_2}{T_1} &= \frac{p_3}{T_3} \Rightarrow T_3 = T_1\frac{p_3}{p_2} = T_1\frac{p_3}{p_1}\frac{p_1}{p_2} \\ &= T_1\left(\frac{V_1}{V_2}\right)^\gamma\left(\frac{V_1}{V_2}\right)^{-1} = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}\end{aligned}$$

5. Calculate the change of entropy of 2.00 moles of ideal gas as it is isothermally compressed to half its initial volume.

Solution

$$\Delta S = \int_i^f \frac{dQ}{T} = \frac{Q}{T}$$

where

$$Q = nRT \ln \frac{V_f}{V_i}$$

so that

$$\Delta S = nR \ln \frac{V_f}{V_i} = -\,(2.00)\,(8.31)\,(\ln 2) = -11.52 \text{ J/K}$$