15-Phys-203

S 2002

Prof. R.A. Serota

Quiz 3 Name ______________________

1. Container A in the Figure, whose volume is $V_A = 0.10\ \text{m}^3$, holds an ideal monatomic gas at a pressure $p_A = 2.0 \times 10^5$ Pa and temperature $T_A = 200$ K. It is connected by a thin tube and a closed valve to a container B of the same volume, $V_B = V_A$, which contains the same gas at a pressure $p_B = 2p_A$ and temperature $T_B = 2T_A$. Both containers (and the tube) are thermally insulated.

   (a) Calculate the internal energy in each of the containers and the total internal energy.

   Hint: Using the formulae,

$$E_{int} = \frac{3}{2} nRT$$
$$pV = nRT$$

   derive the expression for E_{int} in terms of the pressure p and volume V.

$$E_{int} = \frac{3}{2} nRT = \frac{3}{2} pV$$
$$E_A = \frac{3}{2} p_A V_A = 3.0 \times 10^4 \text{ J}$$
$$E_B = \frac{3}{2} p_B V_B = \frac{3}{2} (2p_A) V_A = 2E_A = 6.0 \times 10^4 \text{ J}$$
$$E = E_A + E_B = 3E_A = 9.0 \times 10^4 \text{ J}$$

   The valve is now opened to allow the pressure to equalize.

   (b) What is the internal energy of the gas in the two containers?

   Since the system is insulated and no work is done, the internal energy does not change.

   (c) What is the temperature in the two containers?

$$E = \frac{3}{2} (n_A + n_B) RT = \frac{3}{2} \left(\frac{p_A V_A}{RT_A} + \frac{p_B V_B}{RT_B} \right) RT = \frac{3}{2} \left(\frac{p_A V_A}{T_A} + \frac{(2p_A) V_A}{(2T_A)} \right) T = \frac{3p_A V_A}{T_A} T$$
$$T = \frac{E}{3p_A V_A} T_A = \frac{3E_A}{3p_A V_A} T_A = \frac{(3p_A V_A/2)}{p_A V_A} T_A = \frac{3}{2} T_A = 300 \text{ K}$$

   (d) What is the pressure in the two containers?

$$E = \frac{3}{2} p (V_A + V_B) = \frac{3}{2} p (V_A + V_A) = 3pV_A$$
$$p = \frac{E}{3V_A} = \frac{3E_A}{3V_A} = \frac{(3p_A V_A/2)}{V_A} = \frac{3}{2} p_A = 3.0 \times 10^5 \text{ Pa}$$

Comment: $n_A = n_B$, that is we have equal number of moles of the same gas occupying the same volume. Therefore, the final temperature and pressure will be at the midpoint of the temperatures and pressures respectively.