Problem LL §13

$$M\vec{R} + m\sum \vec{R}_a = 0 \quad \leftarrow \quad \vec{R}_a = \vec{r}_a + \vec{R}$$

$$M\vec{R} + mn\vec{R} + m\sum \vec{r}_a = 0 \quad \leftarrow \quad \mu = M + nm$$

$$\mu\vec{R} + m\sum \vec{r}_a = 0, \qquad \vec{R} = -\frac{m}{\mu}\sum \vec{r}_a$$

$$\dot{\vec{R}}_a = \vec{v}_a + \dot{\vec{R}} = \vec{v}_a - \frac{m}{\mu}\sum \vec{v}_a$$

$$\mathcal{L} = \frac{1}{2}M\dot{\vec{R}}^2 + \frac{1}{2}m\sum \dot{\vec{R}}_a^2 - U$$

$$= \frac{1}{2}M\left(\frac{m}{\mu}\right)^2\left(\sum \vec{v}_a\right)^2 + \frac{1}{2}m\sum\left(\vec{v}_a - \frac{m}{\mu}\sum \vec{v}_a\right)^2 - U$$

$$= \frac{1}{2}m\sum v_a^2 - U$$

$$+ \frac{1}{2}M\left(\frac{m}{\mu}\right)^2\left(\sum \vec{v}_a\right)^2 + \frac{1}{2}m\left(\frac{m}{\mu}\right)^2\left(\sum \vec{v}_a\right)^2 \sum_a 1$$

$$- \frac{m^2}{\mu}\left(\sum \vec{v}_a\right)^2$$

$$= \frac{1}{2}m\sum v_a^2 - U + \left(\sum \vec{v}_a\right)^2\frac{m^2}{\mu}\left[\overbrace{\frac{M}{2\mu} + \frac{mn}{2\mu}}^{1/2} - 1\right]$$

$$= \frac{1}{2}m\sum v_a^2 - \frac{m^2}{2\mu}\left(\sum \vec{v}_a\right)^2 - U$$

Problem LL §41 #3

$$\vec p_a = m\vec v_a - \frac{m^2}{\mu}\sum \vec v_a \; ; \quad \vec v_a = \frac{\vec p_a}{m} + \frac{m}{\mu}\sum \vec v_a$$

apply sum do both sides

$$\sum \vec v_a = \frac{1}{m}\sum \vec p_a + \frac{mn}{\mu}\sum \vec v_a$$

$$\frac{M}{\mu}\sum \vec v_a = \frac{1}{m}\sum \vec p_a \; ; \quad \sum \vec v_a = \frac{\mu}{Mm}\sum \vec p_a$$

$$\vec v_a = \frac{\vec p_a}{m} + \frac{\cancel{m}}{\cancel{\mu}}\,\frac{\cancel{\mu}}{\cancel{m}M}\sum \vec p_a = \frac{\vec p_a}{m} + \frac{1}{M}\sum \vec p_a$$

$$H = \frac{1}{2}m \sum\left(\frac{\vec p_a}{m} + \frac{1}{M}\sum \vec p_a\right)^2 - \frac{1}{2}\frac{m^2}{\mu}\left(\frac{\mu}{Mm}\sum \vec p_a\right)^2 + U$$

$$= \frac{1}{2m}\sum \vec p_a^{\,2} + \frac{\cancel{m}}{\cancel{m}M}\left(\sum \vec p_a\right)^2 + \frac{mn}{2M^2}\left(\sum \vec p_a\right)^2$$

$$- \frac{\mu}{2M^2}\left(\sum \vec p_a\right)^2 + U$$

$$= \frac{1}{2m}\sum \vec p_a^{\,2} + U + \frac{1}{2M}\left(\sum \vec p_a\right)^2\left[2 + \overbrace{\frac{mn}{M} - \frac{\mu}{M}}^{-1}\right]$$

$$= \frac{1}{2m}\sum \vec p_a^{\,2} + \frac{1}{2M}\left(\sum \vec p_a\right)^2 + U$$

roblem 66 §50

$$\left(\frac{\partial S_0}{\partial w}\right)_I = 2I\cos^2 w \qquad (1)$$

$$\frac{\partial q}{\partial \omega} = 0 = \text{const}\left[\frac{\cos w}{\omega^{1/2}}\left(\frac{\partial w}{\partial \omega}\right)_q - \frac{\sin w}{2\omega^{3/2}}\right]$$

$$\left(\frac{\partial w}{\partial \omega}\right)_q = \frac{\tan w}{2\omega} \qquad (2)$$

using (1) and (2)

$$\Lambda = \left(\frac{\partial S_0}{\partial \omega}\right)_{q,I} = \left(\frac{\partial S_0}{\partial w}\right)_I \left(\frac{\partial w}{\partial \omega}\right)_q = \frac{I}{2\omega}\sin 2w$$

$$\lambda \equiv \omega$$

$$\dot{I} = -\left(\frac{\partial \Lambda}{\partial w}\right)_{I,\lambda}\dot{\lambda} = -I\,\frac{\dot{\omega}}{\omega}\cos 2w$$

$$\dot{w} = \omega(I,\lambda) + \left(\frac{\partial \Lambda}{\partial I}\right)_{w,\lambda}\dot{\lambda}$$

$$= \omega + \frac{\dot{\omega}}{2\omega}\sin 2w$$

or preliminaries see Add 1 and
Compare with K&S 13.11