$$S_0 = \int p\,dq = \int \sqrt{2mE - m\omega^2 q^2}\,dq$$

$$= \sqrt{2mE} \int \sqrt{1 - \frac{m\omega^2 q^2}{2E}}\,dq = \sqrt{2mE} \int \sqrt{1 - \left(\frac{q}{A}\right)^2}\,dq$$

$$A^2 = \frac{2E}{m\omega^2}$$

$$= \sqrt{2mE}\,A \int \sqrt{1 - \left(\frac{q}{A}\right)^2}\,d\left(\frac{q}{A}\right)$$

$$= \frac{2E}{\omega} \int \sqrt{1 - \left(\frac{q}{A}\right)^2}\,d\left(\frac{q}{A}\right) \overset{\frac{q}{A} = \sin w}{=} \frac{2E}{\omega} \int \cos^2 w\,dw$$

$$= 2I \int \cos^2 w\,dw$$

$$\sin w = \frac{q}{A} = \frac{q}{\sqrt{\frac{2E}{m\omega^2}}}, \quad q = \sqrt{\frac{2E}{m\omega^2}}\sin w$$

$$= \sqrt{\frac{2I}{m\omega}}\sin w$$

$$p = \sqrt{2mE - m\omega^2 q^2} = \sqrt{2mE}\sqrt{1 - \left(\frac{q}{A}\right)^2}$$

$$= \sqrt{2mE}\cos w$$

$$= \sqrt{2Im\omega}\cos w$$

$$S_0 = I\left(w + \frac{\sin 2w}{2}\right)\Big|_{w = w(q,I)}$$

$$\frac{\partial S_0}{\partial I} = \left[w + \frac{\sin 2w}{2} + I\left(1 + \cos 2w\right)\left(\frac{\partial w}{\partial I}\right)_q\right]\Big|_{w = w(q,I)}$$

$$q = \sqrt{\frac{2I}{m\omega}}\sin w$$

$$\frac{\partial q}{\partial I} = 0 = \frac{1}{2}\sqrt{\frac{2}{Im\omega}}\sin w + \sqrt{\frac{2I}{m\omega}}\cos w\left(\frac{\partial w}{\partial I}\right)_q$$

$$\left(\frac{\partial w}{\partial I}\right)_q = -\frac{1}{2I}\,\frac{\sin w}{\cos w}$$

$$\frac{\partial S_0}{\partial I} = \left[w + \sin w\cos w + I\left(2\cos^2 w\right)\left(-\frac{1}{2I}\frac{\sin w}{\cos w}\right)\right]$$

$$= w \quad !$$