

Problem 4.813

$$M\vec{R} + m \sum \vec{R}_a = 0 \quad \leftarrow \quad \vec{R}_a = \vec{r}_a + \vec{R}$$

$$M\vec{R} + m\vec{R} + m \sum \vec{r}_a = 0 \quad \leftarrow \quad \mu = M + m$$

$$\mu \vec{R} + m \sum \vec{r}_a = 0, \quad \vec{R} = -\frac{m}{\mu} \sum \vec{r}_a$$

$$\dot{\vec{R}}_a = \dot{\vec{v}}_a + \dot{\vec{R}} = \dot{\vec{v}}_a - \frac{m}{\mu} \sum \dot{\vec{v}}_a$$

$$\mathcal{L} = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} m \sum \dot{\vec{R}}_a^2 - U$$

$$= \frac{1}{2} M \left(\frac{m}{\mu}\right)^2 \left(\sum \dot{\vec{v}}_a\right)^2 + \frac{1}{2} m \sum \left(\dot{\vec{v}}_a - \frac{m}{\mu} \sum \dot{\vec{v}}_a\right)^2 - U$$

$$= \frac{1}{2} m \sum \dot{v}_a^2 - U$$

$$+ \frac{1}{2} M \left(\frac{m}{\mu}\right)^2 \left(\sum \dot{\vec{v}}_a\right)^2 + \frac{1}{2} m \left(\frac{m}{\mu}\right)^2 \left(\sum \dot{\vec{v}}_a\right)^2 \left[ \frac{m}{\mu} \right]$$
$$- \frac{m^2}{\mu} \left(\sum \dot{\vec{v}}_a\right)^2$$

$$= \frac{1}{2} m \sum \dot{v}_a^2 - U + \left(\sum \dot{\vec{v}}_a\right)^2 \frac{m^2}{\mu} \left[ \frac{M}{2\mu} + \frac{m}{2\mu} - \frac{m}{\mu} \right]$$

$$= \frac{1}{2} m \sum \dot{v}_a^2 - \frac{m^2}{2\mu} \left(\sum \dot{\vec{v}}_a\right)^2 - U$$

Problem 6b §41 #3

$$\vec{p}_a = m\vec{v}_a - \frac{m^2}{M} \sum \vec{v}_a; \quad \vec{v}_a = \frac{\vec{p}_a}{m} + \frac{m}{M} \sum \vec{v}_a$$

↑  
apply sum to both sides

$$\sum \vec{v}_a = \frac{1}{m} \sum \vec{p}_a + \frac{m}{M} \sum \vec{v}_a$$

$$\frac{M}{m} \sum \vec{v}_a = \frac{1}{m} \sum \vec{p}_a; \quad \sum \vec{v}_a = \frac{M}{Mm} \sum \vec{p}_a$$

$$\vec{v}_a = \frac{\vec{p}_a}{m} + \frac{m}{M} \frac{M}{Mm} \sum \vec{p}_a = \frac{\vec{p}_a}{m} + \frac{1}{M} \sum \vec{p}_a$$

$$H = \frac{1}{2} m \sum \left( \frac{\vec{p}_a}{m} + \frac{1}{M} \sum \vec{p}_a \right)^2 - \frac{1}{2} \frac{m^2}{M} \left( \frac{M}{Mm} \sum \vec{p}_a \right)^2 + U$$

$$= \frac{1}{2m} \sum \vec{p}_a^2 + \frac{m}{M} \left( \sum \vec{p}_a \right)^2 + \frac{m}{2M^2} \left( \sum \vec{p}_a \right)^2$$

$$- \frac{M}{2M^2} \left( \sum \vec{p}_a \right)^2 + U$$

$$= \frac{1}{2m} \sum \vec{p}_a^2 + U + \frac{1}{2M} \left( \sum \vec{p}_a \right)^2 \left[ \frac{m}{M} + \frac{m}{M} - \frac{M}{M} \right]$$

$$= \frac{1}{2m} \sum \vec{p}_a^2 + \frac{1}{2M} \left( \sum \vec{p}_a \right)^2 + U$$