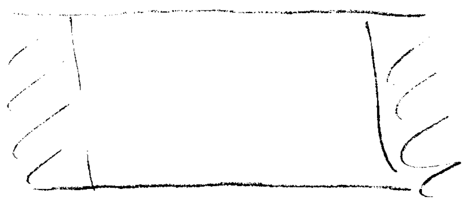
$$\sigma = e^2 D N$$

$$G = e^2 D N L^{d-1} / L = e^2 D N L^{d-2}$$

$$g = \frac{G}{e^2/\hbar} = \hbar D N L^{d-2} = \frac{\hbar D}{L^2} N L^d = \overbrace{\frac{\hbar}{\tau_{diff}}}^{E_c} \nu = \frac{E_c}{\Delta}$$

ν: level density

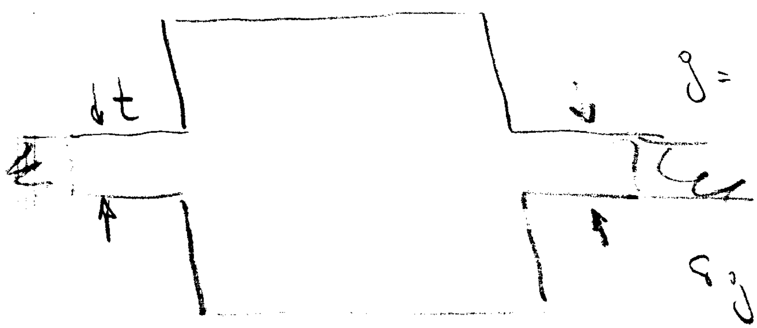
$$\delta g \to \delta D, \quad \delta g \to \delta \nu$$



$$g = \frac{\hbar}{\tau_{diff}} \nu \qquad \tau_{diff} = L^2/D$$

$$\delta g \sim \frac{\hbar}{\tau_{diff}} \delta \nu \sim \frac{\hbar}{\tau_{diff}\,\hbar/\tau_{diff}}$$

$$\sim 1$$



$$g = \frac{\hbar}{\tau_{diff}} \nu \qquad \tau_{diff} = L^2/D \, \frac{L^{d-1}}{t^{d-1}}$$

$$\delta g \sim \frac{\hbar}{\tau_{diff}} \delta \nu \sim \frac{\hbar}{\tau_{diff}\,\hbar/\tau_{diff}}$$

$$\sim 1$$

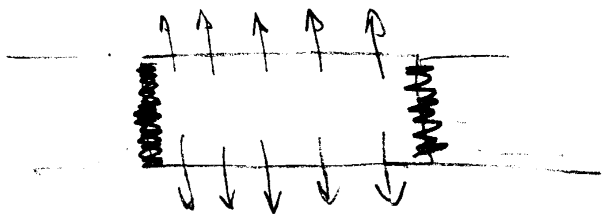
$$\delta g^2 \sim E_c^2 \sum_{\substack{q = \frac{\pi}{L} n \\ n = 1, 2, \ldots}} \frac{1}{(D q^2)^2} \sim \underbrace{\sum_{n=1} \frac{1}{n^4}}_{\zeta(4)}$$

Real space

$P(r,r')$: diffusion propagator

$$D\nabla^2 \xleftarrow{-i\omega} P(r,r') = \delta(r-r')$$

a)



$P(x,y)$

$$P(0,y) = P(L,y) = 0$$

$$\frac{\partial P}{\partial y}(x,0) = \frac{\partial P}{\partial y}(x,L) = 0$$

$$\delta(x-x') = \frac{2}{L}\sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\sin\frac{n\pi x'}{L} = \frac{2}{L}\sum_{n=0}^{\infty}\sin\frac{n\pi x}{L}\sin\frac{n\pi x'}{L}$$

$$\delta(y-y') = \frac{1}{L} + \frac{2}{L}\sum_{n=1}^{\infty}\cos\frac{n\pi y}{L}\cos\frac{n\pi y'}{L} = \frac{2}{L}\sum_{n=0}^{\infty}\varepsilon_n\cos\frac{n\pi y}{L}\cos\frac{n\pi y'}{L}$$

$$\varepsilon_n = \begin{cases} 2, & n=0 \\ 1, & n\neq 0 \end{cases}$$

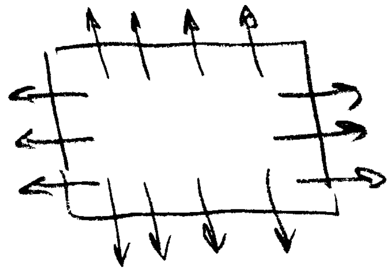
$$P = \sum_{n,m=0}^{\infty} A_{nm}\sin\frac{n\pi x}{L}\sin\frac{n\pi x'}{L}\cos\frac{m\pi y}{L}\cos\frac{m\pi y'}{L}$$

$$-\frac{D\pi^2}{L^2}(n^2+m^2)A_{nm} = \left(\frac{2}{L}\right)^2\varepsilon_m$$

$$P = -\frac{4}{D\pi^2}\sum_{n,m=0}^{\infty}\frac{\varepsilon_m}{\underbrace{n^2+m^2}_{\geq 1}}\sin\frac{n\pi x}{L}\sin\frac{n\pi x'}{L}\cos\frac{m\pi y}{L}\cos\frac{m\pi y'}{L}$$

3)

b)



$$P = -\frac{4}{D\pi^2}\sum_{n,m=0}^{\infty}\frac{\varepsilon_{n,m}}{n^2+m^2 + i\omega (L^2/D)}\cos\frac{n\pi x}{L}\cos\frac{n\pi x'}{L}\cos\frac{m\pi y}{L}\cos\frac{m\pi y'}{L}$$

"zero mode" $n = m = 0$ $\quad P = \text{const}$

$g \sim \frac{E_c}{\Delta}$: number of levels in band of width E_c

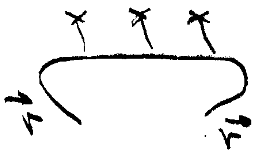
$$\int_{-E_c/2}^{E_c/2} \nu(\varepsilon)\, d\varepsilon = N$$

$$\langle \delta g^2 \rangle_\nu \sim \iint_{-E_c/2}^{E_c/2} \langle \delta\nu(\varepsilon_1)\,\delta\nu(\varepsilon_2)\rangle\, d\varepsilon_1\, d\varepsilon_2 \sim \delta N^2$$

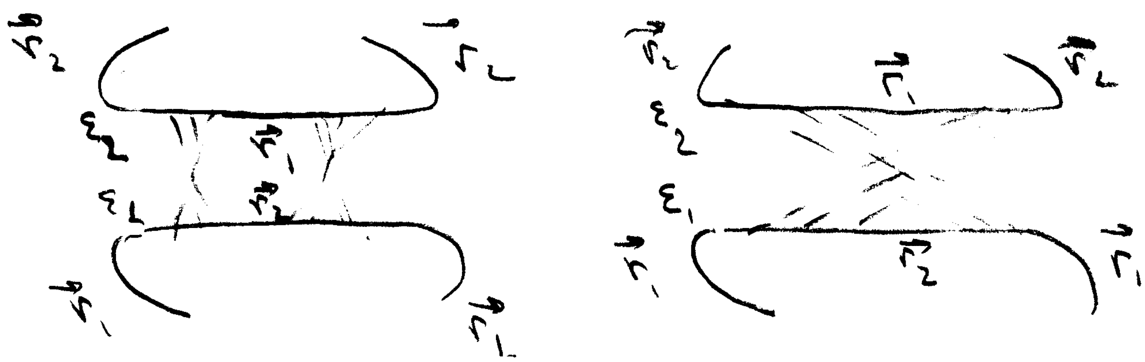
$\delta\nu(\varepsilon) = \nu(\varepsilon) - \langle \nu \rangle$, $\quad \langle \nu \rangle^{-1} = \Delta$: mean interlevel spacing

$$G(r,r';\varepsilon) = \sum_s \frac{\varphi_s^*(\vec r\,')\,\varphi_s(\vec r)}{\varepsilon - \varepsilon_s + i\delta\,\mathrm{sgn}\,\varepsilon_s}$$

$$\int \mathrm{Im}\, G(\vec r,\vec r,\varepsilon)\,d\vec r = \sum_s \delta(\varepsilon-\varepsilon_s) = \nu(\varepsilon)$$



$\langle \delta\nu(\varepsilon_1)\,\delta\nu(\varepsilon_2)\rangle$



$$= \frac{s^2}{2\pi^2}\mathrm{Re}\int d\vec r_1\, d\vec r_2 \left\{ \left[P^{(D)}_{\varepsilon_1-\varepsilon_2}(\vec r_1,\vec r_2)\right]^2 + \left[P^{(C)}_{\varepsilon_1-\varepsilon_2}(\vec r_1,\vec r_2)\right]^2 \right\}$$

$$= -\frac{s^2}{\pi^2}\mathrm{Re}\sum_n \frac{1}{(\varepsilon_1-\varepsilon_2+i\gamma+iDq^2)^2}$$

$q_i^2 = \dfrac{\pi^2 n_i^2}{L_i^2}$ $\qquad n_i = 0, \pm 1, \pm 2 \ldots$ "zero mode"

$$\langle \delta\nu(\varepsilon_1)\,\delta\nu(\varepsilon_2)\rangle = -\frac{s^2}{\pi^2}\,\mathrm{Re}\,\frac{1}{(\varepsilon_1-\varepsilon_2+i\gamma)^2}$$

$$\langle\delta N^2\rangle = -\frac{s^2}{\pi^2}\iint_{-\frac{E_c}{2}}^{E_c/2} d\varepsilon_1\, d\varepsilon_2\, \mathrm{Re}\,\frac{1}{(\varepsilon_1-\varepsilon_2+i\gamma)^2}$$

in the band of width E_c

$$= -\frac{s^2}{\pi^2}\,\mathrm{Re}\int_{-E_c/2}^{E_c/2} d\varepsilon_1 \left(\frac{1}{\varepsilon_1-\frac{E_c}{2}+i\gamma} - \frac{1}{\varepsilon_1+\frac{E_c}{2}+i\gamma}\right)$$

$$= -\frac{s^2}{\pi^2}\,\mathrm{Re}\left\{\ln i\gamma - \ln(E_c+i\gamma) - \ln(E_c+i\gamma) + \ln i\gamma\right\}$$

$$= \frac{2s^2}{\pi^2}\,\mathrm{Re}\,\ln\frac{E_c+i\gamma}{i\gamma} = \frac{s^2}{\pi^2}\ln\frac{E_c^2+\gamma^2}{\gamma^2} \sim 1$$

when open boundaries, no zero mode iD/L^2, $i\gamma = E_c$

(*) 1 instead of s^2

$$\overline{\langle\delta N^2\rangle} \overset{\delta\to 0}{=} -\frac{1^{(*)}}{\pi^2}\iint f(\varepsilon_1)f(\varepsilon_2)\;\mathrm{Re}\,\frac{1}{(\varepsilon_1-\varepsilon_2\pm\mu_B H\mp i\delta)^2}$$

$$\overset{\delta\to 0}{=} -\frac{1}{\pi^2}\,\mathrm{Re}\iint f(\varepsilon_1)\,\big(1-f(-\varepsilon_2)\big)\,\frac{1}{(\varepsilon_1-\varepsilon_2\pm\mu_B H+i\delta)^2}$$

$$= \frac{1}{\pi^2}\,\mathrm{Re}\iint f(\varepsilon_1)\,f(-\varepsilon_2)\,\frac{1}{(\varepsilon_1-\varepsilon_2\pm\mu_B H+i\delta)^2}$$

$$\int d\varepsilon_1\, d\varepsilon_2 \to \int d(\varepsilon_1+\varepsilon_2)\, d(\varepsilon_1-\varepsilon_2)$$

isolade pard in $\overline{\langle\delta N^2\rangle} \propto H^2$

$\gamma \to 0$? a ploblem

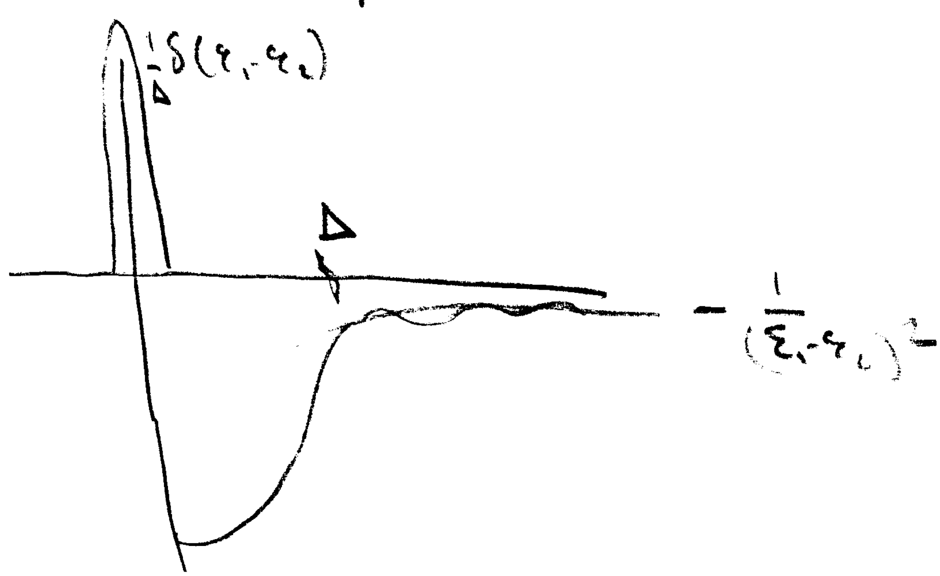
unitary case (Cooperon suppressed)

$$\langle \delta\nu(\varepsilon_1)\delta\nu(\varepsilon_2)\rangle = -\frac{s^2}{2\pi^2}\,\mathrm{Re}\,\frac{1}{(\varepsilon_1-\varepsilon_2+i\gamma)^2}\times\left[1-e^{i\frac{2\pi}{\Delta}(\varepsilon_1-\varepsilon_2+i\gamma)}\right]$$

$$\to -\frac{s^2}{2\pi^2}\,\mathrm{Re}\,\frac{1}{(\varepsilon_1-\varepsilon_2+i\gamma)^2}\,,\quad \gamma \gg \Delta$$

$$\to \left\{\frac{1}{\Delta}\delta(\varepsilon_1-\varepsilon_2)-\frac{\sin^2(\pi(\varepsilon_1-\varepsilon_2)/\Delta)}{\pi^2(\varepsilon_1-\varepsilon_2)^2}\right\}s^2,\quad \gamma\to 0$$

$\sin^2 \to \frac{1}{2}$: perturbadive result



Level repulsion

a) $\int d\omega \langle \delta\nu(\varepsilon+\omega)\,\delta\nu(\varepsilon)\rangle = 0$

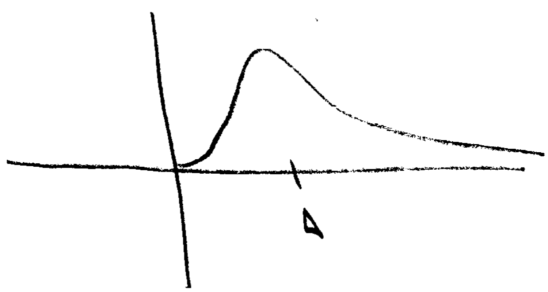
b) From δ-fn, $\delta N^2 \sim N$

cancellation between $\delta(x)$ and $-\frac{\sin^2 x}{x^2}$

on the scale of $\Delta \longrightarrow$ then just $-\frac{1}{x^2}$

and $\ln N$

c) $P(x) \simeq \frac{\pi}{2} x\, e^{-\pi x^2/4}$ (orthogonal)



$x = s/\Delta$

Characteristic of classically chaotic systems

Level clustering

$E_{mn} = \alpha m^2 + n^2$ (algebraic better than transcendental, golden ratio best)

(from $\frac{\hbar^2 \pi^2}{L_x^2}\left(n^2 + m^2 \frac{L_x^2}{L_y^2}\right)^{\alpha}$)

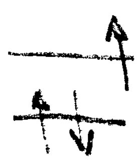
$P(s) = e^{-s} \iff \delta N^2 \sim N$ Characteristic of classically integrable sys

Notice $\int d\omega \langle \delta\nu(\varepsilon+\omega)\delta\nu(\varepsilon)\rangle = \text{const}$ (no "level convers")

Discuss Gutzwiller's formula

Consequences

1) Zeeman response



$M = 2\mu_B \,\text{prob.}\,(s < \mu_B H)$

a) Wigner $\simeq \frac{1}{\Delta^2}\int_0^{\mu_B H} dx\, x = \frac{(\mu_B H)^2}{2\Delta^2} \Rightarrow M = \frac{\mu_B^2}{\Delta}\frac{\mu_B H}{\Delta}$

Notice! $\chi \sim \chi_p \frac{T}{\Delta}$ $\quad (\chi_p H)\frac{T}{\Delta} \xleftarrow{\text{finite } T} = (\chi_p H)\frac{\mu_B H}{\Delta}$

b) Poisson $\simeq \frac{1}{\Delta}\int_0^{\mu_B H} dx = \frac{\mu_B H}{\Delta}$

$\chi \sim \chi_p$

2) $\chi_{vv} = -\frac{\mu_B^2}{V}\sum_k \frac{|\langle k|L_z|0\rangle|}{E_k - E_0}$