Misallocation, Informality, and Human Capital: Understanding the Role of Institutions∗

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Abstract

The aim of this paper is to quantify the role of formal-sector institutions in shaping the demand for human capital and the level of informality. We propose a firm dynamics model where firms face capital market imperfections and costs of operating in the formal sector. Formal firms have a larger set of production opportunities and the ability to employ skilled workers, but informal firms can avoid the costs of formalization. These firm-level distortions give rise to endogenous formal and informal sectors and, more importantly, affect the demand for skilled workers. The model predicts that countries with a low degree of debt enforcement and high costs of formalization are characterized by relatively lower stocks of skilled workers, larger informal sectors, low allocative efficiency, and measured TFP. Moreover, we find that the interaction between entry costs and financial frictions (as opposed to the sum of their individual effects) is the main driver of these differences. This complementarity effect derives from the introduction of skilled workers, which prevents firms from substituting labor for capital and in turn moves them closer to the financial constraint.

Keywords: Financial Structure, Informal Sector, Productivity, Policy Distortions, Human Capital.

JEL Classifications: D24, E26, J24, L11, O16, O17

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1 Introduction

The question this paper addresses is, what role do formal-sector institutions play in the size of the informal sector, the stock of human capital, and measured productivity? In particular, we explore how the demand for human capital interacts with formal-sector institutions (entry costs to the formal sector, tax structure, and efficiency of debt enforcement). In summary, we find that formal-sector entry costs, financial frictions, and taxes are complementary.

We develop a general equilibrium model of firm dynamics with an endogenous demand for human capital.\(^1\) The model also displays endogenous physical capital financing and default decisions and allows for the existence of formal and informal sectors. Although entering and operating in the formal sector is costly, these firms have access to an expanded set of production possibilities and the ability to employ skilled workers. Firms operating in the formal sector face an endogenously lower cost of borrowing because they have access to credit markets with a higher degree of enforcement than those in the informal sector. In our quantitative exercise, we calibrate the model to the US economy and then impose country-specific formal-sector institutions, which are based on those measured by the World Bank as reported in its Doing Business database, to analyze then effects on informality, skill formation, and total factor productivity.\(^2\)

Our focus is on understanding the mechanics of each institution; therefore the main results are presented through a set of counterfactuals where we analyze the individual and joint changes in the costs of formality. We find that the complementary effect of entry costs and financial frictions is the key to understanding the total effect on total factor productivity (TFP), informality, and human capital when moving from the US to developing economies.\(^3\) Individually, neither

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1Our intention is to highlight the role of the demand of human capital. For this reason, we simplify the skill-formation technology (i.e., the supply side) as much as possible. Rich models that focus on the production of human capital include those of Mankiw, Romer and Weil (1992), Manelli and Seshadri (2010), and Erosa, Koreshkova, and Restuccia (2010).

2In order to isolate the effects of institutional differences, we assume that all countries have access to the same production possibilities.

3Relative to the US (which is our benchmark calibration), the model generates up to a 37% decrease in TFP, a 10 times larger informal sector and as large as a 60% decline in the stock of skilled workers when formal sector
the entry cost nor the financial frictions generate significant changes in productivity, informality, or human capital. However, when they are analyzed together, we find that the complementary effect (joint effect of entry costs and financial frictions net of their individual effect) explains a large fraction of the difference between the US and Low Middle Income countries in terms of TFP, informal labor force, and skilled workers (27%, 64%, and 68% of the total difference respectively). We compute the same counterfactual in a model without human capital and show that there is no complementary effect between frictions when human capital is absent.\textsuperscript{4} The intuition is simple. The introduction of relatively expensive skilled workers increases the incentives to substitute away from labor and toward investment in physical capital. However, in the presence of financial frictions, this means that the firms move closer to the financial constraint and effectively pay higher interest rates. These results are in line with what Bergoeing, Loayza and Piguillem (2011) find in a model with technology adoption.

In order to quantify the role of human capital on measured total factor productivity and informality, we run a counterfactual with no skilled workers. We find that the model with human capital generates a drop in measured TFP that is 48% larger than in the model with no human capital. Moreover, we find that the increase in the size of the informal sector is more in line with the data when human capital is present. Finally, we study the role of informality and show that the introduction of the informal sector is quantitatively important as well. The counterfactual with no informal sector generates a reduction in measured TFP, relative to the US, that is 27% smaller than the drop produced by the benchmark model and generates a minimal change in the demand for human capital (as opposed to a 67% drop in our benchmark with informal sector and human capital). In short, when the three formal-sector institutional frictions are introduced together they have a larger impact on TFP, human capital, and informality than the sum of the effects of these frictions when each one is introduced separately.\textsuperscript{5}

\textsuperscript{4}The environment with no human capital is similar to that presented in D’Erasmo and Moscoso Boedo (2012).
\textsuperscript{5}Because only firms in the formal sector are able to hire skilled workers and factor shares are unaffected,
Our work is motivated by the observed cross-country differences in TFP, human capital, and informality and their correlations with formal-sector institutions. There are important differences in human capital in developed versus developing countries. Barro and Lee (2000) document that, in the developing world, in the year 2000, 37% of the population over 25 had no formal schooling and only 27% had some secondary education. On the other hand, in advanced and transition economies, approximately two-thirds of the population over 25 had some secondary education. The correlation between GDP per capita and skills equals 0.642. Other well know fact is the strong and positive relationship between GDP per capita and measured TFP. However, Jones and Romer (2010) document that differences in measured inputs explain less than half of the cross-country differences in per capita GDP. The aim of this paper is to connect institutions in the formal sector across countries to resource misallocation and human capital formation and evaluate their effects on measured TFP.

Informal activity is negatively correlated with aggregate productivity and the stock of human capital (correlations are -0.828 and -0.693 respectively). Agents involved in the informal sector make explicit efforts not to be detected, which makes measuring this sector extremely challenging. The fraction of the labor force that is engaged in production outside of the formal sector ranges from around 10% in developed countries to almost 100% at the low end of the income distribution. Although the measures of informality are extremely noisy, such a large sector of the economy cannot be ignored when analyzing cross-country differences in economic development. This relationship is also supported by firm-level data. Pratap and Quintin (2008)

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6Skills are defined as the percentage of people that completed college as a percentage of the population over 25 years old, were taken from Barro and Lee (2000).

7The measure of informality corresponds the fraction of the labor force in the informal sector. This is measured as the fraction of the labor force not covered by a pension scheme, WDI (2006). We focus on the the share of labor force not covered by pension schemes because it provides a better direct measure of informality for the US, the country we use for our benchmark calibration. However, this measure is highly correlated with most measures of the informal sector, either direct or indirect.

8The size of the informal activity varies according to the measure used, but the empirical correlations remain unaffected. See Schneider and Enste (2000) for direct and indirect approaches to calculating informal activity.
report that the informal sector is characterized by small scale, unskilled, and self-financed activities. Furthermore, in a cross-country study of Latin America countries, Funkhouser (1996) shows that the mean education level in the formal sector is substantially higher than in the informal sector.

The model’s predictions are consistent with the macro and micro facts of the informal sector described above: more specifically, at the calibrated parameters and measured institutions. We find a strong negative correlation between the level of informality, the stock of human capital, and income per capita. Moreover, the informal sector is characterized by very small, relatively unproductive, young firms, whereas the formal sector exhibits ever larger firms in countries with underdeveloped institutions. As we move along the development spectrum, poor countries display a bimodal distribution of firms, with many small and large ones, but not many midsized firms. This feature has been described in the empirical literature as the “missing middle” and is one of the main determinants of the negative relationship between aggregate total factor productivity and income per capita.

Our approach to firm dynamics originates with Hopenhayn (1992) and Hopenhayn and Rogerson (1993), and we add capital markets as in Cooley and Quadrini (2001). Amaral and Quintin (2006) model the informal sector as the endogenous response of managers who are heterogeneous in ability. Their model generates a formal sector that is endogenously skill intensive. In this paper, we contribute to this literature by quantitatively measuring the effects of frictions on informality and the stock of human capital, while uncovering the mechanics of each friction.

Recent related literature on the distributional consequences of frictions follows two approaches in measuring institutional and financial frictions. Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Guner, Ventura, and Xu (2008), Arellano, Bai, and Zhang (2009), and

9As in Rauch (1991) and Loayza (1996), we model the informal sector as an optimal response to the economic environment.
Buera, Kaboski and Shin (2009) back out the implied frictions that firms face to generate the observed distribution of the firms. The second strand of the literature uses the measured frictions documented in the Doing Business data set, as in this paper. Papers in this group include those by Antunes and Cavalcanti (2007), Barseghyan and DiCecio (2010), D’Erasmo and Moscoso Boedo (2012), and Moscoso Boedo and Mukoyama (2012).

The remainder of the paper is organized as follows. Section 2 describes the data, Section 3 presents the model, Section 4 presents the equilibrium, Section 5 explains the calibration for the benchmark case with US level frictions and describes the workings of the model, Section 6 presents the results regarding the effects of each friction on the aggregates, the role of capital-skill complementary, the role of human capital and the role of the informal sector, and Section 7 concludes.

2 Data

2.1 Measured Institutions

We use data from the World Bank Doing Business project to set our institutional differences across countries. This data set provides a quantitative measure of regulations for starting a business, dealing with construction permits, employing workers, registering property, getting credit, protecting investors, paying taxes, trading across borders, enforcing contracts, and closing a business both in terms of time and resources. In this paper, we will focus on the cost of entering the formal sector, the tax rate and the level of tax compliance difficulty (while operating in the formal sector), and the efficiency of the debt-enforcing mechanisms if the firm decides to default on its debt.

The cost of entering the formal sector is constructed as in Moscoso Boedo and Mukoyama

10The construction permits category includes all procedures required for a business in the construction industry to build a standardized warehouse.
(2012). It includes the costs of registering a business and of dealing with licenses to operate a physical locale.\footnote{The data used to generate the cost of dealing with licenses to operate a physical locale are obtained from the World Bank’s Doing Business database as Dealing with Construction Permits. Some of the elements involved in construction permits, such as the cost of connection to basic services, are present when operating a physical locale.} Both consist of a monetary cost and a time cost (which is translated to monetary units by assuming that one worker has to be employed full time in order for the firm to go through the entry process). The cost of entering the formal sector, as a fraction of the wage (denoted by $\omega_n \kappa$), varies greatly across countries, with high levels of $\kappa$ observed only at the very low end of the income distribution. Also, the correlation between the log of entry cost and log GNI per capita is very high at -0.7 and significant at 1%.

In terms of the tax structure, we concentrate on payroll taxes, taxes on profits, and the cost of tax compliance. The tax rates paid on profits, payroll taxes, and cost of tax compliance are respectively denoted by $\tau$, $\tau_\omega$ and $\omega_n c_\tau$, respectively. Cost of tax compliance reflects the time that it takes to pay taxes in each country. We assume there is a full-time unskilled worker during this time who is devoted to the tasks related to tax compliance and therefore we translate time into costs as the worker’s annual wages.

The efficiency of the system in the event of default has two components: a cost component and a recovery rate. The cost of the system $\phi$, reported as a percentage of the estate’s value, includes court fees and the cost of insolvency practitioners, such as legal and accounting fees. The recovery rate $\lambda$ refers to what external lenders obtain once the firm decides to default on its debt. It is effectively zero for many extremely poor countries in sub-Saharan Africa and over 75% in most of the developed economies. It displays a strong correlation of 0.78 with GNI per capita.
2.2 Human Capital and the Informal Sector

The main goal of our paper is to quantify the effects of institutions on the skill distribution, so the definition of “skills” is crucial. It is hard to find an accurate and comprehensive cross-country measure of skills because schooling quality can differ significantly across countries. We follow the standard procedure in the literature and use data on education from Barro and Lee (2000). This data set provides comprehensive coverage for cross-country education attainment up to 1995 and also constructs projections up to the year 2000. Barro and Lee (2000) fill in the missing observations by the perpetual inventory method using the enrollment ratios. The data contains educational attainment data for primary, secondary, and higher levels of education (both completed and not completed) for the population over the age of 25, and the average years of schooling. We define skilled workers as those with completed higher education. According to this definition, skilled individuals account for 30.03% of the population in the US, with the highest level stock of human capital. The lowest level of human capital is in Mozambique with 0.1%.

Informal labor force data is taken from the World Bank Development Indicators (WDI) database (2006), which measures the percentage of the labor force that is not covered by a pension scheme. The share of the labor force not covered by a pension scheme provides a good direct measure of informality for the US, the benchmark country in the calibration and the only direct measure of informality we need for our quantitative exercise. In our sample, all of the countries do have a pension scheme, alleviating the potential drawback of having countries without formal pension schemes. Schneider and Enste (2000) report various alternative measures of the informal sector across countries (highly correlated with our measure), and theirs is the most comprehensive study regarding informality in a cross-country setting. They include indirect estimates of informal output from energy consumption or money demand or from discrepancies between official and actual employment in household surveys.
3 Environment

We build a firm dynamics model, augmenting D’Erasmo and Moscoso Boedo (2012) with human capital. The model is a version of Hopenhayn (1992) that incorporates capital investment and financial frictions as in Cooley and Quadrini (2001). Time is discrete and we set one period to be one year. There are three kinds of entities in the economy: firms, lenders, and consumers. Firms produce the consumption and capital goods. They are the capital owners and pay dividends to the consumers. Each firm chooses to operate in either the formal or the informal sector. Competitive risk-neutral lenders make loans to the corporate sector. Consumers supply both skilled and unskilled labor to the firms. Because our focus is on firm dynamics, we simplify the household problem and skill accumulation as much as possible. We focus on a stationary equilibrium.

3.1 Household Sector

There is an infinitely-lived representative household that maximizes expected utility. Preferences are

\[ U = \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t u(C_t) \right] \]  

(1)

where \( \mathbb{E} \) is the expectation operator, \( C_t \) is aggregate consumption, and \( \beta \in (0, 1) \) is the discount factor.

The household is composed of a unit mass of labor divided between skilled \( (S_t) \) and unskilled workers. Skills can be acquired through home schooling. In order to become skilled, an unskilled worker must remain outside the market for \( x \) years. Given that unskilled labor is the only input in the production of skills, the household decides how to allocate unskilled labor between the production sector and schooling, and the entire stock of skilled workers is allocated to the production sector. We denote the mass of unskilled labor allocated to production by \( N_t \) and
the mass of unskilled labor allocated to schooling by $A_t$. Skilled labor depreciates at the rate $\delta_s$. Thus, the law of motion for human capital is $S_{t+1} = (1 - \delta_s)S_t + (1/x)A_t$.

The wage rate for skilled workers is denoted by $\omega_s$ and the wage rate for the unskilled worker is $\omega_n$. In order to keep the problem as simple as possible, we assume a linear utility function, i.e., $u(C_t) = C_t$. This assumption implies that the skill premium, which would make the household indifferent between allocating resources to the production sector and the formation of new skills, is independent of the wage level. Because we focus on a stationary equilibrium and there is no heterogeneity at the household level, this assumption can be imposed without loss of generality. Also, this implies that the skill premium is constant across countries. In Psacharopoulos and Patrinos (2004) one can see that the skill premium does not display a clear relationship with income, since the countries in the middle of the distribution are the ones with relatively larger premiums for skills.

The consumer is responsible for the creation cost of new firms $c_e$ and consequently owns existing firms in the economy and receives income from the firm’s dividends. Moreover, the household has access to a risk free bond $B_{t+1}$ that is in zero net supply and pays $r_t$ units of the consumption good in the following period. Finally, the household receives a lump sum transfer equal to the total amount of taxes collected.

3.2 Firms and Technology

The unit of production is a single-establishment firm, also understood as a unique investment project. Each project is described by a production function. The production process displays decreasing returns to scale at the firm level. As mentioned earlier, empirical evidence shows that formal-sector workers have more education and earn more as compared to the informal-sector.

\footnote{We refer to firms and establishment interchangeably.}
workers. We assume that the formal sector technology is

\[ f(z, k, s, n) = zk^\alpha s^\epsilon n^\gamma, \]  

(2)

where \( z \) is the exogenous technology shock, \( k \) is physical capital, \( s \) is skilled labor, and \( n \) is unskilled labor.\(^\text{13}\) The informal-sector technology is

\[ g(z, k, n) = zk^\alpha n^\gamma. \]  

(3)

While consistent with the empirical evidence, the main reason for the assumed differences in the use of skilled workers across the formal and informal sector derives from the lack of data on the firm size distribution of informal firms. In one of the counterfactual experiments, we present a version of the model where technology does not differ across sectors and study several levels of capital skill complementarity (See Section 6.2).

There are two processes for \( z \): high (\( h \)) and low (\( l \)). The high productivity process is given by

\[ \ln(z_{t+1}) = (1 - \rho) \ln(\mu_h) + \rho \ln(z_t) + \epsilon_{t+1} \]

with \( \epsilon_{t+1} \sim N(0, (1 - \rho^2)\sigma^2) \), where \( \sigma^2 \) is the variance of \( \ln(z) \), \( \mu_h \) is the mean, and \( \rho \) is the autocorrelation parameter of the process. The use of the high productivity process is restricted to the formal sector. To simplify the exposition of the model, we assume that the low productivity process is a constant given by \( \mu_l \). We also assume that the choice of technology process is irreversible, i.e., once a firm decides on high or low the firm will produce using this process until it exits.\(^\text{14}\) These two processes are calibrated to match the size distribution of formal firms and

\(^{13}\text{This is the simplest production function that abstracts from the interesting implications of assuming different levels of capital-skill complementarity.}\)

\(^{14}\text{This is consistent with the evidence presented in Atkeson and Kehoe (2007), who argue that manufacturing plants needed to be completely redesigned in order to make good use of the new technologies.}\)
the size of the informal sector. Their difference is one of the channels that allows the model to generate capital misallocation together with small establishments operating in the informal sector as observed in the data by Bartelsman et al. (2008) and Perry et al. (2007). Note that the fraction of firms operating under each process is an endogenous outcome of the model and a function of the country-specific frictions. We denote the transition distribution function of $z_t$ by $\eta_j(z_{t+1}, z_t)$ for $j = h, l$.

The assumption of different productivity processes is consistent with the evidence provided by La Porta and Shleifer (2008). They find that firms in the informal sector are fundamentally different from those in the formal sector. They document productivity differences at the firm level between informal firms and small formal firms that range from 100% to 300%. They also find that these differences are permanent and not the result of informal-sector firms operating at a lower scale to avoid detection.\textsuperscript{15,16} Moreover, Bruhn (2011), Bertrand and Kramarz (2001) and McKenzie and Sakho (2007) present empirical evidence that shows that decreases in entry costs do not lead to the formalization of previously informal firms and only generate the creation of new business. Our assumption irreversible technology choice captures this feature. Finally, our assumption that the formal sector is relatively more skill intensive is based on observations by Pratap and Quintin (2008), once we also include the education of the entrepreneur.

Firms maximize expected discounted dividends $d_t$:

$$
E \sum_{t=0}^{\infty} R^t d_t
$$

\textsuperscript{15}For example, differences in sales per worker are much higher (two to three times) than the average entry cost, implying that is not just the barrier to entry the main factor affecting scale, productivity, or the decision to operate informal. Related to this, La Porta and Shleifer (2008) note that, in a sample of developing economies, approximately 91% of registered firms at the time of the survey started as registered firms and do not come from the informal sector. Survey of microfirms in Brazil and Mexico also show that those that start as informal never make any effort to become formal firms.

\textsuperscript{16}Different microfoundations would be consistent with heterogeneous productivity across firms. As in Acemoglu et al. (2007) and Song et al. (2011) we can think of owners of formal firms delegating decision authority to managers (while retaining control) and informal firms not being able to do so due to lack of contract enforcement. Since managers are able to make decisions based on superior information, firms that delegate attain a higher efficiency level.
at the rate $R$. Firms can be created by paying a cost $c_e$. After paying this cost, firms learn their initial level of productivity $z_0$ for the $h$ process. This initial level of productivity is drawn from the distribution $\nu(z_0)$. Draws from this distribution are assumed to be i.i.d across firms. With this knowledge of $z_0$ and $\mu_l$ at hand, firms decide to become formal or informal, or to stay out of the market. If they become formal, firms choose between undertaking either of the two projects available, i.e., $z_0$ or $\mu_l$. If they become informal, they can only undertake the project associated with the low process, i.e., $\mu_l$. Informal firms can choose to formalize in the future. Unimplemented projects go back into the pool.

Formalizing a firm requires an entry cost $\kappa w_n$. The formal-sector incumbent is subject to a proportional tax on profits $\tau$, a cost in unskilled labor units of filing those taxes $c_\tau \omega_n$, and a payroll tax $\tau \omega_n$ for both skilled and unskilled labor.$^{18}$

Operating firms in both the formal and informal sectors pay a random fixed cost of production $c_f$ in every period, measured in units of output, which is iid across firms and over time with distribution $\xi(c_f)$. Establishments own their capital and can borrow from financial intermediaries in the form of non contingent debt $b \geq 0$. They finance investment with either debt or internal funds.

### 3.3 Financial Markets

A competitive credit industry makes loans to firms in the formal and informal sector. Lenders are risk neutral, and have unlimited access to a risk-free asset with return $r_t$. In each period, firms borrow using only one-period non contingent debt denoted by $b$ (i.e., markets are exogenously incomplete). Loans of different sizes for different types of firms are treated as distinct financial assets. Because there is perfect information, the price of the non-contingent bond depends on

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17We only discuss the stationary equilibrium, so without loss of generality we assume that the firms’ discount factor is constant.
18Most countries apply progressive taxes but the tax rates on skilled and unskilled labor might differ. Evaluating the costs as a percentage of the unskilled wage might underestimate the frictions prevailing in the economy.
firms’ characteristics given by their choice of sector (formal or informal), future level of capital \( (k') \), future level of borrowing \( (b') \), and current technology \((z)\). In particular, firms in the formal sector will borrow at price \( q^f_j(k', b', z) \) \( j = h, l \) and firms in the informal sector will borrow at price \( q^i(k', b') \). A default triggers a bankruptcy procedure that liquidates the firm. If a firm defaults in the formal sector, creditors can recover up to a fraction \( \lambda \) of the original loan. The formal bankruptcy procedure has an associated cost equal to a fraction \( \phi \) of the firm’s capital. The values of the recovery rate \( \lambda \) and the bankruptcy cost \( \phi \) are obtained from the Doing Business database. Because the capital of the informal firm is not legally registered, the recovery rate of a loan to an informal-sector firm that defaults is assumed to be zero.

4 Equilibrium

We focus on the stationary equilibrium of the model. In this equilibrium, the wage rates and the schedule of loan prices are constant. Once the wage \( \omega_n \) is solved for, the skill premium equation will determine \( \omega_s \). Every equilibrium function depends on the set of loan prices and the wage rates.

4.1 Consumer Problem

Because we concentrate on the stationary equilibrium, aggregates in the economy are constant. Household maximization implies that the consumer supplies its unit of labor inelastically, that \( \beta = R = \frac{1}{1+r} \), and that aggregate consumption is

\[
C = \omega_n N + \omega_s S + \Pi + T - E + X, \tag{4}
\]

where \( \Pi \) is the total profit, \( T \) is the lump-sum transfer from the income and payroll taxes, \( E \) is the aggregate creation cost, and \( X \) is the exit value of firms.
The unit mass of labor is allocated between unskilled labor, skilled labor, and schooling:

\[ 1 = N + S + A. \]

The benefit to schooling is the discounted value of future skilled wages:

\[ \beta^{x+1} \sum_{t=0}^{\infty} [\omega_s \beta^t (1 - \delta_s)^t]. \]  

(5)

The cost of schooling is the forgone unskilled wages during the \( x \) schooling years:

\[ \sum_{t=0}^{x} \beta^t \omega_n. \]  

(6)

In equilibrium, the household will invest in schooling until he or she is indifferent:

\[ \frac{\omega_s}{\omega_n} = \frac{1 - \beta(1 - \delta_s)}{1 - \beta} \frac{1}{\beta^{x+1}}. \]  

(7)

This equilibrium condition determines the skill premium. Finally, in the steady-state equilibrium, the level of schooling is

\[ \frac{A}{x} = \delta_s S. \]

4.1.1 Formal Sector Incumbent

An incumbent establishment operating in the formal sector with technology \( j \in h, l \) starts the period with physical capital \( k \), debt \( b \), and previous productivity \( z_{-1} \). Then the firm draws the fixed cost of continuing the operation, \( c_f \), and decides to operate the technology, to exit after repayment of debts, or to default and liquidate the firm. The value function of an establishment at this stage is denoted as \( W_j^f(z_{-1}, k, b, c_f) \). The value function of a firm operating in the formal sector is denoted as \( V_j^f(z, k, b, c_f) \).
The incumbent in the formal sector solves the following Bellman equation:

$$W_f^j(z_{-1}, k, b, c_f) = \max \left\{ \int V_f^j(z, k, b, c_f) d\eta_j(z|z_{-1}), \max\{0, (1 - \phi)k - \lambda b\}, k - b \right\}$$ (8)

and

$$V_f^j(z, k, b, c_f) = \max_{n, s, k', b'} d_f^j(z, k, b, c_f) + \beta \int W_f^j(z, k', b', c_f') d\xi(c_f)$$ (9)

s.t.

$$d_f^j(z, k, b, c_f) = (1 - \tau)[f(z, k, s, n) - c_f - \omega_n(1 + \tau\omega)(n + c_f)$$

$$- \omega_s(1 + \tau\omega)s] - k' + (1 - \delta)k + q_f^j(k', b', z) - b \geq 0.$$

The solution to this problem provides the exit decision rule $$\chi_f^j(z_{-1}, k, b, c_f),$$ which takes the value of 0 if the firm continues to operate, 1 if the firm decides to default, and 2 if the firm decides to exit after repayment. We also obtain the optimal capital and debt decision rules, $$k_f^j(z, k, b, c_f)$$ and $$b_f^j(z, k, b, c_f),$$ respectively, for a firm in the formal sector.

Using the exit and default decision rule of the formal firm, we can define the default probability of a formal firm $$p_f^j(k', b', z)$$ as follows:

$$p_f^j(k', b', z) = \int I_{\{\chi_f^j(z, k', b', c_f) = 1\}} d\xi(c_f),$$

where $$I_{\{\cdot\}}$$ is the indicator function that takes value equal to one when the condition in between brackets is true. At a given level of productivity and choices of capital and debt by the formal firm, the default probability integrates over different values of the fixed cost $$c_f$$ to capture those states in which the firm finds optimal to exit by default.
4.1.2 Informal Sector Incumbent

An incumbent establishment operating in the informal sector, after observing the fixed operating cost $c_f$, can choose to stay informal, to pay the formal entry cost $\kappa \omega_n$ and switch operations to the formal sector, or to exit the market after a default. More specifically, the incumbent establishment operating in the informal sector solves the following Bellman equation:

$$ W^i(k, b, c_f) = \max \left\{ V^i(k, b, c_f), \tilde{V}^f_{i}(\mu_l, k, b, c_f), k \right\}, $$

(10)

where the value of staying in the informal sector is

$$ V^i(k, b, c_f) = \max_{n, k', b'} d^i(k, b, c_f) + \beta \int W^i(k', b', c_f') d\xi(c_f) $$

(11)

s.t.

$$ d^i(k, b, c_f) = g(\mu_l, k, n) - c_f - \omega_n n - k' + (1 - \delta)k + q^i(k', b')b' - b \geq 0. $$

(12)

The value of switching to the formal sector is\(^{19}\)

$$ \tilde{V}^f_{i}(z, k, b, c_f) = \max_{n, s, k', b'} \tilde{d}^i(z, k, b, c_f) + \beta \int W^f_{i}(z, k', b', c_f') d\xi(c_f) $$

(13)

s.t.

$$ \tilde{d}^i(z, k, b, c_f) = (1 - \tau)[f(z, k, s, n) - c_f - \omega_n (1 + \tau_s)(n + c_r + \kappa)] $$

$$ - \omega_s (1 + \tau_o) s] - k' + (1 - \delta)k + q^f_{i}(k', b', \mu_l)b' - b \geq 0. $$

The solution to this problem provides the exit decision rule $\chi^i(k, b, c_f)$ which takes the value of 0 if the firm continues to operate in the informal sector, 1 if the firm decides to default, and 2

\(^{19}\)Note that, at this stage, the relevant state is $z = \mu_l$ and $j = l$. We define this function in general form because we will use it as part as the definition of the entry problem.
if it decides to switch its operations to the formal sector. We also obtain the optimal capital and
debt decision rules $k_i'(k, b, c_f)$ and $b_i'(k, b, c_f)$ for a firm operating in the informal sector, as well
as capital and debt decision rules $\tilde{k}_j'(z, k, b, c_f)$ and $\tilde{b}_j'(z, k, b, c_f)$ for a firm that switches from
the informal sector to the formal sector. While the fact that recoveries in the informal sector
are zero affects the bond price negatively it does not prevent these firms from borrowing since
incumbent firms operating in the informal sector have incentives to continue operating and to
borrow in order to invest.

Similar to the definition of the default probability for a formal firm, we can derive the default
probability of an informal firm using the exit decision rules. Specifically, the default probability
of a informal firm $p_i'(k', b')$ is:

$$p_i'(k', b') = \int I\{\chi_i'(k', b', c_f) = 1\} d\xi(c_f).$$

4.1.3 Entrants

To draw from the pool of ideas, potential entrants pay a creation cost given by $c_e$. The value of
a potential entrant $W_e$ is given by

$$W_e = \int \max\{W_i^0(0, 0, 0), \tilde{V}_h^f(z_0, 0, 0, 0)\} d\nu(z_0) - c_e. \quad (14)$$

Effectively, an entrant has no capital, no debt, and the cost of production $c_f$ equals zero. The
entrant chooses between technologies, conditional on the restriction that the high technology
cannot be operated in the informal sector. The sector and technological decision are made after
paying $c_e$ and observing the productivity level $z_0$. Differences in the volatility of the process,
together with differences in initial productivity, will generate differences in the decisions made
by the entrants and by the potential lenders. That introduces differences in behavior as a
function of volatility and contract enforceability. In equilibrium, $W_e = 0$ will hold. The solution
to this problem provides the entry decision rule $\Xi(z_0)$. This determines the entry productivity
threshold to the formal sector $z_0^*$. More specifically, let $z_0^*$ be the value of initial productivity in
the high process such that

$$W^i(0, 0, 0) = \int \tilde{V}^f(z, 0, 0, 0) \eta(z|z_0^*). \tag{15}$$

Then, since it is possible to show that the value of being in the formal sector is increasing in
the level of productivity, the entry decision rule will be $\Xi^e(z_0) = 1$ for $z_0 \geq z_0^*$ and equal to zero
otherwise. The solution to this problem also provides capital and debt decision rules $\tilde{k}'(z, 0, 0, 0)$
and $\tilde{b}'(z, 0, 0, 0)$ for a firm that starts operating in the formal sector.

4.2 Lenders’ Problem

Lenders make loans to establishments operating in the formal and informal sector and take
prices as given. Profit for a loan $b'$ to a firm in the formal sector with future capital $k'$ is

$$\pi^f_j(k', b', z) = -q^f_j(k', b', z)b' + \frac{1 - p^f_j(k', b', z)}{1 + r}b' + \frac{p^f_j(k', b', z)}{1 + r} \min\{\lambda b', (1 - \phi)k'\}, \tag{16}$$

where $p^f_j(k', b')$ denotes the default probability of this borrower. Profit for a loan $b'$ to a firm in
the informal sector with future capital $k'$ is

$$\pi^i(k', b') = -q^i(k', b')b' + \frac{1 - p^i(k', b')}{1 + r}b', \tag{17}$$

where $p^i(k', b')$ denotes the default probability of a firm operating in the informal sector. In
equilibrium, the schedule of prices will adjust so that $\pi^f_j(k', b', z) = 0$ and $\pi^i(k', b') = 0$ for all
(j, k', b', z), that is the equilibrium price schedule is given by

\[
q^f(k', b', z) = \frac{1 - p^f(k', b', z)}{1 + r} + \frac{p^f(k', b', z)}{1 + r} \min\{\lambda b', (1 - \phi)k'\},
\]

(18)

and

\[
q^i(k', b') = \frac{[1 - p^i(k', b')]}{1 + r}.
\]

(19)

The price schedule determines the cost of borrowing for each combination of \(k', b'\) and \(z'\) (conditional on the sector and technology process). Each contract depends on the endogenous default probability (presented above) and the recovery rate in case of default when lending to formal firms. Properties of the exit decision rule (increasing in the level of debt and fixed cost, decreasing in the level of capital and productivity) translate directly into default probabilities and the price schedule.\(^{20}\) When the probability of default is zero (either because the firm chooses not to exit or chooses to exit by full repayment in the case of formal firms), the price of the bond is equal to the opportunity cost of funds for the lender. As the probability of default increases or the recovery rate in the case of default diminishes, the price of the bond decreases (i.e. interest rates increase).

### 4.3 Definition of Equilibrium

A stationary competitive equilibrium is a set of value functions \(W^f_j, W^i, V^f_j, \tilde{V}\), decision rules (physical capital, human capital, debt, default, exit, and sector), the wage rates \(\omega_s\) and \(\omega_n\), a mass of entrants \(M\), and aggregate distributions of firms in the formal \(\vartheta(k, b, z, j; M)\) and informal \(\hat{\vartheta}(k, b; M)\) sectors, such that:

1. Given prices, the value function of the firms and the decision rules are consistent with

\(^{20}\)See Chaterjee, Corbae, Nakajima and Rios Rull (2007) for a formal proof of existence in a similar environment.
firms’ optimization.

2. The free entry condition is satisfied: \( W_e = 0 \).

3. Lenders make zero profit for every type of loan.

4. Distributions \( \vartheta \) and \( \tilde{\vartheta} \) are stationary.

5. Aggregate consumption: \( C = \omega_n N + \omega_s S + \Pi + T - E + X \).

6. The bond market clears: \( B = 0 \).

7. The labor market clears

\[
1 = A + N + S,
\]

\[
S = \sum_j \int s(z, k) d\vartheta(k, b, z, j; M),
\]

\[
N = \sum_j \int n(z, k) d\vartheta(k, b, z, j; M) + \int n(z, k) d\tilde{\vartheta}(k, b; M).
\]

5 Calibration

In this section, we calibrate the model to the US economy. The volatility of the high process \( \sigma \) is set to 0.2305 and the autocorrelation parameter \( \rho \) is set to 0.885 as estimated for the US manufacturing sector by Cooper and Haltiwanger (2006). The process is discretized to obtain the grid for \( z \) and the transition probabilities \( \eta(z'|z) \) following Tauchen (1986). From the transition matrix \( \eta_h(z'|z) \) we can derive the unconditional probabilities \( \eta^*(z) \). We set the distribution of initial shocks to \( \nu(z_0) = \eta^*_h(z) \).\footnote{To correctly identify the entry threshold for each sector and perform the quantitative experiment across countries we need to compute the model with a large number of points in the \( z \) dimension. We set the number of grid points for \( z \) to 100.} We assume that the operating fixed cost can take values of
\{0, \hat{c}_f, +\infty\}.

Table 1: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Moment (US economy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>\beta</td>
<td>0.9615 \hspace{1em} Avg. yearly return 5-yr. T-Note</td>
</tr>
<tr>
<td>Depr. rate for capital</td>
<td>\delta</td>
<td>0.07 \hspace{1em} Manufacturing sector US</td>
</tr>
<tr>
<td>Depr. rate for skilled labor</td>
<td>\delta_s</td>
<td>0.015 \hspace{1em} Avg. return to education</td>
</tr>
<tr>
<td>Years of schooling</td>
<td>x</td>
<td>6 \hspace{1em} Avg. years of schooling in US</td>
</tr>
<tr>
<td>Capital Share</td>
<td>\alpha</td>
<td>0.21 \hspace{1em} Capital share</td>
</tr>
<tr>
<td>Labor Share Informal</td>
<td>\gamma_i</td>
<td>0.64 \hspace{1em} Labor share</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>\sigma</td>
<td>0.2305 \hspace{1em} Manufacturing sector US</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>\rho</td>
<td>0.885 \hspace{1em} Manufacturing sector US</td>
</tr>
<tr>
<td>Skilled labor share</td>
<td>\epsilon_f</td>
<td>0.302 \hspace{1em} Skilled workers as % of labor force</td>
</tr>
<tr>
<td>Labor share formal</td>
<td>\gamma_f</td>
<td>0.338 \hspace{1em} Labor share</td>
</tr>
<tr>
<td>Mean high process</td>
<td>\mu_h</td>
<td>3.162 \hspace{1em} Avg. operating establishment</td>
</tr>
<tr>
<td>Mean low process</td>
<td>\mu_l</td>
<td>0.6871 \hspace{1em} Size of informal sector</td>
</tr>
<tr>
<td>Positive operating cost</td>
<td>\hat{c}_f</td>
<td>8.5 \hspace{1em} Exit rate distribution</td>
</tr>
<tr>
<td>Distribution op. costs</td>
<td>\xi(\hat{c}_f)</td>
<td>0.10 \hspace{1em} Exit rate distribution</td>
</tr>
<tr>
<td></td>
<td>\xi(\infty)</td>
<td>0.042 \hspace{1em} Exit rate distribution</td>
</tr>
<tr>
<td>Creation cost</td>
<td>c_e</td>
<td>0.103 \hspace{1em} Free-entry condition</td>
</tr>
</tbody>
</table>

Following the literature, the risk free interest rate \( r \) is set to 4% per year, which implies that \( \beta = 1/(1 + r) = 0.9615 \). The depreciation rate of skilled labor will be set to \( \delta_s = 0.015 \) to match an average yearly return to college education of 10.5%, as reported by Psacharopoulos and Patrinos (2004). A skilled worker is defined as one with a college degree (16 years of education). By this definition, the fraction of skilled workers in the population in the US equals 30%. From Barro and Lee (2000), the average number of years of schooling in the US is approximately equal to 12. This implies that the average number of years of schooling in the group of unskilled workers equals 10 (i.e., the number of years of education with which each agent in our economy is born). Then, we set \( x \) to 6 to match the number of years of education that are necessary in order to become skilled. The total labor share in each sector is set to 0.64, a standard value. That is, \( \gamma_f + \epsilon_f = \gamma_i = 0.64 \). The value of \( \epsilon_f \) is set to match the equilibrium fraction of skilled workers. The capital share is set such that the degree of decreasing returns to scale at the firm level in
both sectors is consistent with the estimates presented in Restuccia and Rogerson (2008). In particular, we set $\alpha = 0.21$ so that $\alpha + \epsilon_f + \gamma_f = \alpha + \gamma_i = 0.85$. The physical capital depreciation rate $\delta$ is set to 7%, as in Cooper and Haltiwanger (2006). We normalize the unskilled wage rate to 1 and calculate the skilled labor wage rate using equation (7). The value of the entry cost $c_e$ is calibrated as in Hopenhayn and Rogerson (1993), such that, in the benchmark equilibrium with $w = 1$, $c_e$ satisfies the free-entry condition with equality. The parameters $\{\tau, c_\tau, \tau_\omega, \kappa, \lambda, \phi\}$ are taken directly from the values reported in the World Bank Doing Business database (2009) for the U.S. economy. We set the tax rates $\tau = 0.23$, $c_\tau = 0.09$, and $\tau_\omega = 0.20$; the entry cost $\kappa = 0.26$; and the bankruptcy parameters are $\lambda = 0.77$ and $\phi = 0.07$.

We are left with six parameters to calibrate: the mean of the productivity process of the high and low projects $\mu_h$ and $\mu_l$, respectively, the labor share of skilled workers $\epsilon_f$ (which, provided that the total labor share is 0.64, also determines $\gamma_f$), the intermediate operating cost $\hat{c}_f$, and the associated probabilities $\xi(\hat{c}_f)$ and $\xi(\infty)$. To obtain values for these parameters, we target the size of the informal labor force, measured as those workers not covered by a pension scheme (as reported by World Development Indicators, 2006), the average size of establishments operating in the formal sector in the US, the percentage of skilled workers in the labor force, and the exit rate distribution for US establishments. The data regarding the size distribution of establishments (operating in the formal sector) and exit rates in the US come from the Statistics of US Business (SUBS) data set for the years 2003-04.

Table (2) shows the data and the corresponding model moments.
Table 2: Target Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>US Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average formal est.</td>
<td>17.6</td>
<td>17.5</td>
</tr>
<tr>
<td>Informal sector (fraction of labor force)</td>
<td>7.8 %</td>
<td>7.8%</td>
</tr>
<tr>
<td>Skilled labor (fraction of population)</td>
<td>30.03 %</td>
<td>29.51%</td>
</tr>
</tbody>
</table>

Exit rate distribution by employment size:

<table>
<thead>
<tr>
<th>Employment Size</th>
<th>US Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>14.88</td>
<td>12.75</td>
</tr>
<tr>
<td>5-9</td>
<td>6.72</td>
<td>5.88</td>
</tr>
<tr>
<td>10-19</td>
<td>5.57</td>
<td>5.71</td>
</tr>
<tr>
<td>20-49</td>
<td>4.91</td>
<td>4.20</td>
</tr>
<tr>
<td>50-99</td>
<td>4.58</td>
<td>4.20</td>
</tr>
<tr>
<td>100-249</td>
<td>4.16</td>
<td>4.20</td>
</tr>
<tr>
<td>250-499</td>
<td>3.9</td>
<td>4.20</td>
</tr>
<tr>
<td>500+</td>
<td>4.22</td>
<td>4.20</td>
</tr>
</tbody>
</table>

The average size of a establishment operating in the formal sector is 17.6 in the US data, and in our model this figure is 17.5. The model exit rate distribution is very close to what is observed in the data. The amount of skilled labor as a fraction of the population is 30.03% in the US data, and in our model this number is 29.5%. The model is right on target for the size of the informal sector at 7.8%.

Once it is calibrated, we test our model using the size distribution of US establishments operating in the formal sector as reported in Table 3. The model does a good job of generating the correct overall size and age distributions of operating establishments in the formal sector.
Table 3: Distribution of US Formal Establishments by Age and Employment Size

<table>
<thead>
<tr>
<th>Employment Size</th>
<th>Young Data (%)</th>
<th>Young Model (%)</th>
<th>Middle Data (%)</th>
<th>Middle Model (%)</th>
<th>Old Data (%)</th>
<th>Old Model (%)</th>
<th>Total Size Dist. Data (%)</th>
<th>Total Size Dist. Model (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>13.6</td>
<td>0.01</td>
<td>20.6</td>
<td>7.33</td>
<td>14.4</td>
<td>22.99</td>
<td>48.6</td>
<td>30.34</td>
</tr>
<tr>
<td>5-9</td>
<td>2.5</td>
<td>2.43</td>
<td>9.8</td>
<td>9.87</td>
<td>9.5</td>
<td>10.42</td>
<td>21.8</td>
<td>21.42</td>
</tr>
<tr>
<td>10-19</td>
<td>1.2</td>
<td>4.26</td>
<td>6.2</td>
<td>12.24</td>
<td>6.8</td>
<td>5.93</td>
<td>14.2</td>
<td>23.34</td>
</tr>
<tr>
<td>20-49</td>
<td>0.7</td>
<td>0.82</td>
<td>3.9</td>
<td>13.11</td>
<td>5.0</td>
<td>2.97</td>
<td>9.6</td>
<td>17.23</td>
</tr>
<tr>
<td>50-99</td>
<td>0.2</td>
<td>0.09</td>
<td>1.2</td>
<td>4.65</td>
<td>1.8</td>
<td>0.59</td>
<td>3.2</td>
<td>5.35</td>
</tr>
<tr>
<td>100-249</td>
<td>0.1</td>
<td>0.02</td>
<td>0.6</td>
<td>1.86</td>
<td>1.0</td>
<td>0.15</td>
<td>1.8</td>
<td>2.05</td>
</tr>
<tr>
<td>250 +</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.20</td>
<td>0.23</td>
<td>0.01</td>
<td>0.01</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Total Age Dist.</strong></td>
<td><strong>18.4</strong></td>
<td><strong>7.6</strong></td>
<td><strong>42.6</strong></td>
<td><strong>49.3</strong></td>
<td><strong>39.0</strong></td>
<td><strong>42.7</strong></td>
<td><strong>100</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Note: “Formal Establishments” corresponds to establishments operating in the formal sector. Data corresponds to the distribution of establishments by firm size and age for 2004 from Business Dynamics Statistics. “Young” corresponds to 0-1 years in operation, “Middle” corresponds to 2-10 years, and “Old” corresponds to 11 years or more.

Regarding size, the model generates the correct number of small establishments (with fewer than nineteen employees), but misses at the very low end of the distribution (fewer than five employees). With respect to the age distribution of establishments, the model is on target when compared to the fraction of young, middle, and old establishments. A deeper look at the joint distribution shows that the model under-predicts the fraction of young establishments in the smallest size category. The reason for this is that the productivity threshold to enter the formal sector endogenously generates young establishments that are relatively more productive and therefore larger than what is observed in the data. On the other hand, the model yields a distribution of middle and old establishments across sizes that resembles the distribution observed in the data. By construction, the exit and entry rates are the same in the model, and are found to be 7.5%. The entry and exit rates in the data are 11.1% and 10.2%, respectively. Thus, the model average entry and exit rates are 4 and 3 percentage points lower than the US data, respectively.22

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22This distance between the model and data in terms of size distribution of young firms, entry and exit rates is partly due to the way that the data are collected. In the data, establishments are observed at one point in time. Those establishments that are less than one year old are considered entrants. However, the model’s counterpart for entrant establishments is defined as those establishments that are exactly one year old.
5.1 Workings of the Model

In this subsection, we describe the workings of the model. We begin by presenting the determinants of the entry decision rule and what type of firms are initially sorted into the formal sector vs the informal sector. We then move to describe the exit/default decision and the optimal debt to capital ratio as a function capital, debt and productivity and link them with the equilibrium bond price schedule. Finally, we describe the pattern of exit and default on the equilibrium path.

Figure 1 presents the entry decision into formality $\Xi(z_0) \in \{0, 1\}$ as a function of initial productivity $z_0$. Recall that the sector and technological decision are made after paying $c_e$ and observing the productivity $z_0$.

Figure 1: Formal - Informal Entry Decision

![Figure 1: Formal - Informal Entry Decision](image)

Figure 1 makes clear that only firms with relatively high initial productivity will sort into the formal sector (i.e. set $\Xi(z_0) = 1$). For the current parameterization the entry productivity threshold defined in equation (15) equals a value of productivity that is about 1 standard
deviation above of the mean of the $h$ process. Firms that draw an initial productivity below the threshold start operating as informal firms. We also present in the figure the distribution of incumbent formal firms across productivity levels. We observe that some firms that choose to select into informality (due to the entry costs and financial frictions) have higher productivity than many of the incumbent formal firms. Moreover, we also observe that while the initial productivity level has important implications for firm dynamics and the distribution of formal incumbents (as described in Table 3), the nature of the process (i.e. the fact that is not permanent and mean reverting) induces a distribution of incumbents that does not differ greatly from the unconditional probability distribution of the $h$ process.

Understanding the exit/default decision is key to understand firm dynamics in this model. Figure 2 presents the exit decision for formal incumbents as a function of capital and debt for different productivity levels (top panel low productivity, denoted $z_L$; middle panel average productivity, denoted $z_M$ and bottom panel, high productivity denoted $z_H$) evaluated at $c_f = \hat{c}_f$. The low and high productivity levels correspond to minus/plus one standard deviation from the mean. Recall that the exit decision rule can take on three values. That is $x(z_{-1}, k, b, c_f) \in \{0, 1, 2\}$ depending on whether the firm chooses to stay ($x(\cdot) = 0$), the firm chooses to exit by defaulting on its debt ($x(\cdot) = 1$) or the firm chooses to exit by repaying in full ($x(\cdot) = 2$). The different colors in Figure 2 represent each possible value of the exit rule (the white area corresponds to $x(\cdot) = 0$, the grey area corresponds to $x(\cdot) = 1$ and the black area corresponds to $x(\cdot) = 2$).
Figure 2 shows the following properties: 

(i) Exit is decreasing in productivity. Provided productivity in the model is persistent and firms face fixed operating costs, when low levels of productivity realize, the present value of the firm drops below the exit value and firms choose to shut down operations. The top panel shows that all firms whose productivity is $z_L$ will choose to exit (either via repayment or default) while only those with relatively high debt to capital levels will choose to exit in the case of firms with productivity $z_H$. 

(ii) Conditional on the level of productivity and capital, the default decision rule is increasing in the level of debt. As debt level increases, firms choose to exit via default since limited liability provides a floor to losses for the owner of the firm. 

(iii) Conditional on the level of productivity and debt, the default decision is decreasing in the level of capital. At low levels of debt to capital, firms that decide
to exit choose to exit by full repayment since the owner of the firm still recovers something. Since the exit value when the firm does not default is increasing in capital, even firms with intermediate productivity will choose to exit using this option.

We continue analyzing firm behavior by presenting, in Figure 3, the optimal debt to capital ratio for formal firms as a function of capital and debt for different productivity levels. We show the debt to capital ratios only for the relevant region where firms choose not to exit (colors represent different levels of $b'/k'$ and the white area in the bottom right corner of each chart corresponds to a combination of $\{k, b, z\}$ such that the firm chooses to exit).

Figure 3: Debt to Capital Ratios Formal Firms

![Debt to Capital Ratios Formal Firms](image)

Figure 3 shows that firms with higher productivity choose higher levels of debt to capital ratios. The main driving force of firm borrowing is investment. When productivity is high firms
are willing to take on more debt (and potentially pay higher interest rates) since the marginal product of capital is high. This is also consistent with the fact that, conditional on productivity, those firms with initial low debt to capital ratios are also choosing a lower level of debt to capital for the future.

One important determinant of the borrowing level by the firm is the price of the bond they face. Figure 4 shows the price schedule $q(k', b', z)$ as a function of capital and debt for different levels of productivity (as before the plot is shown with a color scale where darker colors represent lower bond prices).

Figure 4: Bond Price Schedule Formal Firms $q_f^j(k', b', z)$

Consistent with the exit/default decision presented in Figure 2, Figure 4 shows that prices are increasing in productivity (i.e. firms with relatively high productivity pay lower interest rates) and that prices are decreasing in debt to capital ratios. This last property is due to
the fact that the default decision is increasing and the recovery rates in the case of default are decreasing in debt to capital ratios.

It is important to note that figures presented above correspond to equilibrium objects (a set of decision rules and the price schedule) but they do not say anything about whether default happens in equilibrium. Since prices are decreasing in debt levels, it is possible that firms are discouraged from borrowing at levels with a positive default probability (i.e. the bond price schedule acts as a borrowing limit). We study this and present, in Table 4, the equilibrium distribution of firms, together with the exit and default fraction (sorted by firm size as in previous tables). This table presents the distribution of all firms in the economy (formal and informal) as well as a split into formal firms (potentially closest to firms that are observed by the US Census Bureau or other data collecting agencies).

Table 4: Firm, Exit and Default Distribution

<table>
<thead>
<tr>
<th>Employment Size</th>
<th>All Firms</th>
<th></th>
<th></th>
<th>Formal Firms</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frac.</td>
<td>Exit</td>
<td>Default</td>
<td>Frac.</td>
<td>Exit</td>
<td>Default</td>
</tr>
<tr>
<td>1 - 4</td>
<td>79.67</td>
<td>14.19</td>
<td>14.18</td>
<td>30.34</td>
<td>12.75</td>
<td>3.98</td>
</tr>
<tr>
<td>5 - 9</td>
<td>10.09</td>
<td>5.88</td>
<td>2.07</td>
<td>21.42</td>
<td>5.88</td>
<td>3.07</td>
</tr>
<tr>
<td>10 - 19</td>
<td>7.10</td>
<td>5.17</td>
<td>3.78</td>
<td>23.34</td>
<td>5.17</td>
<td>2.78</td>
</tr>
<tr>
<td>20 - 49</td>
<td>3.08</td>
<td>4.2</td>
<td>2.59</td>
<td>17.23</td>
<td>4.2</td>
<td>2.59</td>
</tr>
<tr>
<td>50 - 99</td>
<td>0.02</td>
<td>4.2</td>
<td>2.96</td>
<td>5.35</td>
<td>4.2</td>
<td>2.96</td>
</tr>
<tr>
<td>100 - 249</td>
<td>0.01</td>
<td>4.2</td>
<td>3.26</td>
<td>2.04</td>
<td>4.2</td>
<td>3.26</td>
</tr>
<tr>
<td>250 +</td>
<td>0.00</td>
<td>4.2</td>
<td>3.55</td>
<td>0.24</td>
<td>4.2</td>
<td>3.55</td>
</tr>
<tr>
<td><strong>Avg.</strong></td>
<td><strong>14.16</strong></td>
<td><strong>14.14</strong></td>
<td></td>
<td><strong>7.51</strong></td>
<td><strong>2.14</strong></td>
<td></td>
</tr>
</tbody>
</table>

Note: Exit corresponds to firms that decide to stop operating. Default corresponds to firms that exit by not repaying its debt obligations.

This table shows that default is in fact an equilibrium outcome and quantitatively important. The overall default frequency is 14.14% and the default frequency for formal firms is 2.14%. When focusing on the distribution of all firms, we note that most of the smallest firms (those in bins 1-4, 5-9 and 10-19) choose to exit by default, while about half of large firms that exit (i.e. those in bins 20-49 or higher) choose to exit by default. When looking at the distribution
of formal firms, we note that the default frequency is decreasing in firm size and between 1/4 to half of those firms in the smallest bins choose to exit by default. The comparison between the two tables shows that informal firms are not only much smaller than formal firms but also that they are able to borrow while more prone to default.

6 Quantitative Exercise: Country Specific Institutions

We analyze the impact of institutional frictions on the cross-country differences in measured TFP, informality, and the skill distribution. Our focus is on measured differences in the cost of entry to the formal sector, the tax structure, and the efficiency of debt-enforcing mechanisms. The experiment has two parts. The first part can be interpreted as a counterfactual in which the effects of imposing country-specific frictions onto the US economy are measured in the steady state, while the second part goes deeper into the mechanics of each friction, turning them on and off in a series of counterfactual exercises. Due to the high computational burden of the exercise, the number of observations is limited by grouping countries by income level following the World Bank’s definition: High Income Countries (HIC) and Developing Countries, where Developing Countries are classified as Upper Middle Income Countries (UMIC), Lower Middle Income Countries (LMIC) and Low Income Countries (LIC).  

The first experiment can be described as follows. First, calibrate the model to the US economy by using \((\lambda, \phi, \tau, c_r, \tau_w, \kappa)_{US}\). Next, for each income group \((\lambda, \phi, \tau, c_r, \tau_w, \kappa)_g\) are adjusted, where \(g \in \{HIC, UMIC, LMIC, LIC\}\), and a new equilibrium is computed. In

---

\(^{23}\) Roughly, countries are classified as HIC if their GNI per capita is higher than 25% of the US, UMIC if their GNI per capita falls between 8% and 25% of the US, LMIC if their GNI per capita falls between 2% and 8% of the US, and LIC if their GNI per capita is below 2% of the US.

\(^{24}\) In this case, the wage is normalized to one, and then the set of loan prices \(q^f(k’, b’, z)\) and \(q^i(k’, b’)\) are obtained through iteration, until lenders make zero profit on each contract. The mass of potential entrants \(M\) that clears the labor market is found together with the value of entry cost \(c_e\) that satisfies the zero entry condition.

\(^{25}\) More specifically, the wage rate \(w\) and loan prices \(q^f(k’, b’, z)\) and \(q^i(k’, b’)\) are obtained through iteration until lenders make zero profits and the zero entry condition is satisfied (given the \(c_e\) obtained for the US). Finally,
order to implement it, the *Doing Business* database for 2009 is used to obtain the median 
\((\lambda, \phi, \tau, c_r, \tau_w, \kappa)\) for each income group. Table 5 shows parameter values for the US economy 
(used in the benchmark calibration) and those of High, Upper Middle, Lower Middle and Low 
Income countries.

<table>
<thead>
<tr>
<th>Frictions Across Income Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>US 0.77 0.07 0.23 0.09 0.20 0.26</td>
</tr>
<tr>
<td>High (HIC) 0.72 0.08 0.18 0.07 0.28 1.08</td>
</tr>
<tr>
<td>Upper Middle (UMIC) 0.30 0.15 0.17 0.10 0.37 1.33</td>
</tr>
<tr>
<td>Lower Middle (LMIC) 0.25 0.15 0.17 0.14 0.31 5.08</td>
</tr>
<tr>
<td>Low (LIC) 0.15 0.09 0.20 0.13 0.23 7.03</td>
</tr>
</tbody>
</table>

Note: Countries are classified following the World Bank’s income groups. Countries are HIC if their GNI per capita is higher than 
25% of the US, UMIC if their GNI per capita falls between 8% and 25% of the US, LMIC if their GNI per capita falls between 2% 
and 8% of the US, and LIC if their GNI per capita is below 2% of the US. Median values for each group and friction are reported.

We start our analysis by looking at the effects of country-specific institutions on some 
important aggregates. These are the level of aggregate total factor productivity, the size of the 
informal labor force, output per worker, and the fraction of skilled workers. Measured aggregate 
total factor productivity is computed, as is standard in the literature, by using an aggregate 
production function. In particular, we follow cross-country studies such as those by Klenow and 
Rodriguez-Clare (1997) and Hall and Jones (1999) that compute the following equation:

\[
 TFP = \frac{Y}{K^{\hat{\alpha}} H^{1-\hat{\alpha}}},
\]

where \(Y\) denotes aggregate output, \(K\) denotes aggregate capital, \(H\) denotes aggregate labor 
(adjusted for human capital), and \(\hat{\alpha}\) is the capital share. In our model, aggregate output is 
the sum across both formal and informal firms, aggregate capital is the sum of capital across 
establishments in both sectors, and our aggregate measure of labor equals \(1 - A\). We use the 
same capital share as in Hall and Jones (1999), which equals \(1/3\).\(^{26}\) In the data, values of TFP 

\(^{26}\)Including informal-sector output in measures of GDP is line with National Accounts procedures. See
and output per effective worker are obtained by updating Hall and Jones (1999) using Heston, Summers, and Aten (2009) and Barro and Lee (2000). The model TFP is calculated as in Hall and Jones (1999), by calculating the value of human capital given the returns for every level of schooling. The informal labor force is reported by the WDI (2006), as the share of the labor force not covered by a pension scheme. Table 6 displays the main results for each income group and compares the model to the data for the median country in the income group.

Table 6: Overall Effect of Changes in Institutions

<table>
<thead>
<tr>
<th></th>
<th>HIC</th>
<th>UMIC</th>
<th>LMIC</th>
<th>LIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informal Labor Force</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Skilled workers % of population</td>
<td>11.00</td>
<td>28.27</td>
<td>9.00</td>
<td>15.57</td>
</tr>
<tr>
<td>TFP</td>
<td>0.91</td>
<td>0.89</td>
<td>0.70</td>
<td>0.72</td>
</tr>
<tr>
<td>Output per Worker</td>
<td>0.92</td>
<td>0.90</td>
<td>0.45</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Note: TFP and Output per Effective Worker are reported relative to the US value. Data is from the authors’ calculations, based on Hall and Jones (1999). The size of the informal labor force is taken from the World Development Indicators (2006) as the share of the labor force not covered by a pension scheme. Skilled workers are proxied by the percentage of the population over age 25 who have completed college, from Barro and Lee (2000).

Our model accounts for more than two-thirds of the TFP gap between the US and developing economies (a drop of up to 37%). In what follows, we will extensively analyze the sources of observed productivity differences and the role of each friction. In short, we find that allocative inefficiencies, the distribution of human capital, and the share of output produced by firms in the informal sector play a crucial role.

The model accounts for a large fraction of the difference in terms of skilled workers across countries. As in the data, it generates a stock of human capital that is positively correlated with TFP, and income per capita, and negatively related to the size of the informal sector. More specifically, it generates a stock of human capital that is only 52%, 33%, and 40% of the U.S. in UMIC, LMIC, and LIC, respectively. Differences in institutions and the resulting change in the

D’Erasmo and Moscoso Boedo (2012) for an extensive discussion of issues regarding the measurement of aggregate output in the presence of an underground economy.

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size of the informal sector are a driving force of this result. As the size of the informal sector increases, the demand for skilled workers is reduced. Moreover, as frictions increase, formal firms are also prevented from attaining their optimal level of capital, and this in turn also affects the demand for skilled workers.

The model is successful in capturing the drop in human capital between the US and developing economies in the middle of the distribution (UMIC and LMIC). However, there is a discrepancy between the level of human capital generated by the model and the data for HIC (mostly Western Europe). This can be attributed to the fact that these countries are in the early stages of a transition to higher levels of human capital. Enrollment rates in recent years (the fraction of the population enrolled in college) are in line with the value of skilled workers that our model generates. At the very low end of the distribution (LIC), the model also over-predicts the stock of human capital. There are many reasons for this difference. First, we calibrated our model to the US and the calibration resulted in unskilled workers that are endowed with ten years of education (almost completed high school). Second, we simplified our model and assumed that only two levels of skills exist and that the household has linear preferences resulting in a skill premium that is only a function of parameters and thus constant across countries. These assumptions allow us to isolate the role of institutions.

Another important result is that the model delivers an informal labor force comparable with the data. Informality in our model ranges from around 8% in the US to 69% at the low end of the income distribution. Although frictions generate a drop in output per effective worker, in the model output per effective worker in the Low Income Countries is up to four times higher than what is seen in the data. To understand this result, it is crucial to note that we assume no exogenous technological differences across countries, and that the steady state risk-free rate is also equal across countries, generating a similar discrepancy in physical capital per worker ratios (see Table 7).

In Table 7, we present other important aggregate moments across income groups to test our
model along different dimensions.

Table 7: Differences across Income Groups

<table>
<thead>
<tr>
<th></th>
<th>HIC</th>
<th></th>
<th>UMIC</th>
<th></th>
<th>LMIC</th>
<th></th>
<th>LIC</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Avg Employment Formal</td>
<td>11.1</td>
<td>30.63</td>
<td>129.8</td>
<td>48.30</td>
<td>175</td>
<td>90.68</td>
<td>386.4</td>
<td>103.28</td>
</tr>
<tr>
<td>ln (Var employment formal)</td>
<td>10.5</td>
<td>7.1</td>
<td>12.7</td>
<td>8.02</td>
<td>12.7</td>
<td>9.08</td>
<td>13.6</td>
<td>8.71</td>
</tr>
<tr>
<td>Capital per worker</td>
<td>1.05</td>
<td>0.94</td>
<td>0.38</td>
<td>0.64</td>
<td>0.18</td>
<td>0.55</td>
<td>0.04</td>
<td>0.54</td>
</tr>
<tr>
<td>Formal Entry Rate</td>
<td>0.81</td>
<td>0.75</td>
<td>0.65</td>
<td>0.64</td>
<td>0.62</td>
<td>0.58</td>
<td>0.47</td>
<td>0.57</td>
</tr>
<tr>
<td>Business Density</td>
<td>1.62</td>
<td>0.33</td>
<td>0.93</td>
<td>0.28</td>
<td>0.31</td>
<td>0.25</td>
<td>0.03</td>
<td>0.25</td>
</tr>
<tr>
<td>Domestic Credit to Private Sector (% of GDP)</td>
<td>54.9</td>
<td>101.6</td>
<td>21.3</td>
<td>53.64</td>
<td>16</td>
<td>33.42</td>
<td>7.5</td>
<td>33.78</td>
</tr>
</tbody>
</table>

Note: Capital per worker, Formal Entry Rate, Business Density, and Domestic Credit to Private sector are reported relative to the US value. Data on average employment and variance of employment are taken from Alfaro et. al. (2009). Capital per effective worker is from author’s calculations based on Hall and Jones (1999). Data on the Formal Entry Rate and Business Density are taken from the 2008 World Bank Group Entrepreneurship Survey and Database. The model counterpart is obtained as total formal labor force over the average size of establishments operating in the formal sector, which equals the ratio of establishments operating in the formal sector to total population. Domestic Credit to Private Sector (Domestic Credit to GDP) is taken from the World Development Indicators (average 2004-2007). Domestic credit to private sector in the model is computed as the ratio of formal debt to total output.

The model is on target on average size, as reported by Alfaro et al.(2009). Our model predicts that as frictions increase, the exit rate (and the entry rate, by construction) decreases. This implies that, for Low Income Countries, firms choose to operate the limited technology and stay in business for much longer, preventing the natural process of churning of unproductive firms. Also, the model generates a relative business density, measured as the number of registered businesses as a percentage of the active population, which is in line with its observed counterpart. The business density drops to 25% of the US’s for the Low Income Countries. High frictions generate low density, which generates low competitive pressures in the labor markets, which in turn generate low turnover in the formal sector (as observed by the low entry rate in developing economies) and lower average productivity. In terms of domestic credit to GDP, the model is right qualitatively, but predicts more credit to formal firms than observed in the data. In the data, credit to GDP falls to 7.5% of the US value for the Low Income Countries, and the model predicts it to be 33.8% of the US value.
6.1 The Role of Each Friction

In this section, we analyze the effects of each friction separately, as well as the joint effect of entry costs and financial frictions. To this end, we analyze changes in institutions from those of the US to those of LMIC. The LMIC parameter values appear to be a natural benchmark for understanding the effect of institutional differences because they are noticeably different from those of the US (our calibrated economy) but are within the observed range. Panel (a) of Table 8 shows the impact of institutions on the economy with human capital (our benchmark). In order to understand the role of human capital, Panel (b) of Table 8 presents the same counterfactuals in a model without human capital.\textsuperscript{27} To complete the analysis, Table 9 presents the fraction of the total effect that can be assigned to each friction for both models.

\textsuperscript{27}The model with no human capital is calibrated to match the same targets that we use in our benchmark calibration. In particular, $\mu_h$ and $\mu_l$ are adjusted so that the average size of formal firms and the size of the informal sector in the model are consistent with US values.
Table 8: The Role of Each Friction

Panel (a): Model with Human Capital (Benchmark)

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>$\kappa_{LMIC}$</th>
<th>${\lambda, \phi}_{LMIC}$</th>
<th>${\kappa, \lambda, \phi}_{LMIC}$</th>
<th>${\tau, c_r, \tau_w}_{LMIC}$</th>
<th>LMIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informal labor force (%)</td>
<td>7.80</td>
<td>11.70</td>
<td>10.12</td>
<td>53.28</td>
<td>9.85</td>
<td>69.22</td>
</tr>
<tr>
<td>Skilled workers</td>
<td>29.05</td>
<td>28.29</td>
<td>28.79</td>
<td>15.15</td>
<td>28.87</td>
<td>10.03</td>
</tr>
<tr>
<td>TFP</td>
<td>1.00</td>
<td>0.80</td>
<td>0.97</td>
<td>0.67</td>
<td>0.99</td>
<td>0.63</td>
</tr>
<tr>
<td>$var(\log(MPK))$</td>
<td>0.22</td>
<td>0.21</td>
<td>0.27</td>
<td>0.25</td>
<td>0.22</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Panel (b): Model without Human Capital

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>$\kappa_{LMIC}$</th>
<th>${\lambda, \phi}_{LMIC}$</th>
<th>${\kappa, \lambda, \phi}_{LMIC}$</th>
<th>${\tau, c_r, \tau_w}_{LMIC}$</th>
<th>LMIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informal labor force (%)</td>
<td>7.80</td>
<td>78.40</td>
<td>27.10</td>
<td>94.28</td>
<td>17.90</td>
<td>95.8</td>
</tr>
<tr>
<td>Skilled workers</td>
<td>-</td>
<td>0.79</td>
<td>0.94</td>
<td>0.75</td>
<td>0.97</td>
<td>0.75</td>
</tr>
<tr>
<td>TFP</td>
<td>1.00</td>
<td>0.26</td>
<td>0.27</td>
<td>0.32</td>
<td>0.24</td>
<td>0.31</td>
</tr>
<tr>
<td>$var(\log(MPK))$</td>
<td>0.22</td>
<td>0.26</td>
<td>0.27</td>
<td>0.32</td>
<td>0.24</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Note: US and LMIC denote economies where all parameters are set according to the given country. The following columns present the result from models where all the parameters are set to that of the US, except the displayed parameters that are set to LMIC. TFP is reported relative to the US value. Model TFP is calculated as $TFP = \left(\frac{Y}{K^\alpha}H^{1-\alpha}\right)$, where $\hat{\alpha} = 1/3$ is taken from Hall and Jones (1999). See Table 5 for group-specific parameters.

Table 9: Contribution of Each Friction to Overall Effect

<table>
<thead>
<tr>
<th></th>
<th>$\kappa_{LMIC}$</th>
<th>${\lambda, \phi}_{LMIC}$</th>
<th>$\Delta{\kappa, \lambda, \phi}_{LMIC}$</th>
<th>${\tau, c_r, \tau_w}_{LMIC}$</th>
<th>HK</th>
<th>no HK</th>
<th>HK</th>
<th>no HK</th>
<th>HK</th>
<th>no HK</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of Total Effect on</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informal labor force (%)</td>
<td>6.35</td>
<td>80.06</td>
<td>3.78</td>
<td>21.93</td>
<td>63.92</td>
<td>-3.86</td>
<td>3.34</td>
<td>11.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skilled Workers</td>
<td>4.00</td>
<td>-</td>
<td>1.37</td>
<td>-</td>
<td>67.72</td>
<td>-</td>
<td>0.95</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFP</td>
<td>54.05</td>
<td>84.00</td>
<td>8.11</td>
<td>24.00</td>
<td>27.03</td>
<td>-8.00</td>
<td>2.70</td>
<td>12.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: “HK” corresponds to the results of the benchmark model with human capital; “No HK” corresponds to the results of a model with no human capital. Each column provides the fraction of the total effect (i.e., the difference between LMIC and US) that can be assigned to each friction. The column $\Delta\{\kappa, \lambda, \phi\}_{LMIC}$ presents the complementary effect of $\{\kappa, \lambda, \phi\}_{LMIC}$ computed as the joint effect net of the individual effect of $\kappa_{LMIC}$ and $\{\lambda, \phi\}_{LMIC}$.

Entry cost: The third column of Table 8 reports the effects of changing the formal sector entry cost from the US level to the LMIC level. Changes in entry costs affect the productivity threshold that makes firms indifferent between the formal and informal sectors. This change in the productivity threshold causes the informal labor force to increase to 11.7%, and TFP to
fall to 0.80%. The percentage of skilled workers decreases by only 1%. The effects are more pronounced in the model with no human capital and they represent a larger fraction of the overall change in TFP and informality (as can be seen in the first column of Table 9).

**Bankruptcy efficiency:** The fourth column of Table 8 reports the effect of changing the bankruptcy efficiency parameters in the formal sector from US to LMIC values. We change both the recovery rate \( \lambda \) and the cost of bankruptcy proceedings \( \phi \). In this case, TFP drops by only 3% and the percentage of skilled workers falls by less than 1%. These effects are partly due to an increase in informal activity (the informal labor force increases to 10.7% from 7.8%) but also due to an inefficient resource allocation of resources within the formal sector. The variance of the log of the marginal product of capital increases by 23%. As in the case of the entry cost, the fraction of the overall change in TFP and informality explained by the individual effect of bankruptcy efficiency is larger in the model without human capital than in our benchmark.

**Joint Effect of Entry Costs and Bankruptcy Efficiency:** The fifth column of Table 8 reports the effects of changing both the formal sector entry costs and the formal-sector’s bankruptcy efficiency parameters for the formal sector from US to LMIC level (jointly changing \( \kappa, \lambda \) and \( \phi \)). In our benchmark economy, together the change in entry costs and financial frictions cause the size of the informal sector to increase to 53%. This also causes a decline in TFP of 33%, and the stock of skilled workers as a percentage of the population falls to 15.15%. Interestingly, as can be seen from column \( \Delta\{\kappa, \lambda, \phi\}_{LMIC} \) in Table 9, we find that in the model with human capital the complementary effect of entry costs and bankruptcy efficiency (i.e. the joint effect net of the sum of the individual effects) accounts for 27%, 64%, and 68% of the total effect in terms of TFP, informal labor force, and the stock of skilled workers respectively as opposed to a small (and even negative) effect in the case of the model with no human capital.\(^{28}\) This complimentary effect is due to the change in the productivity threshold and the increase in

\(^{28}\)More specifically, the pure complementary effect is taken as the joint effect (reported in column \( \{\kappa, \lambda, \phi\}_{LMIC} \) of Table 8) minus the sum of the individual effects (reported in columns \( \kappa_{LMIC} \) and \( \{\lambda, \phi\}_{LMIC} \)) as a fraction of the overall effect (reported in column LMIC).
the size of the the informal sector. Entry costs and bankruptcy efficiency frictions reinforce the effects. As the entry costs increase, entrants to the formal sector are more productive, and because productivity is not permanent they want to grow as quickly as possible. However, capital investment is costly due to the financial frictions and substituting capital with labor is also expensive due to the presence of skilled workers and the skill premium that they require. This induces them to move closer to a region of the state space where interest rates are high (i.e. the financial frictions become important). On the other hand, in the model with no human capital it is easier to substitute labor for capital. This is also reflected in the larger variance of the log of marginal product of capital. In short, when human capital is incorporated to the analysis, the joint effect is the main driver explaining TFP differences, the level of informality and changes in the stock of skilled workers, and is almost as important as the individual effects of entry costs and bankruptcy efficiency. This is a key result of our paper and is in line with that of Bergoeing, Loayza, and Piguillem (2011) who also find that entry costs and exit frictions jointly explain most of the effects of the frictions on output gaps. In their paper, this is due to a technology adoption mechanism, whereas in our paper it happens through the introduction of human capital.

**Tax structure:** The sixth column of Table 8 reports the effects of changing the tax structure parameters in the formal sector from US to LMIC levels (jointly changing $\tau$, $c_\tau$, $\tau_\omega$). Note that both with and without human capital, the effects are small in magnitude. From Table 9 it can be noted that the individual effect of taxes is bigger in the case without human capital (representing around 12% of the total effect). However, the total impact of taxes is the sum of the individual effect plus the joint effect with the other frictions (represented by the complement of all impacts combined in table 9). In the case without human capital this effect is negligible and has to do with the fact that taxes on dividends are higher in the US than in the LMIC, but taxes on labor are higher in the LMIC than in the US. In the case with human capital the impact of taxes, while small individually, is higher through its joint effect with the other frictions. The total impact
of taxes (individual plus joint) accounts for 10.81%, 25.95% and 26.91% of the gap between
the US and LMIC in terms of TFP, informal labor force and skilled workers respectively. This
change in the impact of taxes with and without human capital is connected with the mechanisms
explained in the case of the joint effect of entry costs and bankruptcy efficiency. With human
capital, firms substitute away labor with capital by getting closer to the financial constraint.
Therefore, taxes that affect the relative price of labor to capital have a differential impact and
the effect of higher labor taxes does not cancel out the one of lower dividend taxes.

In summary, we find that each friction operates through different channels. Entry costs
generate a higher informal sector, whereas bankruptcy efficiency produces bigger changes in
allocative efficiency. The most interesting result we obtain is that, in the model with human
capital, when entry costs and bankruptcy efficiency are considered separately the effects on
the informal sector, TFP and the stock of human capital are small. Once these frictions are
combined, as shown in Tables 8 and 9, we get considerable changes in TFP, the size of the
informal sector, and the stock of human capital. Institutions are highly complementary only in
the presence of human capital. We can see that in the case without human capital the frictions
are additive in their effects on TFP and informality (i.e. the combined effect is the same or
smaller than the sum of the individual effects), while there is a high degree of complementarity
in the case of an economy with human capital.

6.2 Capital-Skill Complementarity

In this section, we explore the implications of having Capital-Skill complementarity in the pro-
duction function. The idea is to simplify the model as much as possible and see what happens
when we minimize the number of assumptions regarding the production technology. In partic-
ular, we assume that there is only one production function, which is of the CES form, and can
be operated in both sectors (formal and informal). The mechanism in mind is the following:
Having Capital-Skill complementarity would increase the incentives to operate in the sector in which financing is easily available (the formal sector). So, changes in the financial environment could generate international differences in informality, TFP, and potentially human capital.

As we see below, these hypothesis fails to materialize under the observed frictions. Even for the US, where the cost of entry to the formal sector is low, and the benefits are high (one of the best financial environments around), the economy becomes 100% informal. In other words, the financial benefits that the formal sector provides, are not enough for firms to choose to pay the entry costs, and taxes related to operating formally. This result remains unchanged as we explore different levels of Capital-Skill complementarity.

To implement this idea, we assume that the production function at the firm level is of the following form,

$$f(z, k, s, n) = z \left\{ a(bk^\theta + (1-b)s^\theta)^{\gamma/\theta} + (1-a)n^\gamma \right\}^{\alpha/\gamma} \quad (20)$$

Where $z$ denotes the firm level productivity, $k$ is the firm level physical capital, $s$ is the employment at the firm level of skilled workers, and $n$ refers to the firm’s employment of unskilled workers.

The parameter of interest determining the Capital-skill complementarity is $\theta$. Note that if $\theta$ is equal to one, then physical capital and skilled workers are perfect substitutes. If $\theta$ equals zero, then physical capital and skilled workers have a capital skill complementarity which is equal to one (this is the Cobb-Douglas case). Finally, for negative values of $\theta$, physical capital and skilled workers are complimentary. As $\theta$ decreases (converges to $-\infty$), these two inputs become more and more complimentary, converging to a Leontieff case.

The parameters are initially set to what Krusell et. al. (2000) report, and are listed in the table below, with the exception of $\theta$. The parameter $\alpha$ is kept at its level from the body of the paper. It is set to a decreasing return to scale equal to 0.85. The share parameterers $a$, and $b$
are set to generate income shares of capital, and labor as observe in the data. Also, the stock of skilled labor force is calibrated to generate values close to what is observed in the HIC countries.

Table 10: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.401</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.850</td>
</tr>
<tr>
<td>$a$</td>
<td>0.425</td>
</tr>
<tr>
<td>$b$</td>
<td>0.420</td>
</tr>
</tbody>
</table>

We then experiment with the Capital-Skill complementarity parameter to even unrealistic values to see if this hypothesis might work. We find that the economy remains at 100% informality and the level of skilled labor force is unchanged, as reported in table 11.

Table 11: Model Results under Different Capital-Skill Complementarity

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>-0.100</th>
<th>-0.495</th>
<th>-0.795</th>
<th>-10.990</th>
</tr>
</thead>
<tbody>
<tr>
<td>informal share</td>
<td>0.999</td>
<td>1.000</td>
<td>1.000</td>
<td>0.998</td>
</tr>
<tr>
<td>output</td>
<td>1.483</td>
<td>1.431</td>
<td>1.400</td>
<td>1.346</td>
</tr>
<tr>
<td>labor</td>
<td>0.987</td>
<td>0.984</td>
<td>0.986</td>
<td>0.986</td>
</tr>
<tr>
<td>output per worker</td>
<td>1.503</td>
<td>1.454</td>
<td>1.420</td>
<td>1.365</td>
</tr>
<tr>
<td>skilled labor share</td>
<td>0.144</td>
<td>0.169</td>
<td>0.153</td>
<td>0.148</td>
</tr>
</tbody>
</table>

Note: Krusell et. al. (2000) report a parameter $\theta$ equal to $-0.495$, our third column.

Note that once the US is completely informal, increasing the frictions (increasing entry costs and decreasing financial development) would only make the forces towards informality stronger. Therefore, we conclude that Capital-Skill complementarity under imperfect financial environments is not a reasonable assumption if the model is to produce international differences in informality, TFP, or human capital under the observed level of frictions.
6.3 The Role of Human Capital

In this section, we analyze the contribution of human capital to changes in aggregate productivity and informality. We compare our benchmark economy with two models without human capital in the formal sector production function. First, we present a model without human capital, calibrated to match the US targets: $\mu_h$ and $\mu_l$ are adjusted so that the average size of formal firms and the size of the informal sector in the model are consistent with the US values. This allows us to make a fair comparison with the benchmark because the starting point (i.e., the US economy) used to evaluate the effects of institutions is quantitatively similar. Second, a model without human capital was computed using the parameters that resulted from our benchmark calibration. This helps us disentangle the endogenous effects versus the effects coming from differences in parameters. Table 12 presents the most relevant statistics.

Table 12: Counterfactual: No Human Capital

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>No HK (1)</th>
<th>No HK (2)</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Process $\mu_h$</td>
<td></td>
<td>1.62</td>
<td>3.16</td>
<td>3.16</td>
</tr>
<tr>
<td>Mean Process $\mu_l$</td>
<td></td>
<td>0.76</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>Moments U.S. Economy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Formal Est.</td>
<td>17.6</td>
<td>17.6</td>
<td>216.28</td>
<td>17.5</td>
</tr>
<tr>
<td>Informal Labor Force (%)</td>
<td>7.8</td>
<td>7.8</td>
<td>0.0</td>
<td>7.8</td>
</tr>
<tr>
<td>Skilled Labor (%)</td>
<td>30.03</td>
<td>-</td>
<td>-</td>
<td>29.51</td>
</tr>
<tr>
<td>Main Results: LMIC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFP</td>
<td>0.44</td>
<td>0.75</td>
<td>0.93</td>
<td>0.63</td>
</tr>
<tr>
<td>Informal Labor Force</td>
<td>71.7</td>
<td>95.8</td>
<td>0.00</td>
<td>69.22</td>
</tr>
<tr>
<td>Output per worker</td>
<td>0.32</td>
<td>0.66</td>
<td>0.84</td>
<td>0.56</td>
</tr>
<tr>
<td>var(ln(MPK)) Formal Sector</td>
<td>-</td>
<td>0.27</td>
<td>0.27</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Note: TFP and Output per worker in LMIC are reported relative to the US value. “No HK (1)” corresponds to the results of a model with no human capital that was calibrated to match the average size of establishments operating in the formal sector in the U.S. and the size of the informal sector; “No HK (2)” corresponds to the results of a model with no human capital computed using the parameters from our benchmark model; “Benchmark” corresponds to the results of the model with human capital.

29 The parameter $\epsilon_f$ is set to zero and the unskilled labor factor share in the formal sector is adjusted so that the total degree of decreasing returns is the same across models.

30 In all of the models, we first compute the equilibrium for the US and recompute the equilibrium using the institutions for LMIC.
Table 12 shows that, without skilled labor in the model, the productivity differences that are needed between the high and low process to match the targets are much smaller (note the different values in columns “No HK(1)” and “Benchmark”). Intuitively, labor is less expensive in the formal sector when firms are not required to hire skilled workers and therefore are not required to pay the skill premium, so the resulting productivity differences to sustain the observed level of informality are smaller.

The model without human capital (No HK (1)) generates a decline in TFP of 25% versus 37% in our benchmark. Thus, incorporating human capital into the model generates a drop in TFP that is 48% larger than a model without human capital accumulation. The third column in the table (No HK (2)) shows that differences in productivity are not the main driving force. A model with no human capital and large productivity differences generates a drop in TFP of only 7%. Note that this model generates no informal sector, so the drop in TFP is coming from the misallocation of resources in the formal sector, as evident from the value of the variance of the marginal product of capital.

Differences in TFP changes between the “No HK (1)” model and our benchmark are not the result of a larger informal sector. In fact, by adding human capital the model is more in line with the data (69.2% in the model versus 71.7% reported for the median LMIC) and generates a fraction of informal labor that is 28% lower than the “No HK (1)” model. At the calibrated productivity differences, a smaller change in the fraction of firms that ends in the informal sector, compared to the case with no human capital, generates a larger effect on measured TFP. Moreover, as presented in the previous section, in the benchmark economy there is a joint effect of entry costs and financial frictions that is not present when human capital is absent. The model with human capital accumulation generates a smaller dispersion of the marginal product of capital.
6.4 The Role of the Informal Sector

In this section, we analyze how the presence of an informal sector affects our results. In particular, we compare our benchmark economy with a model in which firms do not have the option to operate in the informal sector. As in Hopenhayn (1992), potential entrants choose between entering the formal sector or staying out of the market. Table 13 presents the results.

Table 13: Counterfactual: No Informal Sector Model

<table>
<thead>
<tr>
<th></th>
<th>LMIC Data</th>
<th>LMIC Benchmark</th>
<th>LMIC no informal</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0.44</td>
<td>0.63</td>
<td>0.73</td>
</tr>
<tr>
<td>Informal Labor force (%)</td>
<td>71.7</td>
<td>69.22</td>
<td>-</td>
</tr>
<tr>
<td>Output per worker</td>
<td>0.32</td>
<td>0.56</td>
<td>0.71</td>
</tr>
<tr>
<td>Skilled workers (% of pop.)</td>
<td>5.70</td>
<td>10.03</td>
<td>31.93</td>
</tr>
<tr>
<td>var(ln(MPK)) Formal Sector</td>
<td>-</td>
<td>0.24</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Note: TFP, Output per worker are reported relative to the US value. “LMIC Benchmark” corresponds to the results of the model with an informal sector (our benchmark results) and “LMIC no informal” corresponds to the results of a model with no informal sector.

The benchmark exercise leads to a 37% decrease in TFP, whereas it falls by 27% when the informal sector is excluded from the model. Thus, the presence of an informal sector generates a drop in measured aggregate productivity that is 37% larger compared to the model without informality. Similarly, output per worker falls by 46% in the model with an informal sector, whereas in the model without the informal sector this figure only falls by 29%. There are no quantitatively important differences in the var(ln(MPK)) of formal sector firms.

7 Conclusion

The stock of human capital has been related to a country’s level of development. In this paper, we built a firm dynamics model with imperfect capital markets, and measured institutional frictions in order understand the role each formal-sector institution plays in generating the observed informal sector, human capital, and total factor productivity. In our model, entering
and operating in the formal sector is costly, but allows firms to choose from an unrestricted set of technologies while providing them with access to credit markets with better commitment (given by observed recovery rates and associated costs), which leads to sorting into the informal or formal sectors.

We disentangle the effects of each friction (entry costs, taxes, and bankruptcy efficiency), and find that, in a model with endogenous human capital, the main determinant of differences in the stock of skilled workers, informality and total factor productivity is the complementary effect of entry costs and financial frictions. When human capital is absent, there is no such complimentary effect. We also find that incorporating human capital into the production sector generates a drop in TFP that is 48% larger compared to the model without human capital. Finally, the introduction of an informal sector generates a drop in measured aggregate productivity of 37%.

8 References


