tions, cuts in a continuum. But what is a privation? Given that it is not an absolute thing, we might be inclined to suppose that it is a property of something which is an absolute thing. A surface, for example, we might take to be a property of some body—namely, its being extended so far and no farther. But on Ockham's account indivisibles cannot be accidents of absolute things either. Indivisibles, then, cannot fit into any of Aristotle's ten categories. Still, as I have argued, Ockham does want simply to reject the notion of an indivisible as meaningless or incoherent. But what positive account of indivisibles he has in mind or could provide is not clear in these treatises.

Chapter VIII

Infinite Indivisibles and Continuity in Fourteenth-Century Theories of Alteration

Edith Dudley Sylla

1. Introduction

Following Aristotle, most fourteenth-century natural philosophers assumed that there were three or four basic categories of natural motion or change. Not only was there local motion, a change in the place of a body, there was also the motion of augmentation or diminution, a change in the quantity of a body, and the motion of alteration, a change in the qualities of a body. And if alteration was pushed far enough, a fourth kind of change resulted, the generation or destruction of a body. Most fourteenth-century natural philosophers also assumed, again following Aristotle, that within a single motion of any of the first three kinds the place, quantity, or quality acquired or lost was continuous, as was the motion itself. Concerning alteration in particular, the fourteenth-century addition-of-part-to-part theory of alteration made a special point of emphasizing the continuity of the qualities acquired or lost, sometimes carrying this emphasis to the extent of denying altogether the existence of indivisibles in quality.[1]

All this is as one might expect, given the foundation of fourteenth-century natural philosophy in Aristotle's works. There was, however, one fourteenth-century theory of alteration, the so-called succession-of-forms theory, which asserted that the

1. See Sylla 1973, pp. 230–32, 251–62.

qualities acquired in alteration were not continua with parts that could be taken on gradually. According to this theory, every qualitative form that exists in a body is indivisible with respect to degree. When a body is altered, the theory claimed, it takes on an infinite series of indivisible degrees, each old degree in turn being destroyed to be replaced by a new one. This succession theory for qualitative change had similarities to the theories of substantial forms and substantial change propounded by Thomas Aquinas. It was applied to the explanation of change in the Eucharist by Godfrey of Fontaines, and it was elaborated most thoroughly by Walter Burley.[2]

There were in the fourteenth century some "atomists" who asserted that the physical world contains indivisible entities and accepted the deviations from Aristotelian orthodoxy that such an assertion entailed.[3] Walter Burley and the other exponents of the succession-of-forms theory, on the other hand, were not such atomists. While attempting to explain alteration in terms of indivisible degrees of quality, they simultaneously aimed at saving the continuity of motion.

In this chapter I explore some of the problems that emerged in these fourteenth-century efforts to reconcile continuity of motion with indivisible degrees. I will devote most of the chapter to examining selected texts from the works of Walter Burley and of Richard Kilvington insofar as these texts are directed to issues regarding indivisibles and the continuity of motion. Burley's concerns clearly arise from the succession-of-forms theory. His best solutions make use of the generally applicable tools of logic rather than deriving specifically from physics or mathematics. Kilvington, in the texts I will be using, never explicitly states his views concerning theories of alteration—never, for example, supporting either the addition theory or the succession theory. Nevertheless, his work fits well with Burley's and casts light on it because of the similar logical tools Kilvington employed. In addition to directing attention to some fourteenth-century problems of continuous motion and indivisible degrees in the realm of alteration, I hope in

2. Sylla 1973, pp. 230–31, 233–38. For Godfrey of Fontaines, see Sylla forthcoming a.
3. See Murdoch 1974, pp. 11–32. Also Murdoch and Synan 1966, pp. 212–35.

this way to provide a representative case study of fourteenth-century logic in action in the service of natural philosophy.

2. Indivisibles and Aristotelian Continuity

Before turning to the works of Burley and Kilvington, I should make clear a difference between Aristotelian and modern views. When Aristotle's definition of continuity is adhered to, it is impossible to produce a continuum directly from indivisibles. As has been shown in earlier chapters of this volume, Aristotle defined continuity in terms of a nested set of conditions. Entities of a certain sort might have between them other entities of the same sort or of a different sort. If two given entities had no entities of the same sort between them, Aristotle said, they were called "successive"; thus, for instance, houses standing in a row or integers used in counting might be called successive. If successive entities touched each other, they were called "contiguous"; thus, for instance, the air over a lake is contiguous to the water, or muscles are contiguous to bone in the body. If successive contiguous entities had limits that coincided, he said, the entities were called "continuous" and, indeed, "united" or "one"; thus, for instance, one-half of a stick or of a body of water is continuous with, or united to, or one with, the other half.[4]

Two comments might be made about these definitions from a modern point of view. First of all, it should be clear that Aristotle's definition of continuity is physical rather than mathematical. Physical entities such as air and water or bone and muscle can be contiguous without being continuous according to Aristotle's definitions. It is questionable, however, as Averroës very nicely pointed out, whether there can be any difference between continuity and contiguity with respect to mathematical entities, since in mathematics there is no difference of substance that might allow identity of position without physical identity.[5] Thus, from a

4. For these Aristotelian classifications, see *Physics* V 3.
5. See Burley 1501, f. 160ra, "Notanda sunt hic duo secundum Commentatorem. Primo quod contiguatio est duplex, scilicet contiguatio naturalis et contiguatio mathematica. In contiguatione naturali duo ultima remanent duo demonstrata, sed in contiguatione mathematica duo ultima revertuntur in unum. Unde breviter in mathematica duo contigua non habent duo ultima sed unum ultimum.

modern or mathematical point of view, we might consider that Burley has demonstrated mathematical continuity when, from his point of view, he has only argued for contiguity.

A second comment concerns Aristotle's nested criteria for continuity insofar as these might have any applicability to the relations of indivisibles. In Aristotle's view, bodies approach nearer to continuity as they are first successive and then contiguous before they finally become continuous. When one considers the infinitely numerous points of a line in light of these criteria, it is notable that the points not only do not fit the last criterion for continuity but also fail to meet the criteria of contiguity or even of successiveness. The proponents of the succession-of-forms theory assumed that the degrees of quality were extensionless and therefore indivisible, that they were infinite in number, and that they were "mediate,"—i.e., that between any two degrees there were other degrees. So the relation of indivisible degrees to one another was analogous to the relation of points on a line to one another. On the basis of Aristotle's definitions, then, most medieval scholars concluded that entities such as points or indivisible degrees cannot be continuous with each other, since they have no limits or extremes that could possibly touch or become one as required by the definition of continuity.[6] Nevertheless, as modern mathematics represents a line as an infinite set of mediate points, so, from a modern point of view, one might conclude that Burley's infinitely many mediate indivisible degrees form a continuum even when he believes they do not.[7]

Et sic in mathematicis contingua sunt idem quod continua." There is also much else of interest in nearby passages. See also ibid., f. 161rb.

6. See Aristotle, *Physics* VI 1, 231a24–29: "nothing that is continuous can be composed of indivisibles: e.g., a line cannot be composed of points, the line being continuous and the point indivisible. For the extremities of two points can neither be *one* (since of an indivisible there can be no extremity as distinct from some other part) nor *together* (since that which has no parts can have no extremity, the extremity and the thing of which it is the extremity being distinct)." Cf. Burley 1501, f. 172va.

7. As I understand it, modern mathematics uses infinite sets of mediate indivisibles (e.g., points) to represent continua, but these sets of indivisibles must also meet stronger criteria, since mediateness or density alone is recognized as an insufficient condition for continuity.

3. Walter Burley on Continuity and Indivisibles in Alteration

With these comments in mind, then, let me turn to Walter Burley's efforts to reconcile continuity of motion with indivisible degrees. Because of Aristotle's definition of continuity, Burley did not directly assert that indivisibles could be continuous with each other. He did assert, Aristotle to the contrary notwithstanding, that continuous motion could occur over discontinuous magnitudes:

> But a doubt is raised, because the Philosopher in the Sixth Book of this work says that motion takes its continuity from the magnitude over which the motion occurs, therefore motion cannot be continuous unless the magnitude is continuous. I say that we can understand two things by "continuous," namely either that of which the parts are united at a common terminus (and speaking this way of continuity it is not true that motion takes its continuity from magnitude); or in another way we can understand by "continuous" that which is infinitely divisible . . . and in this [second] way it is true that motion takes its continuity from magnitude, because motion is infinitely divisible because the magnitude over which the motion occurs is infinitely divisible. . . .[8]

Thus, according to Burley, for motion to be continuous in the sense of being infinitely divisible, the magnitude over which it occurs must be similarly divisible,[9] but motion can be continuous

8. Burley 1501, f. 160vb, "Sed dubitatur quia Philosophus sexto huius dicit quod motus capit continuitatem a magnitudine super quam est motus, ergo motus non potest esse continuus nisi magnitudo sit continua. Dico quod per continuum possumus duo intelligere, scilicet vel illud cuius partes copulantur ad terminum communem et isto modo loquendo de continuitatem non est verum quod motus capiat continuitatem a magnitudine. Alio modo per continuum possumus intelligere illud quod est divisibile in infinitum per continuitatem divisibilem in infinitum et isto modo est verum quod motus capit continuitatem a magnitudine, quia motus est divisibilis in infinitum propter hoc quia magnitudo super quam est motus est divisibilis in infinitum. Unde concedo quod ad continuitatem motus requiritur continuitas magnitudinis, hoc est divisibilitas magnitudinis in infinitum, et talem continuitatem habet illud quod est unum unitate contiguationis tantum." Cf. ibid., ff. 136va, 173va.

9. In the case of alteration, this infinitely divisible magnitude would be the "latitude of degrees." See Sylla 1973, pp. 226–28, 233–38; see also Burley 1501, f. 163va.

in terms of the unification of its parts when the magnitude over which it occurs is not continuous. Motion, for instance, through air into water can be continuous in Aristotle's sense, according to Burley, even though the air and water are themselves only contiguous.

The problem of alteration was, however, even more difficult for the succession-of-forms theorists, because in alteration there were not even contiguous magnitudes to be traversed, but as I have said, infinitely many mediate indivisible degrees. One context in which Burley attempted to preserve continuous or at least contiguous alteration when infinitely many mediate indivisible degrees were acquired was in his commentary on Aristotle's *Physics*. Aristotle had argued that of all motions only the uniform circular motion of the heavens can be eternal. If any motion is to be eternal, he argued, it must be continuous. But motion is continuous only if the magnitude over which it occurs is continuous. Since an unlimited continuous magnitude over which motion occurs is available only in circular motion, only circular motion can be eternal. Alteration cannot be eternal because continued alteration must eventually or occasionally reverse direction. There is, for instance, a maximum possible degree of heat or of cold, so if continued motion with respect to temperature were to be possible, it would have to involve heating for a while, followed by cooling and then reheating, and so forth. But such reversals would, Aristotle argued, render the motion discontinuous.[10]

In commenting on such an argument, Burley concedes that reversed alterations cannot be continuous, but he maintains that they can be contiguous, something that, as I have commented above, might be considered equivalent from a mathematical point of view. He says:

> . . . when it is said that contrary motions can be contiguous (*habiti*),[11] so that nothing is between them, this may be conceded. And when it is said that then they have the same extreme, I say that this is not true—indeed, they have different extremes. And when it is asked whether these are measured by the same instant or by different instants, I say that they are measured by the same instant. And when it is said that then the extremes of contraries will be in the same thing together (*simul*) at the same instant, I say that the extremes of contraries of which one is intrinsic to one motion and the other is extrinsic to the other, can well be together in the same subject at the same instant, but the extremes of contraries which are intrinsic to them cannot be together at the same instant.
>
> For the understanding of this it should be known that no extreme is intrinsic to motion except the extreme at the end of motion (*ex parte post*) or some changed state (*mutatum esse*) acquired by this motion. Thus, no beginning of motion beginning from rest is intrinsic to that motion. But [by definition] a changed state joining the parts of motion to each other is intrinsic to either part. And then I say that if, when heating is ended, cooling should immediately begin, the extreme of heating and the beginning of cooling are measured by the same instant. And this is possible because the extreme of heating is intrinsic to heating and the beginning of cooling is extrinsic to cooling. Therefore, these can be in the same [thing] at the same instant.[12]

Burley's argument in this passage is that if a body is heated and then cooled, the motions of heating and of cooling can be temporally immediate to each other. This can happen, he believes, because the intrinsic last instant of heating can be the extrinsic beginning limit of the time of cooling.

It should be noted that Burley assumes without proof in this passage of his *Physics* commentary that the alteration is continuous except at the instant of reversal.[13] In fact, the problem of

10. *Physics* VIII 7–8.

11. The terms which Burley uses in this context are *habitus* and *sese* or *sesse*. He takes the term *sese* from Averroës' commentary on the *Physics*. In his own translation Burley had *habitus* where Averroës (in Latin translation) had *sese*. Burley understands *sese* and *habitus* to have meanings similar to "contiguous," implying that the things referred to are successive and touching, but implying further that what is *sese* comes after. In addition, whereas "contiguous" applies only to magnitudes in space, *sese* applies to other sorts of magnitudes, such as motion and time. See Burley 1501, ff. 161ra–b, 164va.

12. See appendix B at the end of this volume.

13. Following Averroës, Burley distinguished between the matter of motion and the form of motion, where "matter" refers to what is gained and "form" refers to the transmutation by which it is gained. (Burley 1501, f. 162ra.) Ockham and those who followed him in trying to minimize the number of entities supposed to exist reduced motion to the first of these senses, arguing that in the outside world there was only the movable object and what it successively acquired, with no further transmutation added. In Burley's succession-of-forms theory, however, motion in the first sense was not continuous, so he was forced to use the second sense to preserve continuity of motion. See Burley 1496, ff. 14rb–va. See also Sylla 1973, pp. 237–38; 279–81.

the continuity of alteration rarely occurs in his work in direct form, perhaps because to a faithful Aristotelian it is a foregone conclusion that most motions are continuous. What Burley needs to argue for, on the other hand, are the infinitely numerous mediate indivisible degrees of the succession-of-forms theory; or more exactly, he needs to argue that these are consistent with continuous alteration. If alteration is continuous, then the parts of such alteration must be in immediate succession. But if the parts of motion are immediate, will it not follow that the degrees acquired by these parts of motion are also immediate and not mediate?

Burley examines this problem both in his *Tractatus primus* (a work which arose out of his attempt to explain the physics of the Eucharist in his commentary on the *Sentences*) and in his *Tractatus secundus*. Suppose that a hot body is cooled. As it is changed from being hot to being cold, there will be a first instant, *A*, at which it is cold. (This is in accordance with the Aristotelian view that every permanent condition or form has a first instant of existence.) Suppose, then, that at precisely that first instant of the body's being cold, something begins to heat it. What can be said about the status of the body immediately after it has begun to be reheated? If it remains cold for a period of time, there will be a degree of cold that is formally the same distance from maximum heat as is some degree of heat—which, Burley claims, implies what he wants to prove: that heat and cold are of the same species of quality. If, however, *A* is the last instant of the body's being cold, what can be said of the degrees of heat that are induced immediately thereafter? If the body is hot, it must have some degree of heat. Every possible degree of heat is indivisible and has a first instant of existence. If, then, the body has some degree of heat, that degree, whatever it may be, will have a first instant of existence, and the body will therefore have a first instant of being hot. But the body also has a last instant of being cold, namely the instant at which it was first cold and at which the reheating agent was applied; so if there is also a first instant of being hot, instants will be immediate to each other.[14]

14. See appendix C at the end of this volume, ff. 209rb–210ra.

In his *Tractatus primus* Burley accepts this argument as valid but rejects the conclusion as inconsistent with the nature of indivisible degrees, arguing that there must be something wrong with one of the assumptions of the case—namely, that cold constitutes a new permanent form. He concludes that heat and cold are not two species of quality but one, and he postulates a latitude of heat and cold having maximum heat at one end and maximum cold at the other, with a middle degree neither hot nor cold but temperate.[15] Then there will be no minimum degree of cold and no first instant of being cold, and so the case falls apart. In his *Tractatus secundus* Burley solves the problem by referring to the motion (alteration) as well as to the degrees acquired.[16] These are not very good arguments on Burley's part, although perhaps he should be excused for some of their defects. The *Tractatus primus* and *secundus* are very clearly the product of live debate and contain many ad hominem arguments; Burley even bases many of his proofs on parts of his opponents' theories that he himself does not accept.[17]

Of greater interest to me here are contexts in which Burley rejects the conclusion of this sort of argument but finds the fault elsewhere than in the argument's assumptions. In these other contexts Burley uses the tools of medieval logic to block inferences of this sort: If immediately after instant *A* the body has some degree of heat, and if every such degree has a first instant of existence, then the body will have a first instant of being hot. He does this by arguing that in a proposition such as "Immediately after instant *A* the body has some degree of heat," the term "some degree of heat" has merely confused supposition because it follows the syncategorematic word "immediately." In such a case one cannot infer that any assignable degree of heat will occur immediately after *A*, even though there is no interval of time such that it will pass before a degree of heat is acquired.[18] Similar

15. His conclusion is quoted in Sylla 1973, p. 236, n. 36.
16. Sylla 1973, pp. 279–81.
17. A sense of the disputational nature of the *Tractatus primus* may be obtained from the excerpts in appendix C below.
18. Burley does not apply this argument in the case of reheating discussed just above, because there it is assumed that heat (and cold) is a separate species of quality for which, according to Aristotle, there must be a first instant of existence. He does use a similar argument for alteration within a single species of quality—

answers are given by many medieval authors to similar problems. The special advantage of these answers lies in the fact that they preserve the sense of "immediately" by asserting that there is no finite interval such that it will pass before the occurrence of an indivisible, while taking account of the fact that the indivisibles in continua are mediate or dense—i.e., that between any two indivisibles there are infinitely many more.

But even if such an answer provides a good resolution of the problem, it does not, at least at first glance, explain the invalidity of the opponent's inference to the conclusion that there will be a first instant of being hot. Perhaps the specious validity of the opponent's argument can be attributed to a difference between finite and infinite sets. If the conditions of the case remained otherwise the same and there were not infinitely many but, say, only three possible degrees of heat, then one could justifiably infer that (since whichever of the three degrees of heat the body has will have a first instant of existence), the body itself will have a first instant of being hot. The plausibility of the opponent's position depends on thinking in terms of a finite set of degrees. When, however, the set of possible degrees of heat is infinite, as Burley assumes it is, the argument becomes invalid. One cannot infer from the fact that each of the infinitely many degrees of quality has a first instant of existence that the body will have a first instant of being hot. But why not? A further cause of perplexity arises from the fact that the acceptable exposition of the proposition "Immediately after instant *A* the body will have some degree of heat" seems intuitively weaker than the original proposition. Does the exposition really capture the sense of the original? Is no stronger exposition possible?

Before I try to answer those questions, I want to spell out in greater detail the position that gives rise to them. Burley takes this position also regarding a case involving only instants rather than the degrees of quality gained in alteration: the issues are the same, but the analysis is simpler. Suppose that *A* is the present instant. What might one say about the instants in the time period immediately after *A*? An opponent might argue:

e.g., for alteration from diminished (remiss) heat to higher degrees of heat. Cf. Sylla 1973, p. 280.

> Between the being and the nonbeing of instant *A* there is no intermediate. But the nonbeing of *A* and the being of an instant other than *A* are the same, interchangeably, because this follows logically: *A* is not; therefore an instant other than *A* is, and vice versa. Therefore between the being of *A* and the being of an instant other than *A* there is no intermediate. Therefore after *A* immediately there will be another instant. Therefore in time instants are immediate.[19]

Burley rejects this argument and explains that when one says, "*A* is, and immediately after *A* there will be another instant" the term "immediately" is a syncategorematic word which gives the term "another instant" following it merely confused supposition. In other words, "immediately" is the sort of word that affects the reference of other terms in a proposition, and in this case it makes the term "another instant" refer to many instants in such a way that no one of them can be picked out. In this interpretation, Burley says, the proposition is true and equivalent to "*A* is, and before every completed interval (*medium completum*) another instant will be." One cannot, however, infer from this that "Some other [given] instant will be without an interval," and so the argument does not prove that instants are immediate.[20]

4. An Analysis of Burley's Position

How does this result follow from the character of the reference assigned to "another instant," its merely confused supposition? In general, medieval authors do not make it entirely clear what one can do with terms having merely confused supposition. They say what one *cannot* do, and they deal with special cases. I believe, however, that many if not all cases of terms labeled as having merely confused supposition can be treated in the following way. Terms that have merely confused supposition often appear in propositions not immediately after but at some distance from the syncategorematic word that is described as bringing about their merely confused supposition. Thus, for instance, in the universal affirmative proposition "Every man is an animal," the predicate term "animal" has merely confused supposition, while the

19. See appendix D at the end of this volume, f. 176ra.
20. See appendix D, f. 177ra.

subject term "man" has distributive confused supposition, and the supposition of these terms is determined by their location relative to the syncategorematic word "every." One may perform a logical descent under a term with distributive confused supposition to a conjunction of singular propositions. For example, from "Every man is an animal" one may descend under its subject term to the conjunction "This man is an animal, and that man is an animal, and that other man is an animal . . . ," indicating every existing man individually.[21] In making this descent we have made no descent under the predicate term of the original proposition, but the supposition of "animal" in the new singular proposition is no longer merely confused but determinate. One can go further, then, in expounding the proposition by descending to a disjunction of the individuals referred to by the predicate term of each of the resultant singular propositions: "This man is this animal or that animal, etc., and that man is this animal or that animal, etc., and that other man is this animal or that animal, etc., . . ." and so on.[22] I suggest, then, that the salient logical characteristic of a term with merely confused supposition is that no logical descent can be made under it as it stands but that after a descent has been made under another (often a previously occurring) term, the supposition of the previously merely confused term is changed and a descent becomes possible under its occurrences in the propositions resultant from the initial descent.

If I am right in this observation, it can be used to help clarify such cases as that described by the proposition "Immediately after *A* there will be another instant." A plausible restatement of this proposition is "Before any interval after *A* is completed there will be another instant." Here "instant" has merely confused supposition, and no logical descent is possible under it unless a descent has previously been made under the term "interval" occurring earlier in the proposition. Descending under "interval," one has "before the five-minute interval after *A* is completed there

21. For one exposition of the permissible descents, see Burley 1955, pp. 20–27.

22. The medieval sources do not say that one can descend under a term with merely confused supposition by a two-step procedure such as the one I have described. They do, however, explain the first descent under a term with a distributive confused supposition, and there are rules of descent that apply to the propositions produced in that way. Cf. Burley 1955, p. 20.

will be another instant, . . . before the four-minute interval after *A* is completed there will be another instant, . . . before the one-second interval after *A* is completed there will be another instant, . . ." and so on infinitely for every completed interval after *A*. And on that basis it is possible to descend under the now-determinate occurrences of "instant" in each of the infinitely many resultant propositions: "before the five-minute interval after *A* is completed there will be . . . the instant four minutes after *A*, or . . . the instant three minutes after *A*, etc., . . . and before the four-minute interval after *A* is completed there will be . . . the instant three minutes after *A*, or . . . the instant two minutes after *A*, etc., . . . and before the one-second interval after *A* is completed there will be . . . the instant one-half second after *A*, or . . . the instant one-fourth second after *A*, etc.," and so on infinitely.[23]

It is clear from this two-stage descent that for different sets of intervals chosen in the first descent, correspondingly different sets of instants correspond to the truth of the propositions resulting from the second descent—which helps to show why the second descent could not be made unless the first descent had already been made. If we put it another way, there is no instant after *A* such that it occurs before the completion of all intervals after *A*, or there is no instant immediately after instant *A*. It is clear, further, that in the standard interpretation of merely confused supposition, one is not committed to saying anything about the set of all the instants after *A* taken collectively but is limited to referring to them individually. In doing so one learns something about the relations of these instants to one another and to the instant *A*, but nothing about the relation to *A* of the whole set of instants after *A*.

Modern mathematics counts infinite sets by putting them into one-to-one correspondence with other infinite sets. Medieval natural philosophers made similar uses of one-to-one correspondences of infinite collections when they supposed, for instance, that if Socrates could count one unit at some instant within each proportional part of an hour (i.e., if he could count one unit in

23. In this descent, obviously, the number of disjuncts within each conjunct as well as the number of conjuncts is infinite; only a selection of each sort of element can be displayed.

the first half-hour, a second unit in the next quarter-hour, a third unit in the next eighth, etc.), then by the end of the hour he could have counted an infinite number of units. But despite their ability to put infinite sets into one-to-one correspondence with each other, many medieval philosophers in effect considered all the points of a line or all the instants of a temporal interval or even all the rational numbers as if they were unspecifiable, even when from a modern point of view the set they had in mind was countable—saying, for instance, that there is no such thing as all the instants in a temporal interval or all the integers—one cannot speak of all of them because there is no limit to them.

In trying to construct continuity from indivisibles, especially given Aristotle's definitions, it would be natural to try to define immediacy of sets of indivisibles. If instants can make up continuous time, then it should be possible to show that the set of instants in the first half of an interval is immediate to the set of instants in the second half of the interval. If the first half of an interval is considered as ending at the middle instant of the interval, can it be shown that this middle instant is immediate to the infinite set of instants that comes after it? How could such immediacy be defined, given that no instant in the set of instants after the middle instant is immediate to the middle instant?

I believe that when Burley relies upon the logical concept of merely confused supposition for the solution of the problem of invisibles and continuity, he in effect renounces the possibility of talking collectively about the set of all the indivisibles. Instead of asserting that one can consider all the instants after instant *A* and show that this set of instants is immediate to *A*, Burley says merely that if one chooses any completed interval after *A*, then it is always possible to find an instant that has occurred before the end of that interval. I will call this resolution of the problem a case-by-case or no-infinite-set resolution. Burley's reliance on a no-infinite-set resolution may explain why his exposition of "immediately" appears weak. But a stronger exposition, admitting the possibility of referring to all the instants of an interval otherwise than in merely confused supposition, might have had other, perhaps unacceptable, results, as I will attempt to explain in my discussion of Kilvington below.

5. Burley on Whole Sets of Degrees

It is interesting that in his *Tractatus primus,* where Burley accepted the inference to a first instant of being hot, which he rejected elsewhere, he also attempted to talk about whole sets of degrees at once, whereas his teacher Thomas Wylton used the no-infinite-set approach implicit in Burley's use elsewhere of merely confused supposition. In the first conclusion of the *Tractatus primus,* Burley stated that a quality by its own power could produce a substantial form.[24] According to the usual medieval theory, when fire heats water, eventually turning it into fire, the heat of the fire first induces greater and greater degrees of heat into the water. Water, it is understood, is naturally cold, but it can be heated and qualified by any degree of heat short of the maximum, the degree of fire itself. Therefore, as the heat of the fire acts on the water, it gives it ever-increasing degrees of heat up to but not including the maximum degree. At the end of the action, the maximum degree of heat is introduced, and simultaneously the water must be destroyed and fire generated. According to the usual theory the heat of the fire causes all the heating of the water up to but not including the maximum degree, but it is the substantial form of fire which finally destroys the water, generating fire from it. That this must be so was supported by the basic principle that a cause must be greater than, or as great as, its effect.

What Burley argues, by contrast, is that if the heat of the fire can produce all the heat in the water up to the maximum degree, then it can produce that degree, too. In the usual picture of alteration, at the last instant of alteration the maximum heat and the substantial form of fire first exist. But when they already exist, Burley argues, then there is no need for them still to be, nor can they be, in process of generation. Thus all the work of producing the maximum degree and substantial form must be done in the interval *before* the final instant. But the heat of the fire can do

24. *Tractatus primus,* MS Vat. lat. 817, f. 203ra. "Qualitas in virtute propria potest producere formam substantialem." My references to the *Tractatus primus* will all be to this manuscript.

everything that is done during that interval. Therefore the heat alone can produce what first appears at that final instant, and the help of the substantial form of fire is not required.[25] Obviously, this argument brings in alteration and causality and various other considerations besides the degrees of heat induced into the water. Nevertheless, it seems to arrive at the conclusion that if something can produce all the degrees of heat short of maximum heat, then it can produce the maximum degree also.[26]

Here Burley's teacher Thomas Wylton raised an objection that seems to involve something closely related to the analysis envisaged by Burley for the case of reversed alteration. Wylton argued that the heat of the fire cannot produce anything necessitating the production of the substantial form of fire, because any accidental form or disposition to fire which it produces before the induction of the substantial form of fire is produced at some instant. Since there must be an interval between the instant when that accidental form is induced and the instant when the substantial form is induced, no two instants being immediate, the former cannot necessitate the latter.[27] Thus Wylton argues that when a no-set approach is used, Burley's conclusion does not follow.

25. Ibid., f. 208rb: "Et dixi quod in illo instanti ultimo temporis mensurantis totam transmutationem elementorum adinvicem non requiritur agens pro tunc inducens formam quia pro tunc forma est inducta. Et pro tunc non requiritur inducens. Inductum enim non oportet inducere sicut non oportet agere acta, sed sufficit quod fuit inducens in toto tempore praecedenti . . ."

26. See ibid., f. 203va, ". . . si aliquid possit (esse) in totam alterationem praecedentem et non in terminum, cum Deus possit suspendere actionem cuiuslibet agentis circa illam materiam, tunc, hoc posito (quod Deus suspendat actionem cuiuslibet alterius agentis), sequitur quod terminus alterationis non inducetur in materia, quia non ab alterante nec ab aliquo alio agente, quia actio cuiuslibet alterius agentis circa istam materiam suspenditur, et ita sequeretur quod calor alterans efficeret alterationem infinitam, quia non terminatam, quod est impossibile, tum quia alterationem repugnat infinitas (sexto Phisicorum in fine) tum quia nulla forma materialis potest in motum infinitum."

27. Ibid., f. 203rb, "Huic rationi respondet Reverendus magister noster dominus Cancellarius Londonensis quod calor in virtute propria non potest in aliquam dispositionem necessitatem ad formam substantialem ignis, quia nulla dispositio praecedens dispositionem formae ignis necessitat ad formam ignis, quod patet quia quaecumque dispositio inducitur ante inductionem formae ignis inducitur in aliquo instanti praecedenti instans in quo inducatur forma ignis et inter illa duo instantia est tempus medium in quo tempore medio non est materia sub forma ignis. Et ideo nulla dispositio praecedens necessitat ad formam ignis ex quo illa inducta non statim inducitur forma ignis. Immo per tempus medium est materia sub privatione formae ignis."

Burley replies by conceding that no heat induced before the induction of the fire necessitates the induction of fire. He says, however, that his argument does not require this, but rather only that the heat be able to produce the whole alteration (*totam alterationem*) preceding the induction of the substantial form.[28] This answer is typical of Burley's so-called realist tendencies—of his tendency to treat the motion of alteration as an effect in addition to the degrees of heat produced.[29] It seems to me that Burley's reference to the alteration in this case is not sufficiently justified. Nevertheless, one can see that there are strong reasons for trying to talk in such a case about all of the alteration or collectively about all the degrees of heat induced, not restricting oneself to the individual degrees. Burley's reference to the alteration substitutes, I think, for talk of whole sets of degrees.

In sum, because of his reliance on Aristotle's definitions of continuity, Burley did not think that the infinite mediate indivisible degrees acquired in alteration or the infinite mediate instants in time formed a continuum. He did, however, argue that the motion of alteration by which such indivisible degrees were acquired could be continuous, and he paid particular attention to reversed alterations, arguing that motions of cooling and heating could be immediate to each other. Accepting the immediacy of alterations, Burley had then to show that immediate alterations did not imply immediate indivisible degrees. He accomplished this using the concept of merely confused supposition. I have tried to argue that the use of merely confused supposition has the effect of substituting a case-by-case or no-set approach for talk about whole sets of indivisible degrees. Where talk about whole sets of degrees might otherwise have been required, Burley sometimes brings in an additional reference to the motion of alteration instead.

28. Ibid., f. 203rb–va, "Bene tamen verum est et necessarium illud quod dicit magister meus quod non est aliquis calor inductus ante inductionem formae ignis necessitans ad formam ignis nec super hoc fundatur ratio. Sed totum fundamentum istius rationis consistit in hoc quod calor in virtute propria potest in totam alterationem praecedentem formam substantialem ignis."

29. See Sylla 1973, appendix I.

6. Richard Kilvington's *Sophismata*

I based my discussion of Burley's ideas mainly on texts taken from his commentary on Aristotle's *Physics* and from his *Tractatus primus,* a work containing the record of a debate that arose out of Burley's lectures on Peter Lombard's *Sentences.* I want now to turn to a work by Richard Kilvington which comes from yet a third intellectual context, namely from the logical disputations *de sophismatibus* in which undergraduates at Oxford had to participate before they could become bachelors of arts.[30] A medieval sophisma is a puzzling proposition—it may be true but appear false, or be false but appear true, or it may have both a plausible proof and a plausible disproof. The subject matter of such a proposition or the proof or disproof may be taken from many different disciplines. Most of Kilvington's sophismata appear to have physical subjects—for example, bodies becoming whiter or traversing distances—but his techniques of analysis and proof and disproof are those developed in medieval logic.

While Kilvington's major interests in his *Sophismata* might be characterized from many different points of view, it would not be wholly wide of the mark to say that the main lesson of the first large segment of the work concerns the proper use and results of the no-infinite-set approach to dealing with infinite collections. The first four sophismata, for example, involve the comparison of infinite sets of mediate degrees of whiteness starting from zero degree as an extrinsic limit.

The first two sophismata—namely, "Socrates is whiter than Plato begins to be white" (*Socrates est albior quam Plato incipit esse albus*) and "Socrates is infinitely whiter than Plato begins to be white" (*Socrates est in infinitum albior quam Plato incipit esse albus*)—are both based on a hypothetical case in which Socrates has the maximum degree of whiteness, while Plato is not white at all at the present instant but now begins to be white, acquiring degrees

30. See Sylla forthcoming b. I have based my study of Kilvington's *Sophismata* on the text and translation of that work by Norman and Barbara Kretzmann (forthcoming). I am grateful to them for providing me with a copy of their edition.

of whiteness as time goes on.[31] The difficulty involved in these two sophismata is that as Plato begins to be white, he initially takes on degrees of whiteness less than any given degree. Does it not therefore follow that Socrates' whiteness, which is finite in degree, will have an infinite ratio to the whiteness with which Plato first begins to be white? And if Plato is white, while Socrates is infinitely whiter, does it not follow that Socrates is infinitely white? In his solution of these sophismata Kilvington grants both propositions and argues that the proposition "Socrates is infinitely whiter than Plato begins to be white" is equivalent to the conjunction "Socrates is white, and Plato will without interval be white, and there will be no whiteness or degree of whiteness without interval in Plato by means of which the whiteness in Plato will without interval be compared to the whiteness in Socrates." From this conjunction it does not follow that Socrates is infinitely white, and so the difficulty is resolved.

The similarity of this solution to Burley's solution of the problem of reversed alteration should be clear. In each case the key point of the solution is the denial that any given degree of quality is immediately acquired. In Kilvington's comparisons, if one could treat all the acquired degrees of whiteness as a single set, one might then take the lower limit of that set and compare Socrates' whiteness to that limit; but this is precisely what Kilvington does not do.

In subsequent sophismata, Kilvington similarly never treats all of a set of infinitely many degrees at once except by means of a singular term with confused supposition. The fifth sophisma sentence is "Socrates will begin to be as white as he himself will be white," which is referred to a hypothetical case in which the whiteness in Socrates increases throughout all of a long life, a life which, as it is later assumed, is terminated not by a last instant of being but rather by a first instant of nonbeing.[32] In interpreting

31. For a discussion and English translation of the first sophisma, see Kretzmann 1977.

32. "Sophisma 5. Socrates incipiet esse ita albus sicut ipsemet erit albus. Posito quod albedo tota in Socrate intendatur per totam vitam Socratis, et vivat Socrates diu post hoc instans. . . . Et ponatur quod A sit primum instans non esse Socratis."

the sophisma sentence, Kilvington assumes that the expression "as he himself will be white" has confused supposition because of the syncategorematic word "as," which he takes as meaning not only that Socrates must be as white as some degree he will have but, more strongly, that he must be as white as all the degrees he will have.[33] He further interprets "Socrates will be as white as he himself will be white" as equivalent to "At some instant Socrates will be as white as he himself will be white." Then the problem is that at no given instant will Socrates be as white as he will be white because, given any instant of his existence, the whiteness he has at that instant will be less than a whiteness he will have later; for he will live past that instant and become whiter for some time, however short, because there is no intrinsic last instant of his existence but only an extrinsic first instant of his nonexistence. Kilvington's solution of the sophisma, therefore, is to deny that Socrates will begin to be as white as he himself will be white on the grounds that there is no instant at which he will begin to be so. Using the two-step explication of merely confused supposition proposed earlier in this chapter, we may say that since the logical descent to specific degrees of whiteness under the clause "Socrates will begin to be (as) white" is made before the descent to degrees of whiteness under the clause "as he himself will be white," a higher degree of whiteness can always be chosen in the second case.

Thus in these sophismata Kilvington takes the same approach that Burley often takes. He deals with infinitely many degrees of form, but he compares them individually and does not refer to all of a set of infinitely many degrees at once except by a term with merely confused supposition. On the whole I think this tech-

33. A way of explaining this with normal supposition theory can be found in Burley 1955. The type of supposition in Burley's theory that fits Kilvington's claims is not merely confused supposition but distributive confused supposition. When a term or expression has distributive confused supposition, according to Burley, descent to any one of the term's supposita is permissible (pp. 24–25). Kilvington himself says that this descent is allowed (in his discussion of sophisma 6). So for Socrates to be as white as he will be white, he would have to be as white as he was in this instant and as white as he was in that instant and as white as he was in a third instant and so on for any instant. Kilvington himself does not use the terms "merely confused supposition" and "distributive confused supposition" but speaks only of supposition being confused. For clarity, I have carried Burley's terminology over to Kilvington's work in my subsequent discussion.

nique provides successful resolutions of the problems Kilvington treats. Kilvington presses the technique farther than Burley does, however, and so reveals more of its consequences. In the remainder of this chapter I want to show that one such consequence is the disclosure of a lack of identity between properties of continua as normally understood and properties of infinitely many mediate indivisibles, when no set of these indivisibles is defined.

7. An Analysis of Kilvington's Sophisma 8

With this in mind, I will consider in some detail Kilvington's sophisma 8, the first in which a comparison of an extrinsically limited series with its limit is made.[34] According to the hypothesis of this sophisma, Plato and Socrates increase equally in whiteness for a period of time *A*, but Socrates exists at the last instant of the time period, whereas Plato is destroyed. Then Socrates lives on after the last instant of time *A* and increases further in whiteness. The sophisma sentence is "Socrates will be precisely as white as Plato will be white in any (*aliquo*) of these [proportional parts of time *A*]." In the proof of the sophisma sentence, the further increase of whiteness in Socrates is used as the basis of an argument that since Socrates is now less white than Plato will be white in any proportional part of time *A*, and since he will be whiter than Plato will be white in any proportional part of time *A*, it follows that he will also be precisely as white as Plato will be white in any proportional part. The instant of his being so will be the instant connecting the interval in which he is less white than Plato to the interval in which he is whiter.

Thus Kilvington interprets the sophisma sentence as meaning simply that there will be at least one instant at which Socrates will be precisely as white as Plato will be white in any proportional part. It does not concern him that Socrates will go on to become whiter, and the further increase in Socrates' whiteness is posited only to assure that Socrates' whiteness will exceed—and so ought also to equal—any whiteness in Plato.

So the question that remains is this: at the last instant of inter-

34. See appendix E at the end of this volume.

val *A* will Socrates be just as white as Plato will be white in any proportional part of *A*, or will he be whiter? Here, clearly, we have an infinite series of degrees of whiteness acquired by Plato of which there is no intrinsic maximum degree. And we have the degree of whiteness that Socrates has at the last instant of *A*, where this degree is the degree which Plato's series of degrees approaches as an extrinsic limit. In favor of the view that the degree Socrates has at the end of *A* is greater than the degree Plato has in any proportional part of *A* is the consideration that Socrates' whiteness is the extrinsic and not the intrinsic limit of the series of whitenesses Plato has. On the other side, however, is the consideration that there is no determinable amount by which Socrates' whiteness will be greater than Plato's.

Faced with these alternatives, Kilvington concludes that the sophisma sentence is false and that at the instant in question Socrates is not precisely as white as Plato is white in any proportional part of *A* but rather whiter. He denies that "By something will Socrates be whiter than Plato will be white in any of these" (where the term "something" has determinate supposition), although he concedes that "Socrates will be whiter by something than Plato will be white in any of these" (where the term "something" has merely confused supposition). And so he can go on to claim that "By nothing, however, will Socrates be whiter than Plato will be white in any of these."[35]

By now a set of propositions of this sort should look very familiar. As in the cases discussed earlier, a comparison is made with infinitely many degrees where these degrees are referred to by means of a term with merely confused supposition and so are taken individually rather than treated all together. When Socrates' whiteness is compared with Plato's degrees of whiteness individually, it is never the case that Socrates is precisely as white as Plato is ever white.

35. In paraphrasing the sophisma I have simplified somewhat by assuming that if Socrates is ever precisely as white as Plato in any proportional part of *A*, it will be at the last instant of *A*. Kilvington makes no such assumption but leaves it open as to when the instant will be which connects the interval during which Socrates is less white than Plato will be in any proportional part of *A* and the interval during which Socrates is whiter than Plato will be in any proportional part of *A*. One might imagine that the instant will be not the last instant of *A* but rather the last instant of Plato's existence. But Plato's existence is extrinsically limited and so has no last instant.

Kilvington could also argue in this case that the difference between Socrates' whiteness at the end of *A* and some degree of whiteness acquired by Plato before the end of *A* is less than any given finite amount. This result appears as his assertion, "By nothing . . . will Socrates be whiter than Plato will be white in any of these." But it is notable that Kilvington nevertheless denies the sophisma sentence. This is a result different from what would be obtained in the case of continua as ordinarily conceived, where two continua that differ by no finite amount are considered equal.

Thus Kilvington's early sophismata are devoted to extrinsically limited dense sets of degrees and the problems these raise—particularly problems of comparison. Repeatedly in the course of his discussions Kilvington uses arguments similar in form to the argument that Socrates' whiteness will be less than Plato's whiteness for a period of time and Socrates' whiteness will be greater than Plato's whiteness for a period of time, and so there must be some instant or period of time when Socrates' whiteness is equal to Plato's. Such an argument ought to be valid if the degrees of whiteness acquired by Socrates form a continuum. (The argument may be construed as an application of the modern intermediate value theorem.) Over and over, however, Kilvington finds reasons why the conclusion of such an argument is to be rejected. In a case like that of sophisma 8 just discussed, Kilvington might argue, for instance, that the last instant of interval *A* is the first instant at which Socrates is whiter than Plato will be white in any proportional part of *A*, and so the interval when Socrates is less white than Plato will be white is connected to the interval when Socrates is whiter than Plato will be white, not by an instant at which Socrates is equally white as Plato ever is, but rather by an instant at which Socrates is whiter. There could not also be an instant at which Socrates equaled the whitenesses Plato has had, because if there were such an instant, it would be immediate to the already mentioned instant, which is impossible.[36]

36. In a later series of sophismata (33. "Socrates movebitur velocius quam Socrates nunc movetur"; 34. "Plato potest moveri uniformiter per aliquod tempus et aequevelociter sicut nunc movetur Socrates"), Kilvington imagines a situation in which he claims Socrates moves with an instantaneous velocity with which no body could move uniformly: "between a uniform motion by which something can move uniformly slower than Socrates now moves and a uniform motion by which

8. Kilvington's Major Interests in Continuity and Indivisibles

From these examples, I think I can summarize what seem to be Kilvington's major interests concerning continuity and indivisibles. Kilvington is extremely interested in the problems raised by sets that have extrinsic limits, such as the degrees of whiteness associated with Plato's beginning to be white. In analyzing such cases he, like Burley, invariably relies on the standard Aristotelian understanding of the relations of indivisibles to continuity. There is, for instance, absolutely no doubt that a continuum is divided into two by only one indivisible. When he compares an extrinsic limit to the degrees that converge to that limit, Kilvington invariably says that the limit is greater or smaller (as the case may require) than the degrees approaching the limit. Thus, if he imagines a continuum of degrees divided at a degree with that degree belonging to the right-hand side, he says that the degrees approaching the division from the left are less than the minimum degree on the right. So, in sophisma 8, at the last instant of *A* Socrates is whiter than Plato is white in any proportional part of *A*. This seems reasonable. Furthermore, all the commonly accepted medieval distinctions between senses of "begin" and "cease" and between intrinsic and extrinsic limits require that the differences involved here not be blurred; if Kilvington did not maintain a difference here, the medieval theories of first and last instants would be rendered meaningless.[37]

Nevertheless, Kilvington's assertion that the whiteness Socrates has at the last instant of *A* is greater than the whiteness Plato has in any of the proportional parts of *A* leads to some results that may be difficult to accept. Kilvington must claim that Socrates cannot be white at a given instant with a degree of whiteness as white as those with which Plato will be white. By not equating a series of degrees with its limit, Kilvington has, therefore, created

something can move uniformly faster than Socrates now moves there is no intermediate uniform motion by which something can move uniformly and precisely as fast as Socrates now moves" (sophisma 34).

37. On first and last instants, see Wilson 1956, pp. 29–56.

intermediate values that appear to break up the continuity of the intervals they occur in. More importantly for the issue that has been my major concern in this chapter, this result is at odds with what would be expected for continua such as lines and so constitutes a reason for denying that the sets of infinitely many degrees imagined by Burley or Kilvington do constitute continua even when Aristotle's requirements for continua are relaxed. This result appears to be at odds with the modern arithmetization of the continuum according to which a line, for instance, can be considered as an infinite set of points.

Nevertheless, there seems to be good reason for denying, with Burley and Kilvington, an equivalence between infinite sets of indivisibles and continua. If indivisibles are admitted, then Kilvington seems justified in concluding that Socrates at the last instant of *A* has a degree of whiteness that Plato never had in any proportional part of *A*. In order to arithmetize the continuum, modern mathematics had, among other things, to deny that the addition or subtraction of an indivisible from a set of indivisibles made any difference.[38] In establishing his one-to-one correspondence of numbers and points on a line, Dedekind recognized that there was a difference such as that accepted by Kilvington but then simply erased it by fiat. Dedekind wrote:

> . . . we ascribe to the straight line completeness, absence of gaps, or continuity. In what, then, does this continuity consist? . . . for a long time I pondered over this in vain, but finally I found what I was seeking. This discovery will, perhaps, be differently estimated by different people; the majority may find its substance very commonplace. It consists of the following. In the preceding section attention was called to the fact that every point *p* of the straight line produces a separation of the same into two portions such that every point of one portion lies to the left of every point of the other. I find the essence of continuity in the converse, i.e. in the following principle: "If all points of the straight line fall into two classes such that every point of the first class lies to the left of every point of the second class, then there exists one and only one point which produces this division of all points into two classes, this severing of the straight line into two portions."
>
> . . . from the last remarks it is sufficiently obvious how the discon-

38. But see Wilson 1956, p. 31.

> tinuous domain R of rational numbers may be rendered complete so as to form a continuous domain. . . . If now any separation of the system R into two classes A_1, A_2 is given which possesses only *this* characteristic property that every number a_1 in A_1 is less than every number a_2 in A_2, then for brevity we shall call such a separation a *cut* [*Schnitt*] and designate it by (A_1, A_2). We can then say that every rational number a produces one cut or, strictly speaking, two cuts, which, however, we shall not look upon as essentially different; this cut possesses, *besides*, the property that either among the numbers of the first class there exists a greatest or among the numbers of the second class a least number. . . .[39]

If there are "two cuts, which, however, we shall not look upon as essentially different," there seems to be a lack of correspondence between real numbers (which might be taken as representative of continuous distances) and indivisible points on a line, a lack of correspondence which modern mathematics has agreed to ignore. If one does admit points and lines or indivisible degrees and latitudes of quality, then it does seem to make a difference whether a segment of the continuum is terminated inclusively or exclusively by an indivisible. Yet the length of that segment of the continuum seems to be the same, whether the indivisible is included or not.[40] For all our willingness to relax Aristotle's rather physical conditions for continuity, we seem to have in the work of Burley and Kilvington a persuasive argument that continua and infinite collections of mediate indivisibles have differing properties.

A modern mathematical logician might formalize the views of Burley and Kilvington on this problem and thereby discover, assuming that the formalization was faithful to the original, the source of the differences between their views and modern views on this problem. I have neither space nor competence to do this, but I suggest that the source of the difference very likely lies in

39. Dedekind 1901, pp. 10–13.

40. Burley already, in a sense, suggested this distinction in an attempt to explain what Averroës may have had in mind in distinguishing mathematical and natural contiguity. (Burley 1501, f. 160rb, quoted in note 7 above): "Vel potest dici quod Commentator dicit quod quantum ad distantiam et ad propinquitatem quas considerat mathematicus non differunt continuatio et contiguatio, quia ultima contiguorum sunt simul et ita ultima contiguorum aequaliter distant a quocumque et aeque propinqua sunt cuicumque. Et ideo mathematicus utitur ultimis contiguorum tanquam uno ultimo. . . ." Cf. ibid., f. 172vb.

the no-infinite-set approach taken by the fourteenth-century philosophers. Remaining closer to natural philosophy than to mathematics, medieval logic operated mainly in terms of familiar physical objects. Modern mathematics has come to different conclusions in part by defining infinite sets as individual entities. Although consideration of God's omniscience and omnipotence sometimes led medieval thinkers to go beyond everyday experience, Walter Burley and Richard Kilvington in this case remained instead within the Aristotelian heritage.