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trary to what they obviously intended.⁹² Was it not, perhaps, for such contemporaries that Ockham wrote his code book?

92. *Exp. Phys.* III, t. 61 (206b27-33), MSS Mert. 293, 72r; Bruges 557, 163r: "Verumtamen aliquando Philosophus ponit unam pro alia [scil. one of two different propositions] non curans multum de verbis et supponens quod addiscentes istam scientiam sunt sufficienter exercitati in logica per quam sciant discernere inter propositiones et advertere quando una ponitur pro alia et quando non. Quod tamen multi moderni ignorant, et ideo, quia frequenter audiunt alias scientias antequam perfecte sciant logicam, multas opiniones erroneas inveniunt et imponunt multa Aristoteli et aliis philosophis quorum contraria ex intentione demonstrant."

CHAPTER VII

Theology and Physics in *De sacramento altaris*: Ockham's Theory of Indivisibles

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1. Introduction

Ockham's theory of indivisibles was very influential in fourteenth-century natural philosophy, accepted even by scholastics who rejected other Ockhamist positions.¹ In articles either recently or about to be published, John Murdoch has presented an account of this theory which leaves it open to serious philosophical objections.² Murdoch's interpretation of Ockham's theory has both a metaphysical and a semantic component. According to Murdoch, Ockham maintains that indivisibles such as points are nonexistent and that one way of dealing with a term referring to an indivisible is to replace it by an expression which does not refer to that indivisible; thus, for instance, the term "point" is said to be equivalent to the expression "line of such and such a length." If we take this account at face value, it is not hard to see how this interpretation makes Ockham's theory of indivisibles appear incoherent. By definition, it is true that

(1) A point is indivisible

I am indebted to Norman Kretzmann and John Bolger for their many helpful comments and suggestions, and I am grateful to John Crosssett, whose efforts on my behalf made this paper possible.

1. See, e.g., Murdoch forthcoming a, forthcoming b, and forthcoming c. Cf. also Eldredge 1979, pp. 95-97.

2. See, e.g., besides the two papers mentioned in n. 1 above, chapter VI. Although I take a critical attitude toward Murdoch's interpretation, I have learned a great deal from his writings on these subjects.

and that

(2) A line (of such and such a length) is divisible;

but if we make the appropriate substitutions warranted by the apparently Ockhamist analysis of the term "point," it will also be true that

(1') A line is indivisible

and that

(2') A point is divisible.

And there are other, similar sorts of serious difficulties with Ockham's theory as Murdoch presents it.³ In the face of such difficulties, it seems worthwhile to reconsider what Ockham says about indivisibles to determine whether his theory really is as open to philosophical objections as Murdoch's account makes it seem. I propose to do so by looking carefully at Ockham's *De sacramento altaris*. However surprising it may at first seem, this treatise contains one of his most elaborate discussions of indivisibles, and Murdoch has relied heavily on it in his account of Ockham's theory.⁴

There are two obstacles to a consideration of Ockham's treatment of indivisibles in that treatise, however. The first has to do with the state of the text and with uncertainties about its historical context. In the only modern edition of *De sacramento altaris*,⁵ which is not a reliable critical edition,⁶ T. Bruce Birch has pre-

3. Some of these are pointed out in Kretzmann forthcoming.

4. There are, of course, other works besides *De sacramento altaris* in which Ockham discusses the nature of indivisibles. Because I am examining Murdoch's interpretation as well as Ockham's theory here, it seems to me important to analyze in detail Ockham's discussion in the work which has more than once constituted the major source for Murdoch's interpretation. For a consideration of Ockham's discussion of the nature of indivisibles or of quantity in works other than *De sacramento altaris*, see, e.g., Weishepl 1963; Sylla 1975; Buescher 1950, pp. 67 ff.; Shapiro 1957 (esp. pp. 24 ff.); Maier 1955, pp. 176 ff.; Lefl 1975, pp. 584 ff.; Zoubov 1961, pp. 48 ff.; and esp. chapter VI.

5. Ockham 1930.

6. For critical reviews of Birch's edition, see, e.g., Anonymous 1932 and 1929-32.

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presented as one work what are really two independent treatises,⁷ and he has given mistaken or at least confusing titles to the two treatises which in his edition constitute two portions of one work. Sorting out the relationships between the two treatises and determining their titles will have to await a critical edition of the texts,⁸ but something tentative can and must be said here.

It is not improbable that only the treatise printed second in Birch's edition is correctly titled *De sacramento altaris* and that the treatise printed first (the one that contains Ockham's elaborate discussion of indivisibles) should be titled *De corpore Christi*.⁹ Simply for the sake of convenience I will refer to the treatises under those titles in the remainder of this chapter. Whether the treatises were written before or during Ockham's stay in Avignon is not clear.¹⁰ But the many references to malicious or sophistical attackers, the frequent citation of ecclesiastical authorities, and the explicit, scrupulous submission to the Church indicate plainly that they were written when Ockham was already under attack for his account of the Eucharist,¹¹ and in fact, Ockham claims that he is writing *De sacramento altaris* not in order to spread novel doctrines about the Eucharist but in order to defend himself from charges made against him on the basis of his earlier writings on the sacrament.¹² There is some suggestion that *De sacramento altaris* was written before *De corpore Christi*,¹³ simply on the basis of the content

7. Cf. Baudry 1936 and Brampton 1964.

8. A critical edition is being prepared as part of the Franciscan Institute edition of Ockham, and I am very grateful to Carlo Grassi for his kindness and generosity in allowing me to see the collations of the manuscripts on which his critical edition will be based. In what follows, the citations to the two treatises will be to Birch's edition; but I have checked Birch's text against Grassi's collated manuscripts, and the Latin given in the footnotes is a corrected version of Birch's edition. See also Solà 1966.

9. See Junghaus 1968, pp. 83-84. What I am calling *De corpore Christi* occupies pp. 1-156 in Birch's edition; what I am calling *De sacramento altaris* runs from p. 158 through p. 500.

10. Junghaus 1968, pp. 84-85.

11. See, e.g., Ockham 1930, pp. 6.13-25; 116.16-18; 126-128.4; 154.13-29; 210.21-22. I have cited only a few of many such passages.

12. *De sacramento altaris*, Ockham 1930, p. 160.8-12.

13. Cf., e.g., Baudry 1936, p. 141, and Maier 1955, pp. 176-77. It has been suggested (in Brampton 1964) that *De corpore Christi* is the precursor to *De sacramento altaris* because in the former Ockham defines terms he means to use in the latter; but this view is mistaken and results from a failure to appreciate the contents of the two treatises.

of the treatises, that suggestion seems plausible. In *De sacramento altaris* Ockham argues for a certain account of transsubstantiation which depends on a particular theory of the nature of quantity. Ockham defends his theory of quantity at length in this treatise, but in a general sort of way, taking into consideration various arguments about the implications of his account for theories of the sacrament and trying to show the philosophical and theological respectability of his position by citing and discussing numerous authorities. It is clear from John Lutterell's accusations, however, and from subsequent fourteenth-century discussion¹⁴ that one of the weakest or most controversial parts of Ockham's account of transsubstantiation is his theory of the nature of quantity. And *De corpore Christi*, which is a short treatise (about half the length of *De sacramento altaris*), is devoted exclusively to a thorough, detailed examination of the nature of quantity, so that (to judge only from its content) it is not implausible that *De corpore Christi* was written to buttress the controversial part of *De sacramento altaris*. But whether or not *De corpore Christi* was written after *De sacramento altaris* and for the sake of supporting it, it is clear from the prologue of *De corpore Christi* that the purpose of its treatment of quantity is to provide a foundation for Ockham's theory of the Eucharist.

Ockham's account of indivisibles, then, is developed within the context of his discussion of the sacrament, and so it is helpful for analyzing Ockham's treatment of indivisibles to know something of his account of transsubstantiation.¹⁵ Indeed, it is the fact that his discussion of indivisibles is deeply enmeshed in his account of transsubstantiation that constitutes the second obstacle to a consideration of Ockham's doctrine of indivisibles in these treatises.

2. Transsubstantiation

The first ten chapters of *De sacramento altaris* are devoted to a presentation of a theory of transsubstantiation current in Ockham's day, a theory Ockham explicitly accepts before going on to present his own interpretation of it. Before the priest's consecra-

14. Cf., e.g., Maier 1955, pp. 199 ff.; and Buescher 1950, pp. xxv-xxvi and 145 ff.

15. Cf. the discussion of the relationship between theology and Ockham's theory of the nature of quantity in Maier 1955, pp. 176 ff., and in Sylva 1975, an excellent paper.

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tion of the bread, Ockham explains, the host consists in the substance of the bread and the accidents (such as taste, color, size, and shape) which inhere in that substance. When the bread is consecrated, the substance of the bread is annihilated, being converted into the substance of Christ's body only—i.e., not into the substance of his blood, or into his soul, or into the accidents inhering in his body, although insofar as these are all natural concomitants of the substance of the body of Christ, when the substance of the body of Christ is present in the host, these are also concomitantly present. But even though the substance of Christ's body is then present in the host, it is not "circumscriptively" but "definitively" present. Ockham means that it is not the case that the whole of Christ's body is present in the whole of the space occupied by the host, each part of his body occupying a part of that space, but rather that the whole of his body is in every part, no matter how small, of the host. Priety alone dictates such a conclusion, since otherwise, for example, in the breaking of the host, the body of Christ would be broken. But there are philosophical reasons for it as well: for instance, it is part of Christian doctrine and experience that the body of Christ in the host is not accessible to human senses, but if the body of Christ were *circumscriptively* present in the host, it is hard to know what would prevent its being seen or otherwise perceived by the senses.

While this doctrine of the definitive rather than circumscriptive presence of Christ's body in the sacrament obviates some problems, it produces a variety of others. For example, if the whole body is present altogether in every part, will there not be a confusion of the parts and of the functions of the body, a result which is philosophically and theologically unacceptable? Of all such problems, the one most focused on by philosophical theologians was a twofold problem having to do with the quantity of the body of Christ in the sacrament. In the first place, since a body is constituted by a certain quantity of matter, and since matter is three-dimensionally extended,

(3a) How can Christ's body in the host be unextended?

as it must be if it is only definitively present there. In the second place, it is part of Christian doctrine that Christ's body is in the

host with all the perfections it has in heaven, so that the body of Christ in the sacrament is in no way less good or complete than it is in heaven. Now, the body of Christ as it is in heaven is extended and so has the characteristic (or accident) of having a certain quantity; hence the body of Christ in the sacrament apparently must also have quantity. But if its presence there is noncircumscriptive and hence unextended,

(3b) How can Christ's body in the host have quantity?

A separate but philosophically related set of issues arises in connection with the substance and accidents of the bread. Doctrine and experience agree in the view that the accidents of the bread remain after the consecration. But if the substance of bread is annihilated when the bread is consecrated,

(4) How can the accidents of the bread exist without their natural subject?

To postulate that the accidents of the bread persist but inhere in no subject requires postulating a number of divine interventions in the natural order, so that God causes each of the remaining accidents to exist independently, as if it were a substance. To postulate that the accidents might inhere in the substance of the body of Christ was considered impious and entails philosophical problems as well.¹⁶ In the face of these difficulties, a solution which had become common by Ockham's time was to say, as Thomas Aquinas had said, that the qualities of the bread after consecration inhere in the quantity of the bread as in a subject, so that one of the accidents serves as the subject in which the other accidents inhere. In that way only one divine intervention in the natural order need be postulated, the one that causes the accident of quantity to exist independently, as if it were a substance.¹⁷

Of these two fundamental problems raised by theories of transubstantiation, it is clear that it is the one expressed in (4) which is Ockham's central concern in both his treatises on the Eucharist.

16. *De sacramento altaris*, Ockham 1930, pp. 202-4.

17. Cf. the discussion in Buescher 1950, pp. 122 ff.

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He says as much in various places,¹⁸ and the table of contents in *De sacramento altaris* also makes it plain. After expounding the basic theory of transubstantiation in chapters 1-10 (I am using Birch's numbering of the chapters) and certain principles fundamental to his account in chapters 11-15, he presents in chapters 16-22 the conclusions he wants to support in the treatise. These all have to do with the quantity of the bread after consecration, and this section of the treatise concludes in chapters 21-22 with an elaborate attack on the notion that after consecration the qualities of the bread inhere in the quantity as in a subject. Ockham's own view is the one requiring more rather than less divine intervention, the view that each of the qualities of the bread remaining after consecration exists independently, without inhering in a subject; and the rest of the treatise, from chapter 23 to the end, is an elaborate defense of this view and the philosophical presuppositions about the nature of quantity on which it rests.¹⁹

3. Ockham's Theory of Quantity

It is Ockham's view that quantity is not an absolute thing, distinct from substance or quality; but, as he points out, this view is not equivalent to the claim that there is no such thing as quantity.²⁰ According to Ockham, something is "an absolute thing" (*res absoluta*) just in case God by his absolute power can make it exist independently of every other thing (always excepting God himself, of course).²¹ Given this definition, motion, for example, is not an absolute thing, since it is logically impossible for motion to exist independently of some moving thing; therefore, even God in his absolute power cannot bring it about that

18. See, e.g., *De corpore Christi*, Ockham 1930, p. 4.9-19; and *De sacramento altaris*, Ockham 1930, p. 240.1-10.

19. Edith Sylla espouses Maier's suggestion that Ockham first developed his theory of quantity in *De sacramento altaris* but thinks that Ockham's theory of quantity was developed to solve problems connected with the quantity of Christ's body in the Eucharist (Sylla 1975, p. 364) rather than with the quantity of the bread; she gives no reasons for her view.

20. See, e.g., *De sacramento altaris*, Ockham 1930, chap. 35, in which Ockham explains how there can be a separate category for quantity even though quantity is not distinct from substance or quality.

21. See Weisheipl 1963, pp. 157-58.

motion exists independently of every other thing, and so motion is not an absolute thing. According to Ockham, only substances (which for Ockham include not only composites of matter and form but also matter as well as form) and qualities are absolute things. It is not unlikely that if it had not been for the doctrine of the Eucharist, Ockham would have admitted only substances as absolute things.²² But in either Aquinas's or Ockham's theory of the sacrament, after consecration at least one accident remains and exists independently of every other thing. And if in the Eucharist the qualities, say, of the bread *do* exist independently, then obviously it is within God's absolute power to bring about their independent existence, and Ockham by his own principles must allow qualities as absolute things.

On Ockham's view, it is not logically possible for any quantity to exist independently of a substance or a quality, and so no quantity is an absolute thing—hence Ockham's claim that quantity is not a thing distinct from substance or quality. Nonetheless, quantity is not to be taken either as identical with substance or quality or as simply nothing. In general, Ockham understands quantity as the condition of having parts capable of local motion toward one another, so that quantity is nothing more than a certain characteristic of a substance or a quality, namely, its extension.²³ Terms of quantity, consequently, are connotative rather than absolute; they signify a substance or a quality and connote that it has parts capable of moving toward one another. On the other hand, though a term of quantity signifies a substance or a quality, a proposition such as

(5) A substance is a quantity

is false because a quantity is not predicabile per se of a substance or a quality. Ockham here takes a proposition (or predication) to be per se only in case the subject connotes just whatever the predicate connotes. In the case of a proposition such as (5), the predicate connotes something, namely, that the parts of whatever

22. *Ibid.*, cf. Sylva 1975, p. 371.

23. See, e.g., *De sacramento altaris*, Ockham 1930, pp. 432-27-434-27. For a fuller treatment of Ockham's theory of the nature of quantity, see, e.g., Buescher 1950, pp. 67 ff., and Maier 1955, pp. 176 ff.

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is denoted (by the predicate term) are capable of motion toward one another, which is not connoted by the subject. Though quantity is not a real thing distinct from substance or quality, the terms "substance" or "quality" are not always interchangeable, *salva veritate*, with the term "quantity."²⁴

Ockham has many arguments for his view of quantity, perhaps the most frequently cited of which is that having to do with condensation and rarefaction. Richard of Middleton had argued that quantity and substance must be really distinct, since in condensation quantity alters, though there is no loss of substance.²⁵ In reply, Ockham maintains that condensation is simply the moving closer together of the parts of some substance, so that the change in quantity is nothing more than a change in the degree of separateness among the parts of a substance. But this separateness is not a real thing distinct from substance, and so neither is quantity. Ockham maintains instead that this separateness of parts is simply a natural, concomitant condition of something's being material—natural but not necessary, since God by his absolute power could condense a material substance to such a degree that it had all its parts in one place, no matter how small.²⁶

Although Ockham's account of quantity in his treatises on the Eucharist is developed as a response to (4), "How can the accidents of the bread exist without their natural subject?" it is plain that it has important implications also for both (3a) and (3b): "How can Christ's body in the host be unextended?" and "How can Christ's body in the host have quantity?" Given Ockham's account of quantity as a natural but not necessary condition of material things, it is possible for God by his absolute power to bring it about that all the parts of Christ's body exist in every part, no matter how small, of the space occupied by the host, so that it is perfectly possible for Christ's body to be only definitively present in the host. This leads Ockham to maintain bluntly, against the Thomistic view, that Christ's body in the host has no

24. See *De sacramento altaris*, Ockham 1930, pp. 464-70; cf. Buescher 1950, pp. 76-77.

25. See Maier 1964, pp. 191 ff. Ockham's discussion of Richard's objections occurs in *De corpore Christi*, Ockham 1930, pp. 116-21-142-13.

26. *De corpore Christi*, Ockham 1930, pp. 128-10 ff. and 142-14 ff.; *De sacramento altaris*, Ockham 1930, pp. 188-96. Cf. Weisheipl 1963, p. 161.

quantity. But, he argues, this claim does not entail that the body of Christ in the host lacks one of the perfections it has in heaven because, in Ockham's view, quantity is not a real thing distinct from substance or quality, and all of the substance and the qualities of Christ's body in heaven are also present in the host.²⁷

4. Ockham's Theory of Indivisibles: Metaphysics

Ockham's theory of indivisibles in *De corpore Christi* is part of his argument for his theory of quantity and needs to be understood in that context. In *De corpore Christi* he begins by briefly presenting an orthodox account of transsubstantiation and then posing two problems, the first having to do with the existence and distinction of the accidents of the bread after consecration and the second having to do with their separation from a subject. Conceding (as universally acknowledged) that the qualities of the bread are really distinct from one another and from a substance, he narrows his investigation to a consideration of quantity and its relation to quality and substance. To do this in a thorough and orderly way, he focuses on three major questions in turn, intending to establish the truth or falsity of these propositions:

- (6) A point is an absolute thing really distinct from a line;
- (7) (a) A line is an absolute thing really distinct from a surface;
(b) A surface is an absolute thing really distinct from a body;
- (8) A body (which is a quantity, on Ockham's view) is an absolute thing really distinct from substance.

He devotes seven or eight times as much space to (6) as to (7), relying heavily in his discussion of (7) on arguments presented in his rejection of (6); and it is plain that in his treatment of indivisibles in this work Ockham is laying the foundation in particular for his subsequent rejection of (8) and in general for his theory of the nature of quantity.

The basic issue in Ockham's discussion of an indivisible, as in his discussion of quantity, is the question whether it is an absolute thing²⁸—that is, whether an indivisible such as a point is

27. See, e.g., *De corpore Christi*, Ockham 1930, pp. 148-4 ff.

28. See, e.g., *De corpore Christi*, Ockham 1930, pp. 6:1-12, 8:1-5, 16:26-30, 22:5-9, 24:25-29, 24:30-26:16, 28:3-25, 28:26-30:11, 72:12-28.

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something which God by his absolute power could cause to exist independently of every other thing. There is something at least misleading, then, in characterizing Ockham's discussion of points here as a discussion about the *existence* of points.²⁹ Ockham himself does not put the issue in those terms here. As he explains it, what is at stake in his treatment of points is not so much whether there are such things as points as what the nature of a point is. The case of a point is in this respect similar to the case of quantity: a demonstration that there is no such thing would entail many absurdities for mathematics and the sciences as well as for ordinary discourse, and Ockham intends no such demonstration. His aim is rather to show that a point, like quantity, is not an absolute thing, an entity which could without logical contradiction be posited to exist separately from every other entity.³⁰

Ockham understands a point primarily as a limit of or a cut in a continuum.³¹ There are such cuts or limits, just as there are absences from loved ones; but a cut or a limit, like an absence, must not be thought of as an independently existing thing. This continues to be Ockham's general strategy also with lines and surfaces—he takes each to be a kind of limit which is not an entity in its own right³²—until he reaches his general conclusion about quantity, namely, that a substance or a quality is not quantified by the addition to it of a separate, independently existing entity which is quantity but instead is quantified simply by itself, by a certain condition and arrangement of its parts.³³

29. As Murdoch does in, e.g., chapter VI as well as in Murdoch forthcoming a and forthcoming b.

30. That this is Ockham's aim is made plain throughout the entire *De corpore Christi* and can be seen even in the basic structure of the treatise, which consists in the treatment of three different but related issues concerning whether or not something is an absolute thing. Furthermore, the overriding purpose of the treatise, as he makes plain in the prologue (Ockham 1930, p. 4:13-19), is to come to some conclusions about the nature of quantity. The principal conclusion he reaches is that quantity is not an absolute thing (see, e.g., p. 92:20-21), and his method for reaching that conclusion is to show that a point is not an absolute thing really distinct from a line, that a line is not an absolute thing really distinct from a surface, that a surface is not an absolute thing really distinct from a body (which, he says, *est quantitas*, p. 92:6) is not an absolute thing really distinct from substance and from quality.

31. Cf., e.g., Ockham's two discussions of the meaning of "point" in *De corpore Christi*, Ockham 1930, pp. 36:22-38:5 and pp. 40:18-42:11.

32. See, e.g., *De corpore Christi*, Ockham 1930, p. 82:12-18, p. 84:1-7, and p. 84:16 ff.

33. See, e.g., *De corpore Christi*, Ockham 1930, p. 92:13-15 and p. 94:4 ff.

In his discussion of points, Ockham is so concerned to demolish the view of points as absolute things that he says repeatedly what at first glance is misleading, namely, that a point is no more a privation than it is a positive entity.³⁴ But if one looks more closely at Ockham's denials in these passages, it becomes apparent that he is trying to guard only against the notion of a point as a hypostasized privative entity which, for instance, must be added to a line in order to terminate it. Thus he says, for example, that "one must not think that a point is some privation totally distinct from a line, as men commonly imagine, because it is impossible to imagine that a point is something, whether privative or positive, a distinct whole in its own right, or not the same as a line."³⁵ Or, again,

this is not to be granted: "A point is a privation," "A line is a privation," "A surface is a privation"; and this in accordance with correct [ordinary] usage and the imagination of those who imagine that a privation is something, considered as a thing, in any way whatever distinct from positive things. Therefore, [these arguments are presented or made] against that imagination which imagines that a point, considered as a thing, is a certain privation distinct in any way imaginable from a line as a whole in its own right.³⁶

And he goes on to add in this same section that although a point is not a privative thing, nonetheless in any nominal definition of the term "point," some negative word or concept must always be included.³⁷ These repeated claims of Ockham's that a point is not a privation do not conflict, then, with his general view of points

34. *De corpore Christi*, Ockham 1930, p. 64.17 ff.

35. *De corpore Christi*, Ockham 1930, p. 64.17-22: "Nec est tamen intelligendum quod punctus sit aliqua privatio distincta totaliter a linea sicut homines communter imaginantur, quia impossibile est imaginari quod punctus sit aliquid vel privatum vel positivum secundum se totum distinctum vel non idem cum linea."

36. *De corpore Christi*, Ockham 1930, p. 70.1-9: "Illa non est concedenda, 'punctus est privatio,' 'linea est privatio,' 'superficies est privatio' et hoc secundum proprietatem sermonis et secundum imaginationem illorum qui imaginantur quod privatio sit aliquid a parte rei quocumque modo distinctum a rebus positivis. Unde argumenta illa sunt recitata sive facta contra illam imaginationem quae imaginatur, quod punctus sit a parte rei, quaedam privatio distincta quocumque modo imaginabili secundum se totum a linea."

37. *De corpore Christi*, Ockham 1930, p. 70.12-15: "Sed non sic ponit illa opinio, nec magis ponit quod punctus est privatio quam positivum; quamvis in diffinitione exprimente quid nominis ipsius debeat poni unum nomen negativum vel conceptus negativus."

as cuts in or limits of a continuum. Rather they are another part of his attempt to show that such a cut or limit is not an entity in its own right, not even some fancied privative entity.³⁸

Ockham uses many different sorts of arguments to show that points are not absolute things; but his basic strategy, which he uses also in connection with his discussion of lines and surfaces, is to show that there is no absolute entity which a point could be, and he constructs a lengthy argument which exhausts the possibilities. He begins by saying that if a point were an absolute entity, it would be either a substance or an accident,³⁹ since in Aristotelian metaphysics these two types exhaust the possibilities for entities. Now, if a point were a substance, Ockham says, it would have to be one of the following: (a) matter, (b) form, (c) a composite of matter and form, or (d) a form abstracted from matter. Three of these possibilities—(a), (c), and (d)—are quickly ruled out, because while points are indivisible, matter is divisible, and so consequently is any composite of matter and form, or any form abstracted from (and thus impossible on) matter. Ockham's reason for ruling out (b) as the sort of substance a point might be is not made clear until later in the treatise in connection with a different argument where Ockham says that the only unextended form is the rational soul.⁴⁰ If a point were a form, it would have to be an indivisible and hence unextended form; consequently if the only unextended form is the rational soul, a point would have to be a rational soul, which is an absurd conclusion and shows that a point cannot be a form. (Ockham gives no hint here why he thinks that the only unextended form is the rational soul; and in the context to assume that the only unextended form is the rational soul is tantamount to begging the question.)

Having disposed in this quick way of the possibility that points are substances, Ockham goes on to consider in great detail

38. It is misleading, then, to claim as Murdoch does, that Ockham intends to reduce points to "macroscopic individuals" or "three-dimensional bodies" (in chapter VI and Murdoch forthcoming a, nn. 29 and 31), since this suggests the erroneous view that Ockham is willing to admit points as real things if we will only take them to be bits of matter, a view that is, moreover, inconsistent with Murdoch's claim that for Ockham indivisibles are nonexistent.

39. *De corpore Christi*, Ockham 1930, p. 8.11 ff.

40. *De corpore Christi*, Ockham 1930, p. 14.16-25.

whether they can be accidents.⁴¹ It seems likely that if it were not for the influence of theology on Ockham's physics, he might have ended his argument with the conclusion that points are not substances. To show that points are not absolute things, however, he is obliged to go on to consider accidents, since in his own view *some* accidents are absolute things because God by his absolute power can make the qualities of the bread after consecration exist independently.

Miracles aside, any accident, Ockham says, must inhere in a subject, and it must do so in one of three ways. Either (i) the whole of the accident exists in the whole of its subject and in every part, no matter how small, of that subject; or (ii) the whole of the accident exists in the whole of its subject and each part of the accident exists in some part of the subject; or (iii) the accident inheres in an indivisible subject. Most of Ockham's arguments are directed against the first and third of these possibilities; the second is easy to rule out, since it requires the accident in question to be divisible into parts, and a point is indivisible. His basic strategy for dispensing with the third possibility is the same as the one he uses in showing that points themselves cannot be substances. If a point is an accident inhering in an indivisible subject, what is the nature of that indivisible subject? It must be (a) matter, (b) form, (c) a composite of both, or (d) form abstracted from matter; but it cannot be (a), (c), or (d), since these are divisible. And it cannot be (b) because, as in the earlier argument, the only indivisible form is the rational soul, but it is absurd to suppose that the rational soul is the subject in which points inhere as accidents. Hence there is no indivisible thing which can be the subject for a point, and the third possibility for points as accidents is eliminated.

Possibility (ii) poses no problem for Ockham because it is the case of a *divisible accident* inhering in a divisible subject; possibility (iii) is that of an indivisible accident inhering in an *indivisible subject*, and its rejection derives straightforwardly from Ockham's views about indivisible substances. But possibility (i) is more difficult.

41. *De corpore Christi*, Ockham 1930, p. 8.25 ff. In what follows, for the sake of brevity I am giving only the skeleton of Ockham's argument, which is more complicated than is indicated here.

cult to dismiss; it is a mixture of the other two cases, and rejecting it requires demonstrating that a point cannot be an *indivisible accident* inhering in a *divisible subject*. To reject it, Ockham takes as his example of a divisible subject a line (segment) *A*, and considers whether it can be a subject for a point *B* which terminates it. If it can be a subject for *B*, he says, then by parity of reasoning it can also be a subject for a point *C*, which terminates *A* at its other extremity. Consequently, *B* and *C* would be in the same subject. But Ockham maintains that accidents which are in the same (primary) subject are not in different places. So if *B* were in *A* as its subject, *B* and *C* would not be in different places—which is absurd. Therefore *B* cannot inhere in *A* as its subject, and so a point cannot be an indivisible accident inhering in a divisible subject. To the natural objection that *B* does not inhere in the *whole* of *A* as its subject but only in a part of *A*, Ockham replies in this way. Let the relevant part of *A*, which is to be the subject for *B*, be called *D*. Then we ask if *B* inheres in *D* as its subject. If it does, then for the same reasons so does point *E*, which terminates part *D* at its other extremity—and the argument continues as before, to the same conclusion. Since there is no divisible part of *A*, no matter how small, for which a similar argument cannot be made, it seems clear that a point cannot be an accident inhering in a divisible subject.

Hence all the possibilities for points are rejected; they can be neither substances nor accidents and consequently cannot be absolute things. As in the case of quantity, which is also not an absolute thing, such a conclusion does not compel us to maintain that there are no points, only that a point is not an independently existing thing.

5. Ockham's Theory of Indivisibles: Semantics

Ockham's theory of the nature of points combined with certain features of his philosophy of language requires him to give a special analysis of terms for indivisibles.⁴² These give him more trouble than do terms for quantity generally, because although he

42. For discussions of relevant portions of Ockham's philosophy of language, see, e.g., Moody 1935 and Loux 1974, esp. pp. 1–46.

denies that any quantity is an absolute thing, he nonetheless does understand quantity as something positive, namely, the extension of an absolute thing; thus he takes "quantity" to signify a substance or a quality and to connote that it has parts which are capable of moving toward one another.⁴³ A point, however, he takes to be simply a certain sort of privation; and it is a privation not even in respect of a substance or a quality but only in respect of a line, which is itself not an absolute thing.

Ockham suggests two methods of dealing with terms for indivisibles. The first he explains in this way:⁴⁴ Some people, he reports, would deal with an abstract term such as "privation" by saying that it is equivalent in signification to the phrase "deprived subject" (*subiectum privatum*). In accordance with this method, as we grant the truth of this proposition

(9) A deprived subject is material (*materia*),

so we ought to grant the truth of this one:

(10) A privation is material.⁴⁵

If any sense is to be made out of this way of dealing with a nonreferential term, I think it must be understood in the following way. A term which appears to signify a privation is to be taken as nothing more than an abbreviated way of signifying a deprived subject. Consequently, talk which is ostensibly about a privation is really only talk about the subject of a privation. Thus in accordance with this method (10) is true, and so the proposition

43. See, e.g., *De sacramento altaris*, Ockham 1930, p. 434.15-27.

44. Ockham's exposition of the first method begins on p. 36.21 of *De corpore Christi*, where he introduces it by saying that we can suppose the term "point" to have the force of a name and not to be equivalent to a composite expression. This exposition continues to p. 40.5, where he introduces the second method by suggesting that we can also suppose the term "point" to be equivalent in signification not to a name but to some composite expression of such a sort that it renders any proposition containing the term "point" figurative. For places in which he speaks of the two methods together, in order to compare or contrast them, see, e.g., *De corpore Christi*, Ockham 1930, p. 66.19-23 and p. 70.16-20.

45. *De corpore Christi*, Ockham 1930, p. 38.13-17: "Unde sicut dicunt aliqui quod hoc nomen 'privatio' aequivallet in significando huic toti 'subiectum privatum', et propter hoc, sicut concedunt istam, 'subiectum privatum est materia', ita concedunt illam 'privatio est materia'."

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(11) A privation is immaterial

is false, not because this method involves a peculiar view of privation, but because it takes the term "privation" as nothing more than a potentially misleading substitute for the longer phrase signifying a subject which is (Ockham evidently thinks) always material. This first method, then, in effect removes the word "privation" from our vocabulary and makes it impossible to mention privation itself by insisting that any occurrence of the term "privation" is simply a disguised occurrence of the phrase "a deprived subject."

If we use this method to construct an analogous account of the term "point," we can say that "a point" is equivalent in signification (*in significando aequivallet*) to "a line not further extended." In accordance with this first method, then, Ockham explains, the proposition

(12) A point is a line

is true, and so is the proposition

(13) A point is divisible.⁴⁶

This method does not provide an analysis of the term "point," however.⁴⁷ The replacement of an analysandum by its analysans

46. *De corpore Christi*, Ockham 1930, p. 38.17 ff: "Ia si hoc nomen 'punctus' in significando aequivallet isti toti 'linea non ulterius protensa' vel alicui consimili . . . , sicut haec conceditur, 'linea est divisibilis', ita illa conceditur de virtute significationis, 'punctus est linea', 'punctus est divisibilis'. Immo quicquid verificatur de hoc nomine 'linea' sumpto significative, verificatur de hoc nomine 'punctus' sumpto significative. . . ."

47. Cf. chapter VI above. See also Murdoch forthcoming a, where he says: "the term 'point' is equivalent to the expression (*complexum*) 'a line of such and such a length' or something of that sort. . . . Of course, the term 'point' occurs everywhere in the propositions of natural philosophy, and there is nothing wrong with using the term 'point' in these propositions; we should merely realize that in so doing we are but employing the term in place of more complex expressions or propositions in which the term 'point' does not appear. . . ." Such an interpretation is not warranted by the text, and it leaves Ockham's theory open to very serious objection. In this interpretation, for example, Ockham's theory has disastrous consequences for both geometry and Aristotelian physics because it will be true to say, for instance, that the center of a circle is a line of such and such a length, and contrary to Aristotle, motion will be possible in an instant, since "instant" (on the analogy of "point") will be equivalent to "temporal interval of such and such a length." Even the laws of logic are altered in drastic ways, in this

within an open context preserves the truth value of the proposition within which the replacement is made. But if we apply this putative analysis within the ordinarily accepted proposition

(14) A point is indivisible,

we produce

(15) A line not further extended is indivisible,

which is absurd. But this method of Ockham's does not *analyze* terms for indivisibles; rather, it is equivalent to removing such terms from our language by insisting that any occurrence of, say, "point" is just a disguised occurrence of the phrase "a line not further extended." Hence, in accordance with this method, ordinarily accepted propositions such as (14) or

(16) Between any two points there is always another point

are actually *false*—not because of some radical Ockhamist geometrical doctrine, but because "point" is simply an abbreviation for "line not further extended," and it is false that a line is indivisible, or that between any two lines (segments) there is always another line (segment).

This is a drastic, even primitive way of dealing with a nonreferential term; and while Ockham does not reject it, he clearly does not favor it, either. Instead he proposes a second method for dealing with the term "point"; and it is this method to which he devotes the most attention and on which he relies throughout the rest of his discussion of points.⁴⁸ It would indeed be peculiar if he were really to adopt the first method, because, as he himself recognizes,⁴⁹ it ultimately entails the truth of propositions such as

(17) A point is an absolute thing.

interpretation, because the law of noncontradiction becomes "Contradictories cannot be true within the same temporal interval (of such and such a length)." But there is no need to saddle Ockham with such a bizarre theory of indivisibles.

48. *De corpore Christi*, Ockham 1930, p. 40.5 ff.

49. See, e.g., *De corpore Christi*, Ockham 1930, p. 66.13-15, p. 66.20-22, and p. 70.16-20.

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That is, if we adopt this first method for dealing with the term "point," we must, to be consistent, adopt it also for "line" and "surface," since lines and surfaces, too, are indivisibles in certain respects. "Line" will then be replaced by "surface not further extended," and "surface" by "body not further extended," so that if we make all the appropriate replacements, it emerges that we are ultimately to replace "point" by "body not further extended." Since in Ockham's view a material object is an absolute thing, (17) would then be true in accordance with this method. Proposition 17, however, is just what Ockham's whole treatment of indivisibles is designed to reject. And Ockham must have recognized that there is something at least confusing and inelegant about adopting a semantic account of "point" which requires us to accept (17), although we are required to reject it in accordance with the metaphysics for points.

According to Ockham's second method for dealing with terms for indivisibles, the term "point" is not properly speaking a name at all and is not equivalent to any noun phrase such as "a line not further extended" but rather does duty for an independent clause such as "a line is not further extended." Hence, it cannot function as a subject term in a proposition. Consequently, propositions containing the term "point" as a subject term must be regarded as metaphorical or figurative and cannot be taken literally in any investigation of the truth.⁵⁰ To illustrate this method, Ockham gives the following example.⁵¹ Suppose that we let *A* stand for the phrase "then correctly" (*tunc bene*). On that hypothesis we are to consider the proposition

(18) *A* is something.

50. *De corpore Christi*, Ockham 1930, pp. 40.5-44.21. Murdoch tends to conflate Ockham's two methods of analyzing terms for indivisibles. In Murdoch forthcoming b, for example, he says, "to give the nominal definition of the term 'point' is to say that the term 'point' signifies the same thing as the phrase 'a line of such and such a length' . . . whenever the term 'point' occurs in a proposition or *oratio*, then the *oratio* is a *locutio figurativa*, not one that functions *secundum proprietatem sermonis*. Still, such *orationes* or propositions can be conceded once we realize that . . . the term 'point' is being used in place of its nominal definition, in place of a phrase equivalent to it."

51. *De corpore Christi*, Ockham 1930, p. 52.16 ff.

Such a proposition has the appearance of being grammatically and logically well formed, but its appearance is deceptive because (18) is simply an abbreviation for

(19) Then correctly is something.

But (19) is neither grammatically nor logically well formed, and it makes no sense to ask about its truth or falsity. The best we can do is to take (19) as figurative and interpret it as if it were

(19') This, which is then correctly, is something

or some equivalent formulation.⁵² According to this second method, then, a proposition in which a term for an indivisible occurs as the subject term is not well formed. In consequence, neither

(20) A point is something

nor

(21) A point is nothing

is either true or false,⁵³ and this claim does not violate the law of noncontradiction because despite appearances, (20) and (21) are not contradictories, any more than (19) and

(22) Then correctly is not something

are contradictories.

In general, in accordance with this second method Ockham tends to understand the term "point" as equivalent in sense to

(23) A line is not further extended,

52. It looks as if the referent of the demonstrative pronoun in (19) is intended to be the expression "then correctly," so that the problem here is a use-mention ambiguity in the interpretation of the arbitrarily assigned designation *A*. His choice of *tunc bene* is probably associated with the fact that it is a familiar expression in medieval disputational exchanges, indicating the correctness of a reply in certain circumstances.

53. *De corpore Christi*, Ockham 1930, p. 58.17 ff. and pp. 64.25-66.6.

rather than to the noun phrase "a line not further extended," as in the first method. But his explicitly stated, theoretical principle is that a proposition including a term for an indivisible must be translated into a well-formed proposition (which can be taken literally) not on the basis of any simple formula but instead on an ad hoc basis, in accordance with the intentions of the author in whose writing the figurative proposition occurs.⁵⁴ Thus, when Aristotle says that

(24) The parts of a line are joined at a point as at a common terminus,

this proposition is to be treated as a figurative expression the literal meaning of which is conveyed in this conjunction:

(24') There is nothing intermediate between the parts of a line, and they themselves form one line per se.⁵⁵

And the proposition

(25) A coming-to-be is in an instant

is to be understood literally as

54. *De corpore Christi*, Ockham 1930, p. 40.18-24: "Sed illae propositiones et consimiles non sunt concedendae secundum proprietatem sermonis, tanquam propriae non excusandae per aliquam figuram grammaticalem; et per consequens non est ex eis arguendum in investigatione veritatis, nisi ipsis acceptis secundum sensum quem habent ex intentione auctorum. . . ."

P. 54.7-12: "Et ideo sicut multa dicta in metris et ornato modo loquendi non sunt recipienda nisi excusentur et glossentur, ita multa dicta causa brevitas non sunt recipienda secundum proprietatem sermonis, sed secundum intentionem auctorum dicentium ea qui figurative propter dictam causam loquebantur."

55. *De corpore Christi*, Ockham 1930, p. 74.5-21: "Et ideo philosophus per illam propositionem, 'partes lineae copulantur ad unum terminum communem,' intelligit illam propositionem, 'inter partes lineae nihil est medium et illae faciunt per se unum,' ita quod duo requiruntur ad hoc quod linea sit continua, videlicet, quod inter partes lineae nihil est medium situalter. Et propter hoc, quando unum corpus est in uno situ et aliud corpus in alio situ et inter illa corpora est aliquod medium situalter, illa duo corpora non continuantur. Secundum requiritur, videlicet, quod faciant per se unum et propter hoc contingit non sunt continua, et ita per illam propositionem, 'partes lineae copulantur ad punctum tanquam ad terminum communem,' et per consimiles intelligit philosophus illam copulativam: 'inter partes lineae nihil est medium et ipsae faciunt per se unam lineam.'"

(25') When something comes to be, it is not the case that one part comes to be before another, but rather the whole comes to be at once.⁵⁶

It is important to see here that Ockham is concerned to remove talk of indivisibles by understanding (24) as (24') and (25) as (25') not because of some belief which requires that indivisibles not be mentioned but rather as a consistent complement to his metaphysical theory of indivisibles. Any accurate helpful gloss on "at once" in (25') will have to mention a cut in the temporal continuum. And if Ockham's concern were to show that all talk of indivisibles is dispensable in virtue of his conviction that indivisibles do not exist,⁵⁷ then such a gloss on "at once" would be embarrassing counterevidence to Ockham's view. But what Ockham wants to demonstrate with his interpretation of (25) is not that instants do not exist and therefore should not be mentioned but rather that semantic considerations do not force us to consider an instant as some sort of real entity *within* which coming-to-be occurs. That this is Ockham's aim is clear from the text:

I say that this proposition "A coming-to-be is in an instant" is not to be understood as it sounds, as if one thing were in another, as the proposition "Water is in a vase" denotes that one thing is in another distinct thing.⁵⁸ Similarly, the proposition "Motion is in time" ought not to be understood as it sounds, according to the form of the proposition, namely, that one thing is in some other thing really distinct from it. Rather it ought to be understood [in the following way]: "When something is moved, it acquires one part before another" [or something similar]. . . .⁵⁹

56. *De corpore Christi*, Ockham 1930, p. 54.16–27: "Unde dico quod illa propositio 'generatio est in instanti' non est recipienda sub intellectu quem sonat, quasi una res sit in alia; sicut per illam propositionem 'aqua est in vase' denotatur quod una res est in alia re distincta; unde discurrendo per omnes modos quibus aequivoce accipitur haec praepositio 'in', de quibus loquitur philosophus, IV. *Physicorum*, patet quod secundum nullum illorum potest dici quod generatio est in instanti; sed illa propositio 'generatio est in instanti' sub illo intellectu debet recipi 'quando aliquid generatur, non generatur pars ante partem sed totum simul generatur' . . ."

57. As Murdoch suggests in chapter VI and Murdoch forthcoming a and forthcoming b.

58. *De corpore Christi*, Ockham 1930, p. 54.16–20 (cf. note 56 above).

59. *De corpore Christi*, Ockham 1930, p. 56.3–8: "Consumitur illa propositio 'motus est in tempore' non debet recipi sub illo intellectu quem sonat secundum

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Ockham's theory of indivisibles, then, is more sophisticated and less open to objection than it has been made to seem. It is part of Ockham's general treatment of quantity, which is itself influenced by Eucharistic doctrine. Ockham understands indivisibles as privations, as cuts in or limits of various kinds of continua; and what he is principally concerned to demonstrate is that, like quantity, an indivisible is not an absolute thing, so that (for example) a point is not a positive, independently existing entity which must be added to a line in order for that line to be terminated or continued. His semantic theory for indivisibles is meant to show that the presence of terms for indivisibles in ordinary language does not require us to understand indivisibles as independently existing entities. In his view, we can in more than one way construct a semantic theory which avoids commitment to the ontological position he repudiates. We can simply remove terms for indivisibles from our language by considering them nothing more than convenient abbreviations for longer phrases in which the indivisible in question is not mentioned. Or as Ockham prefers, we can claim that terms for indivisibles are not really names at all, so that any proposition in which such a term occurs as the subject term is not well formed, although such a proposition can be translated into one which is well formed and which does not commit us to a belief in indivisibles as absolute entities, on the basis of considering the intentions of the author in whose writings the proposition occurs and understanding the nature of indivisibles as limits of various kinds of continua.

It should be clear, however, that to say Ockham's theory of indivisibles is more sophisticated and less open to objection than it has been made to seem is not yet to say it is a good theory of indivisibles. Even in the version I have argued for, the theory is still open to objections. The most important of these, I think, is that Ockham's account of indivisibles in the treatises on the Eucharist is basically negative; it tells us what indivisibles are not. Indivisibles for Ockham cannot be absolute things; they are priva-

formam propositionis, scilicet, quod una res est in re alia distincta realiter ab ea, sed debet recipi sub illo intellectu, quando aliquid movetur, acquirit unam partem ante aliam".

tions, cuts in a continuum. But what is a privation? Given that it is not an absolute thing, we might be inclined to suppose that it is a property of something which is an absolute thing. A surface, for example, we might take to be a property of some body—namely, its being extended so far and no farther. But on Ockham's account indivisibles cannot be accidents of absolute things either. Indivisibles, then, cannot fit into any of Aristotle's ten categories. Still, as I have argued, Ockham does want simply to reject the notion of an indivisible as meaningless or incoherent. But what positive account of indivisibles he has in mind or could provide is not clear in these treatises.

Infinite Indivisibles and Continuity in Fourteenth-Century Theories of Alteration

EDITH DUDLEY SYLLA

1. Introduction

Following Aristotle, most fourteenth-century natural philosophers assumed that there were three or four basic categories of natural motion or change. Not only was there local motion, a change in the place of a body, there was also the motion of augmentation or diminution, a change in the quantity of a body, and the motion of alteration, a change in the qualities of a body. And if alteration was pushed far enough, a fourth kind of change resulted, the generation or destruction of a body. Most fourteenth-century natural philosophers also assumed, again following Aristotle, that within a single motion of any of the first three kinds the place, quantity, or quality acquired or lost was continuous, as was the motion itself. Concerning alteration in particular, the fourteenth-century addition-of-part-to-part theory of alteration made a special point of emphasizing the continuity of the qualities acquired or lost, sometimes carrying this emphasis to the extent of denying altogether the existence of indivisibles in quality.¹

All this is as one might expect, given the foundation of fourteenth-century natural philosophy in Aristotle's works. There was, however, one fourteenth-century theory of alteration, the so-called succession-of-forms theory, which asserted that the

1. See Sylla 1973, pp. 230–32, 251–62.