THE SPATIAL ARROW PARADOX*

BY

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One form of the arrow paradox focuses on place, This version is attributed to Zeno of Elea by Diogenes Laertius: 'the moving thing moves neither in the place where it is nor in the place where it is not'.[1] A spatial version of the Arrow is also attributed to Diodorus Cronus by Sextus Empiricus. The longest account of it occurs at *M* 10.85-86:

> Another weighty reminder of the non-existence of motion is provided by Diodorus Cronus, through which he shows that although nothing is moving, it nonetheless has moved. That it is not moving is a consequence of his hypothesis of indivisibles. For it behooves an indivisible body to be contained in an indivisible place and, on account of this, it is not moving in it [*sc.*, the place where it is]. (For it fills it up; but it is necessary that a moving thing have a larger place [in which to move].) Nor is it moving in the place where it is not; for it is not yet in that place, so as to move in it. But, according to reason, it has moved. For what was formerly observed to be in this place is now observed to be in another place.[2]

In his *Time, Creation and the Continuum* Richard Sorabji raises two related issues concerning the spatial form of the Arrow. The first pertains to the suggestion made by Sextus in the preceding passage that Diodorus' conclusion (the arrow is not moving) 'is a consequence of [Diodorus'] hypothesis of indivisibles': the issue is whether the hypothesis of indivisibles is required for Diodorus' argument to succeed. The second issue is whether the spatial form of the Arrow is sufficiently distinct from its temporal form to constitute a greater threat to the proponents of *continuous* motion. These seem to me to be the most important philosophical questions to be addressed concerning the spatial version of the Arrow. The answers to them that I propose in this essay, however, are the opposite of Sorabji's, who suggests that 'no' is the correct answer to the first question, 'yes' to the second. With respect to Diodorus' spatial arrow, Sorabji comments as follows:

Pacific Philosophical Quarterly 68 (1987) 71 – 77 0031 – 5621/87/0100 – 0071 – $00.70

For if the place of a thing is (as Aristotle held), its *immediate* surroundings, then it cannot move in the place where it is (that place is too tight-fitting to allow movement). But neither, obviously, can it move in the place where it is not. It can best *have* moved from one to the other, with a jerk. The only difference will be that Aristotle is not committed to his jerks being of atomic length, or a multiple of atomic length. In view of this reply, I think Sextus may do Diodorus less than justice when he speaks as if his jerks would follow only from his belief in indivisible bodies and spaces. Aristotle had argued that jerks would follow from that. But Diodorus seems to be responding that jerks, even if not jerks of atomic length, follow also from some more widely held assumptions shared by Aristotle himself.[3]

The spatial version of the Arrow may initially seem rather different from a temporal version, according to which it is argued (a) that a supposedly moving body is not moving (or—a stronger conclusion—is at rest) in 'the now' (*to nun*) and (b) that the preceding proposition (a) entails that a body cannot move at all during the period of time in which it supposedly is in motion.[4] I believe that the difference is superficial, however. A reasonable gloss that would connect the temporal and the spatial versions of the argument is that the moving body does not move in the place where it *now* is nor in the place where it *now* is not. And, in fact, a version of the argument integrating the temporal and spatial aspects is presented by Sextus in the same discussion of Diodorus from which the earlier quotation was extracted:

If something is moving, it is moving now; if it is moving now, it is moving in the present time. If it is moving in the present time, it is therefore moving in an indivisible time. For if the present time is divided, it will certainly be divided into the past and the present and thus will no longer be *present* time. If something is moving in an indivisible time, it is traversing indivisible places. If it is traversing indivisible places it is not moving. For when it is in the first indivisible place, it is not moving because it is still in the first indivisible place. And when it is in the second indivisible place, again it is not moving, but *has moved*. Therefore, it is not the case that anything is moving.[5]

It seems that Diodorus' indivisible temporal minima were true atoms, that is, indivisible temporal *intervals* rather than instants or temporal 'points'.[6] His postulation of temporal minima of this sort precludes Aristotle's well-known response in *Phys.* 6.9 to Zeno's Arrow, namely, his claims (a) that *neither* motion nor rest, each of which presupposes a temporal lapse, can occur with respect to such punctal 'nows' or instants, and (b) that, since time is not constituted of such instants, the fact that motion does not occur at an instant does *not* entail that motion does not occur in a period of time containing such a punctal entity. Since Diodorus' temporal minima *are* intervals or stretches of time, it is entirely intuitive to speak of space and time as constituted (*sungkeimena*) of these atoms. The question of whether the arrow is at rest or in motion *during* such an atom is then a coherent

and apposite question. But the arrow must be at rest during each atom: for it seems that were a body to move during such an interval it would have to traverse different spatial intervals during different temporal subintervals of the interval. But a time *atom* has no such subintervals. The result is that the 'moving' arrow is at rest during one time atom and, during some subsequent time atom, it is at rest, having been displaced by one atomic unit in the direction of its 'motion'. According to Diodorus, although it is true to say that the arrow has moved, it is never true to say that it is moving.[7] With true time (and space) atoms, that is, minima that are indivisible but that have some non-zero magnitude, an Aristotelian strategy for dealing with the Arrow is no longer possible.

It is for this reason, I take it, that Sextus says that Diodorus' conclusion that nothing is moving follows from his hypothesis of indivisibles. If a *to nun* or 'now' is a temporal atom with magnitude, it seems implausible to follow Aristotle in denying (a) that the arrow is either moving or at rest during such a 'now' and (b) that a stretch or period of time is constituted of such atoms as parts. Thus, the Diodorean doctrine of minima vitiates the Aristotelian basis for avoiding the consequence that the supposedly moving arrow is at rest at a 'now' as well the Aristotelian basis for avoiding the inference from the lack of motion of the arrow at a 'now' to the lack of motion of the arrow with respect to the period of time during which it supposedly is moving.

It is possible, I think, that Diodorus viewed this result as a vindication of Zeno, i.e., as eliminating 'motion in the true sense', the *process or kinēsis* of moving. And it does seem that Diodorus has made it impossible to speak of motion *with respect* to an interval or stretch of time in the sense of motion *throughout* that interval: for there are subintervals of any such interval (the atomic ones) at which nothing is moving. However, it is also possible to conceive Diodorus' 'strengthened' Arrow as Sorabji does: as an analysis or 'dissolution' of motion into 'jerks'. What this seems to mean is the following: a body is at rest during a time atom in one position, and during some subsequent time atom it is at rest elsewhere, displaced by one atomic space from its original position. There is, in other words, no process (requiring a stretch of time) of a body's being gradually displaced from its original position and occupying more and more of a position that is a displacement from the original position by one atomic space in the direction of movement. Rather, such a displacement is instantaneous, requiring no time whatsoever.

This brings us back to the second question Sorabji raises with respect to Diodorus and the Arrow: does the Arrow's spatial version pose a greater challenge than its temporal version to advocates (such as Aristotle) of a conception of motion as continuous. In effect, Sorabji argues that the spatial form of the Arrow threatens even continuous, 'Aristotelian' motion with 'dissolution into jerks'. According to Aristotelian doctrine, if a body

moves at all, it moves through a certain distance, which is potentially divisible without limit. This means that if we consider a body moving in a straight line from place X to place Y, there is a potentially infinite collection of (partially overlapping) possible stopping places for the body between X and Y, i.e., *potential* 'places of the thing' between the *actual* place X where it is before it begins to move and the *actual* place Y where it is after it ceases to move. Sorabji's suggestion seems to be that Diodorus' argument shows that the body moving from X to Y cannot move at these or any of the intermediate possible stopping places, that it 'can at best *have* moved from one to the other' such place, and that, consequently, it 'moves by jerks' from one to another place until it reaches the terminal place Y.

In order to see the problem with this suggestion, let us first recall that Aristotle maintains that some duration or 'stretch' of time will be required for the body to move from X to Y, and that this time is potentially divisible without limit in terms of possible stopping times—punctal 'nows'—in just the way that the distance traversed from X to Y is. The collection of possible stopping times can be put into a one-to-one correspondence with the collection of possible stopping places such that for any two possible stopping places P and P' such that P is closer to X than P' and P' is closer to Y than P, there will be corresponding possible stopping times ('nows') t and t' such that t is prior to t'. Furthermore, there will be a distinct possible stopping place P" closer to X than P' and closer to Y than P such that its corresponding stopping time t" will be after t and before t'. Satisfaction of this condition guarantees that the motion does not come in 'fits and stops' but is 'smooth' in the following sense: irrespective of how small a spatial subinterval of the interval XY that we consider, there will always be an *interval* of time *during* which the body is traversing the possible stopping places between the possible stopping places that are the termini of that interval.

Thus, talk of 'jerks', which may be understood as implying discontinuity, does not seem appropriate when considering the passage of the body from X to Y 'through' a densely ordered collection of possible stopping places. Although temporal passage from place X to place Y imparts to the members of this collection of possible stopping places a 'natural' linear ordering (for any two distinct members, the one that is first traversed is closer to X and the one later traversed is closer to Y), it is a dense rather than a discrete ordering: that is, between any two members will be another (and, indeed, an infinite number of others). So there is not any *particular* 'other' place *to which* the body that is continuously moving can be 'jerked' from any 'place where it (potentially or actually) is'.

Aristotle, I believe, has nothing to fear from the spatial version of the Arrow. In fact, it appears that he is *committed* to a certain sort of refutation of it. If the time that it takes the body to move from place X to

place Y is finite and the motion is continuous from X to Y, it is intuitively appealing to maintain that the body does not 'spend any time' at any of the intervening possible stopping places properly between the place X it occupies before it begins to move and the place Y it occupies after it ceases to move. This intuition, which is supported by contemporary set-theoretic considerations,[8] is enunciated by Aristotle in *Phys.* 8.8. There Aristotle claims that a body moving continuously (*sunechōs*) from A to Γ neither has come-to-be at nor has departed from an intermediate place B; it is 'at' B 'only for a "now", not for a [period of] time' (262a29-30). Further on in the chapter Aristotle contrasts continuous motion 'through' a potential stopping place H with rectilinear motion that is reversed at H. In the latter case, Aristotle claims, we cannot truly say that the moving body is only at H at a division or 'cut' (*en tomē(i)*); nor can we say that it neither has come-to-be at H nor has departed from H. Aristotle concludes that, in the former case of a body characterized by continuous rectilinear motion, the moving body is at a mid-point B potentially; but in the latter case of reversed motion, it must be at the turning-point H actually (262b31-32).[9]

Consequently, the response Aristotle makes in *Phys.* 6.8 to a temporal version of the Arrow, where he maintains that the moving arrow is neither moving nor at rest with respect to a punctal 'now', commits him to an analogous response to Diodorus' spatial version of the Arrow. Were Aristotle to allow that the moving arrow is either moving or at rest 'in the place where it is' (i.e., in any of the potential stopping places strictly between X and Y that it traverses in moving from one to the other), he would have to admit that the arrow is either moving or at rest, respectively, at a temporal point or instant. For the arrow must traverse each of these potential stopping places without *actually* spending any time at that place.

To speak of the arrow's being 'jerked' from one of these places to another in a context in which temporal and spatial atoms are not supposed is inappropriate, first, because there is no unique 'next' place to which the arrow could be jerked and, second, because the continuously moving arrow does not spend any time at any of these potential stopping places. The concept of 'jerking' from one place to another presupposes, I think, that the arrow is 'at rest' at such a place *and then* proceeds to another such place, where it rests, etc. The Aristotelian response to Diodorus' spatial version of the Arrow, then, would be that the moving arrow *neither* moves *nor* is at rest 'in the place where it (potentially) is'. Locomotion requires an interval of space, just as it requires an interval of time. In order to speak coherently of locomotion, we must speak of it as occurring *through* an interval of space, which is potentially divisible without limit, according to Aristotle. Only the *terminus a quo* X and the *terminus ad quem* Y are 'places of the object' *energeia(i)* or 'in

actuality'—i.e., places where it spends a *period* of time. The intermediate potential stopping places, which are infinite in number and densely ordered, are 'places of the object' only *dunamei* or 'potentially'. Since the continuously moving object neither arrives at nor departs from any of these places, there is no possibility of the dissolution of such a body's continuous motion into 'jerks'.

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NOTES

* I should like to thank an anonymous referee for some suggestions that, I trust, improved the readability of this paper at several places.

[1] Diogenes Laertius 9.72.

[2] Cf. *M* 10.143 and *PH* 2.245; 3.71.

[3] R. Sorabji, *Time, Creation and the Continuum: Theories in Antiquity and the Early Middle Ages* (London and Ithaca, N.Y., 1983), p. 18.

[4] In *Phys.* 6.9 Aristotle accepts the weaker version of (a), that the moving body is not moving in/at the punctal *to nun*. However he denies (b), apparently for the reason that an interval of time during which a moving body is moving is not composed of such punctal units (239b 8-9). Schofield (in G.S. Kirk, J.E. Raven and M. Schofield, *The Presocratic Philosophers*, 2nd edition [Cambridge, 1983] p. 273) suggests that Aristotle's reason for denying (b) is mistaken, implying that in order to avoid (b) one only need deny that what holds for a period or interval of time (e.g., that a body is moving during that interval) must also hold for 'every moment' of that period of time (e.g., that the body is thus moving at such moments).

[5] *M* 10.119-120.

[6] Although it is widely agreed that Diodorus accepted a doctrine of spatial atoms of this sort, the attribution to him of a doctrine of temporal atoms is more controversial. I find this fact curious in view of Sextus' discussion in *M* 10, where the doctrines of spatial and temporal atoms are closely connected, and the currency of arguments such as the ones Aristotle gives in *Phys.* 6.1, where he maintains that 'by the same reasoning, spatial extent, time, and motion are [all] constituted of indivisibles and are divisible down to these indivisibles, or none of them is' (231b18-20).

[7] This is a special case of Diodorus' claim (*M* 10.97) that the perfective-stative (*suntelestikos*) form of a verb can be truly predicated of a subject without the imperfective-present (*paratatikos*) form ever having been truly predicated of that same subject.

[8] From the set-theoretic perspective, there is a dense and non-denumerable infinity of intervening possible stopping places. Since the motion between X and Y is assumed to be continuous, there seems to be no good reason to hold that the moving body 'spends time' at some of these intervening possible stopping places but not others; it must either spend no time at any of the intervening places or spend time at all of them. But, on pain of inconsistency, the body could spend a non-zero time only at a finite number of the places if the body is moving at a uniform speed, and it could spend a non-zero amount of time at (at most) a denumerable infinity of places if it is not moving at a uniform speed. Therefore, it must spend no time at any of the possible stopping places properly between X and Y.

[9] Of course, as might be expected, the argument that Aristotle gives in this chapter of why a body undergoing reversed rectilinear motion must actually 'spend time' at the turning-point H is not cogent by the lights of contemporary mathematics and physics. However, there is an interesting feature of this chapter that may be significant for the general analysis of locomotion that Aristotle develops in the *Physics*. The implicit equation of a moving body's potentially being in a place with its being in that place 'for a *to nun* (punctal 'now')' and its actually being in a place with its being in that place 'for a (period of) time' seems to represent a modification of Aristotle's earlier discussion in the *Physics* of locomotion. I think that investigation into this modification might permit the development of a more unified interpretation of the *Physics*' theory of motion than is found, for example, in S. Waterlow, *Nature, Change, and Agency in Aristotle's 'Physics'* (Oxford, 1982).