Wheeler is right, we would be unable to give clear sense to the idea of indefinitely small distances in our universe as it is.

Wheeler takes it that time as well as space is atomic. This might be thought to follow simply from the atomicity of space. For any clock depends ultimately on the motion of something across a distance, and if the distances are atomic, so, it might seem, will the times be. But in fact matters are not so simple. For we can imagine two clocks out of phase with each other, so that when one has moved ten atomic spaces, the other is found to be one atomic space ahead. One response would be to reopen the question of whether the spaces are genuinely atomic (might not the faster clock be moving a tenth of a space farther on every beat?). But if it is firmly maintained that the spaces are atomic, does it follow at once that the times are atomic? Not at all; for one hypothesis would be that the faster clock rests between beats for a fraction less time than the slower.[145]

I confess that I do not know Wheeler's own ground for treating time as atomic. It will make a difference if he starts from the Einsteinian idea of space time rather than from the simple idea of space. I hope, however, that the above discussion will show that it is not an easy matter to make sense of the idea of time atoms. One further difficulty would be that atomic times would have to be times during which there was no change.

145. It would make no difference to postulate a maximum speed for the moving parts of clocks, for the argument relies not on indefinitely high speeds but only on indefinitely small differences of speed.

CHAPTER III

Aristotle against the Atomists

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1. Aristotle's Continuum Thesis

Aristotle is deeply committed to the thesis that physical reality is a continuous plenum. In *Physics* VI he argues that all physical magnitude, and hence every movement, is continuous, with a view to arguing in Book VIII that one unmoved mover is responsible for the perpetual movement of the cosmos. For he bases the existence of an unmoved mover on the claim that there is always movement;[1] and he bases the uniqueness of the unmoved mover on the claims that what always is is continuous and that what is continuous is one, so that there is a single cosmic movement which is due to a single mover.[2] This argument presupposes that a movement which lasts for a period of time is essentially continuous and one. A process might consist of different successive movements, as in the case of a relay race in which different runners carry a torch.[3] But since movements are differentiated in terms of differences of moved thing, type (or path) of movement, and time during which there is movement, and since these are the criteria of continuity and unity, Aristotle claims that (in the strict sense of "movement") "every movement is

I am indebted to the studies of David Furley and Richard Sorabji, in spite of important departures from their interpretations. I benefited greatly from Richard Sorabji's perceptive criticisms of an earlier draft of this chapter. My treatment of *stigmē* and *sēmeion* in *De gen. et corr.* I 2 owes much to David Keyt. I also benefited from discussions of *Physics* VI with Christopher Shields and Victor Ten Brink.

1. *Phys.* VIII 6, 258b10–259a6.
2. *Phys.* VIII 6, 259a13–20; 10, 267a21–24.
3. *Phys.* VIII 6, 259a19–20; V 4, 227b3–229a6.

continuous."[4] Hence, Aristotle cannot concede that movement in general is reducible to an unconnected plurality of events without undermining this argument for a prime mover in *Physics* VIII.

The continuum thesis and the arguments with which Aristotle defends it have a philosophical interest apart from the role of the thesis in this argument, "the way *ex motu*," which is open to many familiar objections. In establishing that physical reality is a continuous plenum, Aristotle upholds two principles of fundamental significance for his science of nature: first, the deep structure of movement is the same as that of spatial and temporal magnitude (the thesis of isomorphism); and second, the structure of a continuum, which is shared by movement, space, and time, is not reducible to any deeper structure (the thesis of irreducibility).[5] These principles exerted a long-lasting influence in the history of science, until developments during the last hundred years in subatomic physics and higher mathematics. The possible value of these principles for the future progress of science may not be fully appreciated at present. But in order to understand their true significance for Aristotle, we must take into account the context in which he formed his concept of the continuous: the controversy over the nature and reality of magnitude and movement in which Zeno the Eleatic, the atomists, and the Platonists had become embroiled. In his characteristic manner Aristotle reformulated the old difficulties in his own terms and defined concepts in order to resolve them.

2. The Dilemma of Divisibility

Aristotle's starting point is the arguments of the Eleatics that generation, change, and plurality are unreal. He presents his own theory of the continuum as the only way out of an ancient dilemma which seeks to show the absurdity of continuous magnitudes. In *De generatione et corruptione* I 8 he states the objections of "some ancient men," namely, Parmenides, Melissus, and Zeno, which the atomists, Leucippus and Democritus, had tried to meet. The first familiar objections turn on the alleged unreality of the void,

4. *Phys.* V 4, 228a20.
5. Wieland 1962, pp. 287–88.

but another objection is raised against the view that reality is divisible without containing void:

> In this regard [i.e., in the inability to solve the problem of the one and the many], if one believes that the universe is not continuous but [consists in] what is divided touching, there is no advantage over saying that there are many things, i.e., not one, and the void. For [supposing that the universe is what is divided touching], if it is divisible everywhere, there is no one, and, hence, no many, but the whole is void; whereas if [it is divisible] here but not there, this is like something contrived (*peplasmenō*). For up to what amount [is it divisible], and why is some of the whole thus [indivisible] and a plenum, and part of it divided?[6]

The argument sketched out here, which I shall call "the dilemma of divisibility," presupposes that the theory of the void is set aside. The fundamental question is whether a magnitude is divisible everywhere, i.e., perpetually subdivisible into smaller units, or divisible only down to some atomic magnitude, beyond which subdivision is no longer possible. The first horn of the dilemma starts from the proposition that magnitude is everywhere divisible and argues to the conclusion that the magnitude is thereby reduced to no extension or, more dramatically, to nothing at all. I shall refer to this as "the nihilistic horn" of the dilemma. The other horn, which I shall refer to as "the atomistic horn," starts from the premise that magnitude is not everywhere divisible, leading to the positing of extended but indivisible magnitudes.

What is the sense of "division" at work here? David Furley distinguishes between two different types of division: *physical division*, which is "the division of something in such a way that formerly contiguous parts are separated from each other by a spatial interval," and *theoretical division*, in which "parts can be distinguished within [the thing] by the mind, even if the parts can never be separated from each other by a spatial interval."[7] There is some evidence that Aristotle had an inkling of Furley's distinction between physical and theoretical division at *Metaphysics* IX 9, 1051a21–33: "It is by an activity (*energeia*) also that

6. *De gen. et corr.* I 8, 325a6–12.
7. Furley 1967, p. 4.

geometrical constructions are discovered; for we find them by dividing. If the figures had been already divided, the constructions would have been obvious; but as it is they are present only potentially. . . . Obviously, therefore, the potentially existing constructions are discovered by being brought to actuality; the reason is that the geometer's thinking is an actuality (*energeia*). . . ." (Ross translation). The text presents difficulties,[8] but it does at least suggest that the act by which a geometer divides a triangle in thought into two triangles represents an actualization of a potential, whether or not it is possible to divide the triangle into physical parts.[9] I shall suppose, therefore, that when Aristotle speaks of atoms or indivisibles in the arguments of *Physics* VI, he is thinking of things which cannot be divided even by the geometer in thought.

This dilemma sets the context for much of Aristotle's inquiry into the nature of space, time, and motion. In *De generatione et corruptione* I 2, he recognizes that a reductionist, "punctual" theory of magnitude comes to grief on the nihilistic horn but argues that his own view escapes such difficulties. Throughout *Physics* VI and in some passages in *De caelo* Aristotle presents arguments in keeping with the atomistic horn which show that the hypothesis of indivisible atomism is a contrivance which violates basic assumptions of mathematics. I shall devote the rest of this chapter to the interpretation and criticism of arguments related to this dilemma. My primary concern will be to uncover the underlying assumptions which propel these arguments.

3. The Nihilistic Horn and Aristotle's Escape

In *De generatione et corruptione* Aristotle presents the nihilistic horn of the dilemma as an argument used by the atomists to refute their opponents who hold that magnitude is infinitely divisible. He tries to show that his own theory of magnitude as a continuum escapes the dilemma by distinguishing different senses of the claim that a magnitude is "everywhere divisible." In

8. E.g., *noēsis hē energeia*, which Ross corrects to *hē noēsis energeia*, in Ross 1936.
9. Cf. also *De anima* III 6, 430b20.

one sense, the claim is caught on the nihilistic horn; in the other sense, Aristotle's, the claim escapes.

> Let us now explain that this argument contains a fallacy, and where the fallacy is. Since point is not next to point, there is a sense in which the predicate "divisible everywhere" belongs to magnitudes and a sense in which it does not. When this is asserted [viz., that magnitudes are divisible everywhere], it seems that there is a point anywhere and everywhere, so that the magnitude must have been divided up into nothing—since there is a point everywhere, and so it will be made either of points or of contacts. Yet there is a sense in which there is a point everywhere, in that there is one point anywhere, and all of them are there if you take them one by one. But there is not more than one (since they are not consecutive to each other), and so they are not everywhere.[10]

The philosophical assessment of this passage involves difficulties which are not fully addressed by recent commentators. Furley, for example, offers the following gloss: "It is impossible to divide a magnitude 'at every point,' because points are not next to each other; between *any* two points there is a magnitude. But this does not entail that these are indivisible magnitudes; every magnitude has points on it, at which it may be divided."[11] This interpretation is unobjectionable, as far as it goes; but Aristotle's refutation has the appearance of a non sequitur. Why should the fact that points in a magnitude are not "next" to each other lead to the conclusion that a magnitude cannot be divided "at every point"? Why could not the proponent of the argument concede that there is a magnitude between any two points and still contend that any such magnitude reduces to an aggregate of points at which the line is divisible? Some additional assumptions are required if this argument is to go through. The crucial sentence at 317a7–9, especially, cries out for clarification: "There is a sense in which [this state of affairs] holds everywhere, in that there is one [point] everywhere, and all are everywhere if you take them individually, but there is not more than one (since they are not consecutive), so that [this state of affairs] does not hold everywhere"

10. *De gen. et corr.* I 2, 317a1–9; tr. Furley 1967, p. 92.
11. Furley 1967, p. 92.

(my translation). It is not altogether certain from the context whether the omitted words, here represented by "this state of affairs" in brackets, should be understood as *to diaireton einai*, "the magnitude is divisible," or as *to stigmēn einai*, "there is a point." It remains unclear also what Aristotle means by "there is not more than one" (i.e., not more than one point anywhere), and why he should take this to follow directly from the fact that points are not successive and to lead to the conclusion that points are not everywhere. (If there is *at least* one point anywhere, does it not follow that there *are* points everywhere?)

In the restatement of the atomist's argument, Aristotle makes some important distinctions which are not explicitly used in the refutation but which one would expect to be relevant. The first is between actual and potential senses of "divisible" and "indivisible." It says that a magnitude could be actually indivisible (undivided?) and yet potentially divisible (divided?) at any location (*sēmeion*), "but that it should be divisible everywhere simultaneously (*hama*) in potentiality would seem to be impossible. For if it were possible, it would happen. . . ."[12] It is tempting to suppose that Aristotle's own refutation turns in some way upon the distinction between simultaneous and successive division, and this is certainly suggested by Furley's rendering of the compressed expression *pasai hōs hekastē* (317a8) as "all of them are there if you take them one by one." Joachim goes further and gratuitously translates "simultaneously" at 317a9: "Hence it is not simultaneously divisible" for *hōst' ou pantē*.[13] How does this distinction help the refutation? Aristotle's refutation turns on two senses of "divisible everywhere," *diaireton pantē* or *diaireton hotioun*. In the first sense, "divisible everywhere" applies to magnitude, but the nihilistic conclusion does not follow. The nihilistic conclusion would follow from the second sense, but "divisible

12. *De gen. et corr.* I 2, 316b19–23. The point of the parenthesis b23–25 is to make clear that the relevant implication of the hypothesis that the line is potentially simultaneously everywhere divisible is not that the line is actually both divisible and divisible simultaneously (*ouch hōste hama einai amphō entelecheia, adiaireton kai dierēmenon*); rather the implication is that the line is [actually] divided at every point [simultaneously] (*alla dierēmenon kath' hotioun sēmeion*). The glossator responsible for this passage uses the verbal adjectives as passive participles.

13. Oxford translation; cf. Joachim 1922, p. 85.

everywhere" does not apply to magnitude in this sense. What are these two senses? The interpretations of Ross and Joachim, which seem to me to be only partly correct, place the principal burden of argument upon the sense of *diaireton*, treating the distinction as one between simultaneous and successive divisibility. Although this interpretation is illuminating, in order to understand Aristotle's argument it is also necessary to distinguish correspondingly different uses of *pantē* or *hotioun*.

"*Successively, not simultaneously* divisible." Ross and Joachim suggest rather different reasons why magnitude is supposed to be "divisible everywhere" in the one sense but not the other. Ross says that a body "cannot be divided everywhere at once, for that would mean that it has a finite number of points such that point could be next to point and that the body could be divided at all these points and dissolved away into nothing; whereas it has potentially an infinite number of points, none next to another."[14] This interpretation introduces a distinction between finite and infinite numbers of points, of which there is no suggestion in the refutation itself. There is, rather, an explicit distinction between "one" and "more than one," which this interpretation does not explain. Aristotle does indicate that the atomists had argued that *neither* could the process of bisecting a magnitude be infinite (*aperios*) nor could the magnitude be divided simultaneously at every location.[15] But the second condition is presented as independent of the first: it is not suggested that the reason that the line cannot be so divided is that such divisions would follow from a *finite* process and would involve a finite number of points. Nor does Aristotle's refutation allude to any false assumptions about the finitude of points on a line. Moreover, Ross's interpretation, so far, does not make it clear why the nihilistic conclusion does not follow. Even if it were impossible to perform an infinite number of divisions upon any magnitude, do there not exist an infinite number of points within the magnitude at which division *could* be performed, and is not this all that is needed for the nihilistic horn?

Joachim's interpretation follows the text more closely. He

14. Ross 1923, p. 100.

15. *De gen. et corr.* I 2, 316b29–31.

makes use of the final sentence of Aristotle's refutation of the nihilistic horn.[16] "For if it is divisible at the center, it will also be divisible at the adjoining point; but [this cannot be the case],[17] for there is no place adjoining place or point adjoining point, but this is division and combination."[18] Joachim interprets this as follows: "If, e.g., the given magnitude has been divided at its center, it cannot also be divided at a point *immediately next* to its center: for there is no such point. On the other hand, the magnitude might have been divided at a point *immediately next* to its center, *instead* of at its center: for a point might have been taken *there*, instead of at the center." Joachim's general interpretation of the refutation is this: ". . . though there is a point 'everywhere' in the magnitude, in the sense that a point can be taken 'anywhere' within it, these points (i.e., 'all the points of the magnitude') are not *immediately next* to one another: i.e., they are not 'everywhere' in the sense that *at all places of the magnitude simultaneously* there are points."[19] Joachim's "i.e." implies that the denial that points are immediately next to each other is equivalent to the denial that points exist simultaneously everywhere in a magnitude. But how has this equivalence been shown? What would be absurd about saying that all the places in a magnitude have points *and* that between any two points is at least one other point? The interpretation seemingly makes Aristotle's argument a non sequitur. Joachim's reconstruction of 317a10–12 seems, moreover, to be internally inconsistent, for he says both "the magnitude cannot be divided, immediately next to the midpoint, for *there is no such point*" and "if the magnitude had not been divided at the midpoint, it could have been divided *at the point immediately next to it*." The second statement entails the existence of the point whose existence is denied in the former statement! Joachim seems to mean that for Aristotle, if a magnitude is divided at some point, say *m*, then there exist other points of the magnitude, e.g., the point "immediately next" to *m*, at which the magnitude cannot at that time be divided. But the premise that one can describe a point which cannot exist in a magnitude (with the description "the point immediately next to *m*") and, hence, trivially, a point at which the magnitude could not be divided, does not support the conclusion that for any points which *can exist* in the magnitude, the magnitude is divisible there. Joachim seems to have been no more successful than Ross in recovering a plausible argument from Aristotle's refutation of the nihilistic horn of the dilemma of divisibility. In order to arrive at a satisfactory understanding of Aristotle's refutation, it is necessary to understand the mode in which he refers to points in the expression "divisible *everywhere*."

"Divisible at all potentially, not actually, existing points." In our passage Aristotle uses the phrases *pantē* and *hopēoun* ("everywhere," "anywhere") interchangeably with *kata pan sēmeion* and *kath' hotioun sēmeion* ("at every/any location").[20] Aristotle elsewhere uses *sēmeion* in the same sense as *stigmē* (Bonitz cites several passages), and 317a11–12 leaves no doubt that a *sēmeion* is a pointlike location. Nevertheless, Aristotle understands the expression "anywhere" or "at any location" to refer to such points only in a peculiar mode. He does not think of the domain of the quantifier as consisting of actually existing points on a line. For to concede the actual existence of such pointlike locations prior to any process of division would be already to undermine Aristotle's refutation; the crux of this refutation is that all the points in a magnitude cannot simultaneously exist in actuality.

Aristotle's refutation presupposes a special understanding of the point, which is made more explicit elsewhere. He conceives of the point as a cut (*tomē*) or division (*diairesis*) in a line, just as a line is a cut in a surface, which is, in turn, a cut in a solid.[21] A point is a limit (*peras*) of a line;[22] it is the beginning or end of a line segment.[23] Aristotle denies that points have the primary reality of substances, precisely because points exist as divisions or limits.[24] The denial that points are substances rests, in part, upon a consideration of the mode in which points exist. The operations in

16. This sentence is omitted in Furley 1967 (p. 92) for reasons not given; cf. Joachim 1922, p. 85.
17. Cf. Joachim 1922, p. 86, who attributes this reading to T. W. Allen.
18. *De gen. et corr.* I 2, 317a10–12.
19. Joachim 1922, p. 85.
20. Cf. *De gen. et corr.* I 2, 316b11, 20, 22, 25, 31.
21. *Metaph.* XI 2, 1060b12–19; III 5, 1002a15–b11.
22. *Metaph.* III 5, 1002b10; XI 2, 1060b16; XIV 3, 1090b9.
23. *Phys.* IV 11, 220a10; Ps.-Aristotle, *De lineis insecabilibus* 4, 971a18.
24. *Metaph.* XIV 3, 1090b8–9.

virtue of which points exist or do not exist are *not* processes of coming to be or ceasing to be:

> . . . points and lines and surfaces cannot either come to be or cease to be, when they now exist and now do not exist. For when bodies touch or are divided, their limits become one at once (*hama*) when their bodies touch and two when they are divided; hence when the bodies are combined, the limit does not exist but has perished, and when they have been divided, the limits exist which did not exist before (or it is not the case that the indivisible point is divided into two), and if then limits come to be or cease to be, from what do they come to be?[25]

Aristotle says, in short, "Some things, like points, either exist or do not exist, without coming to be or ceasing to be."[26] Hence, points have *accidental* being.[27] They exist or do not exist in virtue of operations performed upon substances (or, more precisely, upon magnitudes, which are themselves limits or aspects of substances), such as combination or division. Throughout *De generatione et corruptione* I 2, Aristotle also assumes that points come into existence through certain operations by which a magnitude is subdivided, "whether by bisection (*kata to meson*) or by any method in general."[28] Aristotle envisages a process in which a magnitude is divided at some point "into separable magnitudes which are smaller"; these can actually be separated from each other or rejoined at that point.[29] Aristotle has his attention throughout on bisection, e.g., when he says a point or contact (*haphē*) "is always one contact of two things."[30] When a line segment is bisected, we obtain two segments divided (and capable of contact) at one point. Aristotle also assumes, in another argument, that if we can obtain points by dividing magnitude, we should be able to obtain magnitudes by joining together points. Since we cannot do the latter (two points in contact simply coincide and have no extension), we cannot do the former.[31]

A clue to understanding the use of *hotioun sēmeion* in *De generatione et corruptione* I 2 is provided in Aristotle's criticisms of Zeno's dichotomy argument in *Physics* VIII 8: "Further, the potential and the actual are different, so that any location (*hotioun sēmeion*) between the two extremes of the straight line is, potentially, a midpoint (*meson*), but is, actually, not [a midpoint], unless [a moving body] divides it by coming to a stand and starts to move again. Hence, the midpoint is a starting-point and ending-point, a starting-point for the later part and an ending-point for the earlier part."[32] When a moving body moves continuously along a continuous path, Aristotle says, the points along which it moves are potential; he contrasts them with the goal at which it comes to a stop and which is thereby made actual.[33] Such language is comprehensible, provided that points are thought of as potential divisions in magnitudes, actualized by bodies coming to a halt and moving again. Aristotle speaks also of producing points by mental acts such as counting.[34] He concedes the point made by Zeno that any finite magnitude can be repeatedly bisected ad infinitum, so that it contains an infinite number of halves; and since the midpoint at which each half is further bisected is a *sēmeion*, the finite magnitude contains an infinite number of pointlike locations. Nevertheless, Aristotle denies that these *sēmeia* need have actual existence, because a *sēmeion* exists as a boundary between two halves, and for the most part, the halves exist only potentially.[35] This is a crucial distinction for Aristotle's refutation of Zeno's dichotomy paradox, since Aristotle concedes to Zeno only that in order to reach a goal a moving body must pass through a *potentially* infinite number of half-distances.[36] If the body were to traverse an *actually* infinite number of halves, it would have to make an infinite number of stops and starts. It is not a part of the essence (*ousia*) of a finite magnitude that it contains an infinite number of subdistances—or of points.[37] Hence, Aristotle understands *kath' hotioun sēmeion* as "at any potentially existing location" when he concedes that a magni-

25. *Metaph.* III 5, 1002a32–b5; cf. XI 3, 1060b17–19.
26. *Metaph.* VIII 5, 1044b21–22.
27. *Metaph.* VI 2, 1026b22–24.
28. *De gen. et corr.* I 2, 316a19–20.
29. *De gen. et corr.* I 2, 316b28–29.
30. *De gen. et corr.* I 2, 316b6–8.
31. *De gen. et corr.* I 2, 316a29–34.

32. *Phys.* VIII 8, 262a21–26.
33. *Phys.* VIII 8, 262b30–263a1.
34. *Phys.* VIII 8, 262b6–8; cf. 263a25.
35. *Phys.* VIII 8, 263a28–29: *en de tō sunechei enesti men apeira hēmisē, all' ouk entelecheia alla dunamei.*
36. *Phys.* VIII 8, 263b3–5.
37. *Phys.* VIII 8, 263b5–9.

tude can be divided "at any *sēmeion*," in *De generatione et corruptione* I 2. And since he understands the *sēmeion* as becoming actualized only through an operation such as bisection of a line segment, to say that a line is "divisible at any location" is thus to say that any subdivision can itself be further subdivided.

Aristotle refutes the nihilistic horn, used by the atomists, by showing that even though division is possible and a point exists everywhere in the potential mode, it does not follow that magnitude reduces to points. For the existence of every actually existing point is conditional upon the existence of two segments with magnitude into which the subsection is divided. This interpretation makes the best sense of a difficult passage on which both Ross and Joachim founder: "Yet there is a sense in which there is a point [*or:* the magnitude is divisible] everywhere, in that there is one point anywhere, and all of them are there if you take them one by one. But there is not more than one (since they are not consecutive to each other), and so they are [*or:* it is] not everywhere."[38] The words "there is not more than one" mean that in any section of a magnitude *only one* point can be obtained by the process of bisection, viz., the midpoint. There *is* a point "everywhere" in the potential mode: in any section obtained in the process of repeated bisection, a further point can be obtained. But there is *not* a point "everywhere" in the actual mode, because *all* subsections of the given magnitude cannot be simultaneously divided. The latter fact follows from Aristotle's conception of magnitudes and points: at *any* stage a magnitude or section of magnitude is divided only if there are two lesser subsections and a point at which these are divided. Aristotle's reasoning could be evaded only if, at some stage, subsections were divisible not into still smaller subsections but into something else altogether, viz., unextended points. In order to rule out this alternative, Aristotle adds the sentence at 317a10–12: "For, if it is divisible at the center (*kata to meson*) it will also be divisible at the adjoining point; but (this cannot be the case), for there is no location adjoining location or point adjoining point, but this is division and combination." The use of *kata to meson* at 317a10 implies that Aristotle is

38. *De gen. et corr.* I 2, 317a7–9.

envisaging a division which is a stage in the recursive process of division *kata to meson* by which the magnitude is divided up.[39] Aristotle denies that the magnitude could consist of pointlike locations or be divided up into points, because neither of these can be "adjoining" (*echomenē*) or "in succession" (*ephexēs*). The latter claim is broader than the former,[40] if the definitions of *Physics* V 3 are relevant here. Since "adjoining" is defined as "in succession" as well as "touching," to rule out "in succession" is to rule out "adjoining" but not vice versa.[41]

A thing in succession to another, by Aristotle's account, is such that (i) between it and its predecessor there is nothing of the same kind as it and its predecessor, and (ii) it is "after" (*husteron ti*) its predecessor.[42] We might take the relationship between the left half, *A*, and the right half, *B*, of a line segment as a paradigm of succession. Both *A* and *B* are segments, and (i) between *A* and *B* there is no segment, and (ii) *B* is after *A*. Both these conditions are relevant to Aristotle's claim regarding the impossibility of constructing a divisible magnitude out of points, as is shown by his criticism in *Physics* VI 1 of the theory that a line is merely a succession of points. He shows that if a pair of points satisfies either one of these conditions, it necessarily fails the other.[43] If *a* and *b* are points on a line segment and we suppose that *b* is in succession to *a*, then we will have to suppose that (i) between *a* and *b* there is no point, in which case the two points, like the two segments *A* and *B*, will touch. But two points that touch would have to do so "as whole with whole" and thus be entirely coincident,[44] in which case *b* cannot be after *a*. If, on the other hand, we stipulate that (ii) *b* is after *a*, there will be a part of the line segment separating *b* from *a*, and in any part of a line segment there are points.[45] So if condition (ii) is fulfilled, condition (i) is violated. In the case of points, then, the two conditions required for succession are incompatible, so that there can be no

39. Cf. *De gen. et corr.* I 2, 316a19–20.
40. Cf. *De gen. et corr.* I 2, 317a9, 11, 15.
41. *Phys.* V 3, 227a17–b2.
42. *Phys.* V 3, 226b34–227a6.
43. *Phys.* VI 1, 231a29–b10.
44. Cf. *Phys.* VI 1, 231b2.
45. Cf. *Phys.* VI 1, 231b9; V 3, 227a31.

operation by which an extended magnitude could be compounded out of points. By the same token there can be no operation by which an extended magnitude is entirely divided into points. A magnitude can be "divided everywhere" *only* by a process in which a subsection is divided into further subsections, but there is never a stage at which there are no more extended constituents.

In conclusion, a magnitude can be "divided everywhere" only by a process in which a subsection is divided into further subsections. There is never a stage at which the division is completed and the line consists exclusively of unextended constituents. For an actually existing point necessarily presupposes the existence of extended magnitudes which have been divided. Hence, the division of a line must be successive rather than simultaneous, and it occurs "at every point" not in the sense of actually existing points but in the sense of points which could mark further subdivisions. Aristotle's refutation of the nihilistic horn relies upon his own "constructivist" conception of a point as an accidental feature of extended magnitudes undergoing operations.

4. The Atomistic Horn

The difficulty which arises for the atomists in the dilemma of divisibility is that their denial that division can go beyond a certain point is "contrived."[46] In *De caelo* III 4, Aristotle contends that atomism so construed is in conflict with basic principles of mathematics.[47] The source of the difficulty is the atomists' commitment to a "smallest magnitude," *elachiston megethos*.[48] The charge is developed more fully in *Physics* VI. It is important to recognize that *this* is the main objection to the atomic theory, rather than the argument that a magnitude is not composed of "indivisibles," which appears in *Physics* VI 1. Critics have complained that this latter argument contains "loopholes" through which an atomist could slip.[49] Aristotle's argument turns on the

46. *De gen. et corr.* I 8, 325a6–12.
47. *De caelo* III 4, 303a20–24.
48. *De caelo* I 5, 271b9–11.
49. See, e.g., Furley 1967, pp. 114 ff.; also Richard Sorabji, who interprets 231b10–15 as directed against atomism (in chapter II, nn. 57–58).

claims that the indivisibles cannot be in contact with or in succession to each other, and allegedly in both cases the argument begs the question against the atomists by assuming that indivisibles have no magnitude. But it is doubtful whether we should interpret this passage as a refutation of atomism. The doctrine under attack explicitly takes the indivisibles as *points*, and the argument turns on the claim that points cannot be continuous, or touch, or be in succession. It is unlikely that Aristotle would confuse atomic magnitudes with points, for in the argument of *De generatione et corruptione* I 2, which we have been examining, he is considering an argument in which the atomists criticize their opponents for reducing magnitudes to points.[50] And the argument which opens *Physics* VI 1 employs many of the same claims as Aristotle's critique in *De generatione et corruptione* I 2, in particular the claim that points cannot be in succession with points.[51] Moreover, Aristotle himself makes the observation that while pointlike entities cannot be successive, atomic times *can be*[52]—and, given the context, he regards this fact as problematic for atomism! Commentators can be excused for construing Aristotle's argument in *Physics* VI 1 as a refutation of atomism, since Aristotle himself says at VI 2 that it has been proven that it is impossible for something to be [made] out of atoms (*ex atomōn*).[53] Aristotle's reasoning about atomism not infrequently appears muddled because he uses the word *atomos* imprecisely. Here, I suggest, the term *atomos* should be taken as meaning simply "indivisible" and as referring to the points under fire in VI 1. Aristotle elsewhere refers to an *atomon nun*,[54] which is a pointlike instant[55] that should not, I think, be identified with the *atomos chronos* of VIII 8.[56] For instants, like points, cannot be successive, whereas atomic times can be. It is important to draw such conceptual distinctions more sharply than Aristotle's loose terminology would seem to warrant; otherwise we lose sight of significant features of Aristotle's argument. In general, when Aristotle

50. The two positions are clearly distinguished at 317a13–16.
51. Cf. *Phys.* VI 1, 231b6–7, with *De gen. et corr.* I 2, 317a9–11.
52. *Phys.* VIII 8, 264a3–4.
53. *Phys.* VI 2, 232a23–24.
54. *Phys.* IV 13, 222b8; VI 9, 241a25.
55. *Phys.* VI 9, 241a5–6.
56. *Phys.* VIII 8, 264a3–4.

speaks of an "atomic magnitude," *atomos megethos,* in the *Physics*[57] or in *De generatione et corruptione*,[58] he has in view something *with magnitude,* as the terminology indicates; it is a mistake to criticize him for assuming otherwise. This is important for an understanding of Aristotle, because his most important criticisms of atomism are criticisms of atomism *as* a theory of magnitude.

The gist of Aristotle's objection against atomism as a theory of magnitude is identified by Simplicius, who reports that Aristotle is alluding to the theorem, "it is possible to take a magnitude smaller than any given magnitude."[59] As I shall try to show, this principle is closely tied to other principles which Aristotle employs in his arguments against atomism in *Physics* VI: e.g., if a given magnitude is traversed by a moving body in a given time, a *smaller magnitude* will be traversed by a body moving at equal velocity in less time or by a body moving at a lesser velocity in the same time. Aristotle is convinced that he can use these principles to establish important conclusions bearing on the atomic theory. First, he believes that he can establish that spatial magnitude, time, and motion are isomorphic, such that either all three of them have an atomic structure, or all three are continua. Second, on the basis of this thesis (and another premise), he thinks he can establish that one can always take a smaller magnitude than any given magnitude and that the attempt of the atomist to deny this leads to incoherence.

5. The Isomorphism Thesis and the Derivation of Atomic Time

The isomorphism thesis is stated in the following terms: "The same argument applies to magnitude, time, and motion: either they [all] are composed of indivisible things and divided into indivisible things, or none [of them] is."[60] Aristotle believes that he can demonstrate a necessary connection between the continuity (infinite divisibility) of magnitude and time, and hence by con-

57. *Phys.* I 3, 187a3.
58. *De gen. et corr.* I 2, 316b32; 317a1.
59. Diels-Kranz 1968, 68A48a; cf. Furley 1967, p. 88, n. *i.*
60. *Phys.* VI 1, 231b18–20.

traposition, between the atomicity (finite divisibility or ultimate indivisibility) of magnitude and time.[61] The mathematical theorem that it is always possible to take a smaller magnitude than any given magnitude holds for spatial magnitude if and only if it holds also for time and movement. He subsequently argues that the theory of atomic time is inferior to the theory of continuous time because it does not permit a coherent analysis of coming to be.[62] Hence, insofar as atomism is committed to such a view of time, it is vulnerable. Aristotle also develops a general argument that atomism is unable to provide a coherent account of motion, but before considering this general argument, we should assess the arguments for the isomorphism thesis.

The thesis that magnitude, time, and motion all have the same structure is presupposed when Aristotle infers that one of the three is continuous because another is: "For because magnitude is continuous, motion is also continuous, and time because of motion."[63] In asserting that they are continua, Aristotle presupposes an account of the continuous which rules out atomism: "Every continuum is divisible into things which are always divisible (*diaireton eis aei diaireta*)."[64] A continuum is always divisible into other continua. But the isomorphism thesis could be, and evidently was, accepted by Aristotle's atomist opponents, including Epicurus.[65] Let us call such a doctrine, which takes motion and time as well as magnitude to be atomic, *pure atomism.* In chapter II Richard Sorabji suggests another, *mixed* version of atomism, according to which magnitude and motion are atomic but time is continuous. According to this theory, as an object moves from atomic magnitude to atomic magnitude, there is a divisible stretch of time during which it occupies, or lingers at, each atomic magnitude. This is a "cinematographic" theory of motion. Sorabji's distinction is a most important one, for it cannot be assumed that a refutation of pure atomism will be a refutation of mixed atomism.

One strong link in the isomorphism thesis is the claim that if

61. Cf. *Phys.* VI 1, 232a18–22.
62. *Phys.* VIII 8, 263b9–264a6.
63. *Phys.* IV 11, 219a12–13; cf. 219b15–16 and VI 2, 233a11–21.
64. *Phys.* VI 1, 231b16.
65. Cf. Furley 1967, chap. 8, and Sorabji, chapter II, section 7.

magnitudes are indivisible, then motion consists of indivisible moves or "jerks" (*kinēmata*). Aristotle's argument goes: "Let it [the atom] change from *AB* to BC . . . and let *D* be the time in which it is changing in the primary sense. Therefore, with respect to the time when it is changing, it must be either in *AB* or in *BC*, or part of it must be in *AB* and part in *BC*; for everything which is changing is thus. But some of it will not be in each of these; for then there would be a part."[66] In Aristotle's view, one can say that the object "is moving," that is, is undergoing a process, only when it is partly in *AB* and partly in *BC*. For when it is wholly in *AB* it has not yet started, and when it is wholly in *BC*, it has already moved. Hence, there will be no *continuous* process of moving out of *AB* and into *BC*. The object simply occupies one place and then another.

Other links in the thesis are rather more problematic, especially those from atomic magnitude to atomic time and from atomic motion to atomic time. For mixed atomism seems to describe a way in which an object can move in jerks across discontinuous magnitudes in a continuous stretch of time. Aristotle provides arguments which would, if successful, rule out such a theory.

6. The Link between Atomic Magnitude and Atomic Time

The first of these links comes at the end of *Physics* VI 1 and continues into VI 2. The argument as Aristotle sets it forth is certainly not free of difficulties. Nevertheless, if some important qualifications are placed upon the argument, it can be shown to have some merit.

As it stands, the principal argument for the isomorphism thesis in *Physics* VI 2 is unsatisfactory because it is circular. The circularity arises in the following way. In the first part of the argument Aristotle shows that certain commonsense theorems about faster and slower things follow from the assumption that magnitude is continuous;[67] and in the second part of the argument he reasons,

66. *Phys.* VI 10, 240b20–27; cf. VI 1, 231b18–232a17; 4, 234b10–20; and Plato, *Parmenides*, 138C4–139A1.
67. *Phys.* VI 2, 232a23–b20.

on the basis of such commonsense theorems about faster and slower things, that if magnitude is continuous, then so is time.[68] This reasoning is circular, however, because in the first part of his argument Aristotle tacitly assumes that time is continuous so that he can derive his commonsense theorems from the assumption that magnitude is divisible. Consider the two parts of the argument in reverse. In the second part of the argument Aristotle tries to show that time is continuous.[69] He alleges that it has been proved[70] that if *A* is faster than *B*, then if *A* traverses the same magnitude as *B*, *A* will take less time than *B*. He uses this commonsense theorem to show that the time t_1 in which the slower object traverses a magnitude m_1 can be divided, since the faster object takes less time, viz., t_2. He makes implicit use[71] of his other commonsense theorem, that if *A* moves faster than *B* but takes the same time as *B*, then it traverses a greater magnitude than *B*,[72] to show that the magnitude m_1 covered by the faster object *A* in time t_2 can be divided, since the slower object *B* will cover a lesser magnitude, viz., m_2. And so it goes: the former theorem can be used to divide time, and the latter theorem to divide magnitude, in an unending alternation. Hence, since magnitude is continuous, so is time.

Aristotle proves the commonsense theorem that faster things cover the same ground in less time in the first part of the argument as a corollary of the assumption that all magnitude is divisible.[73] He proves this by first establishing the lemma that if *A* is faster than *B*, then given any magnitude which *B* covers in a given time, *A* traverses a greater magnitude in a shorter time.[74] Given that *A* takes less time than *B* to cover a greater magnitude, it trivially follows, from this and the lemma, that if *A* is faster than *B* and covers the same magnitude, then *A* takes less time than *B* to do so. But when Aristotle proves the crucial lemma he simply takes it for granted that time is divisible: "The faster thing

68. *Phys.* VI 2, 232b20–233a12.
69. *Phys.* VI 2, 232b24.
70. *Phys.* VI 2, 232b26: *dedeiketai.*
71. *Phys.* VI 2, 232b31–33.
72. *Phys.* VI 2, 232a25–26, 28–31.
73. *Phys.* VI 2, 232a23–27.
74. Cf. *Phys.* VI 2, 232b6–7.

also covers a greater magnitude in less time, for in the time in which *A* comes to *D*, let *B* come to *E*, since it is slower. So since *A* has come to *D* in the entire time *FG*, it will come to *H in less time than this* (*en elattoni toutou*). Let it be in *FI*, and the magnitude *CH*, which *A* has traversed, is greater than *CE*, but the time *FI* is less than the entire time *FG*, so that it covers more magnitude in less time."[75] Since Aristotle is presupposing that magnitude is continuous, he is begging no questions in assuming he can find a point *H* between *E* and *D*. But since he ultimately intends to prove that time is divisible, he is clearly begging the question in assuming here that he can divide the "entire time" *FG* into a smaller portion *FI* (and its complement, *IG*).

Philosophers typically fall into circularity because they try to prove too much. Aristotle could have argued that given our commonsense beliefs about the faster and the slower, the continuity (or atomicity) of magnitude entails that of motion and time. That is, our prescientific or prephilosophical observations about bodies in motion commit us to the isomorphism thesis. This would be to argue from *ta phainomena*, as Aristotle often does. But Aristotle tries to justify these commonsense beliefs about motion on other grounds, and in this he falls into circularity.

7. The Link between Atomic Motion and Atomic Time

In *Physics* VI 10 Aristotle argues that the same reasoning which commits the atomist to an atomic theory of movement will also commit the atomist to an atomic theory of time. In the course of the argument Aristotle claims that there could be movement in the atomists' sense only if time were composed of partless instants (*ek tōn nun*). This claim is an inference in an argument that atomic bodies cannot move in the strict sense: "So the partless thing cannot be moving or be changing altogether; for there could thus be movement of it only (*monachōs*) if time were composed of instants; for always at an instant it would have moved and have changed, so that never would it be moving but always have moved. But this is impossible, as was shown before, for neither is

75. *Phys.* VI 2, 232a31–b5.

time composed of instants, nor a line of points, nor movement of moves."[76] This argument may be recast as follows:

(1) Always at an instant the partless thing has moved or has changed, so that it never is moving but always has moved.
(2) There is movement of the partless thing only if time consists of instants.
(3) Time does not consist of instants.
(4) The partless thing cannot be moving or be changing altogether.

Aristotle's connectives indicate that (1) is a premise from which (2) follows; and that (3) is an independent premise which, together with (2), establishes the first proposition of the passage, viz. (4), as a conclusion. The "time" in the argument, the wider context shows, is "the primary time in which the thing is [putatively] moving."[77] Just before this passage Aristotle has been arguing that an atom cannot *be moving* from place p_1 to (adjacent) place p_2 because it cannot be in both places without being divisible, having a part in each, which is impossible; and when it is at p_2 it already has moved and thus is not still moving; but when it is at p_1 it is not yet moving and thus is at rest, "for what is in the same place for some time is at rest."

Aristotle can be understood as responding to the atomist who says, "The atom is at atomic place p_1 at instant t_1 and then at atomic place p_2 at instant t_2," by asking the following leading questions: "What happens in the interval between t_1 and t_2?" Aristotle here has his sights on the atomist who answers, "There is no interval between t_1 and t_2." Thus, this atomist seeks to atomize movement into jerks by atomizing time into partless instants. It is for this reason that Aristotle thinks that the atomist who asserts

(1) Always in the instant the partless thing has-moved, so that it never is-moving but always has-moved

is also committed to

(2) There is movement of the partless thing only if time consists of instants.

76. *Phys.* VI 10, 240b30–241a4.
77. *Phys.* VI 10, 240b22–23.

For presumably such an atomist would contend that motion exists over an interval so long as at any instant in the interval one can always say that the object "has moved." One can always apply the *perfect* tense of *kineisthai* to the body. This, of course, departs radically from Aristotle's own view, defended in *Physics* VI 6, that a body has moved to p_2 at t_2 only if during some time interval from t_1 to t_2 it is moving across some distance from p_1 to p_2. Aristotle has two different objections to such a theory. The first is suggested by proposition 1: the present tense of *kineisthai*, "is moving," never applies to the body. This objection, developed in *Physics* VI 1, is that since a body cannot at the same time have moved from one atomic magnitude to the next and be moving from the one to the next, the claim that an atom always *has moved* does not establish that it always *is moving*; in fact, Aristotle thinks, it shows that it is not moving at all.[78] The second objection is central to the argument of *Physics* VI 10: the atomist is committed in proposition 2 to an atomic theory of time—hence, this atomist is committed to a *pure* theory of atomism, which Aristotle rejects with proposition 3.

It should be noted that the mixed atomist fares no better who would answer Aristotle's leading question by saying, "The atom is at p_1 at t_1 and lingers there until it jerks to p_2 at t_2." In the first place the mixed theory does no better than the pure theory in meeting the objection that atoms never actually *are moving*: a body is at rest for a period of time and then, suddenly, it *has moved* to p_2. And in the second place, this theory, in avoiding the pitfalls of atomic time, falls into an even worse difficulty, which is suggested by Aristotle's statement that "what is in the same place for some time (*chronon tina*) is at rest." The mixed theory had led to the disconcerting consequence that the temporal interstices between the instants of the jerks into which movement is analyzed will be periods of rest. Thus, there can be no such thing as a stretch of constant movement. The difficulty is that the mixed theory of motion cannot accommodate the criterion stated by Aristotle: "Movement is always other and other" (*hē kinēsis aei allē kai allē*). This criterion is still invoked by modern philosophers. Thus,

78. *Phys.* VI 1, 231b25–232a17.

Donald Williams states, "Motion is . . . defined and explained in the dimensional manifold as consisting of the presence of the same individual in different places at different times." Richard Gale makes the point, in a somewhat more Aristotelian vein, that "motion is defined as the occupation, by one entity, of a continuous series of places at a continuous series of times."[79] The pure theory of atomism can accommodate Aristotle's criterion of movement: the atomic body occupies a different atomic place at each atomic time. But the mixed theory with its "cinematographic motion" does not satisfy this criterion of motion at all. Like modern cinematography, it substitutes an illusion for the real thing. Hence, it is quite reasonable for Aristotle to suppose that an atomic account of motion entails an atomic account of time.

8. The Refutation of Atomism

We are now in a position to consider precisely how, according to Aristotle, the atomist is impaled upon the second horn of the dilemma of divisibility. Aristotle distinguishes atomic units from pointlike entities, although both are described as "indivisible" and "atomic." I have already suggested that it is essential to the atomist theory he is attacking that atoms, unlike points, have magnitude. His objection is that the atomists treat magnitudes in a contrived way, with the contrivance of a "smallest magnitude," which violates the mathematical theorem that one can always find a magnitude smaller than a given magnitude. The theorem is supported by an appeal to commonsense beliefs about relative velocity. For Aristotle thinks of "taking a smaller magnitude" in terms of an actual operation, such as moving a smaller distance.

79. *Phys.* IV 11, 219b9–10; Williams 1951, pp. 104 f.; Gale 1967, p. 3. Aristotle's criterion should be amplified. The criterion includes "always" (*aei*) because it is not sufficient for an object to be moving throughout a time merely if it is at different places at some of the instants, since this would allow intermittent motion. But if "always" implies that at any different instants the object is at different places, the criterion is too strong: it admits rectilinear movement but excludes the circular movement of heavenly bodies, because such movement is periodic. This difficulty can be met, however. Heavenly motion consists of a continuous succession of revolutions, and each of these does satisfy Aristotle's criterion. Thus, constant motion either satisfies Aristotle's criterion or consists of a succession of motions each of which satisfies the criterion. A pure-atomist account of circular heavenly motion would also satisfy this criterion, but a mixed-atomist account would not.

One statement of the objection appears in VI 2.[80] To see the force of the objection one should note that the argument relies upon two sorts of claims: first, the principles of relative velocity, which Aristotle uses to support the isomorphism thesis; second, the general claim that it is always possible to move faster or slower than any given moving body.[81] The second claim, as well as the first, is required in order to establish the theorem that one can *always* find a smaller magnitude. Both sorts of claims are necessary to carry the day. If either sort of claim were withdrawn, one or the other form of atomism would escape refutation.

Pure atomism can avoid difficulty by challenging the *second* claim (not the first, which it endorses). A pure atomist will hold that an atom *A* moves in an indivisible jerk over an indivisible magnitude in an indivisible time. If this atomist concedes that another atom *B* could move more slowly than *A* and agrees that a slower body covers a smaller magnitude in the same time, he will be driven to the conclusion that there is a smaller magnitude than "the smallest magnitude." The pure atomist can avoid self-contradiction only by refusing to concede that it is always possible to move faster or slower than any given moving body.[82] Epicurus seems to have this issue in mind when he denies that atoms traversing atomic magnitudes can move faster or slower than one another.[83]

On the other hand, mixed atomism, with its cinematographic theory of motion, would attack the principles of relative velocity. The second claim can be accommodated by this theory in a way which causes no difficulty. The atom *A* can move more slowly than *B* by tarrying for a longer stretch of time at each atomic place than *B* does. But the phenomena of relative velocity do create problems for this theory. Consider again the principle that the slower of two moving bodies traverses a smaller magnitude in an equal time. According to mixed atomism, the slower *A* might remain at an atomic unit for two microseconds, while the faster *B* remained there for only one microsecond, after they had arrived simultaneously at their respective destinations. During two microseconds *B* traverses two whole atomic magnitudes while *A* covers only one. But the problem lies with the *first* microsecond, during which *A* and *B* traverse the same magnitude: during this same time the slower *A* does *not* cover less magnitude than *B*. Hence, mixed atomism generates a counterexample to this principle of relative velocity. Thus, the mixed atomist is forced to reject or severely restrict such principles.[84]

Aristotle bases his argument upon certain beliefs about relative velocity. Although they are grounded in ordinary ways of thinking about the world, opponents might call them into question, especially in view of developments in physics since Einstein. A pure atomist might question the claim that it is always possible to move faster than any given motion. But Aristotle has surely identified some deep assumptions in which his thesis of the continuity of magnitude, time, and motion must be anchored.

80. *Phys.* VI 2, 233b19–32.
81. Cf. *Phys.* VI 2, 233b19–20.
82. Cf. *Phys.* VI 2, 233b19–20.
83. Epicurus, *Letter to Herodotus*, sects. 61–62. This interpretation is offered by Furley 1967, pp. 120–22 and 130 n. 9.

84. In section 5 of chapter II Richard Sorabji finds this line of reasoning unconvincing. I am not certain how his suggested distinction between "the time taken to traverse a given space" and "time itself" can save the mixed atomist from the necessity of placing restrictions on commonsense principles concerning relative velocity. Thus, this theory is at variance with the principles advanced by Aristotle.