

Proclus on the Logic of the Ineffable*

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ABSTRACT

A informal introduction to the semantic theory in the Neoplatonic system of Proclus. It is sketched how the Neoplatonic order can be understood as ranked by scalar adjectives as understood in modern linguistics, and how Proclus' exposition, especially his use of privative and hypernegation, presuppose the syllogistic of Aristotle.

1. Introduction

Proclus (412-458) is well known for having detailed doctrines about negative knowledge of the One, and in a vague way he is also thought of as a logician. After all, he advances original proofs in geometry and he employs syllogisms and other logical terminology in his philosophy. What is not clear and what I would like to explore in this essay is how deeply logic runs in his metaphysics. Within the limits of this paper I can only sketch the story, but I shall describe a methodology that employs logical

* This paper, which will appear in the author's collection *Themes in Neoplatonic and Aristotelian Logic: Order, Negation and Abstraction* (Ashgate Publishing Ltd., 2003) is an introduction in non-technical terms to the much fuller investigation in "Proclus and the Neoplatonic Syllogistic," *J. of Philosophical Logic* 30 (2001), (also in the collection). The present account contains some textual support referred to summarily in the longer paper.

concepts only recently studied in modern theory. These concern scalar predicates and negation.

Proclus faced a dilemma. On the one hand, he accepts Plotinus' doctrine that the fundamental structure of reality is that of a linear causal ordering emanating from the One. On the other hand, in working out his ideas and arguing for them, he makes use of logic. Like other philosophers of his time he makes use of the logical theory available, especially the syllogistic and definition *per genus et differentiam*. But these ideas were developed for the radically non-linear metaphysics of Aristotle. In this paper I shall be recounting how Proclus reconciles Aristotle's logic to Neoplatonic ontology. He does so by exploiting properties of what is now known as scalar predicate negation. Using scalar negations he transforms the tree provided by "Aristotelian" definition into a linear "Neoplatonic" structure appropriate for syllogistic reasoning.

The method may be outlined as a series of steps. It begins with Neoplatonic diairesis -- the collection of information in the form of Aristotelian definitions as described by Porphyry in the *Isagoge*. As Proclus sees it, each division in Porphyry's tree is accomplished by means of negation. Beneath *animal*, for example, is *man* and *not-man*. Moreover this negation is not the ordinary classical variety. It is not merely a syntactic marker that lacks any commitment to the true "negativity" in reality of the property picked out by the negated predicate. Rather, Proclus' negation is scalar. It presupposes that there are in reality genuine "positive" and "negative" degrees of comparison among properties. Negations of this sort come in pairs and Proclus uses both. These we shall call *hyper* and *privative* negation, following his usage. The hypernegation of a term moves it up a background scale; its privative moves it down.

In division the set of a node's immediate descendants is ranked by these negations. Apart from the least of the immediate descendants, each is the hypernegation of another in such a way that an order is determined: if x is the hypernegation of y , then $x \leq y$. This "priority" relation corresponds with causation in reality. Privative negation yields the same ranking by moving down the steps of causation. The whole group of a node's immediate descendants is called a *taxon* by Proclus, and the node from which they descend is the immediate cause of the first of the descendants and is called the taxon's *monad*. The tree together with the order on the immediate descendants of each node is called an ordered tree, and an ordered tree determines a line. In Proclus' metaphysics the tree of diairesis with taxa ranked by causation determines the linear order of ontic causation.

What is logically interesting about Proclus' account is this structure and its operations. These force the validity of various formal inferences. The valid moods of Aristotle's syllogistic, for example, are usually thought of as describing structures of Boolean sets. So they are understood in mediaeval logic and in modern model theories for the syllogistic. But the syllogistic moods turn out to be valid also in the linear structures generated by trees ordered by scalar negation. Hence Proclus legitimately may use the syllogistic to expound his philosophy. **A** statements (e.g. *All x is y*) turn out to be non-trivial propositions that express causal order and may be written $x \leq y$. (Here for algebraic reasons 0 is adopted as the ideal supremum and $x \leq y$ is to be read x is *higher than* y or x *causes* y .) **O** statements (e.g. *Some x is y*) express their contradictory opposites, and because the ordering is linear, **O** xy may be written $x \not\leq y$, which in a linear structure is equivalent to $y < x$. In Proclus' linear structure **I** and **E**

statements turn out to be trivially true and false respectively, but this limitation is overcome by the presence of non-traditional hyper and privative negations.

These predicate operators add expressive power that goes well beyond the traditional syllogistic to sanction new valid arguments . By progressive applications of hypernegations to a predicate, the predicate becomes true of progressively higher stages of the causal structure. Likewise, by progressive applications of privative negation, a predicate moves down the order. Hence one may infer "what a point in a higher taxon is not" by knowing "what the point in a lower taxon is." Affirmation, in Proclus' dictum, generates negation: from $x \leq y$, it follows that *hyper-x* \leq *hyper-y*. Knowledge of the higher hypotheses can be obtained by logical inference from predications true of the lower. The *via negativa* literally becomes progressive steps of inference in scalar logic.

The purpose of this short paper is not to amass all the textual evidence that supports this interpretation. Nor will I be able to prove the logical claims. But I shall try to cite enough texts to make the interpretation plausible and to explain the logical ideas in a clear and non-technical in a way.

2. Proclus' Mixed Parentage

Proclus as an Aristotelian

Since they accept Plato's method of diairesis, Neoplatonists understandably also accept its development by Aristotle in his theory of definition. Plotinus himself

occasionally employs diairesis using Aristotle's terminology of genus and species,¹ and in the *Isagoge* Porphyry summarizes the theory in what was to be its standard form in the Middle Ages.² Proclus describes the method as follows:

In demonstrations and definitions the particular (*ton mekpion*) must be subordinate to the universal (*tou kathalou*) and the *definition* (*ton horismon*). Definitions of common features in particular do not take in the particulars as a whole. How, for instance, is the whole of Socrates comprehended by the definition "rational mortal animal," when there exists in him other elements also which make up his so-called 'personal quality'? The reason-principle of Man (*ho tou anthropou logos*) in us comprehends the whole of each particular, for the particular comprehends unitarily all those potencies which are seen as being involved in the individuals. In the case of "animal" and likewise, the instance of it in particulars is less comprehensive than the particulars themselves or the species; for it does not have in actualized form all the differentiae, but only potentially, wherefore it becomes a sort of "matter" to the specifying differentiae that super-impose themselves upon it. The "animal" inherent in us is greater and more comprehensive than "man," for it contains in unified form all the differentiae, not potentially, like the concept, but actualized. If we are, then, to discover the definition which will serve as the beginning of demonstration, the

¹ Plotinus, *Enneads*, trans. A. H. Armstrong, 6 vols., *Loeb Classical Library* (Cambridge, MA: Harvard University Press, 1966-88). Book VI: 3, 9, 1-8, p. 204.

² Porphyry, "Isagoge," in *Five Texts on the Mediaeval Problem of Universals* (Indianapolis, ID: Hackett Publishing Company, 1994). See pp. 5-6.

definition but must of an entity of such a sort as to comprehend everything more particular than itself.³

Proclus is combining Aristotelian definition and demonstration in a single method that he calls logic. This is the first stage of enlightenment, which is followed by philosophy (Platonic intellection) and theology (religious and mystical understanding), and much of his exposition is formulated in these Aristotelian categories.⁴ For example, he reasons about the heavens using Barbara,⁵

All great circles bisect one another

Circles in the Heavens are great circles

Therefore, all circles in the heavens bisect one another

and about the higher hypotheses using Baroco (Proclus treats singular propositions as universal with singular terms as "degenerate" common nouns):⁶

The One is not receptive of multiplicity

The unequal is receptive of multiplicity

Therefore, the One is not unequal.

³ Proclus, "Procli Commentarium in Platonis Parmenidem," in *Procli Philosophi Platonici Operas Inedita* (Paris: Augustus Durand, 1864). Hereafter *IP*. Proclus, *Proclus' Commentary on Plato's Parmenides*, trans. John M. Dillon Glenn R. Morrow (Princeton, NJ: Princeton University of Press, 1987). Hereafter M&D. See *IP* 981:5-27, M&D 335. See also *IP* 767:8-28, M&D 133-34; Proclus, *Théologie Platonicienne*, trans. H.D. Saffrey and L.G. Westerink, vol. I-VI (Paris: Les Belle Lettres, 1968-1997). Hereafter *PT*, S&W. See *PT* IV:3, S&W 13:2--24; *PT* I:10, S&W 43:22-45:18.

⁴ Proclus employs this method throughout both *IP* and *PT*. For general descriptions of his method see *IP* 1070-72, M&D 424-5. *PT* I:4, S&W 17:15-25.

⁵ *IP* 796:37-797:3, M&D 165-116.

⁶ *IP* 1208:11-24, M&D 553. For an explicit Bocardo see *IP* 1208:11-24; M&D 553.

Likewise *modus ponens* and *modus tollens* which a modern logician would symbolize $\forall x(Fx \rightarrow Gx), Fc \vdash Gc$ and $\forall x(Fx \rightarrow Gx), \sim Gc \vdash \sim Fc$ are treated as cases of Baroco and Bocardo respectively, as in the example:⁷

If something is not an animal, it is not a man.

It is not an animal

Therefore, it is not a man

In practice, for reasons we shall touch on below, Proclus eschews **E** and **I** statements in his logical analysis.

Proclus and the Chain of Being

Though Proclus adopts much of Aristotelian logic, he is very much a follower of Plotinus. While he uses diairesis and logical analysis to reveal the tree-like structure of reality, he also uses the metaphor of a chain to describe causation as having what we would call today a "linear" order or, more precisely, a order that is partial (reflexive, transitive, antisymmetric) and total ($x \leq y$ or $y \leq x$). This ordering is the causal progression of hypotheses from the One. He describes it frequently, for example in some of the basic propositions from the *Elements of Theology*. The order has a unique maximal element:

Proposition 11. All that exists proceeds from a single first cause.

It forms a causal line, and the line is partitioned into mutually exclusive non-overlapping *taxa*. Each taxon is also preceded by a first element, its *monad*:

⁷ For *modus ponens* and *modus tollens* see *IP* 1098:2-27, M&D 444-45. *IP* 1055:2 to 1057:4, M&D 413-14. *IP* 1208:11-24, M&D 552-3.

Proposition 14. All that exists is either moved or unmoved; and if the former, either by itself or by another, that is, either intrinsically or extrinsically: so that everything is unmoved, intrinsically moved, or extrinsically moved.

Proposition 21. Every order has its beginning in a monad and proceeds to a manifold co-ordinate therewith; and the manifold in any order may be carried back to a single monad.

Proposition 100. Every series of wholes is referable to an unparticipated first principle and cause; and all unparticipated terms are dependent from the one First Principle of things.

Proposition 147. In any divine rank the highest term is assimilated to the last term of the supra-jacent rank.

Proposition 21. Every order has its beginning in a monad and proceeds to a manifold co-ordinate therewith; and the manifold in any order may be carried back to a single monad.

Proposition 100. Every series of wholes is referable to an unparticipated first principle and cause; and all unparticipated terms are dependent from the one First Principle of things.

Proclus frequently describes the descent in the line as consisting of triads, minimal taxa as it were:

Proposition 67 Every whole is either a whole-before-the parts, a whole-of-the parts, or a whole-in-the-part.⁸

⁸ See also *PT* III:8, S&W 31:20-23, and III:9, S&W 39:20-40:1, and the discussion in Lucan Siorvanes, *Proclus: Neo-Platonic Philosophy of Science* (New Haven: Yale University Press, 1996), pp. 102-105.

He goes on to give a theoretical analysis of why this tripartite breakdown holds as a basic fact of metaphysics:

Proposition 81. All that is participated without loss of separateness is present to the participated through an inseparable potency which it implants.

For if it is itself something from the participant and not contained in it, something which subsists in itself, then they need a mean term to connect them, one which more nearly resembles the participated principle than the participant does, and yet actually resides in the latter.....Accordingly a potency or irradiation, proceeding from the participated to the participant, must link the two; and this medium of participation will be distinct from both.

On the basis of such passages and his many examples of triadic analysis, I think it is fair to attribute to Proclus the view that any point is susceptible to a deeper analysis in which it is followed by a triad further detailing the causal descent. That is, Proclus posits a kind of *causal density*: between any two points x and y such that x causes y , there is a deeper analysis in terms of which there is a triadic descent from x considered as a monad of a taxon the points of which intervene in the causal process between x and y .

It is clear that the points in a triad are ranked. The entire discussion in the *Platonic Theology* is organized around triads at the various levels of the hierarchy, and Proclus carefully explains the ranking within the triad. For example, beneath the gods comes the triad angels, daimons, and heroes. He clarifies their order as follows:

Again we thus come to see that pre-existent properties are indivisibly and uniformly divided among the three fathers, and likewise the demiurgic triad

participates in the demiurge's unity as a result of the monad's unlimited superiority, likewise in its priority the monad contains the triad as befits its causal power.⁹

A central question remains. How can the reality both be a tree and a line? How can the tree of Porphyry also be the chain of being? How can syllogisms be used to reason about hypotheses? Proclus' answer is no less than a reconciliation of Aristotle's logic to Neoplatonism and is to be found in his theory of negation.

3. Scalar Predicates and Negation

Proclus gives names to the points in his causal hierarchy . Indeed he is the father of the doctrine of the "divine names" applied to the higher nodes of the line, and he goes on at great length in the *Platonic Theology* to give names to the lesser nodes of the hierarchy as well. He also describes the hierarchy in relational terms. He uses comparative adjectives for this purpose, e.g. *more real* (*ousiōteron*), *more general* (*katholikōteron*, *holikōtero*, and *merikōteron*), *more causal* (*aitiōteron*), *more perfect* (*teliōtikon*), *holier* (*timiōteron*) , *more powerful* (*dunatōteron*), and *more infinite* (*apeirōteron*). The following passage from the *Elements of Theology* is typical of Proclus' use of comparative adjectives:

...the higher cause (*aitiōterōn*), being the more efficacious (*drastikōteron*), operates sooner upon the participant (for where the same thing is affected by two causes it is affected first by the more powerful (*dunatōteron*); and in the

⁹ *PT* VI VI:7, S&W 32:21-26. Translations from *PT* are the author's.

activity of the secondary the higher is co-operative, because all the effects of the secondary are concomitantly generated by the more determinative cause (*aitiōteron*).

....

All those characters which in the originative causes have higher (*huperteran*) and more universal (*holikōteron*) rank become in the resultant beings, through the irradiations which proceed from them, a kind of substratum for the gifts of the more specific principles (*merikōteron*).¹⁰

Proclus' family of names describing ordered points together with the comparative adjective ranking them form what modern linguists call a family of *scalar adjectives*.

Monadic scalar adjectives have a semantics that presupposes a background ordering, which we shall name \leq , that is referred to by a comparative adjective associated with the family. The ordering forms a line. In algebraic terms it is a total partial ordering. Typically the predicates are ranged along the order from an intuitively identifiable "good" or "positive" extreme to a "bad" or "negative" extreme, and often there is a midpoint predicate. Consider, for example, the heat and happiness families associated with the comparatives *is hotter than* and *is happier than*:

boiling, hot, warm, tepid, cool, cold, freezing

ecstatic, happy, content, so-so, down, sad, miserable

¹⁰ *ET* 66:22-68:2. Such usage of comparatives is frequent. The contexts moreover make it clear that they are meant to refer to the same underlying order. For examples see *ET* 46:19; 58:12; 74:10; 84:14-26; 142:7. In *IP* see 796:14-797:3, M&D 165-166.; 735: 25-29, M&D 110; 892:31-894:34, M&D 253-255; 838:7-14, M&D 211; 1098:3-28, M&D 444-445.

To the logician these families are interesting because of the properties of the presupposed ordering relation \leq . At a minimum, \leq orders all points so that no point is left out; no two points are at the same rank; and every point either comes before or after another. The names and comparatives of Proclus' metaphysics form such a scalar family. Causation is the background relation described in the various comparatives in the previous passage, and the casual stages in the ontic hierarchy are the points that he names in his metaphysical research. But the story is richer.

Since \leq puts items into a rough line, it already imposes a considerable amount of structure. But scalars typically impose additional organization as well, and this is associated with various negative affixes and particles that natural language employs in association with scalar predicates. To each such negative expression in language there corresponds a "negation operations" in the semantic structure of points ordered by \leq . The operation pairs a point with its "negative." Scalars are particularly interesting because there are a number of different negations of this sort. Three are particularly relevant to Proclus.

First there are negations that associate a point to others that are higher or lower in the ordering. The negation that moves to a higher point is called an *intensifier*. In English we have *super* (from Latin) and *hyper* (from Greek), as in *super-happy* and *hyperactive*. Greek has the *alpha-intensivum* recognized in classical grammar and modern linguistics,¹¹ as well as *huper* which in later Neoplatonism (especially pseudo-

¹¹ P. 326. Otto Jespersen, *The Philosophy of Grammar* (London: Allen and Unwin, 1924).. Horn in his important foundation work scalars uses the grammatical and semantic acceptability of such intensifier to

Dionysius) becomes the technical marker for specialized uses of hypernegation in theology. Proclus identifies this operation and calls it *hypernegation*, the term we shall use here. We shall symbolize it by \sim .¹²

The negation that associates a point with one lower is called in traditional grammar the *alpha-privative*. In English we may express it by the all-purpose particle *not*, as in *he is not well*, but we have specialized markers for it as well, like *sub* (from Latin) as in *subnormal* and *sub-par*. Greek has *hupo*. We shall abbreviate it by \neg .

A third variety of scalar negation, which we shall represent by $-$, associates a point corresponding to some degree n to the point $-n$ that is "equidistant" from the midpoint on the opposite extreme. In English we indicate this negation by the prefix *un*. For example, intuitively *unhappy* is synonymous to *sad* which stands for a point as many points below the midpoint of the happiness scale as the point picked out by *happy* is above the midpoint. We shall see that Proclus allows for such an operation relative to the analysis of points within a causal taxon.

4. The Logic of Negation and Syllogisms in Proclus

In two well known passages Proclus distinguishes senses of negation. In the *In Parmenidem* he writes:

Being, after all, is the classic case of assertion whereas Not-Being is of negation.... So then in every class of Being, assertion in general is superior to

identify scalar adjectives. See Horn also for the linguistic background on scalars employed here. Laurence R. Horn, *A Natural History of Negation* (Chicago: University of Chicago Press, 1989).

¹² *IP* 1172:35.

negation. But since not-Being has a number of senses, one superior to Being, another which is of the same rank as Being, and yet another which is the privation of Being, it is clear, surely that we can postulate also three types of negation, one superior to assertion, another inferior to assertion, and another in some way equally balanced by assertion.¹³

Again in the *Platonic Theology* he draws the distinction this way:

In truth my view is that negations come in three sorts, one sort is for beings of a form more fundamental than affirmations. These are generative and perfective of those things generated in affirmation. Another type is placed at the same level as affirmations, and here affirmation is not in any way more worthy than negation. Finally, there are those with a nature inferior to affirmations, namely privations of affirmations.¹⁴

Proclus is referring to hyper and privative negation. The first of the senses he lists is hypernegation; it is appropriate for picking out entities that are "higher" than those in the taxon of Being. The third sense is privative, appropriate for entities "lower." The middle sense is that appropriate for moving up and down within the taxon of Being. As we shall see, both hyper and privative negation are appropriate here as well depending on whether one wants to pick out an entity lower or higher in that series.

The modern semantics for scalars is very abstract and imposes no content on the ordering \leq other than its minimal structural properties. It need not be causation. Nor need it be a genuinely privative process. Privative processes, however, do fit the

¹³ *IP* 1072:28-1073:8, M&D 426

¹⁴ *PT* II:5, S&W 38:18-25.

requirements of a scalar ordering, and privative negation was a important tool in Greek philosophy. Its role in Greek logic, however, was less developed. Aristotle defines it.¹⁵ But investigations of the formal logic of negation were limited to the negative qualities of syllogistic sentences and to Stoic sentential negation, neither of which captures the key ideas of either hypernegation or privation. Neither of the scalar negations, for example, conforms to the laws of double negation or contraposition, but to its own laws:

1. $hyper\text{-}hyper\text{-}happy \leq hyper\text{-}happy \leq happy \leq sub\text{-}happy \leq sub\text{-}sub\text{-}happy$,
and in general, $hyper\text{-}hyper\text{-}happy \neq happy \neq sub\text{-}sub\text{-}happy$.
2. $happy \leq content$ does not logically imply contrapositively either that $hyper\text{-}content \leq hyper\text{-}happy$ or that $sub\text{-}content \leq sub\text{-}happy$.

But in an important sense, as the earlier propositions from the *Elements of Theology* attest, privation is at the heart of Neoplatonic metaphysics. The causal ordering is a process of diminishment. Logic had to wait for Proclus for an exploration of the formal and inferential properties of privatives.

Perhaps the best known logical law that he identifies for these two negations is the one he expresses in saying *negation gives birth to affirmation*.¹⁶

....the first mentioned [forms] are more general, while these latter mentioned are more particular. For this reason by eliminating the earlier ones, he eliminates those that follow them in the hypotheses.

In another passage he states the rule more generally.¹⁷

¹⁵ See the discussion in John N. Martin, "Existence, Negation, and Abstraction in the Neoplatonic Hierarchy," *History and Philosophy of Logic* 16 (1995).

¹⁶ *IP* 1087:2-6, M&D 435.

If, then, the negations generate the affirmations, it is plain that the first negations generate the first and the second the second.

That is, if B is earlier (higher) in the causal ordering than A , then the negation of B is earlier than that of A : $\sim A \leq \sim B$ iff $A \leq B$ iff $\neg A \leq \neg B$. When these laws hold the operations are said in algebra to be *isotonic*.

These laws encapsulate Proclus' version of the *via negativa*.¹⁸ For example he says, "... The soul orders affirmations to negations,"¹⁹ and

....these are the only characteristics that pertain to being *qua* being, the ones which are asserted by the Second Hypothesis and are denied by the First.²⁰

More generally,

To the extent that it pre-exists all others, we celebrate this cause only by negations, while we reveal the high things both negatively and affirmatively, to the extent that it is transcendently superior to inferiors, we reveal the inferiors negatively, but to the extent that they are part of their predecessors, we reveal them affirmatively.²¹

¹⁷ *IP* 1099:32-35.

¹⁸ See the discussion of the doctrine and its role in the exposition of these two works in L.G. Westerink H. D. Saffery, "Introduction," in *Proclus: Théologie Platonicienne* (Paris: Les Belles Lettres, 1968). For statements of the principle in addition to the texts cited below see *IP* 1133:3-5 and *IP* 1056-1060, M&D 413-16.

¹⁹ *PT* I:12; S&W 58. 2-3.

²⁰ *IP* 1086:27-29, M&D 435. See also *IP* 1099:32-35, M&D 446, and *IP* 1208:22-24, M&D 553.

²¹ *PT* IV:11, S&W 37, 21-27. See also *PT* II:12; S&W 66, 7-24.

This discussion of hyper and privative negation then allows us to conclude that Proclus imposes the special structural requirement on scalar ordering that the operations \sim and \neg be isotonic.

The third scalar negation $-$ is logically interesting because when combined with the total ordering \leq , it allows for the development of full sentential logic for the connectives $-$ (*not*), \wedge (*and*), and \vee (*inclusive or*). In order for the semantics of this negation to be well defined, however, it must be possible to associate a numerical rank to the points indicating their distance from the midpoint of \leq . This measurement would then allow for the definition of standard truth-functions for the connectives in a manner first explored by Lukasiewicz and later standardized by Kleene, in his so-called *strong connectives*.

Let the measurement values be set so that the midpoint is 0, higher values being positive, lower negative. In Kleene's semantics $A \wedge B$ has as its value the minimum value (as measured on \leq) of the two values of A and B . The value of $A \vee B$ is the maximum of the two. The value of $-A$ is the negative of the value of A . The resulting logic validates only inferences that are valid in classical two-valued Russellian logic, but fails to validate a special set of classical inferences in which the premises and conclusions violate relevance conditions.²² In the special cases in which a scalar metric can be defined, scalars then have structural properties that lend themselves to the full development of predicate logic with grammatically complex adjectives made up with the operators \wedge , \vee and $-$, as well as by \sim and \neg .

²² John N. Martin, "A Syntactic Characterization of Kleene's Strong Connectives," *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 21 (1975).

Though I do not think there is textual ground for thinking that Proclus identified a single ontic midpoint about which reality pivots, he does often speak of the points relative to a single taxon as causally ordered in a symmetric manner. This is particularly true in the many instances in which he analyzes causal descent in a triad. As I have indicated, there is some ground for believing he held that division is in principle always triadic. In any case, it is clear that he thinks that each taxon of the hierarchy is countably finite.²³

Proposition 149. The entire manifold of divine henads is finite (*peperasmemon*) in number.

Proposition 179. The entire intellectual series is finite (*pas ho noeros arithmos peperastai*).

Thus even if there is no midpoint, each point in a taxon and indeed any point in any taxon at a given distance from the tree's root would have an "opposite" under \neg . That is, relative to a single taxon or, more broadly, relative to the set of all points in any taxon at the same "level" in the tree, maximum and minimum operations would be well defined for Proclus. The result would be a logic for predicate operators \wedge , \vee and \neg in addition to the negations \sim and \neg that Proclus explicitly introduces. The whole would be a perfectly coherent extension of Kleene's logic. Proclus' logic of \sim and \neg accordingly fits nicely within a major school in modern many-valued logic.

²³ It may be remarked that the property of causal density combines with the fact that taxa are finite (that the tree of diairesis is finitely branching) to explain how Proclus can simultaneously hold that the causal ordering \leq is infinite yet every taxon finite. Indeed Proclus seems to require density in order to harmonize these two views.

His logic also harmonizes with Aristotle's. Modern readers of the syllogistic are inclined to interpret it in terms of Boolean structures of sets. In this picture the common nouns and adjectives used as terms in **A**, **E**, **I**, and **O** propositions stand for non-empty sets (the genera and species of apodictic science) that form a tree or a structure of possibly empty sets (if accidental grouping are allowed) within an overarching Boolean algebra on the power set of all objects. It is true that the valid moods, the traditional immediate inferences, and the argument patterns used by Aristotle in his reduction of the moods to Barbara and Celarent can be shown to be sound and completely relative to the set of all interpretations over Boolean structures. It is also true that these structures are not linearly ordered by their ordering relation, which is the subset relation \subseteq .²⁴

However, it is not true that syllogisms are valid only relative to such Boolean structures. In fact, a more accurate model theory for the traditional validities of the syllogistic employs a more abstract characterization. A syllogistic structure in this more abstract sense is any semi-lattice $\langle U, \wedge, 0 \rangle$ such that 0 is the least element of the partial ordering \leq determined by the meet operation \wedge on the lattice. The formal restrictions defining a *semi-lattice* are quite minimal: \wedge must map any pair in U to an element in U , and it must be associative, commutative, and idempotent ($x \wedge x = x$). Then, $x \leq y$ holds iff, $x \wedge y = x$; and for all x in U , $0 \leq x$. The standard completeness proofs for the syllogistic relative to Boolean structures has in fact been generalized to this broader set of semi-

²⁴ For the standard modern natural deduction characterization of the syllogistic in terms of non-empty sets in Boolean structure see Timothy Smiley, "Syllogism and Quantification," *Journal of Symbolic Logic* 27 (1962). John Corcoran, "Completeness of an Ancient Logic," *Journal of Symbolic Logic* 37 (1972).

lattices. What is relevant to Proclus is that lattices of this sort can be totally ordered, and Proclus' scalar structures clearly fit the more abstract notion of acceptable syllogistic semi-lattice. That is, all the traditional Aristotelian argument forms remain valid even if Proclus assumes that reality is linearly ordered. It is also possible to add to syllogistic natural deduction new inference rules and basic deductions governing the new operators \sim , \neg , and $-$, and to add appropriate extra conditions on the definition of acceptable syllogistic semi-lattice so that the resulting logical theory is sound and complete for a syllogistic language augmented to have scalar predicate operators \sim , \neg , and $-$.²⁵

Though syllogistic reasoning is valid in Proclus' scalar universe, it exhibits an oddity: **E** and **I** statements are respectively trivial false and trivially true. If the structure is ordered by a total ("linear") order \leq then an **A** statement *all x are y* may be written $x \leq y$; and an **O** statement *some x is not y*, written as $x \not\leq y$, is by the properties of \leq equivalent to $y < x$. Since for any two points x and y , one is below the other, the **E** statement $x \wedge y = 0$ would be false. Likewise the **I** statement $x \wedge y \neq 0$ is true. As a matter of fact, Proclus does not use **E** and **I** statements in his logical analyses. He does not

²⁵ For the soundness and completeness of the same natural deduction proof theory as that of Corcoran and Smile but characterized in terms of the more general notion of acceptable semi-lattice see John N. Martin, "Aristotle's Natural Deduction Reconsidered," *History and Philosophy of Logic* 18 (1997). The extension to include the logic of scalar negations \sim , \neg , and $-$ consists of introducing appropriate new basic deductions and natural deduction inference rule in the proof theory, and of adding suitable restrictions in the definition of acceptable scalar structure. The soundness completeness proof is then an extension of that previously cited. See , from which the tree diagram below is taken.

because he has an alternative vocabulary for negations ready at hand for what he needs to say, namely \sim and \neg .

5. Negation in Diairesis Determines a Line

Proclus' technique for transforming the information contained in the tree of diairesis into a linear causal order appropriate for syllogistic and scalar reasoning turns on scalar negation. Previous texts illustrate how Proclus posited that the immediate descendants of a node in the tree of diairesis are linearly ranked. Proclus frequently explains this ranking in terms of negation. One node ranks higher than another because it is its hypernegation, or lower because it is its privative. Consider how he ranks the taxon headed by the One:

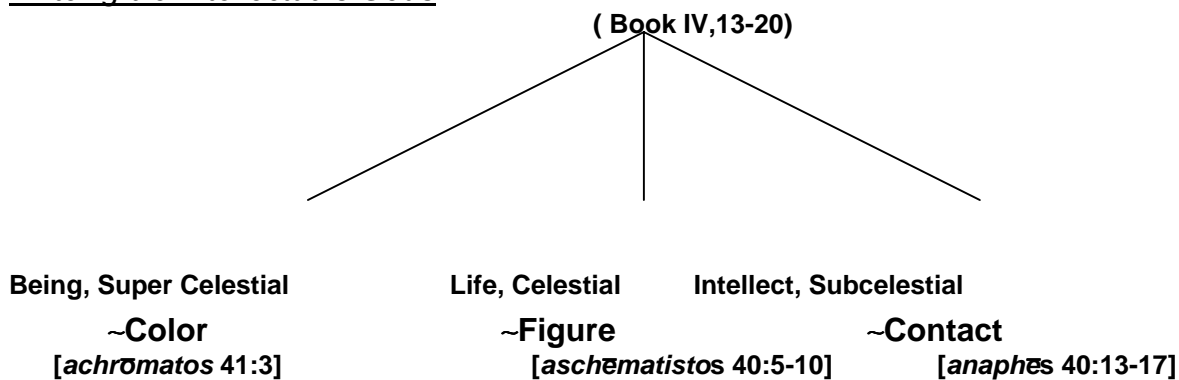
... among the entities engendered after him [the One], at every degree, the cause is totally different from its effects, and that is why nature [*phusis*] is completely incorporeal [*asōmatos*], while being the cause of bodies, the soul is totally eternal but cause of what is engendered, and the intellect immobile [*nous, akinētos*] because it is cause of all that is in movement. If thus for each procession of beings, one negates of the cause the effects which follow from it, it is necessary, I think, to negate them of the universal cause

...

of negations I say that they are not privative of what they bear, but rather make of a kind of contrary [*antikeimenos*], for since the first principle is not many, the many proceeds from him....²⁶

A negative predicate true of the One (if it were not inexpressible) would result from the hypernegation of Intellect, just as *hyper-mobile* is true of Intellect because *mobile* is of Soul, and *eternal* is true of Soul because *sub-eternal* is true of Nature, and *hyper-corporeal* is true of Nature because *corporeal* is true of Body. Saffrey and Westerink point out that the entire discussion of the *Platonic Theology* is organized around progressive negative predications true of the first hypothesis²⁷. Another good example is Proclus' breakdown of the Intelligible-Intellective order. The diverse texts may be summarized in a tree:

Intelligible-Intellectuals Gods



²⁶ *PT* II:10, S&W 62:5-63:17. Note that here and elsewhere (especially *IP*, Book VI) Proclus is careful to make clear that strictly speaking the One is beyond all predication. All references to predications of the One in this reconstruction, even to hypernegations, must be understood as subject to this important Neoplatonic provision.

²⁷ H. D. Saffrey, "Introduction,". See pp. lx-lxxv.

Here hyper negation is used to order the nodes at the first level of division, and these nodes are named both by both negative predicates and lexicalized simple predicates. It is plausible to generalize this practice to all divisions, especially in light of Proclus' view that any node is in principle resolvable into a deeper triadic analysis, and to attribute the view to Proclus that the taxon beneath any node is ranked by its analysis into negative predicates of increasing grammatical complexity. A single linear order describing causation is the result. This line is then the structure appropriate for syllogistic reasoning and valid inferences that turn on the three scalar negations.

6. Conclusion

I hope I have been able to show how Proclus is of interest to logic. It is difficult to overcome the temptation to dismiss Neoplatonic thought as mystical and therefore irrelevant to analytic philosophy, especially logic. Plotinus and his followers, including Proclus, do at some level espouse mysticism. But there is much more as well. It is always disconcerting for the modern reader who knows Neoplatonism only through Plotinus to open Proclus' *Elements of Theology* and find it written in the propositional form of a geometry text. I hope I have been able to explain away that surprise. Proclus uses logic systematically. He not only employs the logic already available, but advances logical theory by identifying and applying in his metaphysics the formal properties of scalar predicates and their negations. Though for mystical reasons the One is beyond language, on the plane of logical analysis the properties of the One are

discoverable by inferences using hypernegations from facts already known about the lower orders of reality. Prolcus believes that in this way logic contributes to theology.

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