Naïve Set Theory. Axioms and Defintitions

Sets

Axioms:

Abstraction: $\forall y(y \in \{x|P[x]\} \leftrightarrow P[y])$ (in its practical form)

Extensionality: $A=B \leftrightarrow \forall x(x \in A \leftrightarrow x \in B)$

or equivalently, $\{x|P[x]\}=\{x|Q[x]\} \leftrightarrow \forall y(y \in \{x|P[x]\} \leftrightarrow y \in \{x|Q[x]\})$

Definitions :			**Technical Name:**
$x \neq y$	x is not identical to y	$\sim(x=y)$	***non-identity*** or ***inequality***
$x \notin A$	x is not an element of set A	$\sim(x \in A)$	***non-membership***
$A \subseteq B$	*Everything in* A *is in* B	. $x(x \in A \rightarrow x \in B)$	A is a ***subset*** of B
$A \subset B$	$A \subseteq B$ *& some* B *is not in* A	$A \subseteq B \wedge \sim A=B$	A is a ***proper subset*** of B
$\varnothing$ or Λ	*set containing nothing*	$\{x\| x \neq x\}$	the ***empty set***
V	*set containing everything*	$\{x\| x=x\}$	the ***universal set***
$A \cap B$	*set of things in both* A *and* B	$\{x\| x \in A \wedge x \in B\}$	the ***intersection*** of A and B
$A \cup B$	*set of things in either* A *or* B	$\{x\| x \in A \vee x \in B\}$	the ***union*** of A and B
$A-B$	*set of things in A but not in B*	$\{x\| x \in A \wedge x \notin B\}$	the ***relative complement*** of B in A
˜ $-A$ or **Error! Bookmark not defined.**Ā	*set of things not in A*	$\{x\| x \notin A\}$	the ***complement*** of A
P(A)	*the set of subsets of* A	$\{B\| B \subseteq A\}$	the ***power set*** of A

Relations

Reduction of *n*-place Relations to Sets of *n*-tuples

Definitions

$<x,y> =_{df} \{x, \{x,y\}\}$ (***ordered pair***)

$<x_1,\ldots,x_n,y>=_{df} <<x_1,\ldots,x_n,>y>$ (***ordered n-tuple***)

Theorems (Propertes of Pairs and *n*-tuples)

$<x,y>=<y,x$ iff $x=y$

$<x_1,\ldots,x_n>=<y_1,\ldots,y_n>$ iff $(x_1=y_1,\ldots,\&\ldots\& x_n=y_n)$

Theorems (Abstraction for Relations)

$\exists R\forall x\forall y\ (<x,y>\in R \leftrightarrow P[x,y])$

$\exists R\forall y_1,\ldots,y_n(<y_1,\ldots,y_n>\in R \leftrightarrow P[y_1,\ldots,y_n])$

$\forall x_1 y(<x,y>\in\{<x,y>|P[x,y_n]\}\leftrightarrow P[x,y])$

$\forall y_1\ldots y_n(<y_1,\ldots,y_n>\in\{<x_1,\ldots,x_n>|P[x_1,\ldots,x_n]\}\leftrightarrow P[y_1,\ldots,y_n])$

Definitions

$AxB = \{<x,y>|\ x\in A\wedge y\in B\}$ ***Cartesian product*** of A and B

$A^2 = AxA$ ***Cartesian product*** of A and A

$A_1x\ldots xA_{n+1} = (A_1x\ldots xA_n)xA_n$

$A^n = A_1x\ldots xA_n$ ***Cartesian product*** of $A_1,\ldots,A_n$

$V^2 = VxV$ ***The universal (binary) relation***

Theorems

$\mathbf{P}(V^2) = \{R \mid R\subseteq V^2\}$ the set of **2-*place relations***

$\mathbf{P}(V^n) = \{R \mid R\subseteq V^n\}$ the set of ***n-place relations***

Theorems (Extensionality for Relations)

If $R,R'\in P(V^2)$, then $R=R' \leftrightarrow \forall x\,y(<x,y>\in R \leftrightarrow <x,y>\in R')$

$\{<x,y>|P[x,y]\}=\{<x,y>|Q[x,y]\}\leftrightarrow\forall x,y(\ P[x,y] \leftrightarrow Q[x,y])$

If $R,R'\in P(V^n)$, then $R=R' \leftrightarrow \forall x_1\ldots x_n(<x_1,\ldots,x_n>\in R \leftrightarrow <x_1,\ldots,x_n>\in R')$

$\{<x_1,\ldots,x_n>|P[x_1,\ldots,x_n]\}=\{<x_1,\ldots,x_n>|Q[x_1,\ldots,x_n]\}\leftrightarrow\forall x_1\ldots x_n(\ P[x_1,\ldots,x_n] \leftrightarrow Q[x_1,\ldots,x_n])$

Definitions

R is a ***binary relation*** $R\subseteq V^2$

R is a **n-place relation** $R\subseteq V^n$

f is a ***1-place function*** $f\subseteq V^2 \&\forall x\forall y\forall z((<x,y>\in f \wedge <x,z>\in f)\rightarrow y=z)$

f is a ***n+1-place function*** $f\subseteq V^n \&\forall x_1\ldots x_n\forall y\forall z((<x_1,\ldots,x_n,y>\in f \wedge <x_1,\ldots,x_n,z>\in f)\rightarrow y=z)$

$f(x)=y$ *means* $<x,y>\in f$

If $f(x)=y$, then x is an ***argument*** of f and y is a ***value***.

Domain(f) = $\{x \mid \exists y f(x)=y\}$
Range(f) = $\{y \mid \exists y f(x)=y\}$
$f^{-1} = \{<y,x,> \mid f(x)=y)\}$ f^{-1} is called the ***inverse*** of f

If f is a n-place function, $f(x_1,\ldots,x_n)=y\}$ means $<x_1,\ldots,x_n,y>$
If $f(x_1,\ldots,x_n)=y$, then $<x_1,\ldots,x_n>$ is an ***argument*** of f and y is a ***value***.
Domain(f) = $\{<x_1,\ldots,x_n> \mid \exists y f(x_1,\ldots,x_n)=y\}$
Range(f) = $\{y \mid \exists y f(x_1,\ldots,x_n)=y\}$

$f(A \xrightarrow{\text{into}} B)$	f is a 1-place function, Domain(f), and Range(f)$\subseteq$B
$f(A \xrightarrow{\text{onto}} B)$	f is a 1-place function, Domain(f), and Range(f)=B
$f(A \xrightarrow{\text{1-1 onto}} B)$	f is a 1-place function, Domain(f), Range(f)=B, and f^{-1} is a 1-place function
f is a ***partial function*** on A	$\exists C \exists B(C \subset A \ \& \ f(C \xrightarrow{\text{into}} B))$