Introduction

These notes concern Aristotle's opinion on the validity of three arguments:

Necessarily all Y is Z	All Y is Z	Necessarily Some X is Y
All X is Y	Necessarily all X is Y	Necessarily some Y is X
Necessarily all X is Z	Necessarily all X is Z	

The first is called *Barbara LXL* because *L,* used to represent *necessarily* (in some theories in modern modal logic), occurs the first premise and the conclusion but not in the second premise. The second argument is called *Barbara* X*LL* for similar reasons. The third argument is called *LI-conversion* because it changes the position (converts) the subject and predicate of a necessary *I*-proposition. Aristotle held the curious view that the first and the third arguments are valid but not the second. A good deal of effort has been made to try to figure out some coherent sense of *necessarily* that would yield Aristotle's view. Here we discuss three accounts. None fully succeeds, and the conclusion may well be that Aristotle just made a mistake.

The discussion has some background. First of all, it takes place assuming the standard definition of *valid* argument. This definition is couched in terms of possible worlds. An argument $P_1,\ldots,P_n/\therefore Q$ is usually defined to be *valid* if and only if for any possible worlds W, if $P_1,\ldots,P_n$ are true in W, then Q is also true in W. Let us assume this definition. The problem then becomes one of explaining the truth-conditions in W of the *A*–proposition *All X is Y* (abbreviated formally as *Axy*), of the *I*–proposition *Some X is Y* (abbreviated *Ixy*), and of necessity propositions of the form *Necessarily* P, where P is an *A*- or *I*-proposition. (We abbreviate *Necessarily* P as *L*P.) That is, we seek explanations of what has to be the case at W to make each of these proposition types true, and we want these explanations to be both intuitively plausible and to yield Aristotle's judgments on the validity of the three argument forms.

The theories share a background framework. In the formal treatment below the terms *X* and *Y,* standing for common nouns or adjectives, are abbreviated by lowercase letters x and *y.* The terms are assigned different interpretations in different "worlds." In any world W, the term *x* is assigned to the set of individuals $V_W(x)$ that exist in that world and of which it is true to say that they are *x*, and *y* is likewise assigned to its set $V_W(y)$. Such sets are assumed by Aristotle to be non-empty. The *A*-statement *Axy* is true if $V_W(x)$ is a subset of $V_W(y)$, and the *I*-statement *Ixy* is true if the intersection of the two sets is non-empty.

The theories, however, differ on their understanding of the truth-conditions of *Necessarily* P. Theories 1 and 2 postulate that there are many worlds, and part of what this assumption means is that different individual things may exist (be in the "domains") of different worlds. Both theories have the metaphysical problem of explaining what kind of entity it is that exists in the domain of a possible world but that does not exist in that of the actual world. What is a possible non-actual entity?

Theories 1 and 2 differ between themselves on how the individuals included under *all X* are followed from world to world. The first says that *Necessarily all X are Y* simply means that the *A*-proposition *All X are Y* is true at all worlds *W* of whatever entities exist at *W*. The view may be put as saying that the necessity attaches to the whole proposition *All X are Y* (it is *de dicto*) in that the proposition *All X are Y* is as a whole is always true, allowing that what counts as *X* and *Y* will differ from world to world. The second says that the necessary proposition is true if all the entities that are *X* in the actual world, if they persist and we follow them to other worlds, will be *Y* in all other worlds. This view is said to be about the things that are *X* in the actual world (it is said to be *de re*) in that it says of the actual *X*'s that they are always *Y.*

Theory 3 is like Aristotle's original (and unlike modern logic) in that it only assumes the existence of actual things. "Worlds" in this theory are simply different ways of assigning terms to those things that actually exist; *X* is assigned to one set of actually existing things in under one assignment (*aka* world) and to another set of actually existing things under another asssssignment. "Worlds" then are all possible "interpretations" of terms over the things that exist.[1] Theory 3 also differs from Theories 1 and 2 in that it captures the doctrine known as Aristotle's essentialism. It assumes that, for a given term *X*, there is both the set of all things that *X* is true of in any sense, either essentially or accidentally (this set is called $V_W(x)$), and that there is a narrower set of things that *X* is true of only essentially and necessarily (this set is called $V_W(x^e)$). Being an *X* is part of the nature of the things in $V_W(x^e)$. It is assumed that $V_W(x^e)$ is a subset of $V_W(x)$.

The three theories differ in their judgments on the validity of the three problematic arguments. In Theory 1 the first two arguments are invalid and third valid. In Theory 2 all are invalid. Theory 3 yields exactly Aristotle's result: the first and third are valid, but the second invalid.

But there is still a problem. The third theory treats the subject terms differently in necessary *A*- and necessary *I*-statements, and it gives no rationale for this difference. It says that in <u>*Necessarily some X*</u> *are Y* it is the necessary *X*'s that are relevant to the truth-value of the sentence, but that in <u>*Necessarily all X*</u> *are Y* it is the wider set of all *X*'s, necessary and otherwise, that is relevant to its truth. Why does the scope of *Necessarily some X* reach over just the things that are necessarily *X,* but that of *Necessarily all X* range over all *X*'s necessary and otherwise? If we reject this different treatment as unmotivated and impose the condition that whatever the range of the subject term in necessity statements it must be the same in both *A*- and *I*-propositions, then there are two alternatives, which we may call Theories 3a and 3b.

[1] This notion of "world" fits Aritotle's practice of testing the validity and invalidity of arguments by keeping the form of the argument constant, replacing terms by letters *X, Y,* and *Z,* and then varying the interpretations of these letters by replacing them by different actual words (from Greek) which stand for sets of things in the world. Under one replacement the terms *X, Y,* and *Z* are replaced by one set of real words with their actual extensions, and under another by a different set of words with their actual extensions. Each such replacement is like a "world" in Theory 3.

In 3a we state the truth-conditions of necessary *A*- and *I*-propositions in terms of the necessary extensions of both subjects and predicates. In 3b we extend the interpretation of the subject term *X* in both necessary *A*- and *I*-propositions to the set of all *X*'s, necessary and otherwise. But then neither theory fits Aristotle's judgments on the initial arguments because in 3a the first argument becomes invalid, and in 3b the third argument fails.

In the end one rather despairs of recapturing any coherent semantics for Aristotle's intuitions on the validity of the three argument forms. The conclusion may be that on this issue there is no coherent theory to recapture because Aristotle was mistaken in his intuitions. The investigation is still interesting because of the various theories constructed. These may be investigated within the philosophy of logic to determine which if any of these has independent merit as a logical theory.

I. Basic Ideas

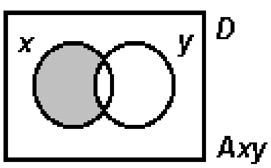
Definition. An *interpretation* V_W of terms relative to a set D_W assigns to each term x a non-empty subset $V_W(x)$ of D_W. Here V_W is the formal proxy of the philosopher's "possible world," and D_W, called its *domain,* takes the role of the set of individuals that exist in that world. It is intended that some domains include things that do not exist in our actual world.

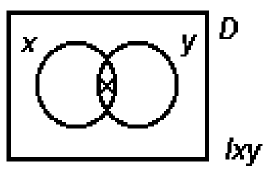
Definition.[2] $V_W(Axy)$=T iff all x's in D_W are also y's, i.e. $V_W(x) \subseteq V_W(y)$,
or for all $d \in D_W$, if $d \in V_W(x)$ then $d \in V_W(y)$

Definition. $V_W(Ixy)$=T iff some x's in D_W is also y's, i.e. $V_W(x) \cap V_W(y) \neq \varnothing$,
or for some $d \in D_W$, $d \in V_W(x)$ and $d \in V_W(y)$

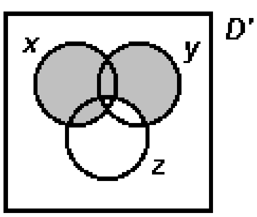
Definition.[3] An argument $P_1,\ldots,P_n/\therefore Q$ is *valid* iff in every interpretation in which the premises are true, the conclusion is true. More formally:

for any W, if $V_W(P_1)$=T,…,$V_W(P_n)$=T, then $V_W(Q)$=T).





Theorem. Barbara is valid.



Barbara	[4]Barbara *L*X*L*	Barbara X*LL*	*LI*-conversion
Ayz	*LAyz*	*Ayz*	*LIxy*
Axy	*Axy*	*LAxy*	*LIyx*
Axz	*LAxz*	*LAxz*	
Valid for Aristotle	Valid for Aristotle	**Invalid** for Aristotle	Valid for Aristotle

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[2] Here "*A*"="∀"="for all" is called the *universal quantifier*. The modern notation for the universal affirmative *Axy* uses the notation $\forall x(Fx \rightarrow Gx)$ ("for all x, if x is F then x is G") where x is a pronoun (variable) and the role of the predicates x and y of *Axy* are taken by the letters F and G. "*I*"="∃"="for some," is the *existential quantifier*, and *Ixy* is $\exists xFx \wedge Gx$ in modern notation ("for some x, x is F and x is G". (In modern logic it is not assumed that F and G stand for at least one thing.)

[3] Here and below $D_{W'}$(with a prime mark ′) is used to range over the domains of all possible interpretations ("worlds") in cases in which multiple "worlds" are referred to.

[4] Here "*L*"="□"="necessarily".

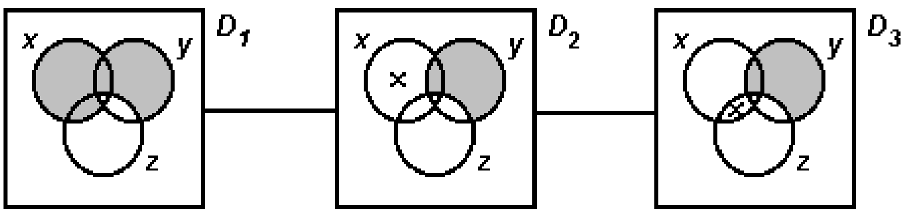
II. Possible World Semantics for Necessity. Entities in possible worlds other than the actual are also posited. This is essentially the standard modern view.

1. *De Dicto (in sensu composito)*

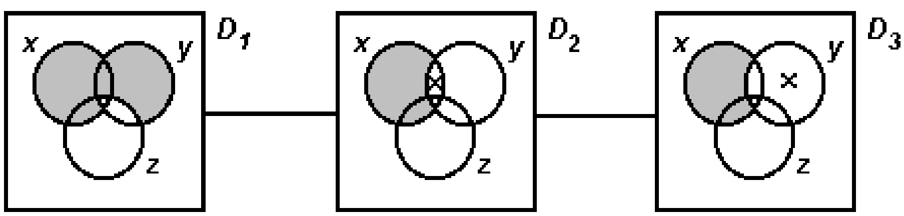
<u>Definition.</u> $V_W(LP)$=T iff for any world and domain $D_{W'}$, $V_{W'}(P)$=T

<u>Theorem.</u>[5] $V_W(LAxy)$=T iff in every world W' (with domain $D_{W'}$), all *x*'s in that world are *y*'s in that world.
(For all W', $V_{W'}(x) \subseteq V_{W'}(y)$, or
for all W' and all $d \in D_{W'}$, if $d \in V_{W'}(x)$ then $d \in V_{W'}(y)$.)

<u>Theorem.</u> Barbara *L*X*L* is invalid.

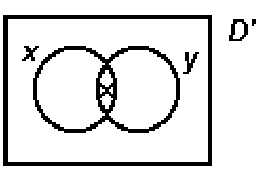


<u>Theorem.</u> Barbara X*LL* is invalid.



<u>Definition.</u>[6] $V_W(LIxy)$=T iff in all possible worlds something is both *x* & *y.*
(For all W', $V_{W'}(x) \cap V_{W'}(y) \neq \varnothing$, or
for all W', there is some $d \in D_{W'}$, $d \in V_{W'}(x)$ and $d \in V_{W'}(y)$.)

<u>Theorem.</u> *LI*-conversion is valid.



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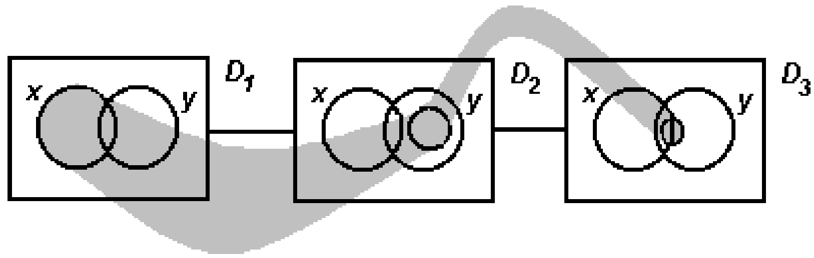
[5] In non-Aristotelian modern logic the usual symbolization of the *de dicto* reading of *LAxy* is $\Box\forall x(Fx \rightarrow Gx)$.

[6] Here again "*I*"="∃"="for some" is the existential quantifier. In non-Aristotelian modern logic the symbolization for the *de dicto* reading of *LIxy* is $\Box\exists x(Fx \wedge Gx)$.

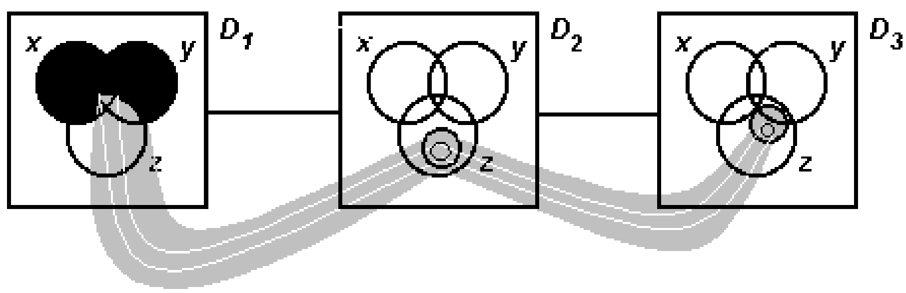
2. *De Re (in sensu diviso)*

<u>Definition.</u>[7] $V_W(LAxy)=T$ iff all (actual) x's are y in every world.
(For all $d \in V_W(x)$ and for any W', $d \in V_{W'}(y)$.)

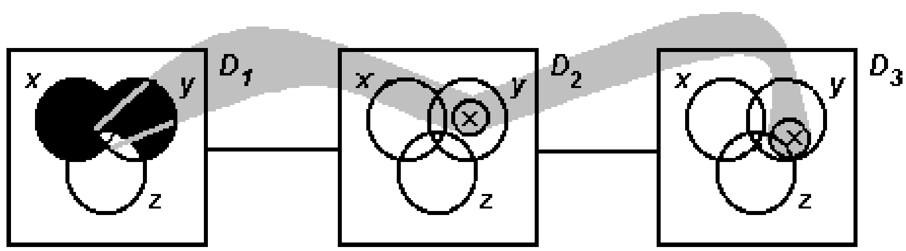


Here *LAxy* is true in V_{W_1} because all the x's in D_1 remain y's in other worlds.

<u>Theorem.</u> Barbara *L*X*L* is valid.

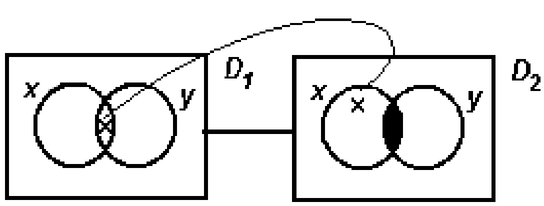


<u>Theorem.</u> Barbara X*LL* is invalid.



<u>Theorem.</u> *LI*-Conversion is invalid

<u>Definition.</u>[8] $V_W(LIxy)=T$ iff some x is y in every world.
(For some $d \in V_W(x)$ and for any W', $d \in V_{W'}(y)$.)



[7] In non-Aristotelian modern logic the notation for the *de re* reading of *LAxy* is $\forall x(Fx \rightarrow \Box Gx)$.
[8] In non-Aristotelian modern logic the notation for the *de re* reading of *LIxy* is $\exists x(Fx \wedge \Box Gx)$.

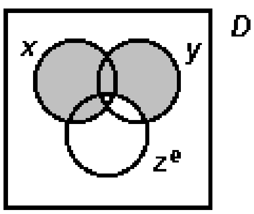
III. Aristotelian Essentialist Modality (*à la* Johnson).[9] Only one domain *D* is posited, the set of "actual individuals." “Worlds” are different possible assignments of terms to set of individuals in *D*. In each assignment *W,* a term x is assigned two sets, $V_W(x)$ or the set of all things it is true of under *W*, and a narrower set $V_W(x^e)$ of all things it is necessarily or “essentially” true of under *W*.

Definition. The *essential interpretation* of terms x relative to an interpretation V_W over a set $_{DW}$, abbreviated $V_W(x^e)$, is some non-empty subset of $V_W(x)$.

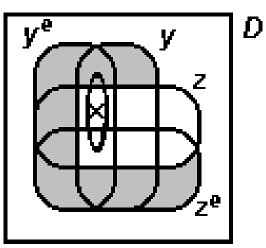
Definition. $V_W(LAxy)$=T iff all x's are essentially y's in $_{DW}$.
$V_W(x)\subseteq V_W(y^e)$, or for all $d\in V_W(x)$, $d\in V_W(y^e)$)

$V_W(LIxy)$=T something in $_{DW}$ is both essentially x and y.
$V_W(x^e)\cap V_W(y^e)\neq\varnothing$

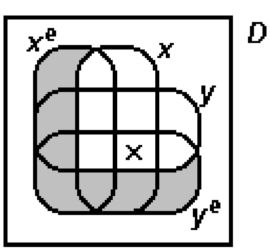
Theorem. Barbara *L*X*L* is valid.



Theorem. Barbara X*LL* is invalid



Theorem. *LI*-Conversion is valid.



[9] Fred Johnson, "Models for Modal Syllogisms," *Notre Dame Journal of Formal Logic*, 30:2 (1989), 271-284.

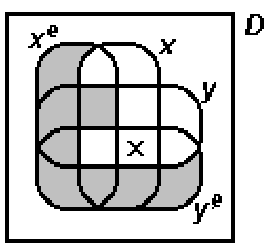
IV. A Problem. Johnson's semantics attributes to Aristotle an unmotivated and seemingly inconsistent reading of the "scope" of "necessarily" in *LA*- and *LI*-statements. On the one hand, in *LA*-statements the essence of only the predicate is being referred to, but in *LI*-statements "essences" are the referents of both the subject and predicate:*

LAxy	means	all *x*'s (essential and otherwise) fall among the things essentially *y*. That is $V_W(x) \subseteq V_W(y^e)$
LIxy	means	there is something that is both essentially *x* and *y*. That is, $V_W(x^e) \cap V_W(y^e) \neq \emptyset$

Solution One. What happens if the essence of both terms is referred to in both *LA*-and *LI*-statements statements? Then, Barbara *LXL* becomes in invalid.

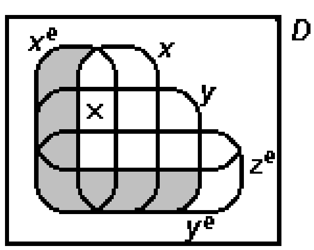
Definition. $V_W(LAxy)$=T iff all *x*'s are essentially *y*'s in D_W.
$V_W(x^e) \subseteq V_W(y^e)$, or for all $d \in V_W(x)$, $d \in V_W(y^e)$

Theorem. Barbara *LXL* is invalid.



Solution Two. What happens if only the predicate is referred to in both *LA*- and *LI*-statements? *LI*-Conversion becomes in invalid.

Theorem. *LI*-Conversion is invalid.



V. Conclusion. It is odd that the order of the premises should matter in *AAA* syllogisms with one necessary conclusion and one necessary premise, especially since Aristotle ignores (freely changes) the order of premises in non-modal syllogisms. (He did not in fact recognize a difference between the 1st and 4th figures placing them all in one group, his 1st figure.) Indeed it is inconsistent with "logical intuition" (our raw intuitions about what seems valid) to make Barbara *XLL* invalid. Perhaps Aristotle's intuitions were confused. It remains now to investigate which of Theoreis 1,1, 3a, or 3b, if any, has merits as a logical theory.

* Note that the argument from *LAxy* to *LIxy* is valid because $V_W(x^e)$ is non-empty.