Definitions

A litteral is an atomic formula or the negation of an atomic formula.

A formulas is in ***negation normal form*** (**NNF**) if its only negations are on its atomic parts.

A formula is in ***disjunctive normal form*** (**DNF**)if it is a disjunction of conjunctions of litterals.

A formula is in ***conjunctive normal form*** (**CNF**) if it is a conjunction of disjunctions of litterals.

Negation Normal Form

Drive $\neg$ inside (from larger parts to smaller) by Demorgan's Laws:

Rewrite $\neg(P\wedge Q)$ as $\neg P\vee\neg Q$
Rewrite $\neg(P\vee Q)$ as $\neg P\wedge\neg Q$
Rewrite $\neg(P\rightarrow Q)$ as $P\wedge\neg Q$
Rewrite $\neg(P\leftrightarrow Q)$ as $(P\wedge\neg Q)\vee(Q\wedge\neg P)$

Algorithm for DNF

1. Eliminate all occurrences of $\leftrightarrow$:
   Rewrite all parts from larger to smaller that are $P\leftrightarrow Q$ as $(P\rightarrow Q)\wedge(Q\rightarrow P)$.
2. Eliminate all occurrences of $\rightarrow$:
   Rewrite all parts from larger to smaller that are $P\rightarrow Q$ as $\neg P\vee Q$.
3. Drive $\neg$ inside (from larger parts to smaller) by Demorgan's Laws:
   Rewrite $\neg(P\wedge Q)$ as $\neg P\vee\neg Q$
   Rewrite $\neg(P\vee Q)$ as $\neg P\wedge\neg Q$
4. Drive $\wedge$ inside by Distribution:
   Rewrite all conjunctions from lager to smaller
   that are $P\wedge(Q\vee R)$ as $(P\wedge Q)\vee(P\wedge R)$, and
   that are $(Q\vee R)\wedge P$ as $(Q\wedge P)\vee(R\wedge P)$.

Theorems

Any formula P has at least one negation, conjunctive and disjunctive normal form.

Any formula and its normal form (of any of the three sorts) are tautologically equivalent.