I. Formal Syntax. Definitions.

Terms: ***Constants***: *a, b, c, ….*
Variables: *t, u, v, x, y, z, ….*

Predicates (of each finte degree):

Degree *1*: $P^1{}_1,\ldots,P^1{}_m,\ldots$
…..
Degree *n*: $P^n{}_1,\ldots,P^n{}_m,\ldots$
….

Atomic Well-formed Formula (atomic wff)
If $t_1,\ldots,t_n$ are terms and $P^n{}_m$ is an *n*-placed predicate then $P^n{}_m(t_1,\ldots,t_n)$ is an ***atomic wff***.

Well-formed Formulas (wff) (an inductive definition)

0. If *A* is an atomic wff, then *A* is a ***wff***.
1. If A is a wff, then $\neg A$ is a ***wff***.
2. If *A* and *B* are wffs, then $(A\wedge B)$ is a ***wff***.
3. If *A* and *B* are wffs, then $(A\vee B)$ is a ***wff***.
4. If *A* and *B* are wffs, then $(A\rightarrow B)$ is a ***wff***.
5. If *A* and *B* are wffs, then $(A\leftrightarrow B)$ is a ***wff***.

(Some of the connectives may be defined in terms of others.)

6. If *A* is a wff and *v* is a variable, then $\forall vA$ is a ***wff*** (and *v* is said to be ***bound*** in $\forall vA$).
7. If *A* is a wff and *v* is a variable, then $\exists vA$ is a ***wff*** (and *v* is said to be ***bound*** in $\exists vA$). (This clause is option if $\exists vA$ is defined as $\neg\forall v\neg A$.)
8. Nothing is a ***wff*** except by clauses 0-7.

Free Variables. A variable *v* is ***free*** in a wff *A* iff it is not bound.
Sentences. A ***sentence*** is any wff without free variables.

II. Formal Semantics (Model Theory). Definitions.

First-order Structures. A "structure" (a.k.a. "model") represents an assignment of meanings in a world to the terms that have fixed meanings in that world, i.e. $\forall$ (for the domain "quantified over"), constants, & predicates.

A ***structure*** or ***model*** is any function $\mathfrak{M}$ assigning values ("interpretations" in $\mathfrak{M}$) to $\forall$, constants, predicates such that:

1. $\mathfrak{M}(\forall)$ is some non-empty set $D^{\mathfrak{M}}$ (know as the ***"domain of quantification** in $\mathfrak{M}$."* These are the objects that ***exist*** in world $\mathfrak{M}$).
2. for any constant *c*, $\mathfrak{M}(c)$ is some element in $D^{\mathfrak{M}}$. (That is, *c* functions like a proper name, and $\mathfrak{M}(c)$ is its "referent" in $\mathfrak{M}$).
3. for any *n*-place predicate *P*, $\mathfrak{M}(P)$, also written $P^{\mathfrak{M}}$, is an *n*-place relation of elements in $D^{\mathfrak{M}}$. If *P* is 1-place, $P^{\mathfrak{M}}$is a set. (Thus, $P^{\mathfrak{M}}$ is the interpretation of the predicate *P* in $\mathfrak{M}$).

(Often the structure/model is identified with the order-pair $<\mathfrak{M}, D^{\mathfrak{M}}>$.)

Variable Assignments. Variables, like pronouns, may have many different meanings within the same "world." Each of the ways to fix the referents of a group of variables in $\mathfrak{M}$ simultaneously is called a "variable assignments."

A ***variable assignment*** for $\mathfrak{M}$ is any function *g* mapping some set of variables into $D^{\mathfrak{M}}$. Intuitively, *g(v)* is the "referent" of v relative to the assignment *g* in the "world" $\mathfrak{M}$. (**Note:** In the text the ***empty assignment*** $g_{\varnothing}$ is defined as the function from the empty set into $D^{\mathfrak{M}}$. It is used there for wffs without free variables (*i.e.* sentences), but is unnecessary here.)

The ***interpretation of an expression*** *E* relative to a model $\mathfrak{M}$ and variable assignment *g*, briefly $\mathfrak{M}_g(E)$, is defined for all terms, predicates and wffs as follows: **Constants.** If *c* is a constant, $\mathfrak{M}_g(c)$ is $\mathfrak{M}(c)$. (Thus, the variable assignment *g* plays not role in fixing the meaning of a constant in $\mathfrak{M}$.)

Variables. If *v* is a variable, $\mathfrak{M}_g(v)$ is just $g(v)$. (Thus, the variable assignment *g* alone determines the meaning of a variable.)

Below we use a the customary notation $[\![t_1]\!]^{\mathfrak{M}}_g$ for $\mathfrak{M}_g(t)$.

Predicates. If *P* is an m-place predicate, $\mathfrak{M}_g(P)$ is $P^{\mathfrak{M}}$. (Thus, the variable assignment *g* is irrelevant to the meaning of a predicate in $\mathfrak{M}$.)

Satisfaction of Wffs. Let *A* be a wff, $\mathfrak{M}$ a model and *g* a variable assignment relative to $\mathfrak{M}$. That *g* ***satisfies*** *A* in $\mathfrak{M}$, abbreviated $\mathfrak{M}\models A[g]$, is defined by cases depending on the grammatical complexity of A:

0. $\mathfrak{M}\models P(t_1,\ldots,t_n))[g]$ iff $[\![t_1]\!]^{\mathfrak{M}}_g,\ldots,[\![t_n]\!]^{\mathfrak{M}}_g$, in that order, are in $P^{\mathfrak{M}}$. (*i.e* iff the term referents in order instantiate the predicated relation or set.)
1. $\mathfrak{M}\models\neg B[g]$ iff, not $\mathfrak{M}\models B[g]$.
2. $\mathfrak{M}\models(B\wedge C)[g]$ iff, $\mathfrak{M}\models B[g]$ and $\mathfrak{M}\models C[g]$
3. $\mathfrak{M}\models(B\vee C)[g]$ iff , $\mathfrak{M}\models B[g]$ or $\mathfrak{M}\models C[g]$
4. $\mathfrak{M}\models(B\rightarrow C)[g]$ iff, not $\mathfrak{M}\models B[g]$ or $\mathfrak{M}\models C[g]$
5. $\mathfrak{M}\models(B\leftrightarrow C)[g]$ iff , $\mathfrak{M}\models B[g]$ iff $\mathfrak{M}\models C[g]$

These clauses are usually summarized by truth-tables.

6. $\mathfrak{M}\models\forall vB[g]$ iff, for all *d* in $D^{\mathfrak{M}}$, $\mathfrak{M}\models B[g(v/d)]$, where *g(v/d)* is that variable assignment like *g* except that it assigns *v to d*.
7. $\mathfrak{M}\models\exists vA[g]$ iff, for some *d* in $D^{\mathfrak{M}}$, $\mathfrak{M}\models A[g(v/d)]$, where *g(v/d)* is that variable assignment like *g* except that it assigns *v to d*. (Optional.)

Truth. Let *A* be sentence (*i.e.* has no free variables). *A* is ***true*** in $\mathfrak{M}$, (abbreviated $\mathfrak{M}\models A$) iff, for any g, $\mathfrak{M}\models A[g]$. (Equivalently, iff $\mathfrak{M}\models A[g_{\varnothing}]$.)

FO-Consequence and **Validity.** Let $A, A_1,\ldots,A_n$ and *B* be sentences.
The argument from $A_1,\ldots,A_n$ to *B* is ***valid*** or a ***FO-consequence*** (briefly, $A_1,\ldots, A_n\models B$) iff, for any $\mathfrak{M}$, if $\mathfrak{M}\models A_1$ and …. and $\mathfrak{M}\models A_n$ then $\mathfrak{M}\models B$.
A is ***valid*** or a ***FO-logical truth*** (briefly $\models A$) iff, for any $\mathfrak{M}$, $\mathfrak{M}\models A$.

III. Explanation of New Symbolism

Symbol:	**Meaning:**
t, u, v, x, y, z, ….	Variables that stand for various objects in the "domain" of a "world" (model), depending on their role in the formula.
a,b,c,	Constants, i.e. proper names in the object language. They stand for objects in the "domain"of a "world" (model).
$P^0{}_1,\ldots,P^0{}_m,\ldots$	These are *0*-place predicates, and are suppose to represent sentential letters without any names, that are simply true or false. They are a technical trick that allows for pure sentential logic to appear as part of the syntax and semantics of first-order logic, and can be idnoged for our purposes.
$P^1{}_1,\ldots,P^1{}_m,\ldots$	The various different 1-place predicates (hence the superscript 1) of the object langauge. There are *m* of these, indicated by the subscripts. These function like English intransitive verbs, common nouns, or adjectives. They stand for sets of objects in the "domain" of a given world (a.k.a. structure or model)
$P^n{}_1,\ldots,P^n{}_m,\ldots$	The various different *n*-place predicates (hence the superscript *n*) of the object langauge. There are *m* of these, indicated by the subscripts. These function like English transitive verbs, comparative adjectives, or verbs plus prepositions. They stand for relations among objects in the "domain" of a given world (a.k.a. structure or model).
$t_1,\ldots,t_n$	These stand for *terms.* A term is a constant or a variable, i.e. an expression that stands for an object in the domain.
$P^n{}_m(t_1,\ldots,t_n)$	This combination of symbols is the atomic formula formed from the *n*-place predicate followed by the *n* terms $t_1,\ldots,t_n$. In says that The objects named by $t_1,\ldots,t_n$ bear, in that order, the relation named by $P^n{}_m$. If the predicate is *1*-place, i.e. if it is some $P^1{}_m$, then $P^1{}_m(t_1)$ says that the object named by t_1 is in the set named by $P^1{}_m$.
$\mathfrak{M}$	The symbol stands for a model. It is also called a *structure.* It is supposed to capture the concept of a "possible world." It does so by assigning to the universal quantifier $\forall$ a "domain" of objects that exist in that world, labled $D^{\mathfrak{M}}$. It also assigns to each constant an object from the domain, and to each *n*-place predicate an *n*-place relation (to a *1*place predicate a set).
$\mathfrak{M}(\forall)$	This is one the notation for the "domain" of objects assigned to $\forall$ by the model $\mathfrak{M}$.
$D^{\mathfrak{M}}$	This is another the notation for the "domain" of objects assigned to $\forall$ by the model $\mathfrak{M}$.
g	*g* is a "variable assignment". That is, *g* is a mapping (function) from the variables of the object language *t, u, v, x, y, z, ….* to the objects in the domain $D^{\mathfrak{M}}$ of the model. Intuitively it assigns a temporary meaning to the variables.
g(v)	This is the notation for "the object in the domain $D^{\mathfrak{M}}$ of the model that is picked out by the variable *v* in the variable assignment *g*.
$g_\varnothing$	This is the notation for the *"empty" variable assignment,* is that defined for the no variables. It is a technical device used in the book but not important here.
E	*E* is any expression of the object language (constant, variable, predicate, or formula)
$\mathfrak{M}_g(E)$	This is the notation that stands for the "entity" that is picked out as the meaning of the expression *E* in the model $\mathfrak{M}$ augmented by the variable assignment *g.*
$\mathfrak{M}_g(c)$	This is the notation for the object in the domain picked out by the constant *c* in the model. It means the same as $\mathfrak{M}(c)$, and the Subscript *g* really has no bearing on the meaning of the constant because it is a constant. Tht is constants are unlike a variables which do not have a meaning unless there is a variable assignment. The meaning of a variable does "vary" depending on which variable assignment is in effect. The referent of a constants however, is fixed by the model $\mathfrak{M}$ and does not shift depending on any variable assignment *g.*
$\mathfrak{M}_g(v)$	This is another notation for *g*(*v*), the object assigned to the variable *v* in the model $\mathfrak{M}$ given the variable assignment *g.*
$\mathfrak{M}_g(t)$	This is a notation representing the object picked out by the term (either a constant or a variable) in relative to *g.* Note that if *t* is a constant, then $\mathfrak{M}_g(t)$ is the same as $_g(t)$ because its meaning does not depend on *g* whose only role is to interprete the variables. If, on the other hand, *t* is a variable, then $\mathfrak{M}_g(t)$ is the same as *g*(*v*), because the meaning of a variable is fixed by the variable assignment *g.*

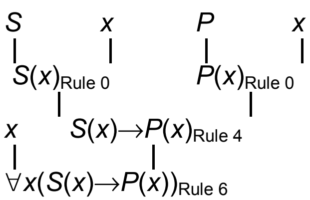
$[\![t]\!]^{\mathfrak{M}}_{g}$	This is another customary notatation for $\mathfrak{M}_g(t)$, i.e. the object picked out by the term *t* in the model $\mathfrak{M}$ with variable assignment *g.*
$\mathfrak{M}(P)$	This is one notation for the relation assigned to the predicate *P* in the model $\mathfrak{M}$. Note that if *P* is a one-place predicate, then is a set of objects from the domain (i.e. $\mathfrak{M}(P)$ is a subset of $D^{\mathfrak{M}}$). If *P* is n-place, then $\mathfrak{M}(P)$ is an *n*-place relation on elements in the domain. In mathematics and advaced logic relations are understood to be sets of ordered *n*-tuples, i.e. sets of *n*-place series if objects in $D^{\mathfrak{M}}$. (In set theory $[D^{\mathfrak{M}}]^n$ is the set of all *n*-tuples of objects in $D^{\mathfrak{M}}$. Hence, $\mathfrak{M}(P^n) \subseteq [D^{\mathfrak{M}}]^n$, a fact we shall not use here.)
$P^{\mathfrak{M}}$	This is a second notation for the relation assigned to the predicate *P* in the model $\mathfrak{M}$.
$\mathfrak{M}_g(P)$	This yet another notation for $P^{\mathfrak{M}}$. Note that the subscript *g* plays no role in fixing the meaning of a predicate *P* because its meaning is fixed by the model $\mathfrak{M}$ itself.
$\mathfrak{M} \models A[g]$	This is the traditional notation for "the formula *A* is true in $\mathfrak{M}$ relative to the variable assignment *g.*"
$\mathfrak{M} \models P(t_1,\ldots,t_n))[g]$	"The atomic formula $P(t_1,\ldots,t_n)$ is true in $\mathfrak{M}$ relative to variable assignment *g*," i.e. the entities $[\![t_1]\!]^{\mathfrak{M}}_{g}$,…., $[\![t_n]\!]^{\mathfrak{M}}_{g}$, in that order, are in relation $P^{\mathfrak{M}}$ picked out by the predicate in the model.
$\mathfrak{M} \models \neg A[g]$	"The formula $\neg A$ is true in $\mathfrak{M}$ relative to variable assignment *g*," i.e. not $\mathfrak{M} \models A[g]$.
$\mathfrak{M} \models (A\wedge B)[g]$	"The formula $A\wedge B$ i is true in $\mathfrak{M}$ relative to variable assignment *g*," i.e. both $\mathfrak{M} \models A[g]$ and $\mathfrak{M} \models B[g]$
$\mathfrak{M} \models (A\vee B)[g]$	"The formula $A\vee B$ i is true in $\mathfrak{M}$ relative to variable assignment *g*," i.e. either $\mathfrak{M} \models A[g]$ or $\mathfrak{M} \models B[g]$
$\mathfrak{M} \models (A\rightarrow B)[g]$	"The formula $A\rightarrow B$ i is true in $\mathfrak{M}$ relative to variable assignment *g*," i.e. either not $\mathfrak{M} \models A[g]$ or $\mathfrak{M} \models B[g]$, or equivalently if $\mathfrak{M} \models A[g]$ then $\mathfrak{M} \models B[g]$.
$\mathfrak{M} \models (A\leftrightarrow B)[g]$	"The formula $A\leftrightarrow B$ i is true in $\mathfrak{M}$ relative to variable assignment *g*," i.e. $\mathfrak{M} \models A[g]$ if and only if $\mathfrak{M} \models B[g]$
g(v/d)	This is the notation for "the variable assignment just like *g* except that it assigns the variable *v* to the object *d* in the domain." It is used to change the object that *g* assigns to *v*, thereby making up a new variable assignment. Note that there is a new assignment of this kind for every object in the domain. If we go through all of them and see what they assign to *v,* we will have reviewed all the objects in the domain. If something holds of all of them, it is a universal truth.
$\mathfrak{M} \models \forall vA[g]$	The formula $\forall vA$ is true in $\mathfrak{M}$ relative to variable assignment *g*, i.e. in every variable assignment *g(v/d)*, which is like *g* except that it assigns the object *d* to *v*, the formula *A* is true in $\mathfrak{M}$ relative to variable assignment *g(v/d)*. The same idea is expressed more precisely in symbols: for all *d* in $D^{\mathfrak{M}}$, $\mathfrak{M} \models A[g(v/d)]$
$\mathfrak{M} \models \exists vA[g]$	The formula $\exists vA$ is true in $\mathfrak{M}$ relative to variable assignment *g*, i.e. there is some variable assignment *g(v/d)*, which is like *g* except that it assigns the object *d* to *v*, the formula *A* is true in $\mathfrak{M}$ relative to variable assignment *g(v/d)*. The same idea is expressed more precisely in symbols: for some *d* in $D^{\mathfrak{M}}$, $\mathfrak{M} \models A[g(v/d)]$
$\mathfrak{M} \models A$	This is the notation for "*A* is ***true*** in $\mathfrak{M}$." Sometimes this is read "*A* is ***true*** in $\mathfrak{M}$ *simpliciter*" meaning that *A* is true throughout all the variable assignment. It happens when *A* is true in all the variable assignments, i.e. for any g, $\mathfrak{M} \models A[g]$). Technically this is equivalent to saying that *A* is true in relative to the empty assignment $g_{\varnothing}$, i.e. i$\mathfrak{M} \models A[g_{\varnothing}]$.
$A_1,\ldots, A_n \models B$	This is the notation for "the argument from sentences $A_1,\ldots,A_n$ to sentence *B* is (*first-order) valid*, or in alternatively the argument is an *FO-consequence.* It holds if in all models in which the premises are ture, the conclusion is, i.e. for any $\mathfrak{M}$, ((if $\mathfrak{M} \models A_1$ and …. and $\mathfrak{M} \models A_n$) then $\mathfrak{M} \models B$).
$\models A$	"The formula *A* is a first-order logical truth." This is also read "*A* is valid (simpliciter). It holds if it is always true: for any $\mathfrak{M}$, $\mathfrak{M} \models A$.

IV. Truth-Conditions. The rules of the syntax and the semantic definitions, particularly the clauses in the definition of $\mathfrak{M}\models\forall vA[g]$, allow us to reformulate in a single "iff" statement what are know as the "truth conditions" of a wff. On the left of the "iff" is the statement "A is true in the model $\mathfrak{M}$" i.e. $\mathfrak{M}\models A$. On the right of the "iff" is an equivalent stating the conditions that must obtain among the entities in the domain D of $\mathfrak{M}$ and the sets and relations picked out by the constants and predicates in $\mathfrak{M}$ in order for A to be true in $\mathfrak{M}$. This biconditional is said to state the ***truth-conditions*** of A, because is records the conditions that must obtain in the world (the right half) in order for the sentence to be true in that world (the left half). There are three examples below, each stating the truth-conditions for one of the lines of a three line argument. The steps working out a sentence's truth-conditions (middle column) parallel the steps in the syntactic construction of the sentence by grammar rules (left column). Reasons for each set are in the right column. In the middle column, each statements is equivalent (by definitions) to the one immediately above and below. In practice it is usually easier to work backward (up) from the statement that $\mathfrak{M}\models A$ to its truth-conditions. That's why the statements are numbered from the bottom up. For example, in the analysis of the argument's 1st line, i.e. $\forall x(S(x)\rightarrow P(x))$, the middle column's bottom line (a. $\mathfrak{M}\models\forall x(S(x)\rightarrow P(x))$) has as its truth-conditional equivalent the middle column's top line (f. for all g, for all d in D, if $d\in S^{\mathfrak{M}}$ then $d\in P^{\mathfrak{M}}$), with its grammar tree in the left column, and the reasons for each step a-f at the right.

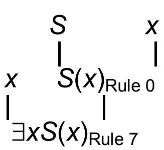
Example 1.
A Version of Modus Pones

All S are P	$\forall x(S(x)\rightarrow P(x))$
<u>There are some S</u>	$\underline{\exists xS(x)}$
There are some P	$\exists xP(x)$

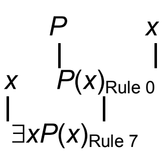
Grammar Tree and Rules | **Analysis of Truth-Conditions (from bottom up)** | **Citations for T-C Steps: Clauses from Def of $\mathfrak{M}\models A[g]$**



Analysis of Truth-Conditions	Citations
f. for all g, for all d in D, if $d\in S^{\mathfrak{M}}$ then $d\in P^{\mathfrak{M}}$	e & Def of $[\![t]\!]^{\mathfrak{M}}_g$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$
e. for all g, for all d in D, if $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\in S^{\mathfrak{M}}$ then $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\in P^{\mathfrak{M}}$	d & Clause 0, Def of $\mathfrak{M}\models P(t)[g]$
d. for all g, for all d in D, if $\mathfrak{M}\models S(x))[g(v/d)]$ then $\mathfrak{M}\models P(x))[g(x/d)]$	c & Clause 4, Def of $\mathfrak{M}\models(A\rightarrow B)[g]$
c. for all g, for all d in D, if $\mathfrak{M}\models(S(x)\rightarrow P(x))[g(x/d)]$	b & Clause 6, Def of $\mathfrak{M}\models\forall vA[g]$
b. for all g, $\mathfrak{M}\models\forall x(S(x)\rightarrow P(x))[g]$	a & Def of $\mathfrak{M}\models A$ (See **Note** with def))
a. $\mathfrak{M}\models\forall x(S(x)\rightarrow P(x))$	



Analysis of Truth-Conditions	Citations
e. for all g, for some d in D, $d\in S^{\mathfrak{M}}$	d & Def of $[\![t]\!]^{\mathfrak{M}}_g$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$
d. for all g, for some d in D, $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\in S^{\mathfrak{M}}$	c & 0, Def of $\mathfrak{M}\models P(t)[g]$
c. for all g, for some d in D, $\mathfrak{M}\models S(x)[g(x/d)]$	b & Clause 7, Def of $\mathfrak{M}\models\exists vA[g]$
b. for all g, $\mathfrak{M}\models\exists xS(x)[g]$	a & Def of $\mathfrak{M}\models A$ (See **Note** with def))
a. $\mathfrak{M}\models\exists xS(x)$	



Analysis of Truth-Conditions	Citations
e. for all g, for some d in D, $d\in P^{\mathfrak{M}}$	d & Def of $[\![t]\!]^{\mathfrak{M}}_g$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$
d. for all g, for some d in D, $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\in P^{\mathfrak{M}}$	c & 0, Def of $\mathfrak{M}\models P(t)[g]$
c. for all g, for some d in D, $\mathfrak{M}\models P(x)[g(x/d)]$	b & Clause 7, Def of $\mathfrak{M}\models\exists vA[g]$
b. for all g, $\mathfrak{M}\models\exists xP(x)[g]$	a & Def of $\mathfrak{M}\models A$ (See **Note** with def))

a. $\mathfrak{M} \models \exists x P(x)$

Metatheorem A First-Order Consequence. From the particular of the antecedent the particular of the consequent follows if the conditional is universal. In symbols:

$$\forall x(S(x)\to P(x)),\ \exists x S(x) \models \exists x P(x)$$

Formal Proof. To show: $\forall x(S(x)\to P(x)),\ \exists x S(x) \models \exists x P(x)$. By def of First-Order Consequence (i.e. the "$\models$" here) , this means we must show: for any $\mathfrak{M}$, if $\mathfrak{M} \models \forall x(S(x)\to P(x))$ and $\mathfrak{M} \models \exists x S(x)$, then $\mathfrak{M} \models \exists x P(x)$. This metatheorem is a universally quantified conditional sentence in the metalanguage. The proof applying Fitch notation in the meta-metalangue to assertions in the metalanguagem would look like this:

1.	| |__ Let $\mathfrak{M}$ be an arbitrary structure and assume $\mathfrak{M} \models \forall x(S(x)\to P(x))$ and $\mathfrak{M} \models \exists x(S(x)$	Hypo. for a Universal Conditional Proof ($\forall$+)
2.	| | $\mathfrak{M} \models \forall x(S(x)\to P(x))$	1. $\wedge$-
3.	| | for all g, for all d in D, if $d\in S^{\mathfrak{M}}$ then $d\in P^{\mathfrak{M}}$	2 . and steps a-e of the 1st T-C Analysis, previous page
4.	| | $\mathfrak{M} \models \exists x S(x)$	1. $\wedge$-
5.	| | for all g, for some d in D, $d\in S^{\mathfrak{M}}$	3 and steps a-d of the 2 nd T-C Analysis previous page
6.	| | |__ Let g be an arbitrary variable assignment	Hypo for $\forall$+
7.	| | | for some d in D, $d\in S^{\mathfrak{M}}$	5. $\forall$-
8.	| | | |__ Let d be a name of convenience and assume $d\in S^{\mathfrak{M}}$	Hypo. For $\exists$-
9.	| | | | for all d in D, if $d\in S^{\mathfrak{M}}$ then $d\in P^{\mathfrak{M}}$	3. $\forall$-
10.	| | | | if $d\in S^{\mathfrak{M}}$ then $d\in P^{\mathfrak{M}}$	11. $\forall$-
11.	| | | | $d\in P^{\mathfrak{M}}$	8, 10 . $\to$-
12.	| | | | for some d in D, $d\in P^{\mathfrak{M}}$	11. $\exists$+
13.	| | | for some d in D, $d\in P^{\mathfrak{M}}$	8-12. $\exists$-
14.	| | for all g, for some d in D, $d\in P^{\mathfrak{M}}$	6-12. $\forall$+
15.	| | $\mathfrak{M} \models \exists x P(x)$	16 and steps a-d of the 3rd T-C Analysis, previous page
16.	| for any $\mathfrak{M}$, if $\mathfrak{M} \models \forall x(S(x)\to P(x))$ and $\mathfrak{M} \models \exists x S(x)$, then $\mathfrak{M} \models \exists x P(x)$	1-17. $\forall$+
17.	| $\forall x(S(x)\to P(x)),\ \exists x S(x) \models \exists x P(x)$.	18 and Def of First-Order Consequence

Informal Proofs. Normally, such a proof would be written in English, in paragraph form, as an "informal proof". The reason for sketching this "formal proof" (which would usually be done on "scratch paper" and then recast as an "informal proof") is to demonstrate how you should work through preparing to write the sort of informal proof logicians expect. Now, let us see what the informal proof looks like. It is written so the reader might be able to reconstruct the sort of formal proof above from the descriptions in the informal proof. In the informal version the writer must make sure that the reader understands all the steps using the quantifiers in the metalanguage (*e.g..* steps about *all g, some d, any* $\mathfrak{M}$). In addition, in the informal proof it is customary to actually provide all the steps of the truth-conditional analysis for all the sentences used. That is, step 3 above would be expanded to the steps a-g for premise 1 of the previous page, step 5 would be expanded to the a-f for premise 2 on the prvious page, and a-g for the conclusion likewise would all be exhibited. Look over the proof on the nest page until you see how it is an informal summar of the proof above and the truth-conditional analyses of the previous page.

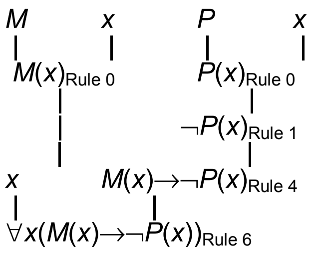
Metatheorem. $\forall x(S(x)\rightarrow P(x)), \exists xS(x) \models \exists xP(x)$.

Informal Proof. To show: $\forall x(S(x)\rightarrow P(x)), \exists xS(x) \models \exists xP(x)$. By def of First-Order Consequence (i.e. the "$\models$" here) , this means we must show : for any $\mathfrak{M}$, if $\mathfrak{M}\models\forall x(S(x)\rightarrow P(x))$ and $\mathfrak{M}\models \exists x(S(x)$, then $\mathfrak{M}\models\exists xP(x)$. This is a universally quantified conditional sentence in the metalanguage. Thus we assume an arbitrary $\mathfrak{M}$, and assume for conditional proof that $\mathfrak{M}\models\forall x(S(x)\rightarrow P(x)$ and $\mathfrak{M}\models \exists xS(x)$. Let us consider the first of these conjuncts first. Now, by truth-conditional analysis, $\mathfrak{M}\models \forall x(S(x)\rightarrow P(x))$ iff for all g, $\mathfrak{M}\models\forall x(S(x)\rightarrow P(x))[g]$ [by Def of $\mathfrak{M}\models A$] iff for all g, for all d in D, $\mathfrak{M}\models (S(x)\rightarrow P(x))[g(v/d)]$ [by Def of $\mathfrak{M}\models\forall vA[g]$ iff for all g, for all d in D, $\mathfrak{M}\models S(x))[g(x/d)]$ only if $\mathfrak{M}\models P(x))[g(x/d)]$ [by Def of $\mathfrak{M}\models (A\rightarrow B)[g]]$ iff for all g, for all d in D, if $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \in S^{\mathfrak{M}}$ then $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \in P^{\mathfrak{M}}$ [by Def of $\mathfrak{M}\models P(t)[g]$] iff for all g, for all d in D, if $d\in S^{\mathfrak{M}}$ then $d\in P^{\mathfrak{M}}$ [by Def of $[\![t]\!]^{\mathfrak{M}}_{g}$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$]. Hence, for all g, for all d in D, if $d\in S^{\mathfrak{M}}$ then $d\in M^{\mathfrak{M}}$. Let us instantiate this. Since it is true for all assignment functions, let it be true for the arbitrary assignment g. Further since it is true for all elements of D, let us instantiate for an individual element d of D. Hence, if $d\in S^{\mathfrak{M}}$ then $d\in P^{\mathfrak{M}}$. Let us now return to the second conjunct. That is we know $\mathfrak{M}$ $\mathfrak{M}\models\exists xS(x)$. But by truth-conditional analysis we know $\mathfrak{M}\models\exists xS(x)$ iff for all g, $\mathfrak{M}\models\exists xS(x)[g]$ {by Def of $\mathfrak{M}\models A$] iff for all g, for some d in D, if $\mathfrak{M}\models S(x)[g(v/d)]$ [by Def of $\mathfrak{M}\models\exists vA[g]]$ iff for all g, for some d in D, $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \in S^{\mathfrak{M}}$ [by Def of $\mathfrak{M}\models P(t)[g]]$ iff for all g, for some d in D, $d\in S^{\mathfrak{M}}$ [by Def of $[\![t]\!]^{\mathfrak{M}}_{g}$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$]. By universal instantiation over variable assignments and by existential instantiation over elements of D, choosing d as a name of convenience for this instantiation, we deduce $d\in S^{\mathfrak{M}}$. By truth-tables (modus pones) then $d\in P^{\mathfrak{M}}$. Hence by existential generalization, for some d in D, $d\in P^{\mathfrak{M}}$, and then by universal generalization (we are general in g) for all g, for some d in D, $d\in P^{\mathfrak{M}}$. But by the analysis of truth-conditions we know that for all g, for some d in D, $d\in P^{\mathfrak{M}}$ iff for all g, for some d in D, $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \in P^{\mathfrak{M}}$ [by Def of $[\![t]\!]^{\mathfrak{M}}_{g}$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$] iff for all g, for some d in D, $\mathfrak{M}\models P(x))[g(x/d)]$) [by Def of $\mathfrak{M}\models P(t)[g]]$ iff for all g, $\mathfrak{M}\models\exists xP(x)[g]$ [by Def of $\mathfrak{M}\models\exists vA[g]]$ iff $\mathfrak{M}\models\exists xP(x)$ [by Def of $\mathfrak{M}\models A$]. Thus by conditional proof we know that if $\mathfrak{M}\models\forall x(S(x)\rightarrow P(x)$ and $\mathfrak{M}\models \exists x(S(x)$, then $\mathfrak{M}\models\exists xP(x)$. Furthermore since we are general in $\mathfrak{M}$, we can universally generalize: for any $\mathfrak{M}$, if $\mathfrak{M}\models\forall x(S(x)\rightarrow P(x)$ and $\mathfrak{M}\models \exists x(S(x)$, then $\mathfrak{M}\models\exists xP(x)$. Thus by definition, $\forall x(S(x)\rightarrow P(x)), \exists xS(x)\models\exists xP(x)$. QED.

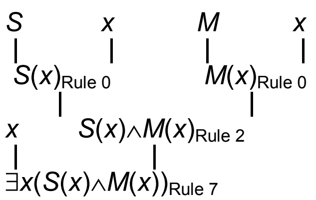
Example 2. **Ferio**

No M are P	$\forall x(M(x)\rightarrow\neg P(x))$
Some S are M	$\exists x(S(x)\wedge M(x)$
Some S are not P	$\exists x(S(x)\wedge P(x))$

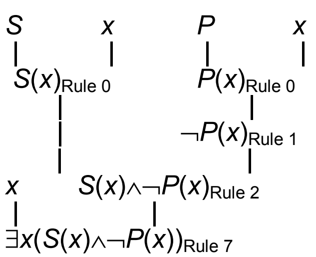
Grammar Tree and Rules | **Analysis of Truth-Conditions (from bottom up)** | **Citations for T-C Steps: Clauses from Def of $\mathfrak{M}\models A[g]$**



Analysis of Truth-Conditions	Citations
g. for all g, for all d in D, if $d\in M^{\mathfrak{M}}$ then $d\notin P^{\mathfrak{M}}$	f & Def of $[\![t]\!]^{\mathfrak{M}}_{g}$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$
f. for all g, for all d in D, if $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\in M^{\mathfrak{M}}$ then $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\notin P^{\mathfrak{M}}$	e & Clause 0, Def of $\mathfrak{M}\models P(t)[g]$
e. for all g, for all d in D, if $\mathfrak{M}\models M(x))[g(x/d)]$ then not($\mathfrak{M}\models P(x))[g(x/d)]$)	d & Clause 1, Def of $\mathfrak{M}\models\neg A[g]$
d. for all g, for all d in D, if $\mathfrak{M}\models M(x))[g(v/d)]$ then $\mathfrak{M}\models\neg P(x))[g(x/d)]$	c & Clause 4, Def of $\mathfrak{M}\models(A\rightarrow B)[g]$
c. for all g, for all d in D, if $\mathfrak{M}\models(M(x)\rightarrow\neg P(x))[g(x/d)]$	b & Clause 6, Def of $\mathfrak{M}\models\forall vA[g]$
b. for all g, $\mathfrak{M}\models\forall x(M(x)\rightarrow\neg P(x))[g]$	a & Def of $\mathfrak{M}\models A$ (See **Note** with def))
a. $\mathfrak{M}\models\forall x(M(x)\rightarrow\neg P(x))$	



Analysis of Truth-Conditions	Citations
f. for all g, for some d in D, $d\in S^{\mathfrak{M}}$ and $d\in M^{\mathfrak{M}}$	e & Def of $[\![t]\!]^{\mathfrak{M}}_{g}$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$
e. for all g, for some d in D, $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\in S^{\mathfrak{M}}$ and $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\in M^{\mathfrak{M}}$	d & 0, Def of $\mathfrak{M}\models P(t)[g]$
d. for all g, for some d in D, $\mathfrak{M}\models S(x))[g(x/d)]$ and $\mathfrak{M}\models M(x))[g(x/d)]$	c & Clause 2, Def of $\mathfrak{M}\models(A\wedge B)[g]$
c. for all g, for some d in D, $\mathfrak{M}\models(S(x)\wedge M(x))[g(x/d)]$	b & Clause 7, Def of $\mathfrak{M}\models\exists vA[g]$
b. for all g, $\mathfrak{M}\models\exists x(S(x)\wedge M(x))[g]$	a & Def of $\mathfrak{M}\models A$ (See **Note** with def))
a. $\mathfrak{M}\models\exists x(S(x)\wedge M(x))$	



Analysis of Truth-Conditions	Citations
g. for all g, for some d in D, $d\in S^{\mathfrak{M}}$ and $d\notin P^{\mathfrak{M}}$	f & Def of $[\![t]\!]^{\mathfrak{M}}_{g}$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$
f. for all g, for some d in D, $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\in S^{\mathfrak{M}}$ and $[\![x]\!]^{\mathfrak{M}}_{g(x/d)}\notin P^{\mathfrak{M}}$	e & Clause 0, Def of $\mathfrak{M}\models P(t)[g]$
e. for all g, for some d in D, $\mathfrak{M}\models S(x))[g(v/d)]$ and not($\mathfrak{M}\models P(x))[g(x/d)]$)	d & Clause 1, Def of $\mathfrak{M}\models\neg A[g]$
d. for all g, for some d in D, $\mathfrak{M}\models S(x))[g(v/d)]$ and $\mathfrak{M}\models\neg P(x))[g(x/d)]$	c & Clause 2, Def of $\mathfrak{M}\models(A\wedge B)[g]$
c. for all g, for some d in D, $\mathfrak{M}\models(S(x)\wedge\neg P(x))[g(x/d)]$	b & Clause 7, Def of $\mathfrak{M}\models\exists vA[g]$
b. for all g, $\mathfrak{M}\models\exists x(S(x)\wedge\neg P(x))[g]$	a & Def of $\mathfrak{M}\models A$ (See **Note** with def))
a. $\mathfrak{M}\models\exists x(S(x)\wedge\neg P(x))$	

Metatheorem ***Ferio*** is a First-Order Consequent (i.e. a valid argument in First-Order Logic). In symbols:

$$\forall x(M(x)\rightarrow\neg P(x)),\ \exists x(S(x)\wedge M(x))\ \models\exists x(S(x)\wedge\neg P(x))$$

Formal Proof. To show: $\forall x(M(x)\rightarrow\neg P(x))$, $\exists x(S(x)\wedge M(x))\models\exists x(S(x)\wedge\neg P(x))$. By def of First-Order Consequence (i.e. the $\mathfrak{M}\models$" here) , this means we must show: for any $\mathfrak{M}$, if $\mathfrak{M}\models\forall x(M(x)\rightarrow\neg P(x))$ and $\mathfrak{M}\models x(S(x)\wedge M(x))$, then $\mathfrak{M}\models\exists x(S(x)\wedge\neg P(x))$. This is a universally quantified conditional sentence in the metalanguage. The proof in Fitch notation, with the introduction and elimination rules applied to the quantifiers and connectives of the metalanguage, is:

1.	ǀ ǀ__ Let $\mathfrak{M}$ be an arbitrary structure and assume $\mathfrak{M}\models\forall x(M(x)\rightarrow\neg P(x))$ and $\mathfrak{M}\models\exists x(S(x)\wedge M(x))$	Hypo. for a Universal Conditional Proof (∀+)
2.	ǀ ǀ $\mathfrak{M}\models\forall x(M(x)\rightarrow\neg P(x))$	1. ∧-
3.	ǀ ǀ for all *g*, for all *d* in *D*, if $d\in M^{\mathfrak{M}}$ then $d\notin P^{\mathfrak{M}}$	2 . and steps a-g of the 1st T-C Analysis, previous page
4.	ǀ ǀ $\mathfrak{M}\models\exists x(S(x)\wedge M(x)$	1. ∧-
5.	ǀ ǀ for all *g*, for some *d* in *D*, $d\in S^{\mathfrak{M}}$ and $d\in M^{\mathfrak{M}}$	3 and steps a-f of the 2 nd T-C Analysis previous page
6.	ǀ ǀ ǀ__ Let *g* be an arbitrary variable assignment	Hypo. For ∀+
7.	ǀ ǀ ǀ for some *d* in *D*, $d\in S^{\mathfrak{M}}$ and $d\in M^{\mathfrak{M}}$	5. ∀-
8.	ǀ ǀ ǀ ǀ__ Let *d* be a name of convenience and assume $d\in S^{\mathfrak{M}}$ and $d\in M^{\mathfrak{M}}$	Hypo. For ∃-
9.	ǀ ǀ ǀ ǀ $d\in S^{\mathfrak{M}}$	8. ∧-
10.	ǀ ǀ ǀ ǀ $d\in M^{\mathfrak{M}}$	8. ∧-
11.	ǀ ǀ ǀ ǀ for all *d* in *D*, if $d\in M^{\mathfrak{M}}$ then $d\notin P^{\mathfrak{M}}$	3. ∀-
12.	ǀ ǀ ǀ ǀ if $d\in M^{\mathfrak{M}}$ then $d\notin P^{\mathfrak{M}}$	11. ∀-
13.	ǀ ǀ ǀ ǀ $d\notin P^{\mathfrak{M}}$	10, 12 . →-
14.	ǀ ǀ ǀ ǀ $d\in S^{\mathfrak{M}}$ and $d\notin P^{\mathfrak{M}}$	9,13. ∧+
15.	ǀ ǀ ǀ ǀ for some *d* in *D*, $d\in S^{\mathfrak{M}}$ and $d\notin P^{\mathfrak{M}}$	14. ∃+
16.	ǀ ǀ ǀ for some *d* in *D*, $d\in S^{\mathfrak{M}}$ and $d\notin P^{\mathfrak{M}}$	8-14. ∃-
17.	ǀ ǀ for all *g*, for some *d* in *D*, $d\in S^{\mathfrak{M}}$ and $d\notin P^{\mathfrak{M}}$	6-16. ∀+
18.	ǀ ǀ $\mathfrak{M}\models\exists x(S(x)\wedge\neg P(x))$	17 and steps a-g of the 3rd T-C Analysis, previous page
19.	ǀ for any $\mathfrak{M}$, if $\mathfrak{M}\models\forall x(M(x)\rightarrow\neg P(x))$ and $\mathfrak{M}\models\exists x(S(x)\wedge M(x))$, then $\mathfrak{M}\models\exists x(S(x)\wedge\neg P(x))$	1-18. ∀+
20.	ǀ $\forall x(M(x)\rightarrow\neg P(x)),\ \exists x(S(x)\wedge M(x))\models\exists x(S(x)\wedge\neg P(x))$.	19 and Def of First-Order Consequence

Metatheorem. $\forall x(M(x)\rightarrow\neg P(x)), \exists x(S(x)\wedge M(x)) \models \exists x(S(x)\wedge\neg P(x))$.

Informal Proof. To show: $\forall x(M(x)\rightarrow\neg P(x)), \exists x(S(x)\wedge M(x)) \models \exists x(S(x)\wedge\neg P(x))$. By def of First-Order Consequence (i.e. the "$\models$" here) , this means we must show : for any $\mathfrak{M}$, if $\mathfrak{M} \models \forall x(M(x)\rightarrow\neg P(x))$ and $\mathfrak{M} \models \exists x(S(x)\wedge M(x))$, then $\mathfrak{M} \models \exists x(S(x)\wedge\neg P(x))$. This is a universally quantified conditional sentence in the metalanguage. Thus we assume an arbitrary $\mathfrak{M}$, and assume for conditional proof that $\mathfrak{M} \models \forall x(M(x)\rightarrow\neg P(x))$ and $\mathfrak{M} \models \exists x(S(x)\wedge M(x))$. Let us consider the second of these conjuncts first. Now, by truth-conditional analysis, $\mathfrak{M} \models \exists x(S(x)\wedge M(x))$ iff for some g, $\mathfrak{M} \models \exists x(S(x)\wedge M(x))[g]$ [by Def of $\mathfrak{M} \models A$] iff for all g, for some d in D, $\mathfrak{M} \models (S(x)\wedge M(x))[g(v/d)]$ [by Def of $\mathfrak{M} \models \exists vA[g]$ iff for all g, for some d in D, $\mathfrak{M} \models S(x))[g(v/d)]$ and $\mathfrak{M} \models M(x))[g(v/d)]$ [by Def of $\mathfrak{M} \models (A\wedge B)[g]$] iff for all g, for some d in D, $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \in S^{\mathfrak{M}}$ and $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \in M^{\mathfrak{M}}$ [by Def of $\mathfrak{M} \models P(t)[g]$] iff for all g, for some d in D, $d\in S^{\mathfrak{M}}$ and $d\in M^{\mathfrak{M}}$ [by Def of $[\![t]\!]^{\mathfrak{M}}_{g}$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$]. Hence, for all g, for some d in D, $d\in S^{\mathfrak{M}}$ and $d\in M^{\mathfrak{M}}$. Let us instantiate this. Since it is true for all assignment functions, let it be true for the arbitrary assignment g. Further since it is true of some element of D, let us existentially instantiate it for one such element, giving it the temporary name of convenience d. Hence, $d\in S^{\mathfrak{M}}$ and $d\in M^{\mathfrak{M}}$. Let us now return to the first conjunct. That is, we know $\mathfrak{M} \models \forall x(M(x)\rightarrow\neg P(x))$. But by truth-conditional analysis we know $\mathfrak{M} \models \forall x(M(x)\rightarrow\neg P(x))$ iff for all g, $\mathfrak{M} \models \forall x(M(x)\rightarrow\neg P(x))[g]$ {by Def of $\mathfrak{M} \models A$] iff for all g, for all d in D, if $\mathfrak{M} \models (M(x)\rightarrow\neg P(x))[g(x/d)]$ [by Def of $\mathfrak{M} \models \forall xA[g]$] iff for all g, for all d in D, if $\mathfrak{M} \models M(x))[g(x/d)]$ then $\mathfrak{M} \models \neg P(x))[g(x/d)]$ [by Def of $\mathfrak{M} \models (A\rightarrow B)[g]$] iff for all g, for all d in D, if $\mathfrak{M} \models M(x))[g(x/d)]$ then not($\mathfrak{M} \models P(x))[g(x/d)]$) [by Def of $\mathfrak{M} \models \neg A[g]$] iff for all g, for all d in D, if $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \in M^{\mathfrak{M}}$ then $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \notin P^{\mathfrak{M}}$ [by Def of $\mathfrak{M} \models P(t)[g]$] iff for all g, for all d in D, if $d\in M^{\mathfrak{M}}$ then $d\notin P^{\mathfrak{M}}$ [by Def of $[\![t]\!]^{\mathfrak{M}}_{g}$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$]. By universal instantiation for both of these universal quantifiers, we then know if $d\in M^{\mathfrak{M}}$ then $d\notin P^{\mathfrak{M}}$. By truth-tables (hypothetical syllogism)we then know $d\in S^{\mathfrak{M}}$ and $d\notin P^{\mathfrak{M}}$. Hence by existential generalization, for some d in D, $d\in S^{\mathfrak{M}}$ and $d\notin P^{\mathfrak{M}}$. and then by universal generalization (we are general in g) we know, for all g, for some d in D, $d\in S^{\mathfrak{M}}$ and $d\notin P^{\mathfrak{M}}$. But by the analysis of truth-conditions we know that for all g, for some d in D, $d\in S^{\mathfrak{M}}$ and $d\notin P^{\mathfrak{M}}$ iff for all g, for some d in D, $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \in S^{\mathfrak{M}}$ and $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} \notin P^{\mathfrak{M}}$ [by Def of $[\![t]\!]^{\mathfrak{M}}_{g}$, i.e. $[\![x]\!]^{\mathfrak{M}}_{g(x/d)} = g(x/d)(x) = d$] iff for all g, for some d in D, $\mathfrak{M} \models S(x))[g(v/d)]$ and not($\mathfrak{M} \models P(x))[g(x/d)]$) [by Def of $\mathfrak{M} \models P(t)[g]$] iff for all g, for some d in D, $\mathfrak{M} \models S(x))[g(x/d)]$ and $\mathfrak{M} \models \neg P(x))[g(x/d)]$ [by Def of $\mathfrak{M} \models \neg A[g]$] iff for all g, for some d in D, $\mathfrak{M} \models (S(x)\wedge\neg P(x))[g(x/d)]$ [by Def of $\mathfrak{M} \models (A\wedge B)[g]$] iff for all g, $\mathfrak{M} \models \exists x(S(x)\wedge\neg P(x))[g]$ [by Def of $\mathfrak{M} \models \exists xA[g]$] iff $\mathfrak{M} \models \exists x(S(x)\wedge\neg P(x))$ [by Def of $\mathfrak{M} \models A$]. Thus by conditional proof we know that if $\mathfrak{M} \models \forall x(M(x)\rightarrow\neg P(x))$ and $\mathfrak{M} \models \exists x(S(x)\wedge M(x))$, then $\mathfrak{M} \models \exists x(S(x)\wedge\neg P(x))$. Furthermore since we are general in $\mathfrak{M}$, we can universally generalize: for any $\mathfrak{M}$, if $\mathfrak{M} \models \forall x(M(x)\rightarrow\neg P(x))$ and $\mathfrak{M} \models \exists x(S(x)\wedge M(x))$, then $\mathfrak{M} \models \exists x(S(x)\wedge\neg P(x))$. Thus by definition, $\forall x(M(x)\rightarrow\neg P(x), \exists x(S(x)\wedge M(x) \models \exists x(S(x)\wedge\neg P(x)$. QED.

V. Exercise

A Version of Disjunctive Syllogism

Everything is either S or P	$\forall x(S(x)\vee P(x))$
There are some not-P	$\exists x\neg P(x)$
There are some S	$\exists xS(x)$

Using the examples above as models, show that this argument is a first-order consequence.

1. Give a grammatical tree for each of the sentence in the argument, showing how each part of the sentence is constructed from more basic parts until the construction terminates with variables, constants and predicates . Give three trees, one for each sentence in the argument.

2. State the truth-conditions for each of the wffs in the argument. For each of the three sentences *A* in the argument, list a series of equivalencies ("iff's") working backward from the statement that *A* is true in a model $\mathfrak{M}$ (in symbols, $\mathfrak{M}\models A$) to a final equivalent that describes what must be true of the way elements in the domain stand to the sets and relations picked out by the predicates in *A* when *A* is true in $\mathfrak{M}$. There should be three such breakdowns, one for each sentence in the argument. (Do Parts 1 and 2 on the same page, next to each other as in the examples. Write neatly.)

3. Give a formal proof in Fitch style notation that the argument is valid. Set the proof up as a universal generalization of a conditional metalinguistic statement that quantifies over all models, and prove the conditional that if the premises are all true in an arbitrary model $\mathfrak{M}$ then the conclusion is true. In the conditional proof ($\rightarrow$+) assume as premises of a sub-proof that the premises are true (in the arbitrary $\mathfrak{M}$) and conclude the subproof with final line that the conclusion is true in $\mathfrak{M}$. Work forward from the premises of the subproof, and backward from its conclusion by applying the truth-conditional analyses you did in Part 2 above. (Do Part 3 on a single page as in the examples.)

4. Rewrite your formal proof of Part 3 above as an informal proof in paragraph form, being sure to make clear to the reader the strategy of the proof: what is being proved, and when temporary assumptions are being introduced and when the are finally "discharged." (Do Part 4 on a single page as in the examples.)