Resources Available in Informal Proofs

General Proof Strategies

Reduction to the Absurd. Assume the opposite of what it to be proved and deduce a falsehood. (Also called *indirect proof,* and *negation introduction* in Fitch)

Proof by Cases. Divide the subject matter into exhaustive cases and show that what is to be proved follows from each. (Called *disjunction introduction* in Fitch.)

Analytic Truths and Consequences

If a sentence is true by the definition or by the "meanings" of the predicates, you can just assert it as true justifying your claim by citing as a reason "by definition" or "analytically true".
Likewise if an argument is valid by the definition or by the "meanings" of the predicates, you can just assert it as true or valid justifying your claim by citing as a reason "by definition" or "analytically valid," or, as in our text, "an analytical consequent."

Later in the text in Fitch there is a special rule called **ana con** ("analytic consequence") that can be used to justify lines that state such truths and are derived by inference steps. Note however, that **ana con** is not a genuine logical rule, but is rather a "reason" from informal logic. Thus, **ana con** is not acceptable in a strictly formal proof. It is unacceptable in homework unless the problem explicitly says you can use it.

It is useful to refer to some of the definitional truths and related consequences of relational predicates by citing the traditional name in mathematics of the relational property in question. Let R be a (two-place) relation.

R is *transitive* iff, if [(R(x,y) &R(y,z)) then R(x,z)]
R is *reflexive* iff, R(x,x)
R is *symmetric* iff, [R(x,y) iff R(y,x)]
R is asymmetric iff, [R(x,y)then notR(y,x)]
R is *antisymmetric* iff, [(R(x,y) & R(y,x)) then x=y]
R is *total* or *complete* iff, [either R(x,y) or R(y,x)]

Identity

Numerical identity, indicated by the sysmbol =, has special logical properties that have long been recognized both in logic and mathematics. Let P[*t*] be any sentence containing the (free) term *t* and let P[*s*] be like P[*t*] except that it contains *s* in one or more places wherever P[*s*] contains the (free) term *t*.

Self-Identity
t=*t* (Called = *introduction* in Fitch)

The Substitutivity of Identity
If *t*=*s* and P[*t*], then P[*s*] (Called = *elimination* in Fitch)