Resources Available in Informal Proofs

General Proof Strategies

Reduction to the Absurd. Assume the opposite of what it to be proved and deduce a falsehood.

Proof by Cases. Divide the subject matter into exhaustive cases and show that what is to be proved follows from each.

Analytic Truths

If something is true by definition or by the "meanings" of the predicates or relations in a formula, you can just assert it as true "by definition".

It is useful to refer to some of the definitional truths of relational predicates by citing the name of the property in question:

R is *transitive* $\leftrightarrow$ $(R(x,y) \wedge R(y,z)) \rightarrow R(x,z))$
R is *reflexive* $\leftrightarrow$ $R(x,x)$
R is *symmetric* $\leftrightarrow$ $R(x,y) \leftrightarrow R(y,x)$
R is asymmetric $\leftrightarrow$ $R(x,y) \rightarrow \neg R(y,x)$
R is *antisymmetric* $\leftrightarrow$ $(R(x,y) \wedge R(y,x)) \rightarrow x=y)$
R is *total* or *complete* $\leftrightarrow$ $R(x,y) \vee R(y,x)$

Truths of Propositonal Logic

Tautologies. You can just assert that any tautology is true, and refer the reader to truth-tables.

Substitution of Equivalents. If one proposition is equivalent (by truth-tables) to another you may just replace one by the other.

You can also just reformulate them citing their traditional names:

Double Negation $P \leftrightarrow \neg\neg P$	
DeMorgan's Laws	$\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$
	$\neg(P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$
Commutation	$(P \wedge Q) \leftrightarrow (Q \wedge P)$
	$(P \vee Q) \leftrightarrow (Q \vee P)$
Associativity	$(P \wedge (Q \wedge R)) \leftrightarrow ((P \wedge Q) \wedge R)$
	$(P \vee (Q \vee R)) \leftrightarrow ((P \vee Q) \vee R)$
Distribution	$(P \wedge (Q \vee R)) \leftrightarrow ((P \wedge Q) \vee (P \wedge R))$
	$(P \vee (Q \wedge R)) \leftrightarrow ((P \vee Q) \wedge (P \vee R))$
Idempotence	$P \leftrightarrow (P \wedge P) \leftrightarrow (P \vee P)$
Implication	$(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$
Equivalence	$(P \leftrightarrow Q) \leftrightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)) \leftrightarrow ((P \wedge Q) \vee (\neg P \wedge \neg Q))$

Laws of Identity

Self Identity. You can always just assert the law of self identity : x=x

Substitution. You can always substitute identities.

Quantifiers

Universal Instantiation. You can always instantiate a universal law for a particular case.

Universal Generalization. If one case is true and that case is "typical of everything", you may generalize to a universal law.

Existential Generalization. To show that there is some entity having a property, construct one.

Existential Instantiation. If you know there is something with that property, then within the course of your proof you can if it is helpful give that entity a name and proceed as if it has that name. But you must be sure that you have not already used that name for something else and that you eliminate the name from what you are proving by the time you reach the end of the proof.

Set Theory

Principle of Comprehension or Abstraction. A set, relation, or function exists if you can define or "construct" it.

Principle of Extensionality. Two sets are identical if they have the same members (i.e. each is a subset of the other).

Two relations are the same if they hold of the same relata.

Two functions are the same if they have the always have same value for the same argument.