THE COLLECTED WORKS OF THOMAS HOBBES

Collected and Edited by
SIR WILLIAM MOLESWORTH

Volume I

With a new Introduction by
Dr G.A.J. Rogers
University of Keele

Publisher's Note

INTRODUCTION

The English Works of Thomas Hobbes are dedicated
to the great scholar George Grote. It was Grote who
had first drawn Molesworth's attention to Hobbes and
had expressed the opinion that it was to be very much
regretted that Hobbes's works were so scarce, and so
much less read and studied than they deserved to be.

In part because of Molesworth's edition, first
published over 150 years ago, not only is Hobbes's
reputation secure but also his works are widely read.
Any reprint, therefore, of what is still the standard
English edition must be warmly welcomed. Further-
more, the completion of a full critical edition of
Hobbes's works by Oxford University Press is some
distance in the future and for many years to come the
standard reference to his English writings will remain
the Molesworth edition. For this reason alone it is an
indispensable work of reference for all Hobbes
scholars.

In his original edition Molesworth offered his
readers no account of the order in which Hobbes
wrote his works or how they fitted into his life. In this
Introduction, to meet that obvious gap, I give some
brief remarks under that head. In these I also indicate
the volume of this edition in which the particular
works mentioned may be found. In addition I make
some few comments on the state of contemporary
Hobbes scholarship. Finally, I have added a Contents
List for the eleven volumes, again something missing
from the original edition.

Outline of Hobbes's Life and Writings

Hobbes's father was a minor clergyman in the village of Westport, then just outside the Wiltshire town of Malmesbury. Hobbes was born there in 1588, and he tells us that his mother gave premature birth to him on rumour of the Spanish Armada approaching English shores ('fear and I were born twins!'). He revealed himself to be a clever boy at the local grammar school, mastering Latin and Greek and showing considerable ability as a translator of poetry as well as prose. These were skills that remained with him all his life. At the age of fourteen he went to Magdalen Hall in Oxford and was there for five years until his graduation in 1608. He was recommended as a suitable tutor to the son of the Earl of Devonshire and so began a connection with the Cavendish family which was to last, with some important breaks, for the whole of Hobbes's life.

From 1610 to 1615 Hobbes travelled on the continent with his pupil. This was the first of three such tours, the others following in 1628–31 and in 1634–37. On them he was to become acquainted with many of the leading intellectual figures of Europe, including Gassendi, Mersenne and Galileo. Back in England in 1614, he met through the Cavendish family Francis Bacon. He went to Bacon's estate at St Albans to act as his secretary and they often strolled through the gardens together deep in conversation. Meanwhile Hobbes continued with his classical studies which led in 1629 to the publication of his first work, a translation of Thucydides' *Eight Books of the Peloponnesian War* (in this edition, Vols VIII and IX).

It was on his second trip to the continent that Hobbes, according to his biographer, Aubrey, first discovered Euclid's geometry. He was so impressed by the logical power of Euclid's system that, so Aubrey

tells us, he fell 'in love with geometry', and it was to remain a dominant intellectual force, powerfully present in all his original writings, which he was now to begin to compose. The first of these to show this impact was a manuscript work of 1640, *The Elements of Law*, not published until 1889. For this reason it does not appear in the Molesworth edition in that form, though versions were to appear in 1649 and 1650 as *Human Nature* and *De Corpore Politico* (Vol. IV).

In November 1640 Hobbes fled from England to Paris, fearful of the hostility of parliamentary forces to the argument of *The Elements of Law*, manuscript copies of which were in circulation. He was to remain there throughout the Civil War and beyond, not returning to England until 1651.

In Paris Hobbes lived in a convent presided over by Mersenne and the latter invited him to write objections to Descartes' *Meditationes* which were to be published with the original. His contribution to the *Objectiones* (1641), written anonymously, were Hobbes's first published philosophical work. In the following year appeared *De Cive*, the third part of his planned trilogy, of which the first two were intended to be the already mentioned *De Corpore*, and *De Homine*. *De Cive* appeared in Hobbes's English translation in 1651 (Vol. II).

In Paris, as well as his political writings, and acting as mathematics tutor to the exiled Charles II, Hobbes was much engaged in natural science, especially optics, an enquiry which engaged him at many points in his life. He also wrote, in 1642–3 the long Latin manuscript, first published in 1973, which under its English title is known as *Thomas White's 'De Mundo' Examined*. In 1650 *Human Nature* (Vol. IV) was published in London, the second part of his philosophical system. At the same time he was

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completing his most famous work, *Leviathan* (Vol. III), which was published in London on his return in 1651. The storm which it engendered was to continue long after Hobbes's death.

One of the many controversial issues which *Leviathan* raised was the problem of free will. Hobbes's apparent commitment to some kind of determinism was the subject of private debate between himself and Bishop Bramhall. Hobbes's response to Bramhall, *Of Liberty and Necessity*, (Vol. IV; cf. also Vol. V) was published without Hobbes's consent in 1654, and led Bramhall and Hobbes into exchanges in print.

In 1655 he published what was intended as the first part of his comprehensive philosophical scheme, *De Corpore*, with an English translation, the *Elements of Philosophy* (Vol. I) in the following year. At about the same time Hobbes entered another controversy, this time with John Wallis and Seth Ward, the professors of mathematics at Oxford, over the possibility of squaring the circle (Vol. VII). It was a debate that Hobbes would have done well not to join. Meanwhile he was preparing *De Homine* for publication, and it duly appeared in 1658. A few years later he composed his account of the Civil War, *Behemoth* (Vol. VI), but the King forbade its publication. It appeared after Hobbes's death in 1680. In his last years, because he had nothing better to do, so he tells us, he translated Homer's *Iliad* and *Odyssey* (Vol. X). Hobbes died in 1679, at the age of 91, at the Cavendish Derbyshire home of Hardwick. He is buried in its church.

Contemporary Hobbes Scholarship

Interest in Hobbes has been growing rapidly in recent years. One of the most important aspects of that

interest is that is an expanding one, not only in the English-speaking world but throughout Europe and Japan.

There are a number of aspects to this strong surge. The first perhaps is the general rising interest in the history of philosophy. Secondly, there has been the new style in which the history of philosophy is increasingly being practised. Another factor has been the new translations of Hobbes's works and the publication of hitherto unprinted manuscripts, often in critical editions, which themselves both reflect and encourage interest in his work. Finally, and no doubt most important, we might note the actual quality and range of Hobbes's thought and its ever increasing recognition by scholars from many intellectual and cultural backgrounds.

The rising interest in the history of philosophy generally has many manifestations. Almost weekly there is information on publishers' new series which are either on the history of philosophy or which relate to that history. In the past few years many journals have begun with similar briefs, and others are promised.

It is inappropriate in this Introduction to attempt a detailed survey of publications which are specifically on Hobbes. However, we should notice that large publishing projects are on hand in several European countries to produce critical editions and translations of Hobbes's work, and that much of the best research on Hobbes's scientific writings as well as his political works has been and is being done in continental Europe, with France, Italy and Holland especially prominent. Although there is much to be done, the critical edition of Hobbes's *Works* already mentioned, of which the first two volumes have appeared, expresses a major commitment in Britain. We should

also note the speed of these developments. It was not until 1971, for example, that a French translation of *Leviathan* appeared. It remains the only one which compares the 1651 text with Hobbes's Latin version of 1668. Mention must also be made of the International Hobbes Association which does much to unite Hobbes scholarship around the world.

Much of this recent interest has been linked with a new conception of how the history of philosophy and intellectual history generally should be done. It is a style that first became prominent amongst historians of science in the late 1950s and 1960s. It came to be much more fully appreciated than hitherto that to understand a major intellectual work it was necessary to understand the context of its creation. Thus, to make sense of Galileo or Newton it was necessary to comprehend the complex intellectual forces both within and outside natural philosophy which shaped what is now thought of as the scientific revolution of the seventeenth century. Many of those interested in the history of philosophy took this message from the historians of science and the lesson learnt is now amply finding fruitful expression. Hobbes studies have been enormously enriched by this movement, undoubtedly aided by the fact that Hobbes himself so richly crosses the boundaries between science, philosophy, history, and political thought. There is every reason to suppose that this is a process set to continue for some time to come. The reissue of Molesworth's *The English Works of Thomas Hobbes* can only advance that enterprise.

G.A.J. Rogers
University of Keele
1992

CONTENTS

THE

ENGLISH WORKS
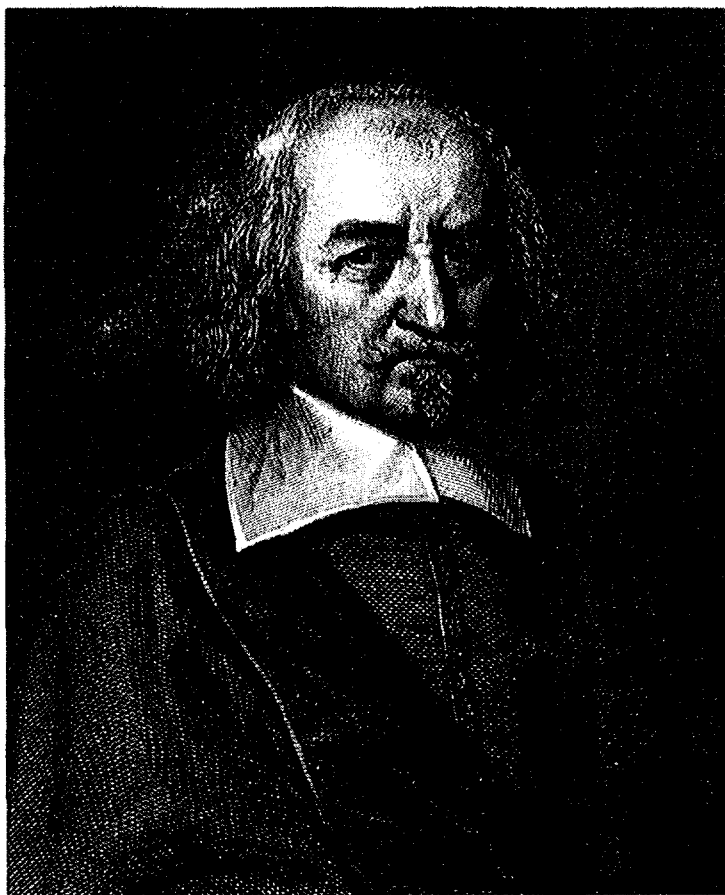
OF

THOMAS HOBBES.

THOMAS HOBBES.

THE

ENGLISH WORKS

OF

THOMAS HOBBES

OF MALMESBURY;

NOW FIRST COLLECTED AND EDITED

BY

SIR WILLIAM MOLESWORTH, BART.

VOL. I.

LONDON·

JOHN BOHN,

HENRIETTA STREET, COVENT GARDEN.

MDCCCXXXIX.

TO

GEORGE GROTE, ESQ.

M.P. FOR THE CITY OF LONDON.

―――――――

Dear Grote,

I dedicate to you this edition of
the Works of Hobbes ; first, because I know
you will be well pleased to see a complete
collection of all the writings of an Author
for whom you have so high an admiration.
Secondly, because I am indebted to you for
my first acquaintance with the speculations of
one of the greatest and most original thinkers
in the English language, whose works, I have
often heard you regret, were so scarce, and so
much less read and studied than they deserved
to be. It now, therefore, gives me great satis-

faction to be able to gratify a wish, you have frequently expressed, that some person, who had time and due reverence for that illustrious man, would undertake to edite his works, and bring his views again before his countrymen, who have so long and so unjustly neglected him. And likewise, I am desirous, in some way, to express the sincere regard and respect that I feel for you, and the gratitude that I owe you for the valuable instruction, that I have obtained from your society, and from the friendship with which you have honoured me, during the many years we have been companions in political life.

Yours, truly,

WILLIAM MOLESWORTH.

February 25th, 1839.
79, Eaton Square, London.

ELEMENTS OF PHILOSOPHY.

THE FIRST SECTION,

CONCERNING BODY,

WRITTEN IN LATIN

BY

THOMAS HOBBES OF MALMESBURY,

AND

TRANSLATED INTO ENGLISH.

TRANSLATOR TO THE READER.

IF, when I had finished my translation of this first section of the Elements of Philosophy, I had presently committed the same to the press, it might have come to your hands sooner than now it doth. But as I undertook it with much diffidence of my own ability to perform it well; so I thought fit, before I published it, to pray Mr. Hobbes to view, correct, and order it according to his own mind and pleasure. Wherefore, though you find some places enlarged, others altered, and two chapters, xviii and xx, almost wholly changed, you may nevertheless remain assured, that as now I present it to you, it doth not at all vary from the author's own sense and meaning. As for the Six Lessons to the Savilian Professors at Oxford, they are not of my translation, but were written, as here you have them in English, by Mr. Hobbes himself; and are joined to this book, because they are chiefly in defence of the same.*

* They will be published in a separate volume, with other works of a similar description. W. M.

THE AUTHOR'S EPISTLE DEDICATORY,

RIGHT HONORABLE, MY MOST HONORED LORD,

WILLIAM, EARL OF DEVONSHIRE.

THIS first section of the Elements of Philosophy, the monument of my service and your Lordship's bounty, though, after the Third Section published, long deferred, yet at last finished, I now present, my most excellent Lord, and dedicate to your Lordship. A little book, but full; and great enough, if men count well for great; and to an attentive reader versed in the demonstrations of mathematicians, that is, to your Lordship, clear and easy to understand, and almost new throughout, without any offensive novelty. I know that that part of philosophy, wherein are considered lines and figures, has been delivered to us notably improved by the ancients; and withal a most perfect pattern of the logic by which they were enabled to find out and demonstrate such excellent theorems as they have done. I know also that the

hypothesis of the earth's diurnal motion was the invention of the ancients; but that both it, and astronomy, that is, celestial physics, springing up together with it, were by succeeding philosophers strangled with the snares of words. And therefore the beginning of astronomy, except observations, I think is not to be derived from farther time than from Nicolaus Copernicus; who in the age next preceding the present revived the opinion of Pythagoras, Aristarchus, and Philolaus. After him, the doctrine of the motion of the earth being now received, and a difficult question thereupon arising concerning the descent of heavy bodies, Galileus in our time, striving with that difficulty, was the first that opened to us the gate of natural philosophy universal, which is the knowledge of the nature of *motion*. So that neither can the age of natural philosophy be reckoned higher than to him. Lastly, the science of *man's body*, the most profitable part of natural science, was first discovered with admirable sagacity by our countryman Doctor Harvey, principal Physician to King James and King Charles, in his books of the *Motion of the Blood*, and of the *Generation of Living Creatures;* who is the only man I know, that conquering envy, hath established a new doctrine in his life-time. Before these, there was nothing certain in natural philosophy.

but every man's experiments to himself, and the
natural histories, if they may be called certain, that
are no certainer than civil histories.   But since these,
astronomy and natural philosophy in general have,
for so little time, been extraordinarily advanced by
Joannes Keplerus, Petrus Gassendus, and Marinus
Mersennus; and the science of human bodies in
special by the wit and industry of physicians, the
only true natural philosophers, especially of our most
learned men of the College of Physicians in London.
Natuial Philosophy is therefore but young; but
Civil Philosophy yet much younger, as being no older
(I say it provoked, and that my detractors may know
how little they have wrought upon me) than my own
book *De Cive*.   But what? were there no philoso-
phers natural nor civil among the ancient Greeks?
There were men so called; witness Lucian, by whom
they are derided; witness divers cities, from which
they have been often by public edicts banished.   But
it follows not that there was *philosophy*.   There
walked in old Greece a certain phantasm, for super-
ficial gravity, though full within of fraud and filth, a
little like philosophy; which unwary men, thinking
to be it, adhered to the professors of it, some to one,
some to another, though they disagreed among them-
selves, and with great salary put their children to

them to be taught, instead of wisdom, nothing but
to dispute, and, neglecting the laws, to determine
every question according to their own fancies. The
first doctors of the Church, next the Apostles, born
in those times, whilst they endeavoured to defend
the Christian faith against the Gentiles by natural
reason, began also to make use of philosophy, and
with the decrees of Holy Scripture to mingle the
sentences of heathen philosophers; and first some
harmless ones of Plato, but afterwards also many
foolish and false ones out of the physics and meta-
physics of Aristotle; and bringing in the enemies,
betrayed unto them the citadel of Christianity. From
that time, instead of the worship of God, there entered
a thing called *school divinity*, walking on one foot
firmly, which is the Holy Scripture, but halted on
the other rotten foot, which the Apostle Paul called
vain, and might have called *pernicious philosophy*;
for it hath raised an infinite number of controversies
in the Christian world concerning religion, and from
those controversies, wars. It is like that *Empusa* in
the Athenian comic poet, which was taken in Athens
for a ghost that changed shapes, having one brazen
leg, but the other was the leg of an ass, and was sent,
as was believed, by Hecate, as a sign of some ap-
proaching evil fortune. Against this *Empusa* I think

there cannot be invented a better exorcism, than to distinguish between the rules of religion, that is, the rules of honouring God, which we have from the laws, and the rules of philosophy, that is, the opinions of private men; and to yield what is due to religion to the Holy Scripture, and what is due to philosophy to natural reason. And this I shall do, if I but handle the Elements of Philosophy truly and clearly, as I endeavour to do. Therefore having in the Third Section, which I have published and dedicated to your Lordship, long since reduced all power ecclesiastical and civil by strong arguments of reason, without repugnance to God's word, to one and the same sovereign authority; I intend now, by putting into a clear method the true foundations of natural philosophy, to fright and drive away this metaphysical *Empusa* ; not by skirmish, but by letting in the light upon her. For I am confident, if any confidence of a writing can proceed from the writer's fear, circumspection, and diffidence, that in the three former parts of this book all that I have said is sufficiently demonstrated from definitions; and all in the fourth part from suppositions not absurd. But if there appear to your Lordship anything less fully demonstrated than to satisfy every reader, the cause was this, that I professed to write not all to

all, but some things to geometricians only.   But that your Lordship will be satisfied, I cannot doubt.

There remains the second section, which is concerning *Man*.   That part thereof, where I handle the *Optics*, containing six chapters, together with the tables of the figures belonging to them, I have already written and engraven lying by me above these six years.   The rest shall, as soon as I can, be added to it; though by the contumelies and petty injuries of some unskilful men, I know already, by experience, how much greater thanks will be due than paid me, for telling men the truth of what men are.   But the burthen I have taken on me I mean to carry through; not striving to appease, but rather to revenge myself of envy, by encreasing it.   For it contents me that I have your Lordship's favour, which, being all you require, I acknowledge ; and for which, with my prayers to Almighty God for your Lordship's safety, I shall, to my power, be always thankful.

Your Lordship's most humble servant,

THOMAS HOBBES.

London,
April 23, 1655.

To the Right Honorable
my very good Lord
ye Earle of Devonshire

London

Right Honorable and my very good Lord.

I haue seen the Nottinghamshire petition against
BBs. In it there are reckoned vp abondance of
abuses committed by Ecclesiasticall persons and their
Officers, wch can neyther be denyed nor excused. But
that they proceed from ye Episcopacy it selfe, is not
so euidently proued. Howeuer since the Contentiousnes
and Superstitious behauior of th persons, haue made
the people weary of that forme, I see nothing to be
misliked in the new way propounded. If it displease
any that there are to be so many Lay Commissioners
for church matters and so few ministers, I thinke
it will be those that haue most desired the change
and made account to haue the Episcopall authority
diuided amongst them. I am of the opinion, that
Ministers ought to minister rather then gouerne, at
least that all Church gouernment will depend on the
state, and authority of the kingdom, wthout wch there
can be no vnity in ye church. Your Lop may perhaps
thinke this opinion, but a fancy of Philosophy. but I am
sure that Experience teaches, thus much, that the
dispute for [power?] betwext the spirituall and ciuill powr, has
of late more then any other thing in the world, bene
the cause of ciuill warrs, in all places of Christendom.

[left margin, rotated text:]
Your most humble and most
obedient seruant
W. Hobbes
to th Earle of Deuonshire

from July 23
Aug. 2. 1641.

AUTHOR'S EPISTLE TO THE READER.

THINK not, Courteous Reader, that the philosophy, the elements whereof I am going to set in order, is that which makes philosophers' stones, nor that which is found in the metaphysic codes; but that it is the natural reason of man, busily flying up and down among the creatures, and bringing back a true report of their order, causes and effects. Philosophy, therefore, the child of the world and your own mind, is within yourself; perhaps not fashioned yet, but like the world its father, as it was in the beginning, a thing confused. Do, therefore, as the statuaries do, who, by hewing off that which is superfluous, do not make but find the image. Or imitate the creation: if you will be a philosopher in good earnest, let your reason move upon the deep of your own cogitations and experience; those things that lie in confusion must be set asunder, distinguished, and every one stamped with its own name set in order; that is to say, your method must resemble that of the creation. The order of the creation was, *light, distinction of day and night,* the *firmament,* the *luminaries, sensible creatures, man ;* and, after the creation, the *commandment.* Therefore the order of contemplation will be, *reason, definition, space,* the *stars, sensible quality, man ;* and after man is grown up, *subjection to command.* In the first part of this section, which is entitled Logic, I set up the light of reason. In the second, which hath for title

the Grounds of Philosophy, I distinguish the most common notions by accurate definition, for the avoiding of confusion and obscurity. The third part concerns the expansion of space, that is Geometry. The fourth contains the Motion of the Stars, together with the doctrine of sensible qualities.

In the second section, if it please God, shall be handled *Man*. In the third section, the doctrine of *Subjection* is handled already. This is the method I followed; and if it like you, you may use the same; for I do but propound, not commend to you anything of mine. But whatsoever shall be the method you will like, I would very fain commend philosophy to you, that is to say, the study of wisdom, for want of which we have all suffered much damage lately. For even they, that study wealth, do it out of love to wisdom ; for their treasures serve them but for a looking-glass, wherein to behold and contemplate their own wisdom. Nor do they, that love to be employed in public business, aim at anything but place wherein to show their wisdom. Neither do voluptuous men neglect philosophy, but only because they know not how great a pleasure it is to the mind of man to be ravished in the vigorous and perpetual embraces of the most beauteous world. Lastly, though for nothing else, yet because the mind of man is no less impatient of empty time than nature is of empty place, to the end you be not forced for want of what to do, to be troublesome to men that have business, or take hurt by falling into idle company, but have somewhat of your own wherewith to fill up your time, I recommend unto you to study philosophy. Farewell.

T. H.

TITLES OF THE CHAPTERS.

PART FIRST,

OR LOGIC.

PART SECOND,

OR THE FIRST GROUNDS OF PHILOSOPHY.

PART THIRD,

OF THE PROPORTIONS OF MOTIONS AND MAGNITUDES.

PART FOURTH,

OF PHYSICS, OR THE PHENOMENA OF NATURE.

COMPUTATION OR LOGIC.

CHAPTER I.

OF PHILOSOPHY.

PHILOSOPHY seems to me to be amongst men now, in the same manner as corn and wine are said to have been in the world in ancient time. For from the beginning there were vines and ears of corn growing here and there in the fields; but no care was taken for the planting and sowing of them. Men lived therefore upon acorns; or if any were so bold as to venture upon the eating of those unknown and doubtful fruits, they did it with danger of their health. In like manner, every man brought Philosophy, that is, Natural Reason, into the world with him; for all men can reason to some degree, and concerning some things: but where there is need of a long series of reasons, there most men wander out of the way, and fall into error for want of method, as it were for want

of sowing and planting, that is, of improving their reason. And from hence it comes to pass, that they who content themselves with daily experience, which may be likened to feeding upon acorns, and either reject, or not much regard philosophy, are commonly esteemed, and are, indeed, men of sounder judgment than those who, from opinions, though not vulgar, yet full of uncertainty, and carelessly received, do nothing but dispute and wrangle, like men that are not well in their wits. I confess, indeed, that that part of philosophy by which magnitudes and figures are computed, is highly improved. But because I have not observed the like advancement in the other parts of it, my purpose is, as far forth as I am able, to lay open the few and first Elements of Philosophy in general, as so many seeds from which pure and true Philosophy may hereafter spring up by little and little.

I am not ignorant how hard a thing it is to weed out of men's minds such inveterate opinions as have taken root there, and been confirmed in them by the authority of most eloquent writers; especially seeing true (that is, accurate) Philosophy professedly rejects not only the paint and false colours of language, but even the very ornaments and graces of the same; and the first grounds of all science are not only not beautiful, but poor, arid, and, in appearance, deformed. Nevertheless, there being certainly some men, though but few, who are delighted with truth and strength of reason in all things, I thought I might do well to take this pains for the sake even of those few. I proceed therefore to the matter, and take my beginning

from the very definition of philosophy, which is PART I.
this.   1.

2. PHILOSOPHY *is such knowledge of effects or* Definition of Philosophy
appearances, as we acquire by true ratiocination explained.
from the knowledge we have first of their causes
or generation: And again, of such causes or gene-
rations as may be from knowing first their effects.

For the better understanding of which definition,
we must consider, first, that although Sense and
Memory of things, which are common to man and
all living creatures, be knowledge, yet because they
are given us immediately by nature, and not gotten
by ratiocination, they are not philosophy.

Secondly, seeing Experience is nothing but me-
mory; and Prudence, or prospect into the future
time, nothing but expectation of such things as
we have already had experience of, Prudence also
is not to be esteemed philosophy.

By RATIOCINATION, I mean *computation.* Now
to compute, is either to collect the sum of many
things that are added together, or to know what
remains when one thing is taken out of another.
Ratiocination, therefore, is the same with *addition*
and *substraction;* and if any man add *multiplica-*
tion and *division,* I will not be against it, seeing
multiplication is nothing but addition of equals one
to another, and division nothing but a substraction
of equals one from another, as often as is possible.
So that all ratiocination is comprehended in these
two operations of the mind, addition and substrac-
tion.

3. But how by the *ratiocination* of our mind, Ratiocination of the Mind.
we add and substract in our silent thoughts, with-
out the use of words, it will be necessary for me

to make intelligible by an example or two. If
therefore a man see something afar off and ob-
scurely, although no appellation had yet been given
to anything, he will, notwithstanding, have the
same idea of that thing for which now, by im-
posing a name on it, we call it *body*. Again, when,
by coming nearer, he sees the same thing thus and
thus, now in one place and now in another, he
will have a new idea thereof, namely, that for
which we now call such a thing *animated*. Thirdly,
when standing nearer, he perceives the figure,
hears the voice, and sees other things which are
signs of a rational mind, he has a third idea,
though it have yet no appellation, namely, that for
which we now call anything *rational*. Lastly,
when, by looking fully and distinctly upon it, he
conceives all that he has seen as one thing, the
idea he has now is compounded of his former ideas,
which are put together in the mind in the same
order in which these three single names, *body*,
animated, *rational*, are in speech compounded into
this one name, *body-animated-rational*, or *man*.
In like manner, of the several conceptions of *four
sides, equality of sides, and right angles,* is com-
pounded the conception of a *square*. For the
mind may conceive a figure of four sides without
any conception of their equality, and of that equa-
lity without conceiving a right angle; and may
join together all these single conceptions into one
conception or one idea of a square. And thus we
see how the conceptions of the mind are com-
pounded. Again, whosoever sees a man standing
near him, conceives the whole idea of that man;
and if, as he goes away, he follow him with his

eyes only, he will lose the idea of those things which were signs of his being rational, whilst, nevertheless, the idea of a body-animated remains still before his eyes, so that the idea of rational is substracted from the whole idea of man, that is to say, of body-animated-rational, and there remains that of body-animated; and a while after, at a greater distance, the idea of animated will be lost, and that of body only will remain ; so that at last, when nothing at all can be seen, the whole idea will vanish out of sight. By which examples, I think, it is manifest enough what is the internal ratiocination of the mind without words.

We must not therefore think that computation, that is, ratiocination, has place only in numbers, as if man were distinguished from other living creatures (which is said to have been the opinion of *Pythagoras*) by nothing but the faculty of numbering; for *magnitude, body, motion, time, degrees of quality, action, conception, proportion, speech and names* (in which all the kinds of philosophy consist) are capable of addition and substraction. Now such things as we add or substract, that is, which we put into an account, we are said to *consider*, in Greek λογίζεσθαι, in which language also συλλογίζεσθαι signifies to *compute, reason,* or *reckon.*

4. But *effects* and the *appearances* of things to sense, are faculties or powers of bodies, which make us distinguish them from one another ; that is to say, conceive one body to be equal or unequal, like or unlike to another body ; as in the example above, when by coming near enough to any body, we perceive the motion and going of the same, we distinguish it thereby from a tree, a

Properties, what they are.

column, and other fixed bodies ; and so that motion or going is the *property* thereof, as being proper to living creatures, and a faculty by which they make us distinguish them from other bodies.

5. How the knowledge of any effect may be gotten from the knowledge of the generation thereof, may easily be understood by the example of a circle : for if there be set before us a plain figure, having, as near as may be, the figure of a circle, we cannot possibly perceive by sense whether it be a true circle or no ; than which, nevertheless, nothing is more easy to be known to him that knows first the generation of the propounded figure. For let it be known that the figure was made by the circumduction of a body whereof one end remained unmoved, and we may reason thus ; a body carried about, retaining always the same length, applies itself first to one *radius*, then to another, to a third, a fourth, and successively to all ; and, therefore, the same length, from the same point, toucheth the circumference in every part thereof, which is as much as to say, as all the *radii* are equal. We know, therefore, that from such generation proceeds a figure, from whose one middle point all the extreme points are reached unto by equal *radii*. And in like manner, by knowing first what figure is set before us, we may come by ratiocination to some generation of the same, though perhaps not that by which it was made, yet that by which it might have been made ; for he that knows that a circle has the property above declared, will easily know whether a body carried about, as is said, will generate a circle or no.

6. The *end* or *scope* of philosophy is, that we may make use to our benefit of effects formerly seen; or that, by application of bodies to one another, we may produce the like effects of those we conceive in our mind, as far forth as matter, strength, and industry, will permit, for the commodity of human life. For the inward glory and triumph of mind that a man may have for the mastering of some difficult and doubtful matter, or for the discovery of some hidden truth, is not worth so much pains as the study of Philosophy requires; nor need any man care much to teach another what he knows himself, if he think that will be the only benefit of his labour. The end of knowledge is power; and the use of theorems (which, among geometricians, serve for the finding out of properties) is for the construction of problems; and, lastly, the scope of all speculation is the performing of some action, or thing to be done.

7. But what the *utility* of philosophy is, especially of natural philosophy and geometry, will be best understood by reckoning up the chief commodities of which mankind is capable, and by comparing the manner of life of such as enjoy them, with that of others which want the same. Now, the greatest commodities of mankind are the arts; namely, of measuring matter and motion; of moving ponderous bodies; of architecture; of navigation; of making instruments for all uses; of calculating the celestial motions, the aspects of the stars, and the parts of time; of geography, &c. By which sciences, how great benefits men receive is more easily understood than expressed. These benefits are enjoyed by almost all the people of

Europe, by most of those of Asia, and by some of Africa : but the Americans, and they that live near the Poles, do totally want them. But why ? Have they sharper wits than these ? Have not all men one kind of soul, and the same faculties of mind ? What, then, makes this difference, except philosophy ? Philosophy, therefore, is the cause of all these benefits. But the utility of moral and civil philosophy is to be estimated, not so much by the commodities we have by knowing these sciences, as by the calamities we receive from not knowing them. Now, all such calamities as may be avoided by human industry, arise from war, but chiefly from civil war ; for from this proceed slaughter, solitude, and the want of all things. But the cause of war is not that men are willing to have it ; for the will has nothing for object but good, at least that which seemeth good. Nor is it from this, that men know not that the effects of war are evil ; for who is there that thinks not poverty and loss of life to be great evils ? The cause, therefore, of civil war is, that men know not the causes neither of war nor peace, there being but few in the world that have learned those duties which unite and keep men in peace, that is to say, that have learned the rules of civil life sufficiently. Now, the knowledge of these rules is moral philosophy. But why have they not learned them, unless for this reason, that none hitherto have taught them in a clear and exact method ? For what shall we say ? Could the ancient masters of Greece, Egypt, Rome, and others, persuade the unskilful multitude to their innumerable opinions concerning the nature of their gods, which they

themselves knew not whether they were true or false, and which were indeed manifestly false and absurd; and could they not persuade the same multitude to civil duty, if they themselves had understood it? Or shall those few writings of geometricians which are extant, be thought sufficient for the taking away of all controversy in the matters they treat of, and shall those innumerable and huge volumes of *ethics* be thought unsufficient, if what they teach had been certain and well demonstrated? What, then, can be imagined to be the cause that the writings of those men have increased science, and the writings of these have increased nothing but words, saving that the former were written by men that knew, and the latter by such as knew not, the doctrine they taught only for ostentation of their wit and eloquence? Nevertheless, I deny not but the reading of some such books is very delightful; for they are most eloquently written, and contain many clear, wholesome and choice sentences, which yet are not universally true, though by them universally pronounced. From whence it comes to pass, that the circumstances of times, places, and persons being changed, they are no less frequently made use of to confirm wicked men in their purposes, than to make them understand the precepts of civil duties. Now that which is chiefly wanting in them, is a true and certain rule of our actions, by which we might know whether that we undertake be just or unjust. For it is to no purpose to be bidden in every thing to do right, before there be a certain rule and measure of right established, which no man hitherto hath established. Seeing,

therefore, from the not knowing of civil duties, that is, from the want of moral science, proceed civil wars, and the greatest calamities of mankind, we may very well attribute to such science the production of the contrary commodities. And thus much is sufficient, to say nothing of the praises and other contentment proceeding from philosophy, to let you see the utility of the same in every kind thereof.

Subject of
Philosophy.

8. The *subject* of Philosophy, or the matter it treats of, is every body of which we can conceive any generation, and which we may, by any consideration thereof, compare with other bodies, or which is capable of composition and resolution; that is to say, every body of whose generation or properties we can have any knowledge. And this may be deduced from the definition of philosophy, whose profession it is to search out the properties of bodies from their generation, or their generation from their properties; and, therefore, where there is no generation or property, there is no philosophy. Therefore it excludes *Theology*, I mean the doctrine of God, eternal, ingenerable, incomprehensible, and in whom there is nothing neither to divide nor compound, nor any generation to be conceived.

It excludes the doctrine of *angels*, and all such things as are thought to be neither bodies nor properties of bodies; there being in them no place neither for composition nor division, nor any capacity of more and less, that is to say, no place for ratiocination.

It excludes *history*, as well *natural* as *political*, though most useful (nay necessary) to philosophy;

because such knowledge is but experience, or authority, and not ratiocination.

It excludes all such knowledge as is acquired by Divine inspiration, or revelation, as not derived to us by reason, but by Divine grace in an instant, and, as it were, by some sense supernatural.

It excludes not only all doctrines which are false, but such also as are not well-grounded ; for whatsoever we know by right ratiocination, can neither be false nor doubtful ; and, therefore, *astrology*, as it is now held forth, and all such divinations rather than sciences, are excluded.

Lastly, the doctrine of *God's worship* is excluded from philosophy, as being not to be known by natural reason, but by the authority of the Church ; and as being the object of faith, and not of knowledge.

9. The principal parts of philosophy are two. Parts of
Philosophy. For two chief kinds of bodies, and very different from one another, offer themselves to such as search after their generation and properties ; one whereof being the work of nature, is called a *natural body*, the other is called a *commonwealth*, and is made by the wills and agreement of men. And from these spring the two parts of philosophy, called *natural* and *civil*. But seeing that, for the knowledge of the properties of a commonwealth, it is necessary first to know the dispositions, affections, and manners of men, civil philosophy is again commonly divided into two parts, whereof one, which treats of men's dispositions and manners, is called *ethics ;* and the other, which takes cognizance of their civil duties, is called *politics,* or simply *civil philosophy.* In the first place, there-

fore (after I have set down such premises as appertain to the nature of philosophy in general), I will discourse of *bodies natural ;* in the second, of the *dispositions and manners of men ;* and in the third, of the *civil duties of subjects.*

Epilogue.

10. To conclude; seeing there may be many who will not like this my definition of philosophy, and will say, that, from the liberty which a man may take of so defining as seems best to himself, he may conclude any thing from any thing (though I think it no hard matter to demonstrate that this definition of mine agrees with the sense of all men); yet, lest in this point there should be any cause of dispute betwixt me and them, I here undertake no more than to deliver the elements of that science by which the effects of anything may be found out from the known generation of the same, or contrarily, the generation from the effects; to the end that they who search after other philosophy, may be admonished to seek it from other principles.

CHAPTER II.

OF NAMES.

1. The necessity of sensible Moniments or Marks for the help of Memory : a Mark defined.—2. The necessity of Marks for the signification of the conceptions of the Mind.—3. Names supply both those necessities.—4. The Definition of a Name.— 5. Names are Signs not of Things, but of our Cogitations.— 6. What it is we give Names to.—7. Names Positive and Negative.—8. Contradictory Names.—9. A Common Name.— 10. Names of the First and Second Intention.—11. Universal, Particular, Individual, and Indefinite Names.—12. Names Univocal and Equivocal.—13. Absolute and Relative Names.— 14. Simple and Compounded Names.—15. A Predicament described.—16. Some things to be noted concerning Predicaments.

1. How unconstant and fading men's thoughts are, and how much the recovery of them depends upon chance, there is none but knows by infallible experience in himself. For no man is able to remember quantities without sensible and present measures, nor colours without sensible and present patterns, nor number without the names of numbers disposed in order and learned by heart. So that whatsoever a man has put together in his mind by ratiocination without such helps, will presently slip from him, and not be revocable but by beginning his ratiocination anew. From which it follows, that, for the acquiring of philosophy, some sensible moniments are necessary, by which our past thoughts may be not only reduced, but

PART I.
2.

Necessity of sensible Moniments or Marks for the help of Memory.

also registered every one in its own order.   These moniments I call MARKS, namely, sensible things taken at pleasure, that, by the sense of them, such thoughts may be recalled to our mind as are like those thoughts for which we took them.

Necessity of
Marks for the
signification of
the conceptions
of the Mind.

2. Again, though some one man, of how excellent a wit soever, should spend all his time partly in reasoning, and partly in inventing marks for the help of his memory, and advancing himself in learning ; who sees not that the benefit he reaps to himself will not be much, and to others none at all?   For unless he communicate his notes with others, his science will perish with him.   But if the same notes be made common to many, and so one man's inventions be taught to others, sciences will thereby be increased to the general good of mankind.   It is therefore necessary, for the acquiring of philosophy, that there be certain signs, by which what one man finds out may be manifested and made known to others.   Now, those things we call SIGNS are the *antecedents of their consequents, and the consequents of their antecedents, as often as we observe them to go before or follow after in the same manner.*   For example, a thick cloud is a sign of rain to follow, and rain a sign that a cloud has gone before, for this reason only, that we seldom see clouds without the consequence of rain, nor rain at any time but when a cloud has gone before.   And of signs, some are *natural*, whereof I have already given an example, others are *arbitrary*, namely, those we make choice of at our own pleasure, as a bush hung up, signifies that wine is to be sold there ; a stone set in the ground signifies the bound of a field ; and

words so and so connected, signify the cogitations
and motions of our mind. The difference, there-
fore, betwixt marks and signs is this, that we
make those for our own use, but these for the use
of others.

3. Words so connected as that they become
signs of our thoughts, are called SPEECH, of which
every part is a *name*. But seeing (as is said) both
marks and signs are necessary for the acquiring of
philosophy, (marks by which we may remember
our own thoughts, and signs by which we may
make our thoughts known to others), names do
both these offices; but they serve for marks before
they be used as signs. For though a man were
alone in the world, they would be useful to him
in helping him to remember; but to teach others,
(unless there were some others to be taught) of
no use at all. Again, names, though standing
singly by themselves, are marks, because they serve
to recal our own thoughts to mind; but they can-
not be signs, otherwise than by being disposed and
ordered in speech as parts of the same. For ex-
ample, a man may begin with a word, whereby the
hearer may frame an idea of something in his
mind, which, nevertheless, he cannot conceive to
be the idea which was in the mind of him that
spake, but that he would say something which
began with that word, though perhaps not as by
itself, but as part of another word. So that the
nature of a name consists principally in this, that
it is a mark taken for memory's sake; but it serves
also by accident to signify and make known to
others what we remember ourselves, and, therefore,
I will define it thus:

4. *A* NAME *is a word taken at pleasure to serve for a mark, which may raise in our mind a thought like to some thought we had before, and which being pronounced to others, may be to them a sign of what thought the speaker had, or had not before in his mind.* And it is for brevity's sake that I suppose the original of names to be arbitrary, judging it a thing that may be assumed as unquestionable. For considering that new names are daily made, and old ones laid aside; that diverse nations use different names, and how impossible it is either to observe similitude, or make any comparison betwixt a name and a thing, how can any man imagine that the names of things were imposed from their natures? For though some names of living creatures and other things, which our first parents used, were taught by God himself; yet they were by him arbitrarily imposed, and afterwards, both at the Tower of Babel, and since, in process of time, growing everywhere out of use, are quite forgotten, and in their room have succeeded others, invented and received by men at pleasure. Moreover, whatsoever the common use of words be, yet philosophers, who were to teach their knowledge to others, had always the liberty, and sometimes they both had and will have a necessity, of taking to themselves such names as they please for the signifying of their meaning, if they would have it understood. Nor had mathematicians need to ask leave of any but themselves to name the figures they invented, *parabolas, hyperboles, cissoeides, quadratices,* &c. or to call one magnitude A, another B.

5. But seeing names ordered in speech (as is defined) are signs of our conceptions, it is manifest they are not signs of the things themselves; for that the sound of this word *stone* should be the sign of a stone, cannot be understood in any sense but this, that he that hears it collects that he that pronounces it thinks of a stone. And, therefore, that disputation, whether names signify the matter or form, or something compounded of both, and other like subtleties of the *metaphysics*, is kept up by erring men, and such as understand not the words they dispute about.

6. Nor, indeed, is it at all necessary that every name should be the name of something. For as these, a *man*, a *tree*, a *stone*, are the names of the things themselves, so the images of a man, of a tree, and of a stone, which are represented to men sleeping, have their names also, though they be not things, but only fictions and phantasms of things. For we can remember these; and, therefore, it is no less necessary that they have names to mark and signify them, than the things themselves. Also this word *future* is a name, but no future thing has yet any being, nor do we know whether that which we call future, shall ever have a being or no. Nevertheless, seeing we use in our mind to knit together things past with those that are present, the name *future* serves to signify such knitting together. Moreover, that which neither is, nor has been, nor ever shall, or ever can be, has a name, namely, *that which neither is nor has been*, &c.; or more briefly this, *impossible*. To conclude; this word *nothing* is a name, which yet cannot be the name of any thing: for when, for

example, we substract 2 and 3 from 5, and so
nothing remaining, we would call that substrac-
tion to mind, this speech *nothing remains,* and in
it the word *nothing* is not unuseful.   And for the
same reason we say truly, *less than nothing* re-
mains, when we substract more from less; for the
mind feigns such remains as these for doctrine's
sake, and desires, as often as is necessary, to call
the same to memory.   But seeing every name has
some relation to that which is named, though that
which we name be not always a thing that has a
being in nature, yet it is lawful for doctrine's sake
to apply the word *thing* to whatsoever we name ;
as if it were all one whether that thing be truly
existent, or be only feigned.

Names Positive
and Negative.

7. The first distinction of names is, that some
are *positive,* or *affirmative,* others *negative,* which
are also called *privative* and *indefinite.   Positive*
are such as we impose for the likeness, equality,
or identity of the things we consider; *negative,*
for the diversity, unlikeness, or inequality of the
same.   Examples of the former are, *a man, a
philosopher ;* for a *man* denotes any one of a
multitude of men, and *a philosopher,* any one of
many philosophers, by reason of their similitude ;
also, *Socrates* is a positive name, because it sig-
nifies always one and the same man.   Examples of
negatives are such positives as have the negative
particle *not* added to them, as *not-man, not-
philosopher.*   But positives were before negatives ;
for otherwise there could have been no use at all
of these.   For when the name of *white* was
imposed upon certain things, and afterwards upon
other things, the names of *black, blue, trans-*

parent, &c. the infinite dissimilitudes of these
with *white* could not be comprehended in any one name, save that which had in it the negation of white, that is to say, the name *not-white*, or some other equivalent to it, in which the word *white* is repeated, such as *unlike to white, &c.* And by these *negative names*, we take notice ourselves, and signify to others what we have not thought of.

8. Positive and negative names are *contra-* Contradictory *dictory* to one another, so that they cannot both names. be the name of the same thing. Besides, of contradictory names, one is the name of anything whatsoever ; for whatsoever is, is either man, or not-man, white or not-white, and so of the rest. And this is so manifest, that it needs no farther proof or explication ; for they that say *the same thing cannot both be, and not be,* speak obscurely; but they that say, *whatsoever is, either is, or is not,* speak also absurdly and ridiculously. The certainty of this axiom, viz. *of two contradictory names, one is the name of anything whatsoever, the other not,* is the original and foundation of all ratiocination, that is, of all philosophy ; and therefore it ought to be so exactly propounded, that it may be of itself clear and perspicuous to all men ; as indeed it is, saving to such, as reading the long discourses made upon this subject by the writers of *metaphysics* (which they believe to be some egregious learning) think they understand not, when they do.

9. Secondly, of names, some are *common* to A common many things, as a *man, a tree ;* others *proper* to name. one thing, as *he that writ the Iliad, Homer, this man, that man.* And a common name, being the

name of many things severally taken, but not collectively of all together (as man is not the name of all mankind, but of every one, as of Peter, John, and the rest severally) is therefore called an *universal name ;* and therefore this word *universal* is never the name of any thing existent in nature, nor of any idea or phantasm formed in the mind, but always the name of some word or name; so that when *a living creature, a stone, a spirit*, or any other thing, is said to be *universal*, it is not to be understood, that any man, stone, &c. ever was or can be universal, but only that these words, *living creature, stone, &c.* are *universal names*, that is, names common to many things ; and the conceptions answering them in our mind, are the images and phantasms of several living creatures, or other things. And therefore, for the understanding of the extent of an universal name, we need no other faculty but that of our imagination, by which we remember that such names bring sometimes one thing, sometimes another, into our mind. Also of common names, some are more, some less common. *More common*, is that which is the name of more things; *less common*, the name of fewer things; as *living creature* is more common than *man,* or *horse*, or *lion*, because it comprehends them all : and therefore a more common name, in respect of a less common, is called the *genus*, or a *general name ;* and this in respect of that, the *species*, or a *special name.*

Names of the first and second intention.

10. And from hence proceeds the third distinction of names, which is, that some are called names of the *first*, others of the *second intention.*

Of the *first intention* are the names of things, PART I. a *man, stone, &c.*: of the *second* are the names 2. of names and speeches, as *universal, particular, genus, species, syllogism*, and the like. But it is hard to say why those are called names of the *first*, and these of the *second intention*, unless perhaps it was first intended by us to give names to those things which are of daily use in this life, and afterwards to such things as appertain to science, that is, that our second intention was to give names to names. But whatsoever the cause hereof may be, yet this is manifest, that *genus, species, definition, &c.* are names of words and names only; and therefore to put *genus* and *species* for things, and *definition* for the nature of any thing, as the writers of *metaphysics* have done, is not right, seeing they be only significations of what we think of the nature of things.

11. Fourthly, some names are of *certain* and Universal, *determined*, others of *uncertain* and *undetermined* particular, individual, signification. Of *determined* and *certain* signifi- and indefinite cation is, first, that name which is given to any names. one thing by itself, and is called an *individual name*; as *Homer, this tree, that living creature*, &c. Secondly that which has any of these words, *all, every, both, either*, or the like added to it; and it is therefore called an universal name, because it signifies every one of those things to which it is common; and of *certain* signification for this reason, that he which hears, conceives in his mind the same thing that he which speaks would have him conceive. Of *indefinite* significa- tion is, first, that name which has the word *some*, or the like added to it, and is called a *particular*

name ; secondly, a common name set by itself without any note either of universality or particularity, as *man, stone*, and is called an *indefinite name ;* but both *particular* and *indefinite* names are of uncertain signification, because the hearer knows not what thing it is the speaker would have him conceive ; and therefore in speech, particular and indefinite names are to be esteemed equivalent to one another.   But these words, *all, every, some, &c.* which denote universality and particularity, are not names, but parts only of names ; so that *every man*, and *that man which the hearer conceives in his mind*, are all one ; and *some man*, and *that man which the speaker thought of*, signify the same.   From whence it is evident, that the use of signs of this kind, is not for a man's own sake, or for his getting of knowledge by his own private meditation (for every man has his own thoughts sufficiently determined without such helps as these) but for the sake of others ; that is, for the teaching and signifying of our conceptions to others ; nor were they invented only to make us remember, but to make us able to discourse with others.

12. Fifthly, names are usually distinguished into *univocal* and *equivocal*.   *Univocal* are those which in the same train of discourse signify always the same thing ; but *equivocal* those which mean sometimes one thing and sometimes another. Thus, the name *triangle* is said to be *univocal*, because it is always taken in the same sense ; and *parabola* to be *equivocal*, for the signification it has sometimes of allegory or similitude, and sometimes of a certain geometrical figure.   Also every

metaphor is by profession *equivocal*. But this distinction belongs not so much to names, as to those that use names, for some use them properly and accurately for the finding out of truth ; others draw them from their proper sense, for ornament or deceit.

13. Sixthly, of names, some are *absolute*, others *relative*. *Relative* are such as are imposed for some comparison, as *father, son, cause, effect, like, unlike, equal, unequal, master, servant, &c.* And those that signify no comparison at all are *absolute names*. But, as it was noted above, that universality is to be attributed to words and names only, and not to things, so the same is to be said of other distinctions of names ; for no things are either *univocal* or *equivocal*, or *relative* or *absolute*. There is also another distinction of names into *concrete* and *abstract ;* but because abstract names proceed from proposition, and can have no place where there is no affirmation, I shall speak of them hereafter.

14. Lastly, there are *simple* and *compounded names*. But here it is to be noted, that a name is not taken in philosophy, as in grammar, for one single word, but for any number of words put together to signify one thing ; for among philosophers *sentient animated body* passes but for one name, being the name of every living creature, which yet, among grammarians, is accounted three names. Also a *simple name* is not here distinguished from a *compounded name* by a preposition, as in grammar. But I call a *simple name*, that which in every kind is the most common or most universal ; and that a *compounded name*, which

by the joining of another name to it, is made less universal, and signifies that more conceptions than one were in the mind, for which that latter name was added.   For example, in the conception of *man* (as is shown in the former chapter.)   First, he is conceived to be something that has extension, which is marked by the word *body*.   *Body*, therefore, is a *simple name*, being put for that first single conception ; afterwards, upon the sight of such and such motion, another conception arises, for which he is called an *animated body ;* and this I here call a *compounded name*, as I do also the name *animal*, which is equivalent to an *animated body*.   And, in the same manner, an *animated rational body*, as also a *man*, which is equivalent to it, is a more compounded name. And by this we see how the composition of conceptions in the mind is answerable to the composition of names ; for, as in the mind one idea or phantasm succeeds to another, and to this a third ; so to one name is added another and another successively, and of them all is made one *compounded* name.   Nevertheless we must not think bodies which are without the mind, are compounded in the same manner, namely, that there is in nature a body, or any other imaginable thing existent, which at the first has no magnitude, and then, by the addition of magnitude, comes to have quantity, and by more or less quantity to have density or rarity ; and again, by the addition of figure, to be figurate, and after this, by the injection of light or colour, to become lucid or coloured ; though such has been the philosophy of many.

15. The writers of *logic* have endeavoured to
digest the names of all the kinds of things into
certain *scales* or degrees, by the continual subor-
dination of names less common, to names more
common. In the scale of *bodies* they put in the
first and highest place *body* simply, and in the
next place under it less common names, by which
it may be more limited and determined, namely
animated and *inanimated,* and so on till they
come to *individuals.* In like manner, in the
scale of *quantities,* they assign the first place to
quantity, and the next to *line, superficies,* and
solid, which are names of less latitude ; and these
orders or scales of names they usually call *predi-
caments* and *categories.* And of this ordination
not only positive, but negative names also are
capable ; which may be exemplified by such forms
of the *predicaments* as follow :

THE FORM OF THE PREDICAMENT OF BODY.

Not-Body, or
   Accident.

Body
       { Not ani-
         mated.
       { Animated { Not living
                  Creature.
                  Living { Not Man.
                  Creature { Man { Not Peter.
                                    { Peter.

Both Accident and Body { Absolutely, as { Quantity, or so much.
  are considered {   { Quality, or such,
                      or
                 Comparatively, which is called
                   their Relation.

THE FORM OF THE PREDICAMENT OF QUANTITY.

A predicament
described.

Quantity
- Not continual, as Number.
- Continual
  - Of itself, as
    - Line.
    - Superficies.
    - Solid.
  - By accident, as
    - Time, by Line.
    - Motion, by Line and Time.
    - Force, by Motion and Solid.
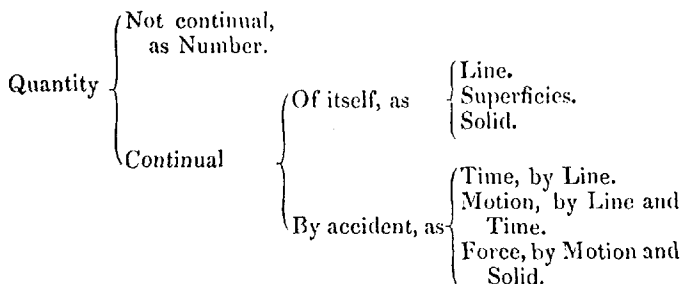
Where, it is to be noted, that *line, superficies,* and *solid,* may be said to be of such and such quantity, that is, to be originally and of their own nature capable of equality and inequality ; but we cannot say there is either majority or minority, or equality, or indeed any quantity at all, in *time,* without the help of *line* and *motion* ; nor in *motion,* without *line* and *time* ; nor in *force,* otherwise than by *motion* and *solid.*

THE FORM OF THE PREDICAMENT OF QUALITY.

Quality
- Perception by Sense
  - Primary
    - Seeing.
    - Hearing.
    - Smelling.
    - Tasting.
    - Touching.
  - Secondary
    - Imagination.
    - Affection
      - pleasant.
      - unpleasant.
- Sensible Quality
  - By Seeing, as Light and Colour.
  - By Hearing, as Sound.
  - By Smelling, as Odours.
  - By Tasting, as Savours.
  - By Touching, as Hardness, Heat, Cold, &c.
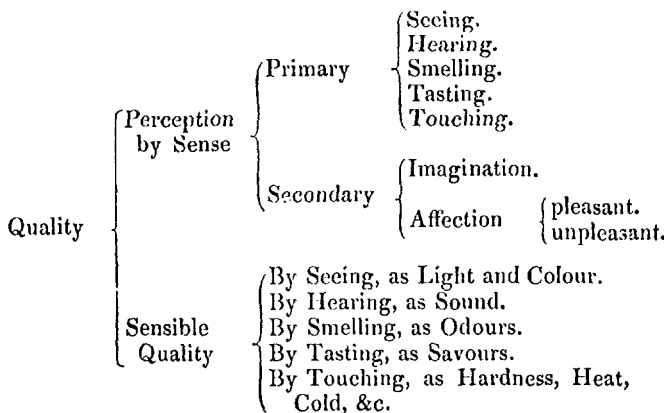
The Form of the Predicament of Relation.

Relation of
- Magnitudes, as Equality and Inequality.
- Qualities, as Likeness and Unlikeness.
- Order
  - Together
    - In Place.
    - In Time.
  - Not together
    - In Place { Former. Later.
    - In Time { Former. Later.

16. Concerning which predicaments it is to be noted, in the first place, that as the division is made in the first predicament into contradictory names, so it might have been done in the rest. For, as there, *body* is divided into *animated* and *not-animated*, so, in the second predicament, *continual quantity* may be divided into *line* and *not-line*, and again, *not-line* into *superficies* and *not-superficies*, and so in the rest; but it was not necessary.

Some things to be noted concerning predicaments.

Secondly, it is to be observed, that of *positive* names the former comprehends the latter; but of *negatives* the former is comprehended by the latter. For example, *living-creature* is the name of every man, and therefore it comprehends the name *man*; but, on the contrary, *not-man* is the name of everything which is not-living-creature, and therefore the name *not-living-creature*, which is put first, is comprehended by the latter name, *not-man*.

Thirdly, we must take heed that we do not think, that as names, so the diversities of things themselves may be searched out and determined by such distinctions as these; or that arguments

may be taken from hence (as some have done ridiculously) to prove that the kinds of things are not infinite.

Fourthly, I would not have any man think I deliver the forms above for a true and exact ordination of names ; for this cannot be performed as long as philosophy remains imperfect ; nor that by placing (for example) *light* in the predicament of *qualities,* while another places the same in the predicament of bodies, I pretend that either of us ought for this to be drawn from his opinion ; for this is to be done only by arguments and ratiocination, and not by disposing of words into *classes.*

Lastly, I confess I have not yet seen any great use of the predicaments in philosophy. I believe *Aristotle* when he saw he could not digest the things themselves into such orders, might nevertheless desire out of his own authority to reduce words to such forms, as I have done ; but I do it only for this end, that it may be understood what this ordination of words is, and not to have it received for true, till it be demonstrated by good reason to be so.

CHAPTER III.

OF PROPOSITION.

1. Divers kinds of speech.—2. Proposition defined.—3. Subject, predicate, and copula, what they are; and abstract and concrete what. The use and abuse of names abstract.—5. Proposition, universal and particular.—6. Affirmative and negative. —7. True and false.—8. True and false belongs to speech, and not to things.—9. Proposition, primary, not primary, definition, axiom, petition.—10. Proposition, necessary and contingent.— 11. Categorical and hypothetical. — 12. The same proposition diversely pronounced.—13. Propositions that may be reduced to the same categorical proposition, are equipollent.—14. Universal propositions converted by contradictory names, are equipollent.—15. Negative propositions are the same, whether negation be before or after the copula.— 16. Particular propositions simply converted, are equipollent. —17. What are subaltern, contrary, subcontrary, and contradictory propositions.—18. Consequence, what it is.—19. Falsity cannot follow from truth.—20. How one proposition is the cause of another.

1. FROM the connexion or contexture of names arise divers kinds of speech, whereof some signify the desires and affections of men ; such are, first, *interrogations*, which denote the desire of knowing : as, *Who is a good man ?* In which speech there is one name expressed, and another desired and expected from him of whom we ask the same. Then *prayers*, which signify the desire of having something ; *promises, threats, wishes, commands, complaints,* and other significations of other affections. Speech may also be absurd and insignificant ; as when there is a succession of

PART I.
3.

Divers kinds of speech.

words, to which there can be no succession of
thoughts in mind to answer them ; and this hap-
pens often to such, as, understanding nothing in
some subtle matter, do, nevertheless, to make
others believe they understand, speak of the same
incoherently ; for the connection of incoherent
words, though it want the end of speech (which
is signification) yet it is speech ; and is used by
writers of *metaphysics* almost as frequently as
speech significative.   In philosophy, there is but
one kind of speech useful, which some call in Latin
dictum, others *enuntiatum et pronunciatum ;* but
most men call it *proposition*, and is the speech of
those that affirm or deny, and expresseth truth or
falsity.

Proposition
defined.

2. *A* PROPOSITION *is a speech consisting of
two names copulated, by which he that speaketh
signifies he conceives the latter name to be the
name of the same thing whereof the former is
the name ; or* (which is all one) *that the former
name is comprehended by the latter*. For example,
this speech, *man is a living creature,* in which
two names are copulated by the verb *is*, is a *pro-
position*, for this reason, that he that speaks it
conceives both *living creature* and *man* to be
names of the same thing, or that the former name,
man, is comprehended by the latter name, *living
creature*.   Now the former name is commonly
called the *subject*, or *antecedent*, or the *contained
name*, and the latter the *predicate*, *consequent*,
or *containing name*. The sign of connection
amongst most nations is either some word, as the
word *is* in the proposition *man is a living creature*,
or some case or termination of a word, as in this

proposition, *man walketh* (which is equivalent to this, *man is walking*) ; the termination by which it is said he *walketh*, rather than he *is walking*, signifieth that those two are understood to be copulated, or to be names of the same thing.

But there are, or certainly may be, some nations that have no word which answers to our verb *is*, who nevertheless form propositions by the position only of one name after another, as if instead of *man is a living creature*, it should be said *man a living creature;* for the very order of the names may sufficiently show their connection ; and they are as apt and useful in philosophy, as if they were copulated by the verb *is*.

3. Wherefore, in every proposition three things *Subject,* are to be considered, *viz.* the two names, which *predicate, and copula,* are the *subject*, and the *predicate*, and their *what they are,* *copulation;* both which names raise in our mind *and concrete* the thought of one and the same thing ; but the *what.* copulation makes us think of the cause for which those names were imposed on that thing. As, for example, when we say *a body is moveable*, though we conceive the same thing to be designed by both those names, yet our mind rests not there, but searches farther what it is *to be a body*, or *to be moveable*, that is, wherein consists the difference betwixt these and other things, for which these are so called, others are not so called. They, therefore, that seek what it is *to be* any thing, as *to be moveable, to be hot, &c.* seek in things the causes of their names.

And from hence arises that distinction of names (touched in the last chapter) into *concrete* and *abstract*. For *concrete* is the name of any thing

which we suppose to have a being, and is therefore called the *subject*, in Latin *suppositum*, and in Greek ὑποκείμενον; as *body, moveable, moved, figurate, a cubit high, hot, cold, like, equal, Appius, Lentulus,* and the like; and, *abstract* is that which in any subject denotes the cause of the concrete name, as *to be a body, to be moveable, to be moved, to be figurate, to be of such quantity, to be hot, to be cold, to be like, to be equal, to be Appius, to be Lentulus, &c.* Or names equivalent to these, which are most commonly called *abstract* names, as *corporiety, mobility, motion, figure, quantity, heat, cold, likeness, equality,* and (as Cicero has it) *Appiety* and *Lentulity.* Of the same kind also are infinitives; for *to live* and *to move* are the same with *life* and *motion,* or *to be living* and *to be moved.* But *abstract names* denote only the causes of *concrete names,* and not the things themselves. For example, when we see any thing, or conceive in our mind any visible thing, that thing appears to us, or is conceived by us, not in one point, but as having parts distant from one another, that is, as being extended and filling some space. Seeing therefore we call the thing so conceived *body,* the cause of that name is, that that thing is *extended,* or the *extension* or *corporiety* of it. So when we see a thing appear sometimes here, sometimes there, and call it *moved* or *removed,* the cause of that name is that it *is moved* or the *motion* of the same.

And these causes of names are the same with the causes of our conceptions, namely, some power of action, or affection of the thing con-

ceived, which some call the manner by which any thing works upon our senses, but by most men they are called *accidents* ; I say accidents, not in that sense in which accident is opposed to necessary; but so, as being neither the things themselves, nor parts thereof, do nevertheless accompany the things in such manner, that (saving extension) they may all perish, and be destroyed, but can never be abstracted.

4. There is also this difference betwixt *concrete* and *abstract* names, that those were invented before propositions, but these after; for these could have no being till there were propositions, from whose *copula* they proceed. Now in all matters that concern this life, but chiefly in philosophy, there is both great use and great abuse of *abstract names ;* and the use consists in this, that without them we cannot, for the most part, either reason, or compute the properties of bodies ; for when we would multiply, divide, add, or substract heat, light, or motion, if we should double or add them together by concrete names, saying (for example) hot is double to hot, light double to light, or moved double to moved, we should not double the properties, but the bodies themselves that are hot, light, moved, &c. which we would not do. But the abuse proceeds from this, that some men seeing they can consider, that is (as I said before) bring into account the increasings and decreasings of quantity, heat and other accidents, without considering their bodies or subjects (which they call *abstracting*, or making to exist apart by themselves) they speak of accidents, as if they might be separated from all bodies. And

from hence proceed the gross errors of writers of metaphysics; for, because they can consider thought without the consideration of body, they infer there is no need of a thinking-body; and because quantity may be considered without considering body, they think also that quantity may be without body, and body without quantity; and that a body has quantity by the addition of quantity to it. From the same fountain spring those insignificant words, *abstract substance, separated essence,* and the like; as also that confusion of words derived from the Latin verb *est,* as *essence, essentiality, entity, entitative;* besides *reality, aliquiddity, quiddity, &c.* which could never have been heard of among such nations as do not copulate their names by the verb *is,* but by adjective verbs, as runneth, readeth, &c. or by the mere placing of one name after another; and yet seeing such nations compute and reason, it is evident that philosophy has no need of those words *essence, entity,* and other the like barbarous terms.

5. There are many distinctions of propositions, whereof the first is, that some are *universal,* others *particular,* others *indefinite,* and others *singular;* and this is commonly called the distinction of *quantity.* An *universal* proposition is that whose subject is affected with the sign of an universal name, as *every man is a living creature. Particular,* that whose subject is affected with the sign of a particular name, as *some man is learned.* An *indefinite* proposition has for its subject a common name, and put without any sign, as *man is a living creature, man is learned.*

And a *singular* proposition is that whose subject is a singular name, as *Socrates is a philosopher*, *this man is black*.

6. The second distinction is into *affirmative* and *negative*, and is called the distinction of *quality*. An *affirmative* proposition is that whose predicate is a positive name, as *man is a living creature*. *Negative*, that whose predicate is a negative name, as *man is not a stone*.

7. The third distinction is, that one is *true*, another *false*. A *true* proposition is that, whose predicate contains, or comprehends its subject, or whose predicate is the name of every thing, of which the subject is the name ; as *man is a living creature* is therefore a true proposition, because whatsoever is called *man*, the same is also called *living creature ;* and *some man is sick*, is true, because *sick* is the name of *some man*. That which is not true, or that whose predicate does not contain its subject, is called a *false* proposition, as *man is a stone*.

Now these words *true, truth*, and *true proposition*, are equivalent to one another; for truth consists in speech, and not in the things spoken of; and though *true* be sometimes opposed to *apparent* or *feigned*, yet it is always to be referred to the truth of proposition ; for the image of a man in a glass, or a ghost, is therefore denied to be a very man, because this proposition, *a ghost is a man,* is not true ; for it cannot be denied but that a ghost is a very ghost. And therefore truth or verity is not any affection of the thing, but of the proposition concerning it. As for that which the writers of metaphysics say, that *a thing, one*

PART I.
3.

thing, and *a very thing,* are equivalent to one another, it is but trifling and childish ; for who does not know, that *a man, one man,* and *a very man,* signify the same.

True & false belongs to speech, and not to things.

8. And from hence it is evident, that truth and falsity have no place but amongst such living creatures as use speech. For though some brute creatures, looking upon the image of a man in a glass, may be affected with it, as if it were the man himself, and for this reason fear it or fawn upon it in vain ; yet they do not apprehend it as true or false, but only as like ; and in this they are not deceived. Wherefore, as men owe all their true ratiocination to the right understanding of speech ; so also they owe their errors to the mis-understanding of the same ; and as all the orna-ments of philosophy proceed only from man, so from man also is derived the ugly absurdity of false opinions. For speech has something in it like to a spider's web, (as it was said of old of *Solon's* laws) for by contexture of words tender and delicate wits are ensnared and stopped; but strong wits break easily through them.

From hence also this may be deduced, that the first truths were arbitrarily made by those that first of all imposed names upon things, or received them from the imposition of others. For it is true (for example) that *man is a living creature,* but it is for this reason, that it pleased men to impose both those names on the same thing.

Proposition, primary, not primary, definition, axiom, petition.

9. Fourthly, propositions are distinguished into *primary* and *not primary.* *Primary* is that wherein the subject is explicated by a predicate of many names, as *man is a body, animated,*

rational ; for that which is comprehended in the
name *man,* is more largely expressed in the names
body, animated, and *rational,* joined together;
and it is called *primary,* because it is first in ratio-
cination ; for nothing can be proved, without
understanding first the name of the thing in
question. Now *primary* propositions are nothing
but definitions, or parts of definitions, and these
only are the principles of demonstration, being
truths constituted arbitrarily by the inventors of
speech, and therefore not to be demonstrated.
To these propositions, some have added others,
which they call *primary* and *principles,* namely,
axioms, and *common notions ;* which, (though
they be so evident that they need no proof) yet,
because they may be proved, are not truly prin-
ciples ; and the less to be received for such, in
regard propositions not intelligible, and some-
times manifestly false, are thrust on us under the
name of *principles* by the clamour of men, who
obtrude for evident to others, all that they them-
selves think true. Also certain petitions are com-
monly received into the number of principles ; as,
for example, *that a straight line may be drawn
between two points,* and other petitions of the
writers of geometry ; and these are indeed the
principles of art or construction, but not of science
and demonstration.

10. Fifthly, propositions are distinguished into Proposition
necessary, that is, necessarily true ; and true, but necessary &
contingent.
not necessarily, which they call *contingent.* A
necessary proposition is when nothing can at any
time be conceived or feigned, whereof the subject
is the name, but the predicate also is the name of

the same thing ; as *man is a living creature* is a necessary proposition, because at what time soever we suppose the name *man* agrees with any thing, at that time the name *living-creature* also agrees with the same. But a *contingent* proposition is that, which at one time may be true, at another time false ; as *every crow is black ;* which may perhaps be true now, but false hereafter. Again, in every *necessary* proposition, the predicate is either equivalent to the subject, as in this, *man is a rational living creature ;* or part of an equivalent name, as in this, *man is a living creature,* for the name *rational-living-creature,* or *man*, is compounded of these two, *rational* and *living-creature*. But in a *contingent* proposition this cannot be ; for though this were true, *every man is a liar*, yet because the word *liar* is no part of a compounded name equivalent to the name *man*, that proposition is not to be called *necessary*, but *contingent*, though it should happen to be true always. And therefore those propositions only are *necessary*, which are of sempiternal truth, that is, true at all times. From hence also it is manifest, that truth adheres not to things, but to speech only, for some truths are eternal ; for it will be eternally true, *if man, then living-creature ;* but that any *man*, or *living-creature*, should exist eternally, is not necessary.

11. A sixth distinction of propositions is into *categorical* and *hypothetical*. A *categorical* proposition is that which is simply or absolutely pronounced, as *every man is a living-creature, no man is a tree ;* and *hypothetical* is that which is pronounced conditionally, as, *if any thing be a*

man, the same is also a living-creature, if any-
thing be a man, the same is also not-a-stone.

A *categorical* proposition, and an *hypothe-*
tical answering it, do both signify the same, if the
propositions be necessary ; but not if they be con-
tingent. For example, if this, *every man is a
living-creature*, be true, this also will be true, *if
any thing be a man, the same is also a living-
creature ;* but in contingent propositions, though
this be true, *every crow is black*, yet this, *if any
thing be a crow, the same is black*, is false. But
an *hypothetical* proposition is then rightly said
to be true, when the consequence is true, as *every
man is a living-creature*, is rightly said to be a
true proposition, because of whatsoever it is truly
said *that is a man*, it cannot but be truly said also,
the same is a living creature. And therefore
whensoever an *hypothetical* proposition is true,
the *categorical* answering it, is not only true, but
also necessary ; which I thought worth the noting,
as an argument, that philosophers may in most
things reason more solidly by *hypothetical* than
categorical propositions.

12. But seeing every proposition may be, and
uses to be, pronounced and written in many forms,
and we are obliged to speak in the same manner
as most men speak, yet they that learn philosophy
from masters, had need to take heed they be not
deceived by the variety of expressions. And
therefore, whensoever they meet with any obscure
proposition, they ought to reduce it to its most
simple and categorical form ; in which the copu-
lative word *is* must be expressed by itself, and not
mingled in any manner either with the subject or

predicate, both which must be separated and clearly distinguished one from another. For example, if this proposition, *man can not sin*, be compared with this, *man cannot sin*, their difference will easily appear if they be reduced to these, *man is able not to sin*, and, *man is not able to sin*, where the predicates are manifestly different. But they ought to do this silently by themselves, or betwixt them and their masters only ; for it will be thought both ridiculous and absurd, for a man to use such language publicly. Being therefore to speak of *equipollent* propositions, I put in the first place all those for *equipollent*, that may be reduced purely to one and the same categorical proposition.

Propositions that may be reduced to the same categorical proposition, are equipollent.

13. Secondly, that which is categorical and necessary, is *equipollent* to its hypothetical proposition ; as this categorical, *a right-lined triangle has its three angles equal to two right angles*, to this hypothetical, *if any figure be a right-lined triangle, the three angles of it are equal to two right angles*.

Universal propositions converted by contradictory names, are equipollent.

14. Also, any two universal propositions, of which the terms of the one (that is, the subject and predicate) are contradictory to the terms of the other, and their order inverted, as these, *every man is a living creature*, and *every thing that is not a living-creature is not a man*, are *equipollent*. For seeing *every man is a living creature* is a true proposition, the name *living creature* contains the name *man* ; but they are both positive names, and therefore (by the last article of the precedent chapter) the negative name *not man*, contains the negative name *not living-creature* ;

wherefore *every thing that is not a living-crea-*
ture, is not a man, is a true proposition. Likewise
these, *no man is a tree, no tree is a man,* are
equipollent. For if it be true that *tree* is not the
name of any *man*, then no one thing can be signi-
fied by the two names *man* and *tree*, wherefore
no tree is a man is a true proposition. Also to
this, *whatsoever is not a living-creature is not a
man*, where both the terms are negative, this
other proposition is equipollent, *only a living crea-
ture is a man.*

15. Fourthly, negative propositions, whether Negative
the particle of negation be set after the copula as propositions
are the same,
some nations do, or before it, as it is in Latin and whether the
negation be
Greek, if the terms be the same, are equipollent : before or after
the copula.
as, for example, *man is not a tree*, and, *man is
not-a-tree*, are equipollent, though Aristotle deny
it. Also these, *every man is not a tree*, and *no
man is a tree*, are equipollent, and that so mani-
festly, as it needs not be demonstrated.

16. Lastly, all particular propositions that have Particular
their terms inverted, as these, *some man is blind*, simply con-
some blind thing is a man, are equipollent; for verted, are
equipollent.
either of the two names, is the name of some one
and the same man ; and therefore in which soever
of the two orders they be connected, they signify
the same truth.

17. Of propositions that have the same terms, What are sub-
and are placed in the same order, but varied either altern, con-
trary, subcon-
by quantity or quality, some are called *subaltern*, trary, and
contradictory
others *contrary*, others *subcontrary*, and others propositions.
contradictory.

Subaltern, are universal and particular propo-
sitions of the same quality ; as, *every man is a*

living creature, some man is a living creature; or, *no man is wise, some man is not wise.* Of these, if the universal be true, the particular will be true also.

Contrary, are universal propositions of different quality; as, *every man is happy, no man is happy.* And of these, if one be true, the other is false: also, they may both be false, as in the example given.

Subcontrary, are particular propositions of different quality; as, *some man is learned, some man is not learned;* which cannot be both false, but they may be both true.

Contradictory are those that differ both in quantity and quality; as, *every man is a living creature, some man is not a living-creature;* which can neither be both true, nor both false.

Consequence,
what it is.
18. A proposition is said to *follow* from two other propositions, when these being granted to be true, it cannot be denied but the other is true also. For example, let these two propositions, *every man is a living creature,* and, *every living creature is a body,* be supposed true, that is, that *body* is the name of *every living creature,* and *living creature* the name of *every man.* Seeing therefore, if these be understood to be true, it cannot be understood that *body* is not the name of *every man,* that is, that *every man is a body* is false, this proposition will be said to *follow* from those two, or to be necessarily *inferred* from them.

Falsity
cannot follow
from truth.
19. That a true proposition may follow from false propositions, may happen sometimes; but false from true, never. For if these, *every man is a stone,* and *every stone is a living creature,*

(which are both false) be granted to be true, it is granted also that *living creature* is the name of *every stone*, and *stone* of *every man*, that is, that *living creature* is the name of *every man;* that is to say, this proposition *every man is a living creature*, is true, as it is indeed true. Wherefore a true proposition may sometimes follow from false. But if any two propositions be true, a false one can never follow from them. For if true follow from false, for this reason only, that the false are granted to be true, then truth from two truths granted will follow in the same manner.

20. Now, seeing none but a true proposition will follow from true, and that the understanding of two propositions to be true, is the cause of understanding that also to be true which is deduced from them ; the two antecedent propositions are commonly called the causes of the inferred proposition, or conclusion. And from hence it is that logicians say, the *premises* are causes of the *conclusion ;* which may pass, though it be not properly spoken ; for though understanding be the cause of understanding, yet speech is not the cause of speech. But when they say, the cause of the properties of any thing, is the thing itself, they speak absurdly. For example, if a figure be propounded which is triangular ; seeing every triangle has all its angles together equal to two right angles, from whence it follows that all the angles of that figure are equal to two right angles, they say, for this reason, that that figure is the cause of that equality. But seeing the figure does not itself make its angles, and therefore cannot be said to be the *efficient-cause*, they

call it the *formal-cause* ; whereas indeed it is no
cause at all ; nor does the property of any figure
follow the figure, but has its being at the same
time with it ; only the knowledge of the figure
goes before the knowledge of the properties ;
and one knowledge is truly the cause of another
knowledge, namely the *efficient cause*.

And thus much concerning *proposition ;* which
in the progress of philosophy is the first step,
like the moving towards of one foot. By the
due addition of another step I shall proceed to
syllogism, and make a complete pace. Of which
in the next chapter.

CHAPTER IV.

OF SYLLOGISM.

1. The definition of syllogism.—2. In a syllogism there are but
three terms.—3. Major, minor, and middle term ; also major
and minor proposition, what they are.—4. The middle term in
every syllogism ought to be determined in both the propositions
to one and the same thing.—5. From two particular propo-
sitions nothing can be concluded.—6. A syllogism is the col-
lection of two propositions into one sum.—7. The figure of a
syllogism, what it is.—8. What is in the mind answering to a
syllogism.—9. The first indirect figure, how it is made.—
10. The second indirect figure, how made.—11. How the third
indirect figure is made.—12. There are many moods in every
figure, but most of them useless in philosophy.—13. An
hypothetical syllogism when equipollent to a categorical.

Definition
of syllogism.

1. A SPEECH, consisting of three propositions,
from two of which the third follows, is called a
SYLLOGISM ; and that which follows is called the
conclusion ; the other two *premises*. For example,

this speech, *every man is a living creature, every living creature is a body*, therefore, *every man is a body*, is a *syllogism*, because the third proposition follows from the two first ; that is, if those be granted to be true, this must also be granted to be true.

2. From two propositions which have not one term common, no conclusion can follow ; and therefore no *syllogism* can be made of them. For let any two premises, *a man is a living creature, a tree is a plant*, be both of them true, yet because it cannot be collected from them that *plant* is the name of a *man*, or *man* the name of a *plant*, it is not necessary that this conclusion, *a man is a plant*, should be true. Corollary : therefore, in the *premises* of a *syllogism* there can be but three *terms*.

In a syllogism there are but three terms.

Besides, there can be no term in the *conclusion*, which was not in the *premises*. For let any two premises be, *a man is a living creature, a living creature is a body*, yet if any other term be put in the conclusion, as *man is two-footed ;* though it be true, it cannot follow from the premises, because from them it cannot be collected, that the name *two-footed* belongs to a *man ;* and therefore, again, in every *syllogism* there can be but three *terms*.

3. Of these terms, that which is the *predicate* in the conclusion, is commonly called the *major ;* that which is the *subject* in the conclusion, the *minor*, and the other is the *middle term ;* as in this syllogism, *a man is a living creature, a living creature is a body*, therefore, *a man is a body, body* is the *major, man* the *minor*, and

Major, minor and middle term; also major & minor proposition, what they are.

living creature the *middle term*. Also of the premises, that in which the *major term* is found, is called the *major proposition*, and that which has the *minor term*, the *minor proposition*.

The middle term in every syllogism to be determined in both propositions to one and the same thing.

4. If the middle term be not in both the premises determined to one and the same singular thing, no conclusion will follow, nor syllogism be made. For let the minor term be *man*, the middle term *living creature*, and the major term *lion;* and let the premises be, *man is a living creature, some living creature is a lion*, yet it will not follow that *every* or *any man is a lion*. By which it is manifest, that in every syllogism, that proposition which has the middle term for its *subject*, ought to be either *universal* or *singular*, but not *particular* nor *indefinite*. For example, this syllogism, *every man is a living creature, some living creature is four-footed*, therefore *some man is four-footed*, is therefore faulty, because the middle term, *living creature*, is in the first of the premises determined only to *man*, for there the name of *living creature* is given to man only, but in the latter premise it may be understood of some other living creature besides man. But if the latter premise had been *universal*, as here, *every man is a living creature, every living creature is a body*, therefore *every man is a body*, the syllogism had been true; for it would have followed that *body* had been the name of *every living creature*, that is of *man;* that is to say, the conclusion *every man is a body* had been true. Likewise, when the middle term is a *singular* name, a syllogism may be made, I say a true syllogism, though useless in philosophy, as this, *some man is Socrates, Socrates*

is a philosopher, therefore, *some man is a philo-* *sopher ;* for the premises being granted, the conclusion cannot be denied.

5. And therefore of two premises, in both which the middle term is particular, a syllogism cannot be made ; for whether the middle term be the *subject* in both the premises, or the *predicate* in both, or the *subject* in one, and the *predicate* in the other, it will not be necessarily determined to the same thing. For let the premises be,

Some man is blind, } In both which the middle
Some man is learned, } term is the *subject*,

it will not follow that *blind* is the name of any *learned man,* or *learned* the name of any *blind man,* seeing the name *learned* does not contain the name *blind,* nor this that ; and therefore it is not necessary that both should be names of the same man. So from these premises,

Every man is a living-creature, } In both which the middle
Every horse is a living-creature, } term is the *predicate*,

nothing will follow. For seeing *living creature* is in both of them *indefinite,* which is equivalent to *particular,* and that *man* may be one kind of *living creature,* and *horse* another kind, it is not necessary that *man* should be the name of *horse,* or *horse* of *man.* Or if the premises be,

Every man is a living- } In one of which the middle-
creature, } term is the *subject*, and in
Some living creature is } the other the *predicate*,
four-footed, }

the conclusion will not follow, because the name

living creature being not determined, it may in one of them be understood of *man*, in the other of *not-man*.

6. Now it is manifest from what has been said, that a syllogism is nothing but a collection of the sum of two propositions, joined together by a common term, which is called the *middle term*. And as proposition is the addition of two names, so syllogism is the adding together of three.

7. Syllogisms are usually distinguished according to their diversity of *figures*, that is, by the diverse position of the middle term. And again in figure there is a distinction of certain *moods*, which consist of the differences of propositions in *quantity* and *quality*. The first figure is that, in which the terms are placed one after another according to their latitude of signification ; in which order the *minor term* is first, the *middle term* next, and the *major* last ; as, if the minor term be *man*, the middle term, *living creature*, and the major term, *body*, then, *man is a living-creature, is a body,* will be a syllogism in the first figure : in which, *man is a living creature* is the minor proposition ; the major, *living creature is a body,* and the conclusion, or sum of both, *man is a body*. Now this figure is called *direct*, because the terms stand in direct order ; and it is varied by *quantity* and *quality* into four *moods :* of which the first is that wherein all the terms are *positive*, and the minor term *universal*, as *every man is a living creature, every living creature is a body :* in which all the propositions are affirmative, and universal. But if the major term be a negative name, and the minor an universal name,

the *figure* will be in the second *mood*, as, *every man is a living creature, every living creature is not a tree*, in which the major proposition and conclusion are both universal and negative. To these two, are commonly added two more, by making the minor term particular. Also it may happen that both the major and middle terms are negative terms, and then there arises another *mood*, in which all the propositions are negative, and yet the syllogism will be good; as, if the minor term be *man*, the middle term *not a stone*, and the major term *not a flint*, this syllogism, *no man is a stone, whatsoever is not a stone is not a flint*, therefore, *no man is a flint*, is true, though it consist of three negatives. But in philosophy, the profession whereof is to establish universal rules concerning the properties of things, seeing the difference betwixt negatives and affirmatives is only this, that in the former the subject is affirmed by a negative name, and by a positive in the latter, it is superfluous to consider any other *mood* in direct *figure*, besides that, in which all the propositions are both universal and affirmative.

8. The thoughts in the mind answering to a direct syllogism, proceed in this manner; first, there is conceived a phantasm of the thing named, with that accident or quality thereof, for which it is in the minor proposition called by that name which is the subject; next, the mind has a phantasm of the same thing with that accident, or quality, for which it hath the name, that in the same proposition is the predicate; thirdly, the thought returns of the same thing as having that

accident in it, for which it is called by the name, that is the predicate of the major proposition; and lastly, remembering that all those are the accidents of one and the same thing, it concludes that those three names are also names of one and the same thing; that is to say, the conclusion is true. For example, when this syllogism is made, *man is a living creature, a living creature is a body*, therefore, *man is a body*, the mind conceives first an image of a man speaking or discoursing, and remembers that that, which so appears, is called *man;* then it has the image of the same man moving, and remembers that that, which appears so, is called *living creature;* thirdly, it conceives an image of the same man, as filling some place or space, and remembers that what appears so is called *body;* and lastly, when it remembers that that thing, which was extended, and moved and spake, was one and the same thing, it concludes that the three names, *man, living creature,* and *body*, are names of the same thing, and that therefore *man is a living creature* is a true proposition. From whence it is manifest, that living creatures that have not the use of speech, have no conception or thought in the mind, answering to a syllogism made of universal propositions; seeing it is necessary to think not only of the thing, but also by turns to remember the divers names, which for divers considerations thereof are applied to the same.

The first indirect figure how made. 9. The rest of the figures arise either from the inflexion, or inversion of the first or direct figure; which is done by changing the major, or minor, or both the propositions, into converted propositions equipollent to them.

From whence follow three other figures; of which, two are *inflected*, and the third *inverted*. The first of these three is made by the conversion of the major proposition. For let the minor, middle, and major terms stand in direct order, thus, *man is a living creature, is not a stone,* which is the first or direct figure ; the inflection will be by converting the major proposition in this manner, *man is a living creature, a stone is not a living creature* ; and this is the second figure, or the first of the indirect figures ; in which the conclusion will be, *man is not a stone.* For (having shown in the last chapter, art. 14, that universal propositions, converted by contradiction of the terms, are equipollent) both those syllogisms conclude alike ; so that if the major be read (like Hebrew) backwards, thus, *a living creature is not a stone,* it will be direct again, as it was before. In like manner this direct syllogism, *man is not a tree, is not a pear-tree,* will be made indirect by converting the major proposition (by contradiction of the terms) into another equipollent to it, thus, *man is not a tree, a pear-tree is a tree* ; for the same conclusion will follow, *man is not a pear-tree.*

But for the conversion of the direct figure into the first indirect figure, the major term in the direct figure ought to be negative. For though this direct, *man is a living creature, is a body,* be made indirect, by converting the major proposition, thus,

> *Man is a living creature,*
> *Not a body is not a living creature,*
> Therefore, *Every man is a body;*

Yet this conversion appears so obscure, that

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this mood is of no use at all. By the conversion of the major proposition, it is manifest, that in this figure, the middle term is always the predicate in both the premises.

Second indirect figure how made.
10. The second indirect figure is made by converting the minor proposition, so as that the middle term is the subject in both. But this never concludes universally, and therefore is of no use in philosophy. Nevertheless I will set down an example of it ; by which this direct

> *Every man is a living creature,*
> *Every living creature is a body,*

by conversion of the minor proposition, will stand thus,

> *Some living creature is a man,*
> *Every living creature is a body,*
> Therefore, *Some man is a body.*

For *every man is a living creature* cannot be converted into this, *every living creature is a man :* and therefore if this syllogism be restored to its direct form, the minor proposition will be *some man is a living creature,* and consequently the conclusion will be *some man is a body,* seeing the minor term *man,* which is the subject in the conclusion, is a particular name.

How the third indirect figure is made.
11. The third indirect or inverted figure, is made by the conversion of both the premises. For example, this direct syllogism,

> *Every man is a living creature,*
> *Every living creature is not a stone,*
> Therefore, *Every man is not a stone,*

being inverted, will stand thus,

Every stone is not a living creature,
Whatsoever is not a living creature, is not a man,
Therefore, *Every stone is not a man ;*

which conclusion is the converse of the direct conclusion, and equipollent to the same.

The figures, therefore, of syllogisms, if they be numbered by the diverse situation of the middle term only, are but three ; in the first whereof, the middle term has the middle place ; in the second, the last ; and in the third, the first place. But if they be numbered according to the situation of the terms simply, they are four ; for the first may be distinguished again into two, namely, into direct and inverted. From whence it is evident, that the controversy among logicians concerning the fourth figure, is a mere λογόμαχια, or contention about the name thereof; for, as for the thing itself, it is plain that the situation of the terms (not considering the quantity or quality by which the moods are distinguished) makes four differences of syllogisms, which may be called figures, or have any other name at pleasure.

12. In every one of these figures there are many moods, which are made by varying the premises according to all the differences they are capable of, by quantity and quality ; as namely, in the direct figure there are six moods ; in the first indirect figure, four ; in the second, fourteen; and in the third, eighteen. But because from the direct figure I rejected as superfluous all moods besides that which consists of universal propositions, and whose minor proposition is affirmative, I do, together with it, reject the moods of the rest

There are many moods in every figure, but most of them useless in philosophy.

of the figures which are made by conversion of
the premises in the direct figure.

An hypotheti-
cal syllogism
when equipol-
lent to a cate-
gorical.

13. As it was showed before, that in necessary
propositions a categorical and hypothetical propo-
sition are equipollent; so likewise it is manifest
that a categorical and hypothetical syllogism are
equivalent. For every categorical syllogism, as
this,

> *Every man is a living creature,*
> *Every living creature is a body,*
> Therefore, *Every man is a body,*

is of equal force with this hypothetical syllogism :

> *If any thing be a man, the same is also a living creature,*
> *If any thing be a living creature, the same is a body,*
> Therefore, *If any thing be a man, the same is a body.*

In like manner, this categorical syllogism in an
indirect figure,

> *No stone is a living creature,*
> *Every man is a living creature,*
> Therefore, *No man is a stone,*
> Or, *No stone is a man,*

is equivalent to this hypothetical syllogism :

> *If any thing be a man, the same is a living creature,*
> *If any thing be a stone, the same is not a living creature,*
> Therefore, *If any thing be a stone, the same is not a man,*
> Or, *If any thing be a man, the same is not a stone.*

And thus much seems sufficient for the nature
of syllogisms ; (for the doctrine of moods and
figures is clearly delivered by others that have
written largely and profitably of the same). Nor
are precepts so necessary as practice for the
attaining of true ratiocination ; and they that
study the demonstrations of mathematicians, will

sooner learn true logic, than they that spend time in reading the rules of syllogizing which logicians have made ; no otherwise than little children learn to go, not by precepts, but by exercising their feet. This, therefore, may serve for the first pace in the way to Philosophy.

In the next place I shall speak of the faults and errors into which men that reason unwarily are apt to fall ; and of their kinds and causes.

CHAPTER V.

OF ERRING, FALSITY, AND CAPTIONS.

1. Erring and falsity how they differ. Error of the mind by itself without the use of words, how it happens.—2. A seven-fold incoherency of names, every one of which makes always a false proposition.—3. Examples of the first manner of inco-herency.—4. Of the second.—5. Of the third.—6. Of the fourth.—7. Of the fifth.—8. Of the sixth.—9. Of the seventh. 10. Falsity of propositions detected by resolving the terms with definitions continued till they come to simple names, or names that are the most general of their kind.—11. Of the fault of a syllogism consisting in the implication of the terms with the copula.—12. Of the fault which consists in equivo-cation.—13. Sophistical captions are oftener faulty in the matter than in the form of syllogisms.

1. MEN are subject to *err* not only in affirming and denying, but also in perception, and in silent cogitation. In affirming and denying, when they call any thing by a name, which is not the name thereof ; as if from seeing the sun first by reflec-tion in water, and afterwards again directly in the

Erring & fal-sity how they differ. Error of the mind by itself, without the use of words, how it happens.

firmament, we should to both those appearances give the name of sun, and say there are two suns ; which none but men can do, for no other living creatures have the use of names. This kind of error only deserves the name of *falsity*, as arising, not from sense, nor from the things themselves, but from pronouncing rashly ; for names have their constitution, not from the species of things, but from the will and consent of men. And hence it comes to pass, that men pronounce *falsely*, by their own negligence, in departing from such appellations of things as are agreed upon, and are not deceived neither by the things, nor by the sense ; for they do not perceive that the thing they see is called sun, but they give it that name from their own will and agreement. Tacit errors, or the errors of sense and cogitation, are made, by passing from one imagination to the imagination of another different thing ; or by feigning that to be past, or future, which never was, nor ever shall be ; as when, by seeing the image of the sun in water, we imagine the sun itself to be there ; or by seeing swords, that there has been or shall be fighting, because it uses to be so for the most part ; or when from promises we feign the mind of the promiser to be such and such; or lastly, when from any sign we vainly imagine something to be signified, which is not. And errors of this sort are common to all things that have sense ; and yet the deception proceeds neither from our senses, nor from the things we perceive; but from ourselves while we feign such things as are but mere images to be something more than images. But neither things, nor imaginations of

things, can be said to be false, seeing they are truly what they are ; nor do they, as signs, promise any thing which they do not perform ; for they indeed do not promise at all, but we from them ; nor do the clouds, but we, from seeing the clouds, say it shall rain. The best way, therefore, to free ourselves from such errors as arise from natural signs, is first of all, before we begin to reason concerning such conjectural things, to suppose ourselves ignorant, and then to make use of our ratiocination ; for these errors proceed from the want of ratiocination ; whereas, errors which consist in affirmation and negation, (that is, the falsity of propositions) proceed only from reasoning amiss. Of these, therefore, as repugnant to philosophy, I will speak principally.

2. Errors which happen in reasoning, that is, in syllogizing, consist either in the falsity of the premises, or of the inference. In the first of these cases, a syllogism is said to be faulty in the *matter* of it ; and in the second case, in the *.form*. I will first consider the matter, namely, how many ways a proposition may be false ; and next the form, and how it comes to pass, that when the premises are true, the inference is, notwithstanding, false.

Seeing, therefore, that proposition only is true, (chap. III, art. 7) in which are copulated two names of one and the same thing; and that always false, in which names of different things are copulated, look how many ways names of different things may be copulated, and so many ways a false proposition may be made.

Now, all things to which we give names, may be

reduced to these four kinds, namely, *bodies, acci-dents, phantasms,* and *names* themselves; and there-fore, in every true proposition, it is necessary that the names copulated, be both of them names of *bodies,* or both names of *accidents,* or both names of *phantasms,* or both names of *names.* For names otherwise copulated are incoherent, and constitute a false proposition. It may happen, also, that the name of a *body,* of an *accident,* or of a *phantasm,* may be copulated with the name of a *speech.* So that copulated names may be incoherent seven manner of ways.

1. If the name of a Body		the name of an Accident.
2. If the name of a Body		the name of a Phantasm.
3. If the name of a Body	be copulated with	the name of a Name.
4. If the name of an Accident		the name of a Phantasm.
5. If the name of an Accident		the name of a Name.
6. If the name of a Phantasm		the name of a Name.
7. If the name of a Body, Accident, or Phantasm		the name of a Speech.

Of all which I will give some examples.

3. After the first of these ways propositions are false, when abstract names are copulated with concrete names; as (in Latin and Greek) *esse est ens, essentia est ens,* τὸ τί ἦν εἶναι (*i.*); *quidditas est ens,* and many the like, which are found in Aristotle's *Metaphysics.* Also, the *understanding worketh,* the *understanding understandeth,* the *sight seeth;* a *body* is *magnitude,* a *body* is *quantity,* a *body* is *extension;* to *be a man* is a *man, whiteness* is a *white thing,* &c.; which is as if one should say, the *runner* is the *running,* or the *walk walketh.* Moreover, *essence* is *sepa-rated, substance* is *abstracted:* and others like these, or derived from these, (with which common

philosophy abounds.) For seeing no *subject* of
an *accident* (that is, no *body*) is an *accident :* no
name of an *accident* ought to be given to a *body*,
nor of a *body* to an *accident*.

4. False, in the second manner, are such propo-
sitions as these; a *ghost is a body*, or a *spirit*,
that is, a thin body ; *sensible species fly up and
down in the air*, or *are moved hither and thither*,
which is proper to bodies; also, a *shadow is
moved*, or *is a body ; light is moved*, or *is a
body ; colour is the object of sight, sound of
hearing ; space* or *place is extended;* and innu-
merable others of this kind. For seeing ghosts,
sensible species, a shadow, light, colour, sound,
space, &c. appear to us no less sleeping than
waking, they cannot be things without us, but
only phantasms of the mind that imagines them ;
and therefore the names of these, copulated with
the names of bodies, cannot constitute a true
proposition.

5. False propositions of the third kind, are such
as these ; *genus est ens, universale est ens, ens
de ente prædicatur*. For *genus*, and *universale*,
and *predicare*, are names of names, and not of
things. Also, *number is infinite*, is a false propo-
sition ; for no number can be infinite, but only
the word *number* is then called an indefinite name
when there is no determined number answering to
it in the mind.

6. To the fourth kind belong such false propo-
sitions as these, *an object is of such magnitude or
figure as appears to the beholders ; colour, light,
sound, are in the object ;* and the like. For the
same object appears sometimes greater, sometimes

lesser, sometimes square, sometimes round, accor-
ding to the diversity of the distance and medium;
but the true magnitude and figure of the thing
seen is always one and the same ; so that the
magnitude and figure which appears, is not the
true magnitude and figure of the object, nor any-
thing but phantasm ; and therefore, in such pro-
positions as these, the names of accidents are
copulated with the names of phantasms.

The fifth.   7. Propositions are false in the fifth manner,
when it is said that *the definition is the essence of
a thing ; whiteness,* or some other accident, *is
the genus, or universal.* For definition is not the
essence of any thing, but a speech signifying
what we conceive of the essence thereof ; and so
also not whiteness itself, but the word whiteness,
is a genus, or an universal name.

The sixth.   8. In the sixth manner they err, that say the
idea of anything is universal; as if there could
be in the mind an image of a man, which were
not the image of some one man, but a man simply,
which is impossible ; for every idea is one, and of
one thing ; but they are deceived in this, that they
put the *name* of the thing for the *idea* thereof.

The seventh.   9. They err in the seventh manner, that make
this distinction between things that have being,
that some of them *exist by themselves,* others by
accident ; namely, because *Socrates is a man* is
a necessary proposition, and *Socrates is a musi-
cian* a contingent proposition, therefore they say
some things exist necessarily or by themselves,
others contingently or by accident ; whereby,
seeing *necessary, contingent, by itself, by accident,*
are not names of things, but of propositions, they

that say *any thing that has being, exists by acci-
dent*, copulate the name of a proposition with the
name of a thing.   In the same manner also, they
err, which place some ideas in the understanding,
others in the fancy ; as if from the understanding
of this proposition, *man is a living creature*, we
had one idea or image of a man derived from
sense to the memory, and another to the under-
standing ; wherein that which deceives them is
this, that they think one idea should be answerable
to a name, another to a proposition, which is
false ; for proposition signifies only the order of
those things one after another, which we observe
in the same idea of man ; so that this proposition,
man is a living creature raises but one idea in
us, though in that idea we consider that first, for
which he is called man, and next that, for which
he is called living creature.   The falsities of pro-
positions in all these several manners, is to be
discovered by the definitions of the copulated
names.

10. But when names of bodies are copulated
with names of bodies, names of accidents with
names of accidents, names of names with names of
names, and names of phantasms with names of
phantasms, if we, nevertheless, remain still doubt-
ful whether such propositions are true, we ought
then in the first place to find out the definition of
both those names, and again the definitions of
such names as are in the former definition, and so
proceed by a continual resolution till we come to
a simple name, that is, to the most general or
most universal name of that kind ; and if after all

Falsity of
propositions
detected by
resolving the
terms with
definitions.

this, the truth or falsity thereof be not evident, we must search it out by philosophy, and ratiocination, beginning from definitions. For every proposition, universally true, is either a definition, or part of a definition, or the evidence of it depends upon definitions.

Of the fault of a syllogism consisting in implication of the terms with the copula.
11. That fault of a syllogism which lies hid in the form thereof, will always be found either in the implication of the copula with one of the terms, or in the equivocation of some word; and in either of these ways there will be four terms, which (as I have shewn) cannot stand in a true syllogism. Now the implication of the copula with either term, is easily detected by reducing the propositions to plain and clear predication; as (for example) if any man should argue thus,

> *The hand toucheth the pen,*
> *The pen toucheth the paper,*
> Therefore, *The hand toucheth the paper ;*

the fallacy will easily appear by reducing it, thus :

> *The hand, is, touching the pen,*
> *The pen, is, touching the paper,*
> Therefore, *The hand, is, touching the paper ;*

where there are manifestly these four terms, *the hand, touching the pen, the pen,* and *touching the the paper*. But the danger of being deceived by sophisms of this kind, does not seem to be so great, as that I need insist longer upon them.

Of the fault which consists in equivocation.
12. And though there may be fallacy in equivocal terms, yet in those that be manifestly such, there is none at all; nor in metaphors, for they profess the transferring of names from one thing

to another. Nevertheless, sometimes equivocals (and those not very obscure) may deceive; as in this argumentation :—*It belongs to metaphysics to treat of principles ; but the first principle of all, is, that the same thing cannot both exist and not exist at the same time ; and therefore it belongs to metaphysics to treat whether the same thing may both exist and not exist at the same time ;* where the fallacy lies in the equivocation of the word principle ; for whereas Aristotle in the beginning of his *Metaphysics,* says, that *the treating of principles belongs to primary science,* he understands by principles, causes of things, and certain existences which he calls primary ; but where he says *a primary proposition is a principle,* by principle, there, he means the beginning and cause of knowledge, that is, the understanding of words, which, if any man want, he is incapable of learning.

13. But the *captions* of sophists and sceptics, by which they were wont, of old, to deride and oppose truth, were faulty for the most part, not in the form, but in the matter of syllogism ; and they deceived not others oftener than they were themselves deceived. For the force of that famous argument of Zeno against motion, consisted in this proposition, *whatsoever may be divided into parts, infinite in number, the same is infinite ;* which he, without doubt, thought to be true, yet nevertheless is false. For to be divided into infinite parts, is nothing else but to be divided into as many parts as any man will. But it is not necessary that a line should have parts infinite in

PART 1.
5.

number, or be infinite, because I can divide and subdivide it as often as I please; for how many parts soever I make, yet their number is finite; but because he that says parts, simply, without adding how many, does not limit any number, but leaves it to the determination of the hearer, therefore we say commonly, a line may be divided Conclusion. infinitely; which cannot be true in any other sense.

And thus much may suffice concerning syllogism, which is, as it were, the first pace towards philosophy; in which I have said as much as is necessary to teach any man from whence all true argumentation has its force. And to enlarge this treatise with all that may be heaped together, would be as superfluous, as if one should (as I said before) give a young child precepts for the teaching of him to go; for the art of reasoning is not so well learned by precepts as by practice, and by the reading of those books in which the conclusions are all made by severe demonstration. And so I pass on to the way of philosophy, that is, to the method of study.

CHAPTER VI.

OF METHOD.

1. Method and science defined.—2. It is more easily known concerning singular, than universal things, that they are ; and contrarily, it is more easily known concerning universal, than singular things, why they are, or what are their causes.—3. What it is philosophers seek to know.—4. The first part, by which principles are found out, is purely analytical.—5. The highest causes, and most universal in every kind, are known by themselves.—6. Method from principles found out, tending to science simply, what it is.—7. That method of civil and natural science, which proceeds from sense to principles, is analytical ; and again, that, which begins at principles, is synthetical.—8. The method of searching out, whether any thing propounded be matter or accident.—9. The method of seeking whether any accident be in this, or in that subject. 10. The method of searching after the cause of any effect propounded.—11. Words serve to invention, as marks ; to demonstration, as signs.—12. The method of demonstration is synthetical.—13. Definitions only are primary and universal propositions.—14. The nature and definition of a definition. 15. The properties of a definition.—16. The nature of a demonstration.—17. The properties of a demonstration, and order of things to be demonstrated.—18. The faults of a demonstration.—19. Why the analytical method of geometricians cannot be treated of in this place.

1. For the understanding of *method,* it will be necessary for me to repeat the definition of philosophy, delivered above (Chap. i, art. 2.) in this manner, *Philosophy is the knowledge we acquire, by true ratiocination, of appearances, or apparent effects, from the knowledge we have of some possible production or generation of the same ; and*

PART I.
6.
Method and
science defined.

*of such production, as has been or may be, from
the knowledge we have of the effects.* METHOD,

therefore, in the study of philosophy, *is the
shortest way of finding out effects by their known
causes, or of causes by their known effects.* But
we are then said to know any effect, when we
know *that there be causes of the same,* and *in
what subject those causes are,* and *in what sub-
ject they produce that effect,* and *in what manner
they work the same.* And this is the science of
causes, or, as they call it, of the δίότι. All other
science, which is called the ὅτι, is either percep-
tion by sense, or the imagination, or memory
remaining after such perception.

The first beginnings, therefore, of knowledge,
are the phantasms of sense and imagination ; and
that there be such phantasms we know well enough
by nature ; but to know why they be, or from
what causes they proceed, is the work of ratioci-
nation ; which consists (as is said above, in the
1st Chapter, Art. 2) in *composition,* and *division*
or *resolution.* There is therefore no method, by
which we find out the causes of things, but is
either *compositive* or *resolutive,* or *partly com-
positive,* and *partly resolutive.* And the resolutive
is commonly called *analytical* method, as the
compositive is called *synthetical.*

It is easier
known con-
cerning singu-
lar than uni-
versal things,
that they are ;
and contrarily
it is easier
known con-
cerning uni-
versal than sin-

2. It is common to all sorts of method, to pro-
ceed from known things to unknown ; and this is
manifest from the cited definition of philosophy.
But in knowledge by sense, the whole object is
more known, than any part thereof ; as when we
see a man, the conception or whole idea of that
man is first or more known, than the particular

ideas of his being *figurate*, *animate*, and *rational;* that is, we first see the whole man, and take notice of his being, before we observe in him those other particulars. And therefore in any knowledge of the ὅτι, or that any thing *is*, the beginning of our search is from the whole idea; and contrarily, in our knowledge of the διότι, or of the causes of any thing, that is, in the sciences, we have more knowledge of the causes of the parts than of the whole. For the cause of the whole is compounded of the causes of the parts ; but it is necessary that we know the things that are to be compounded, before we can know the whole compound. Now, by parts, I do not here mean parts of the thing itself, but parts of its nature ; as, by the parts of man, I do not understand his head, his shoulders, his arms, &c. but his figure, quantity, motion, sense, reason, and the like; which accidents being compounded or put together, constitute the whole nature of man, but not the man himself. And this is the meaning of that common saying, namely, that some things are more known to us, others more known to nature; for I do not think that they, which so distinguish, mean that something is known to nature, which is known to no man ; and therefore, by those things, that are more known to us, we are to understand things we take notice of by our senses, and, by more known to nature, those we acquire the knowledge of by reason ; for in this sense it is, that the *whole*, that is, those things that have universal names, (which, for brevity's sake, I call *universal)* are more known to us than the *parts*, that is, such things as have names less universal,

F 2

(which I therefore call *singular*) ; and the causes of the parts are more known to nature than the cause of the whole ; that is, universals than singulars.

What it is philosophers seek to know.
3. In the study of philosophy, men search after science either simply or indefinitely ; that is, to know as much as they can, without propounding to themselves any limited question ; or they enquire into the cause of some determined appearance, or endeavour to find out the certainty of something in question, as what is the cause of *light*, of *heat*, of *gravity*, of a *figure* propounded, and the like ; or in what *subject* any propounded *accident* is inherent ; or what may conduce most to the *generation* of some propounded *effect* from many *accidents* ; or in what manner particular causes ought to be compounded for the production of some certain effect. Now, according to this variety of things in question, sometimes the *analytical method* is to be used, and sometimes the *synthetical*.

The first part, by which principles are discovered, is purely analytical.
4. But to those that search after science indefinitely, which consists in the knowledge of the causes of all things, as far forth as it may be attained, (and the causes of singular things are compounded of the causes of universal or simple things) it is necessary that they know the causes of universal things, or of such accidents as are common to all bodies, that is, to all matter, before they can know the causes of singular things, that is, of those accidents by which one thing is distinguished from another. And, again, they must know what those universal things are, before they can know their causes. Moreover, seeing universal

things are contained in the nature of singular things, the knowledge of them is to be acquired by reason, that is, by resolution. For example, if there be propounded a conception or *idea* of some singular thing, as of a *square*, this square is to be resolved into a *plain, terminated with a certain number of equal and straight lines and right angles*. For by this resolution we have these things universal or agreeable to all matter, namely, *line, plain,* (which contains *superficies*) *terminated, angle, straightness, rectitude,* and *equality;* and if we can find out the causes of these, we may compound them altogether into the cause of a square. Again, if any man propound to himself the conception of *gold,* he may, by resolving, come to the ideas of *solid, visible, heavy,* (that is, tending to the centre of the earth, or downwards) and many other more universal than gold itself; and these he may resolve again, till he come to such things as are most universal. And in this manner, by resolving continually, we may come to know what those things are, whose causes being first known severally, and afterwards compounded, bring us to the knowledge of singular things. I conclude, therefore, that the method of attaining to the universal knowledge of things, is purely *analytical*.

5. But the causes of universal things (of those, at least, that have any cause) are manifest of themselves, or (as they say commonly) known to nature; so that they need no method at all; for they have all but one universal cause, which is motion. For the variety of all figures arises out of the variety of those motions by which they are

The highest causes, and most universal in every kind, are known by themselves.

made; and motion cannot be understood to have any other cause besides motion; nor has the variety of those things we perceive by sense, as of *colours, sounds, savours,* &c. any other cause than motion, residing partly in the objects that work upon our senses, and partly in ourselves, in such manner, as that it is manifestly some kind of motion, though we cannot, without ratiocination, come to know what kind. For though many cannot understand till it be in some sort demonstrated to them, that all mutation consists in motion; yet this happens not from any obscurity in the thing itself, (for it is not intelligible that anything can depart either from rest, or from the motion it has, except by motion), but either by having their natural discourse corrupted with former opinions received from their masters, or else for this, that they do not at all bend their mind to the enquiring out of truth.

Method from principles found out, tending to science simply, what it is.

6. By the knowledge therefore of universals, and of their causes (which are the first principles by which we know the διότι of things) we have in the first place their definitions, (which are nothing but the explication of our simple conceptions.) For example, he that has a true conception of *place,* cannot be ignorant of this definition, *place is that space which is possessed or filled adequately by some body ;* and so, he that conceives *motion* aright, cannot but know that *motion is the privation of one place, and the acquisition of another.* In the next place, we have their generations or descriptions; as (for example) that *a line is made by the motion of a point, superficies by the motion of a line,* and *one motion by another*

motion, &c. It remains, that we enquire what
motion begets such and such effects ; as, what
motion makes a straight line, and what a circular ;
what motion thrusts, what draws, and by what
way ; what makes a thing which is seen or heard,
to be seen or heard sometimes in one manner,
sometimes in another. Now the method of this
kind of enquiry, is *compositive.* For first we are
to observe what effect a body moved produceth,
when we consider nothing in it besides its motion;
and we see presently that this makes a line, or
length ; next, what the motion of a long body
produces, which we find to be superficies ; and so
forwards, till we see what the effects of simple
motion are ; and then, in like manner, we are to
observe what proceeds from the addition, multipli-
cation, subtraction, and division, of these motions,
and what effects, what figures, and what properties,
they produce ; from which kind of contemplation
sprung that part of philosophy which is called
geometry.

From this consideration of what is produced by
simple motion, we are to pass to the consideration
of what effects one body moved worketh upon
another ; and because there may be motion in all
the several parts of a body, yet so as that the
whole body remain still in the same place, we
must enquire first, what motion causeth such and
such motion in the whole, that is, when one body
invades another body which is either at rest or in
motion, what way, and with what swiftness, the
invaded body shall move ; and, again, what motion
this second body will generate in a third, and so
forwards. From which contemplation shall be

PART I.
6.

Method from
principles
found out,
tending to
science simply,
what it is.

Method from
principles
found out,
tending to
science simply,
what it is.

drawn that part of philosophy which treats of motion.

In the third place we must proceed to the enquiry of such effects as are made by the motion of the parts of any body, as, how it comes to pass, that things when they are the same, yet seem not to be the same, but changed. And here the things we search after are sensible qualities, such as *light, colour, transparency, opacity, sound, odour, savour, heat, cold,* and the like; which because they cannot be known till we know the causes of sense itself, therefore the consideration of the causes of *seeing, hearing, smelling, tasting,* and *touching,* belongs to this third place; and all those qualities and changes, above mentioned, are to be referred to the fourth place; which two considerations comprehend that part of philosophy which is called *physics.* And in these four parts is contained whatsoever in natural philosophy may be explicated by demonstration, properly so called. For if a cause were to be rendered of natural appearances in special, as, what are the motions and influences of the heavenly bodies, and of their parts, the reason hereof must either be drawn from the parts of the sciences above mentioned, or no reason at all will be given, but all left to uncertain conjecture.

After *physics* we must come to *moral philosophy ;* in which we are to consider the motions of the mind, namely, *appetite, aversion, love, benevolence, hope, fear, anger, emulation, envy, &c.;* what causes they have, and of what they be causes. And the reason why these are to be considered after *physics* is, that they have

their causes in sense and imagination, which are
the subject of *physical* contemplation. Also the
reason, why all these things are to be searched
after in the order above-said, is, that physics
cannot be understood, except we know first what
motions are in the smallest parts of bodies; nor
such motion of parts, till we know what it is that
makes another body move; nor this, till we know
what simple motion will effect. And because all
appearance of things to sense is determined, and
made to be of such and such quality and quantity
by compounded motions, every one of which has a
certain degree of velocity, and a certain and
determined way; therefore, in the first place, we
we are to search out the ways of motion simply
(in which geometry consists); next the ways of
such generated motions as are manifest; and,
lastly, the ways of internal and invisible motions
(which is the enquiry of natural philosophers).
And, therefore, they that study natural philosophy,
study in vain, except they begin at geometry;
and such writers, or disputers thereof, as are
ignorant of geometry, do but make their readers
and hearers lose their time.

7. *Civil* and *moral philosophy* do not so adhere That method
of civil and na-
to one another, but that they may be severed. tural science,
For the causes of the motions of the mind are proceeding
from sense to
known, not only by ratiocination, but also by the principles, is
analytical;
experience of every man that takes the pains to and again, that
observe those motions within himself. And, which begins
at principles
therefore, not only they that have attained the is synthetical.
knowledge of the passions and perturbations of
the mind, by the *synthetical method*, and from
the very first principles of philosophy, may by

PART I.
6.

That method
of civil and na-
tural science,
proceeding
from sense
to principles,
is analytical;
and again that
which begins
at principles,
is synthetical.

proceeding in the same way, come to the causes
and necessity of constituting commonwealths, and
to get the knowledge of what is natural right, and
what are civil duties ; and, in every kind of
government, what are the rights of the common-
wealth, and all other knowledge appertaining to
civil philosophy ; for this reason, that the princi-
ples of the politics consist in the knowledge of
the motions of the mind, and the knowledge of
these motions from the knowledge of sense and
imagination ; but even they also that have not
learned the first part of philosophy, namely,
geometry and *physics*, may, notwithstanding,
attain the principles of civil philosophy, by the
analytical method. For if a question be pro-
pounded, as, *whether such an action be just or
unjust;* if that *unjust* be resolved into *fact against
law*, and that notion *law* into the *command* of him
or them that have *coercive power ;* and that
power be derived from the *wills* of men that con-
stitute such power, to the end they may live in
peace, they may at last come to this, that the
appetites of men and the passions of their minds
are such, that, unless they be restrained by some
power, they will always be making war upon one
another ; which may be known to be so by any
man's experience, that will but examine his own
mind. And, therefore, from hence he may pro-
ceed, by compounding, to the determination of
the justice or injustice of any propounded action.
So that it is manifest, by what has been said, that
the method of philosophy, to such as seek science
simply, without propounding to themselves the
solution of any particular question, is partly

analytical, and partly synthetical; namely, that
which proceeds from sense to the invention of
principles, analytical; and the rest synthetical.

8. To those that seek the cause of some certain The method
of searching
out, whether
anything pro-
pounded be
matter or ac-
cident.
and propounded appearance or effect, it happens,
sometimes, that they know not whether the thing,
whose cause is sought after, be matter or body, or
some accident of a body. For though in geometry,
when the cause is sought of magnitude, or propor-
tion, or figure, it be certainly known that these
things, namely magnitude, proportion, and figure,
are accidents; yet in natural philosophy, where all
questions are concerning the causes of the phan-
tasms of sensible things, it is not so easy to
discern between the things themselves, from which
those phantasms proceed, and the appearances of
those things to the sense; which have deceived
many, especially when the phantasms have been
made by light. For example, a man that looks
upon the sun, has a certain shining idea of the
magnitude of about a foot over, and this he calls
the sun, though he know the sun to be truly a
great deal bigger; and, in like manner, the phan-
tasm of the same thing appears sometimes round,
by being seen afar off, and sometimes square, by
being nearer. Whereupon it may well be doubted,
whether that phantasm be matter, or some body
natural, or only some accident of a body; in the ex-
amination of which doubt we may use this method.
The properties of matter and accidents already
found out by us, by the synthetical method, from
their definitions, are to be compared with the idea
we have before us; and if it agree with the pro-
perties of matter or body, then it is a body; other-

wise it is an accident. Seeing, therefore, matter cannot by any endeavour of ours be either made or destroyed, or increased, or diminished, or moved out of its place, whereas that idea appears, vanishes, is increased and diminished, and moved hither and thither at pleasure; we may certainly conclude that it is not a body, but an accident only. And this method is *synthetical*.

9. But if there be a doubt made concerning the subject of any known accident (for this may be doubted sometimes, as in the precedent example, doubt may be made in what subject that splendour and apparent magnitude of the sun is), then our enquiry must proceed in this manner. First, matter in general must be divided into parts, as, into object, medium, and the sentient itself, or such other parts as seem most conformable to the thing propounded. Next, these parts are severally to be examined how they agree with the definition of the subject; and such of them as are not capable of that accident are to be rejected. For example, if by any true ratiocination the sun be found to be greater than its apparent magnitude, then that magnitude is not in the sun; if the sun be in one determined straight line, and one determined distance, and the magnitude and splendour be seen in more lines and distances than one, as it is in reflection or refraction, then neither that splendour nor apparent magnitude are in the sun itself, and, therefore, the body of the sun cannot be the subject of that splendour and magnitude. And for the same reasons the air and other parts will be rejected, till at last nothing remain which can be the subject of that splendour and mag-

nitude but the sentient itself. And this method,
in regard the subject is divided into parts, is
analytical; and in regard the properties, both of
the subject and accident, are compared with the
accident concerning whose subject the enquiry is
made, it is synthetical.

10. But when we seek after the cause of any
propounded effect, we must in the first place get
into our mind an exact notion or idea of that
which we call cause, namely, that *a cause is the
sum or aggregate of all such accidents, both in
the agents and the patient, as concur to the
producing of the effect propounded; all which
existing together, it cannot be understood but
that the effect existeth with them; or that
it can possibly exist if any one of them be
absent.* This being known, in the next place we
must examine singly every accident that accom-
panies or precedes the effect, as far forth as it
seems to conduce in any manner to the production
of the same, and see whether the propounded
effect may be conceived to exist, without the
existence of any of those accidents; and by this
means separate such accidents, as do not concur,
from such as concur to produce the said effect;
which being done, we are to put together the
concurring accidents, and consider whether we
can possibly conceive, that when these are all
present, the effect propounded will not follow;
and if it be evident that the effect will follow,
then that aggregate of accidents is the entire
cause, otherwise not; but we must still search out
and put together other accidents. For example,
if the cause of light be propounded to be sought

PART 1.
6.

Method of
searching for
the cause of
any effect,
propounded.
out; first, we examine things without us, and find that whensoever light appears, there is some principal object, as it were the fountain of light, without which we cannot have any perception of light; and, therefore, the concurrence of that object is necessary to the generation of light. Next we consider the medium, and find, that unless it be disposed in a certain manner, namely, that it be transparent, though the object remain the same, yet the effect will not follow; and, therefore, the concurrence of transparency is also necessary to the generation of light. Thirdly, we observe our own body, and find that by the indisposition of the eyes, the brain, the nerves, and the heart, that is, by obstructions, stupidity, and debility, we are deprived of light, so that a fitting disposition of the organs to receive impressions from without is likewise a necessary part of the cause of light. Again, of all the accidents inherent in the object, there is none that can conduce to the effecting of light, but only action (or a certain motion), which cannot be conceived to be wanting, whensoever the effect is present; for, that anything may shine, it is not requisite that it be of such or such magnitude or figure, or that the whole body of it be moved out of the place it is in (unless it may perhaps be said, that in the sun, or other body, that which causes light is the light it hath in itself; which yet is but a trifling exception, seeing nothing is meant thereby but the cause of light; as if any man should say that the cause of light is that in the sun which produceth it); it remains, therefore, that the action, by which light is generated, is motion only in the parts of the

object. Which being understood, we may easily conceive what it is the medium contributes, namely, the continuation of that motion to the eye ; and, lastly, what the eye and the rest of the organs of the sentient contribute, namely, the continuation of the same motion to the last organ of sense, the heart. And in this manner the cause of light may be made up of motion continued from the original of the same motion, to the original of vital motion, light being nothing but the alteration of vital motion, made by the impression upon it of motion continued from the object. But I give this only for an example, for I shall speak more at large of light, and the generation of it, in its proper place. In the mean time it is manifest, that in the searching out of causes, there is need partly of the analytical, and partly of the synthetical method ; of the analytical, to conceive how circumstances conduce severally to the production of effects ; and of the synthetical, for the adding together and compounding of what they can effect singly by themselves. And thus much may serve for the method of invention. It remains that I speak of the method of teaching, that is, of demonstration, and of the means by which we demonstrate.

11. In the method of invention, the use of *Words serve to invention* words consists in this, that they may serve for *as marks; to* marks, by which, whatsoever we have found out *demonstration as signs.* may be recalled to memory ; for without this all our inventions perish, nor will it be possible for us to go on from principles beyond a syllogism or two, by reason of the weakness of memory. For example, if any man, by considering a triangle

set before him, should find that all its angles together taken are equal to two right angles, and that by thinking of the same tacitly, without any use of words either understood or expressed; and it should happen afterwards that another triangle, unlike the former, or the same in different situation, should be offered to his consideration, he would not know readily whether the same property were in this last or no, but would be forced, as often as a different triangle were brought before him (and the difference of triangles is infinite) to begin his contemplation anew; which he would have no need to do if he had the use of names, for every universal name denotes the conceptions we have of infinite singular things. Nevertheless, as I said above, they serve as *marks* for the help of our memory, whereby we register to ourselves our own inventions; but not as *signs* by which we declare the same to others; so that a man may be a philosopher alone by himself, without any master; Adam had this capacity. But to teach, that is, to demonstrate, supposes two at the least, and syllogistical speech.

The method of demonstration is synthetical. 12. And seeing teaching is nothing but leading the mind of him we teach, to the knowledge of our inventions, in that track by which we attained the same with our own mind; therefore, the same method that served for our invention, will serve also for demonstration to others, saving that we omit the first part of method which proceeded from the sense of things to universal principles, which, because they are principles, cannot be demonstrated; and seeing they are known by nature, (as was said above in the 5th article) they

need no demonstration, though they need expli-
cation. The whole method, therefore, of demon-
stration, is *synthetical*, consisting of that order of
speech which begins from primary or most
universal propositions, which are manifest of
themselves, and proceeds by a perpetual com-
position of propositions into syllogisms, till at
last the learner understand the truth of the
conclusion sought after.

13. Now, such principles are nothing but defi-
nitions, whereof there are two sorts; one of
names, that signify such things as have some con-
ceivable cause, and another of such names as
signify things of which we can conceive no cause
at all. Names of the former kind are, *body*, or
matter, *quantity*, or *extension*, *motion*, and what-
soever is common to all matter. Of the second
kind, are *such a body, such and so great motion,
so great magnitude, such figure*, and whatsoever
we can distinguish one body from another by.
And names of the former kind are well enough
defined, when, by speech as short as may be, we
raise in the mind of the hearer perfect and clear
ideas or conceptions of the things named, as when
we define motion to be *the leaving of one place,
and the acquiring of another continually ;* for
though no thing moved, nor any cause of motion
be in that definition, yet, at the hearing of that
speech, there will come into the mind of the
hearer an *idea* of motion clear enough. But
definitions of things, which may be understood to
have some cause, must consist of such names as
express the cause or manner of their generation,
as when we define a circle to be a figure made by

PART 1.
6.

Definitions
only are
primary,
& universal
propositions.

the circumduction of a straight line in a plane, &c. Besides definitions, there is no other proposition that ought to be called primary, or (according to severe truth) be received into the number of principles. For those *axioms of Euclid,* seeing they may be demonstrated, are no principles of demonstration, though they have by the consent of all men gotten the authority of principles, because they need not be demonstrated. Also, those *petitions,* or *postulata,* (as they call them) though they be principles, yet they are not principles of demonstration, but of construction only ; that is, not of science, but of power; or (which is all one) not of *theorems,* which are speculations, but of *problems,* which belong to practice, or the doing of something. But as for those common received opinions, *Nature abhors vacuity. Nature doth nothing in vain,* and the like, which are neither evident in themselves, nor at all to be demonstrated, and which are oftener false than true, they are much less to be acknowledged for principles.

To return, therefore, to definitions ; the reason why I say that the cause and generation of such things, as have any cause or generation, ought to enter into their definitions, is this. The end of science is the demonstration of the causes and generations of things ; which if they be not in the definitions, they cannot be found in the conclusion of the first syllogism, that is made from those definitions ; and if they be not in the first conclusion, they will not be found in any further conclusion deduced from that ; and, therefore, by proceeding in this manner, we shall never come to

science ; which is against the scope and intention of demonstration.

14. Now, seeing definitions (as I have said) are principles, or primary propositions, they are there-fore speeches ; and seeing they are used for the raising of an *idea* of some thing in the mind of the learner, whensoever that thing has a name, the definition of it can be nothing but the expli-cation of that name by speech ; and if that name be given it for some compounded conception, the definition is nothing but a resolution of that name into its most universal parts. As when we define man, saying *man is a body animated, sentient, rational,* those names, *body animated, &c.* are parts of that whole name *man;* so that definitions of this kind always consist of *genus* and *difference;* the former names being all, till the last, *general ;* and the last of all, *difference.* But if any name be the most universal in its kind, then the defini-tion of it cannot consist of *genus* and *difference,* but is to be made by such circumlocution, as best explicateth the force of that name. Again, it is possible, and happens often, that the *genus* and *difference* are put together, and yet make no definition ; as these words, *a straight line,* contain both the *genus* and *difference;* but are not a definition, unless we should think a straight line may be thus defined, *a straight line is a straight line :* and yet if there were added another name, consisting of different words, but signifying the same thing which these signify, then these might be the definition of that name. From what has been said, it may be understood how a defini-tion ought to be defined, namely, *that it is a*

G 2

PART I.
6.

proposition, whose predicate resolves the subject, when it may ; and when it may not, it exemplifies the same.

Properties of
a definition.

15. The properties of a definition are:

First, that it takes away equivocation, as also all that multitude of distinctions, which are used by such as think they may learn philosophy by disputation. For the nature of a definition is to define, that is, to determine the signification of the defined name, and to pare from it all other signification besides what is contained in the definition itself; and therefore one definition does as much, as all the distinctions (how many soever) that can be used about the name defined.

Secondly, that it gives an universal notion of the thing defined, representing a certain universal picture thereof, not to the eye, but to the mind. For as when one paints a man, he paints the image of some man; so he, that defines the name man, makes a representation of some man to the mind.

Thirdly, that it is not necessary to dispute whether definitions are to be admitted or no. For when a master is instructing his scholar, if the scholar understand all the parts of the thing defined, which are resolved in the definition, and yet will not admit of the definition, there needs no further controversy betwixt them, it being all one as if he refused to be taught. But if he understand nothing, then certainly the definition is faulty; for the nature of a definition consists in this, that it exhibit a clear idea of the thing defined; and principles are either known by themselves, or else they are not principles.

Fourthly, that, in philosophy, definitions are

before defined names. For in teaching philosophy,
the first beginning is from definitions; and all pro-
gression in the same, till we come to the knowledge
of the thing compounded, is compositive. Seeing,
therefore, definition is the explication of a com-
pounded name by resolution, and the progression
is from the parts to the compound, definitions
must be understood before compounded names;
nay, when the names of the parts of any speech
be explicated, it is not necessary that the definition
should be a name compounded of them. For
example, when these names, *equilateral, quadri-
lateral, right-angled,* are sufficiently understood,
it is not necessary in geometry that there should
be at all such a name as *square;* for defined
names are received in philosophy for brevity's
sake only.

Fifthly, that compounded names, which are de-
fined one way in some one part of philosophy,
may in another part of the same be otherwise
defined; as a *parabola* and an *hyperbole* have
one definition in geometry, and another in rhetoric;
for definitions are instituted and serve for the
understanding of the doctrine which is treated of.
And, therefore, as in one part of philosophy, a
definition may have in it some one fit name for
the more brief explanation of some proposition in
geometry; so it may have the same liberty in
other parts of philosophy; for the use of names is
particular (even where many agree to the settling
of them) and arbitrary.

Sixthly, that no name can be defined by any
one word; because no one word is sufficient for
the resolving of one or more words.

PART I.
6.

Seventhly, that a defined name ought not to be repeated in the definition. For a defined name is the whole compound, and a definition is the resolution of that compound into parts ; but no total can be part of itself.

Nature of a demonstration.

16. Any two definitions, that may be compounded into a syllogism, produce a conclusion ; which, because it is derived from principles, that is, from definitions, is said to be demonstrated ; and the derivation or composition itself is called a demonstration. In like manner, if a syllogism be made of two propositions, whereof one is a definition, the other a demonstrated conclusion, or neither of them is a definition, but both formerly demonstrated, that syllogism is also called a demonstration, and so successively. The definition therefore of a demonstration is this, *a demonstration is a syllogism, or series of syllogisms, derived and continued, from the definitions of names, to the last conclusion.* And from hence it may be understood, that all true ratiocination, which taketh its beginning from true principles, produceth science, and is true demonstration. For as for the original of the name, although that, which the Greeks called ἀπόδειξις, and the Latins *demonstratio*, was understood by them for that sort only of ratiocination, in which, by the describing of certain lines and figures, they placed the thing they were to prove, as it were before men's eyes, which is properly ἀποδεικνύειν, or to *shew* by the figure ; yet they seem to have done it for this reason, that unless it were in geometry, (in which only there is place for such figures) there was no ratiocination certain, and ending in

science, their doctrines concerning all other things
being nothing but controversy and clamour;
which, nevertheless, happened, not because the
truth to which they pretended could not be made
evident without figures, but because they wanted
true principles, from which they might derive
their ratiocination; and, therefore, there is no
reason but that if true definitions were premised
in all sorts of doctrines, the demonstrations also
would be true.

17. It is proper to methodical demonstration, Properties of a
First, that there be a true succession of one demonstration,
reason to another, according to the rules of syllo- things to be
gizing delivered above. demonstrated.

Secondly, that the premises of all syllogisms be
demonstrated from the first definitions.

Thirdly, that after definitions, he that teaches
or demonstrates any thing, proceed in the same
method by which he found it out; namely, that
in the first place those things be demonstrated,
which immediately succeed to universal definitions
(in which is contained that part of philosophy
which is called *philosophia prima*). Next, those
things which may be demonstrated by simple
motion (in which geometry consists). After
geometry, such things as may be taught or shewed
by manifest action, that is, by thrusting from, or
pulling towards. And after these, the motion or
mutation of the invisible parts of things, and the
doctrine of sense and imaginations, and of the
internal passions, especially those of men, in which
are comprehended the grounds of civil duties, or
civil philosophy; which takes up the last place.
And that this method ought to be kept in all sorts of
philosophy, is evident from hence, that such things

PART I.
6.

as I have said are to be taught last, cannot be demonstrated, till such as are propounded to be first treated of, be fully understood. Of which method no other example can be given, but that treatise of the elements of philosophy, which I shall begin in the next chapter, and continue to the end of the work.

Faults of a demonstration.

18. Besides those *paralogisms,* whose fault lies either in the falsity of the premises, or the want of true composition, of which I have spoken in the precedent chapter, there are two more, which are frequent in demonstration; one whereof is commonly called *petitio principii;* the other is the supposing of a *false cause;* and these do not only deceive unskilful learners, but sometimes masters themselves, by making them take that for well demonstrated, which is not demonstrated at all. *Petitio principii* is, when the conclusion to be proved is disguised in other words, and put for the definition or principle from whence it is to be demonstrated; and thus, by putting for the cause of the thing sought, either the thing itself or some effect of it, they make a circle in their demonstration. As for example, he that would demonstrate that the earth stands still in the centre of the world, and should suppose the earth's gravity to be the cause thereof, and define gravity to be a quality by which every heavy body tends towards the centre of the world, would lose his labour; for the question is, what is the cause of that quality in the earth? and, therefore, he that supposes gravity to be the cause, puts the thing itself for its own cause.

Of a *false cause* I find this example in a certain treatise where the thing to be demonstrated

is the motion of the earth. He begins, therefore, with this, that seeing the earth and the sun are not always in the same situation, it must needs be that one of them be locally moved, which is true ; next, he affirms that the vapours, which the sun raises from the earth and sea, are, by reason of this motion, necessarily moved, which also is true; from whence he infers the winds are made, and this may pass for granted ; and by these winds he says, the waters of the sea are moved, and by their motion the bottom of the sea, as if it were beaten forwards, moves round ; and let this also be granted ; wherefore, he concludes, the earth is moved ; which is, nevertheless, a paralogism. For, if that wind were the cause why the earth was, from the beginning, moved round, and the motion either of the sun or the earth were the cause of that wind, then the motion of the sun or the earth was before the wind itself ; and if the earth were moved, before the wind was made, then the wind could not be the cause of the earth's revolution ; but, if the sun were moved, and the earth stand still, then it is manifest the earth might remain unmoved, notwithstanding that wind ; and therefore that motion was not made by the cause which he allegeth. But paralogisms of this kind are very frequent among the writers of *physics*, though none can be more elaborate than this in the example given.

19. It may to some men seem pertinent to treat in this place of that art of the geometricians, which they call *logistica*, that is, the art, by which, from supposing the thing in question to be true, they proceed by ratiocination, till either they come to something known, by which they may

PART I.
6.

Why the analytical method of geometricians cannot be treated of in this place.

demonstrate the truth of the thing sought for ; or to something which is impossible, from whence they collect that to be false, which they supposed true. But this art cannot be explicated here, for this reason, that the method of it can neither be practised, nor understood, unless by such as are well versed in geometry ; and among geometricians themselves, they, that have most theorems in readiness, are the most ready in the use of this *logistica ;* so that, indeed, it is not a distinct thing from geometry itself; for there are, in the method of it, three parts ; the first whereof consists in the finding out of equality betwixt known and unknown things, which they call equation ; and this equation cannot be found out, but by such as know perfectly the nature, properties, and transpositions of proportion, as also the addition, subtraction, multiplication, and division of lines and superficies, and the extraction of roots ; which are the parts of no mean geometrician. The second is, when an equation is found, to be able to judge whether the truth or falsity of the question may be deduced from it, or no ; which yet requires greater knowledge. And the third is, when such an equation is found, as is fit for the solution of the question, to know how to resolve the same in such manner, that the truth or falsity may thereby manifestly appear ; which, in hard questions, cannot be done without the knowledge of the nature of crooked-lined figures ; but he that understands readily the nature and properties of these, is a complete geometrician. It happens besides, that for the finding out of equations, there is no certain method, but he is best able to do it, that has the best natural wit.

PART II.

FÍRST GROUNDS OF PHILOSOPHY.

CHAPTER VII.

OF PLACE AND TIME.

1. In the teaching of natural philosophy, I cannot begin better (as I have already shewn) than from *privation* ; that is, from feigning the world to be annihilated. But, if such annihilation of all things be supposed, it may perhaps be asked, what would remain for any man (whom only I except from this universal annihilation of things) to consider as the subject of philosophy, or at all to reason upon ; or what to give names unto for ratiocination's sake.

PART II.
7.

Things that have no existence, may nevertheless be understood and computed.

I say, therefore, there would remain to that man ideas of the world, and of all such bodies as he had, before their annihilation, seen with his eyes, or perceived by any other sense; that is to say, the memory and imagination of magnitudes, motions, sounds, colours, &c. as also of their order and parts. All which things, though they be nothing but ideas and phantasms, happening internally to him that imagineth; yet they will appear as if they were external, and not at all depending upon any power of the mind. And these are the things to which he would give names, and subtract them from, and compound them with one another. For seeing, that after the destruction of all other things, I suppose man still remaining, and namely that he thinks, imagines, and remembers, there can be nothing for him to think of but what is past; nay, if we do but observe diligently what it is we do when we consider and reason, we shall find, that though all things be still remaining in the world, yet we compute nothing but our own phantasms. For when we calculate the magnitude and motions of heaven or earth, we do not ascend into heaven that we may divide it into parts, or measure the motions thereof, but we do it sitting still in our closets or in the dark. Now things may be considered, that is, be brought into account, either as internal accidents of our mind, in which manner we consider them when the question is about some faculty of the mind; or as species of external things, not as really existing, but appearing only to exist, or to have a being without us. And in this manner we are now to consider them.

2. If therefore we remember, or have a phantasm of any thing that was in the world before the supposed annihilation of the same; and consider, not that the thing was such or such, but only that it had a being without the mind, we have presently a conception of that we call *space :* an imaginary space indeed, because a mere phantasm, yet that very thing which all men call so. For no man calls it space for being already filled, but because it may be filled; nor does any man think bodies carry their places away with them, but that the same space contains sometimes one, sometimes another body; which could not be if space should always accompany the body which is once in it. And this is of itself so manifest, that I should not think it needed any explaining at all, but that I find space to be falsely defined by certain philosophers, who infer from thence, one, that the world is infinite (for taking *space* to be the extension of bodies, and thinking extension may encrease continually, he infers that bodies may be infinitely extended); and, another, from the same definition, concludes rashly, that it is impossible even to God himself to create more worlds than one; for, if another world were to be created, he says, that seeing there is nothing without this world, and therefore (according to his definition) no space, that new world must be placed in nothing; but in nothing nothing can be placed; which he affirms only, without showing any reason for the same; whereas the contrary is the truth : for more cannot be put into a place already filled, so much is empty space fitter than that, which is full, for the receiving of new bodies.

PART II.
7.

Having therefore spoken thus much for these men's sakes, and for theirs that assent to them, I return to my purpose, and define *space* thus : SPACE *is the phantasm of a thing existing without the mind simply ;* that is to say, that phantasm, in which we consider no other accident, but only that it appears without us.

Time.

3. As a body leaves a phantasm of its magnitude in the mind, so also a moved body leaves a phantasm of its motion, namely, an idea of that body passing out of one space into another by continual succession. And this idea, or phantasm, is that, which (without receding much from the common opinion, or from *Aristotle's* definition) I call *Time.* For seeing all men confess a year to be time, and yet do not think a year to be the accident or affection of any body, they must needs confess it to be, not in the things without us, but only in the thought of the mind. So when they speak of the times of their predecessors, they do not think after their predecessors are gone, that their times can be any where else than in the memory of those that remember them. And as for those that say, days, years, and months are the motions of the sun and moon, seeing it is all one to say, motion *past* and motion *destroyed,* and that *future* motion is the same with motion which *is not yet begun,* they say that, which they do not mean, that there neither is, nor has been, nor shall be any time : for of whatsoever it may be said, *it has been* or *it shall be,* of the same also it might have been said heretofore, or may be said hereafter, *it is.* What then can days, months, and years, be, but the names of such

computations made in our mind ? *Time* therefore
is a phantasm, but a phantasm of motion, for if we would know by what moments time passes away, we make use of some motion or other, as of the sun, of a clock, of the sand in an hour-glass, or we mark some line upon which we imagine something to be moved, there being no other means by which we can take notice of any time at all. And yet, when I say *time* is a phantasm of motion, I do not say this is sufficient to define it by; for this word *time* comprehends the notion of *former* and *latter*, or of *succession* in the motion of a body, in as much as it is first *here* then *there*. Wherefore a complete definition of *time* is such as this, TIME *is the phantasm of before and after in motion;* which agrees with this definition of *Aristotle, time is the number of motion according to former and latter ;* for that numbering is an act of the mind; and therefore it is all one to say, *time is the number of motion according to former and latter ;* and *time is a phantasm of motion numbered.* But that other definition, *time is the measure of motion,* is not so exact, for we measure time by motion and not motion by time.

4. One space is called *part* of another space, and one time *part* of another time, when this contains that and something besides. From whence it may be collected, that nothing can rightly be called a PART, but that which is compared with something that contains it.

5. And therefore to *make parts*, or to *part* or DIVIDE space or *time,* is nothing else but to consider one and another within the same ; so that

if any man *divide* space or time, the diverse conceptions he has are more, by one, than the parts he makes; for his first conception is of that which is to be divided, then of some part of it, and again of some other part of it, and so forwards as long as he goes on in dividing.

But it is to be noted, that here, by *division*, I do not mean the severing or pulling asunder of one space or time from another (for does any man think that one hemisphere may be separated from the other hemisphere, or the first hour from the second?) but diversity of consideration ; so that division is not made by the operation of the hands but of the mind.

One.

6. When space or time is considered among other spaces or times, it is said to be ONE, namely *one of them ;* for except one space might be added to another, and subtracted from another space, and so of time, it would be sufficient to say space or time simply, and superfluous to say one space or one time, if it could not be conceived that there were another. The common definition of *one,* namely, that *one is that which is undivided,* is obnoxious to an absurd consequence ; for it may thence be inferred, that whatsoever is divided is many things, that is, that every divided thing, is divided things, which is insignificant.

Number.

7. NUMBER is *one* and *one,* or *one one* and *one,* and so forwards ; namely, *one* and *one* make the number *two,* and *one one* and *one* the number *three ;* so are all other numbers made ; which is all one as if we should say, *number is unities.*

Composition.

8. To COMPOUND space of spaces, or time of times, is first to consider them one after another,

and then altogether as one ; as if one should
reckon first the head, the feet, the arms, and the
body, severally, and then for the account of them
all together put *man*. And that which is so put
for all the severals of which it consists, is called
the WHOLE ; and those severals, when by the
division of the whole they come again to be
considered singly, are parts thereof; and therefore
the *whole* and *all the parts taken together* are
the same thing. And as I noted above, that in
division it is not necessary to pull the parts
asunder ; so in *composition*, it is to be understood,
that for the making up of a whole there is no
need of putting the parts together, so as to make
them touch one another, but only of collecting
them into one sum in the mind. For thus all men,
being considered together, make up the whole of
mankind, though never so much dispersed by time
and place ; and twelve hours, though the hours of
several days, may be compounded into one number
of twelve.

9. This being well understood, it is manifest, The whole.
that nothing can rightly be called a whole, that is
not conceived to be compounded of parts, and that
it may be divided into parts ; so that if we deny
that a thing has parts, we deny the same to be a
whole. For example, if we say the soul can have
no parts, we affirm that no soul can be a whole
soul. Also it is manifest, that nothing has parts
till it be divided ; and when a thing is divided,
the parts are only so many as the division makes
them. Again, that a part of a part is a part of
the whole; and thus any part of the number *four*,
as *two*, is a part of the number *eight ;* for *four* is

made of *two* and *two ;* but *eight* is compounded of *two, two,* and *four*, and therefore *two*, which is a part of the part *four*, is also a part of the whole *eight*.

10. Two spaces are said to be CONTIGUOUS, when there is no other space betwixt them. But two times, betwixt which there is no other time, are called immediate, as A B, B C. And any two spaces, as well as

$$\overline{\text{A} \qquad \text{B} \qquad \text{C}}$$

times, are said to be CONTINUAL, when they have one common part, as A C, B D, where the part B C is common;

$$\overline{\text{A} \quad \text{B} \quad \text{C} \quad \text{D}}$$

and more spaces and times are continual, when every two which are next one another are continual.

11. That part which is between two other parts, is called a MEAN; and that which is not between two other parts, an EXTREME. And of extremes, that which is first reckoned is the BEGINNING, and that which last, the END; and all the means together taken are the WAY. Also, *extreme parts* and *limits* are the same thing. And from hence it is manifest, that *beginning* and *end* depend upon the order in which we number them; and that to *terminate* or *limit* space and time, is the same thing with *imagining their beginning and end;* as also that every thing is FINITE or INFINITE, according as we imagine or not imagine it *limited* or *terminated* every way; and that the *limits* of any number are *unities*, and of these, that which is the first in our numbering is the *beginning*, and that which we number last, is the *end.* When we say number is *infinite*, we mean only that no number is expressed; for when we

speak of the numbers *two, three, a thousand,* &c. they are always *finite*. But when no more is said but this, *number is infinite*, it is to be understood as if it were said, this name *number* is an *indefinite* name.

12. Space or time is said to be *finite in power*, or *terminable*, when there may be assigned a number of finite spaces or times, as of paces or hours, than which there can be no greater number of the same measure in that space or time; and *infinite in power* is that space or time, in which a greater number of the said paces or hours may be assigned, than any number that can be given. But we must note, that, although in that space or time which is infinite in power, there may be numbered more paces or hours than any number that can be assigned, yet their number will always be finite; for every number is finite. And therefore his ratiocination was not good, that undertaking to prove the world to be finite, reasoned thus; *If the world be infinite, then there may be taken in it some part which is distant from us an infinite number of paces : but no such part can be taken ; wherefore the world is not infinite ;* because that consequence of the major proposition is false; for in an infinite space, whatsoever we take or design in our mind, the distance of the same from us is a finite space; for in the very designing of the place thereof, we put an end to that space, of which we ourselves are the beginning ; and whatsoever any man with his mind cuts off both ways from infinite, he determines the same, that is, he makes it finite.

Of infinite space or time, it cannot be said that

PART II.
7.

it is a *whole* or *one:* not a *whole,* because not compounded of parts ; for seeing parts, how many soever they be, are severally finite, they will also, when they are all put together, make a whole finite : nor *one,* because nothing can be said to be one, except there be another to compare it with ; but it cannot be conceived that there are two spaces, or two times, infinite. Lastly, when we make question whether the world be finite or infinite, we have nothing in our mind answering to the name *world ;* for whatsoever we imagine, is therefore finite, though our computation reach the fixed stars, or the ninth or tenth, nay, the thousandth sphere. The meaning of the question is this only, whether God has actually made so great an addition of body to body, as we are able to make of space to space.

Division proceeds not to the least.

13. And, therefore, that which is commonly said, that space and time may be divided infinitely, is not to be so understood, as if there might be any infinite or eternal division ; but rather to be taken in this sense, *whatsoever is divided, is divided into such parts as may again be divided ;* or thus, *the least divisible thing is not to be given ;* or, as geometricians have it, *no quantity is so small, but a less may be taken ;* which may easily be demonstrated in this manner. Let any space or time, that which was thought to be the least divisible, be divided into two equal parts, A and B. I say either of them, as A, may be divided again. For suppose the part A to be contiguous to the part B of one side, and of the other side to some other space equal to B. This whole space, therefore, being greater than the

space given, is divisible. Wherefore, if it be
divided into two equal parts, the part in the middle, which is A, will be also divided into two equal parts; and therefore A was divisible.

CHAPTER VIII.

OF BODY AND ACCIDENT.

1. Body defined.—2. Accident defined.—3. How an accident may be understood to be in its subject.—4. Magnitude, what it is.—5. Place, what it is, and that it is immovable.— 6. What is full and empty.—7. Here, there, somewhere, what they signify.—8. Many bodies cannot be in one place, nor one body in many places.—9. Contiguous and continual, what they are.—10. The definition of motion. No motion intelligible but with time.—11. What it is to be at rest, to have been moved, and to be moved. No motion to be conceived, without the conception of past and future.—12. A point, a line, superficies and solid, what they are.—13. Equal, greater, and less in bodies and magnitudes, what they are.—14. One and the same body has always one and the same magnitude. 15. Velocity, what it is.—16. Equal, greater, and less in times, what they are.—17. Equal, greater, and less, in velocity, what. 18. Equal, greater, and less, in motion, what.—19. That which is at rest, will always be at rest, except it be moved by some external thing; and that which is moved, will always be moved, unless it be hindered by some external thing.— 20. Accidents are generated and destroyed, but bodies not so. 21. An accident cannot depart from its subject.—22. Nor be moved.— 23. Essence, form, and matter, what they are. 24. First matter, what.—25. That the whole is greater than any part thereof, why demonstrated.

1. HAVING understood what imaginary space is, Body defined. in which we supposed nothing remaining without us, but all those things to be destroyed, that, by

existing heretofore, left images of themselves in our minds; let us now suppose some one of those things to be placed again in the world, or created anew. It is necessary, therefore, that this new-created or replaced thing do not only fill some part of the space above mentioned, or be coincident and coextended with it, but also that it have no dependance upon our thought. And this is that which, for the extension of it, we commonly call *body*; and because it depends not upon our thought, we say is *a thing subsisting of itself*; as also *existing*, because without us; and, lastly, it is called the *subject*, because it is so placed in and *subjected* to imaginary space, that it may be understood by reason, as well as perceived by sense. The definition, therefore, of *body* may be this, a *body is that, which having no dependance upon our thought, is coincident or coextended with some part of space.*

Accident defined.
2. But what an *accident* is cannot so easily be explained by any definition, as by examples. Let us imagine, therefore, that a body fills any space, or is coextended with it; that coextension is not the coextended body: and, in like manner, let us imagine that the same body is removed out of its place; that removing is not the removed body: or let us think the same not removed; that not removing or rest is not the resting body. What, then, are these things? They are *accidents* of that body. But the thing in question is, *what is an accident?* which is an enquiry after that which we know already, and not that which we should enquire after. For who does not always and in the same manner understand him that says any

thing is extended, or moved, or not moved ? But
most men will have it be said that *an accident is*
something, namely, some part of a natural thing,
when, indeed, it is no part of the same. To satisfy
these men, as well as may be, they answer best
that define an *accident* to be *the manner by which
any body is conceived;* which is all one as if they
should say, *an accident is that faculty of any
body, by which it works in us a conception of
itself.* Which definition, though it be not an
answer to the question propounded, yet it is an
answer to that question which should have been
propounded, namely, *whence does it happen that
one part of any body appears here, another
there ?* For this is well answered thus : *it happens
from the extension of that body.* Or, *how comes
it to pass that the whole body, by succession, is
seen now here, now there ?* and the answer will be,
by reason of its motion. Or, lastly, *whence is it
that any body possesseth the same space for
sometime ?* and the answer will be, *because it is
not moved.* For if concerning the name of a
body, that is, concerning a concrete name, it be
asked, *what is it ?* the answer must be made by
definition ; for the question is concerning the
signification of the name. But if it be asked
concerning an abstract name, *what is it ?* the
cause is demanded why a thing appears so or so.
As if it be asked, *what is hard ?* The answer
will be, hard is that, whereof no part gives place,
but when the whole gives place. But if it be
demanded, *what is hardness ?* a cause must be
shewn why a part does not give place, except the

whole give place. Wherefore, I define an *accident* to be *the manner of our conception of body.*

How an accident may be understood to be in its subject.

3. When an *accident* is said *to be in a body*, it is not so to be understood, as if any thing were contained in that body ; as if, for example, redness were in blood, in the same manner, as blood is in a bloody cloth, that is, as a part in the whole ; for so, an accident would be a body also. But, as magnitude, or rest, or motion, is in that which is great, or which resteth, or which is moved, (which, how it is to be understood, every man understands) so also, it is to be understood, that every other accident *is in* its subject. And this, also, is explicated by *Aristotle* no otherwise than negatively, namely, that *an accident is in its subject, not as any part thereof, but so as that it may be away, the subject still remaining ;* which is right, saving that there are certain accidents which can never perish except the body perish also; for no body can be conceived to be without extension, or without figure. All other accidents, which are not common to all bodies, but peculiar to some only, as *to be at rest, to be moved, colour, hardness,* and the like, do perish continually, and are succeeded by others ; yet so, as that the body never perisheth. And as for the opinion that some may have, that all other accidents are not in their bodies in the same manner that extension, motion, rest, or figure, are in the same ; for example, that colour, heat, odour, virtue, vice, and the like, are otherwise in them, and, as they say, *inherent ;* I desire they would suspend their judgment for the present, and expect a little, till it be found out

by ratiocination, whether these very accidents are not also certain motions either of the mind of the perceiver, or of the bodies themselves which are perceived; for in the search of this, a great part of natural philosophy consists.

4. The *extension* of a body, is the same thing with the *magnitude* of it, or that which some call *real space*. But this *magnitude* does not depend upon our cogitation, as imaginary space doth; for this is an effect of our imagination, but *magnitude* is the cause of it; this is an accident of the mind, that of a body existing out of the mind.

5. That space, by which word I here understand imaginary space, which is coincident with the magnitude of any body, is called the *place* of that body; and the body itself is that which we call the *thing placed*. Now *place*, and the *magnitude* of the thing placed, differ. First in this, that a body keeps always the same *magnitude*, both when it is at rest, and when it is moved; but when it is moved, it does not keep the same *place*. Secondly in this, that *place* is a phantasm of any body of such and such quantity and figure; but *magnitude* is the peculiar accident of every body; for one body may at several times have several places, but has always one and the same magnitude. Thirdly in this, that *place* is nothing out of the mind, nor *magnitude* any thing within it. And lastly, *place* is feigned extension, but *magnitude* true extension; and a placed body is not extension, but a thing extended. Besides, *place is immovable;* for, seeing that which is moved, is understood to be carried from place to place, if place were moved, it would also be carried from place to

Place, what it
is, and that it
is immovable.

place, so that one place must have another place, and that place another place, and so on infinitely, which is ridiculous. And as for those, that, by making *place* to be of the same nature with *real space*, would from thence maintain it to be immovable, they also make place, though they do not perceive they make it so, to be a mere phantasm. For whilst one affirms that place is therefore said to be immovable, because space in general is considered there ; if he had remembered that nothing is general or universal besides names or signs, he would easily have seen that that space, which he says is considered in general, is nothing but a phantasm, in the mind or the memory, of a body of such magnitude and such figure. And whilst another says: real space is made immovable by the understanding ; as when, under the super-ficies of running water, we imagine other and other water to come by continual succession, that superficies fixed there by the understanding, is the *immovable place* of the river : what else does he make it to be but a phantasm, though he do it obscurely and in perplexed words? Lastly, the nature of *place* does not consist in the *superficies of the ambient,* but in *solid space ;* for the whole placed body is coextended with its whole place, and every part of it with every answering part of the same place ; but seeing every placed body is a solid thing, it cannot be understood to be coex-tended with superficies. Besides, how can any whole body be moved, unless all its parts be moved together with it? Or how can the internal parts of it be moved, but by leaving their place ? But the internal parts of a body cannot leave the

superficies of an external part contiguous to it; and, therefore, it follows, that if place be the superficies of the ambient, then the parts of a body moved, that is, bodies moved, are not moved.

6. Space, or place, that is possessed by a body, is called *full,* and that which is not so possessed, is called *empty.*

7. *Here, there, in the country, in the city,* and other the like names, by which answer is made to the question *where is it ?* are not properly names of place, nor do they of themselves bring into the mind the place that is sought ; for *here* and *there* signify nothing, unless the thing be shewn at the same time with the finger or something else ; but when the eye of him that seeks, is, by pointing or some other sign, directed to the thing sought, the place of it is not hereby defined by him that answers, but found out by him that asks the question. Now such shewings as are made by words only, as when we say, *in the country,* or *in the city,* are some of greater latitude than others, as when we say, *in the country, in the city, in such a street, in a house, in the chamber, in bed,* &c. For these do, by little and little, direct the seeker nearer to the proper place ; and yet they do not determine the same, but only restrain it to a lesser space, and signify no more, than that the place of the thing is within a certain space designed by those words, as a part is in the whole. And all such names, by which answer is made to the question *where ?* have, for their highest *genus,* the name *somewhere.* From whence it may be understood, that whatsoever is somewhere, is in some place properly so called, which place is part of

PART II.
8.

that greater space that is signified by some of these names, *in the country, in the city,* or the like.

Many bodies cannot be in one place, nor one body in many places.

8. A body, and the magnitude, and the place thereof, are divided by one and the same act of the mind ; for, to divide an extended body, and the extension thereof, and the idea of that extension, which is place, is the same with dividing any one of them ; because they are coincident, and it cannot be done but by the mind, that is by the division of space. From whence it is manifest, that neither two bodies can be together in the same place, nor one body be in two places at the same time. Not two bodies in the same place ; because when a body that fills its whole place is divided into two, the place itself is divided into two also, so that there will be two places. Not one body in two places ; for the place that a body fills being divided into two, the placed body will be also divided into two ; for, as I said, a place and the body that fills that place, are divided both together ; and so there will be two bodies.

Contiguous and continual, what they are.

9. Two bodies are said to be *contiguous* to one another, and *continual,* in the same manner as spaces are ; namely, *those are contiguous, between which there is no space.* Now, by space I understand, here as formerly, an idea or phantasm of a body. Wherefore, though between two bodies there be put no other body, and consequently no magnitude, or, as they call it, real space, yet if another body may be put between them, that is, if there intercede any imagined space which may receive another body, then those bodies are not contiguous. And this is so easy to be understood, that I should wonder at some men, who being

otherwise skilful enough in philosophy, are of a
different opinion, but that I find that most of those
that affect metaphysical subtleties wander from
truth, as if they were led out of their way by an
ignis fatuus.   For can any man that has his
natural senses, think that two bodies must
therefore necessarily touch one another, because
no other body is between them ?   Or that there
can be no *vacuum,* because *vacuum* is nothing, or
as they call it, *non ens ?*   Which is as childish, as
if one should reason thus ; no man can fast,
because to fast is to eat nothing ; but nothing
cannot be eaten.   *Continual, are any two bodies
that have a common part ; and more than two are
continual, when every two, that are next to one
another, are continual.*

   10. MOTION *is a continual relinquishing of* The definition
one place, and acquiring of another ; and that of motion. No
motion intelli-
place which is relinquished is commonly called the gible but with
time.
terminus a quo, as that which is acquired is called
the *terminus ad quem ;* I say a continual relin-
quishing, because no body, how little soever, can
totally and at once go out of its former place into
another, so, but that some part of it will be in a
part of a place which is common to both, namely,
to the relinquished and the acquired places.   For
example, let any body be in the
place A C B D; the same body can-
not come into the place B D E F,
but it must first be in G H I K,
whose part G H B D is common to
both the places A C B D, and G H I K, and
whose part B D I K, is common to both the places
G H I K, and B D E F.   Now it cannot be con-

ceived that any thing can be moved without time; for time is, by the definition of it, a phantasm, that is, a conception of motion ; and, therefore, to conceive that any thing may be moved without time, were to conceive motion without motion, which is impossible.

What it is to be at rest, to have been moved, and to be moved. No motion to be conceived without the conception of past and future.

11. *That is said to be at rest, which, during any time, is in one place ; and that to be moved, or to have been moved, which, whether it be now at rest or moved, was formerly in another place than that which it is now in.* From which definitions it may be inferred, first, that *whatsoever is moved, has been moved ;* for if it be still in the same place in which it was formerly, it is at rest, that is, it is not moved, by the definition of *rest ;* but if it be in another place, it has been moved, by the definition of *moved.* Secondly, that *what is moved, will yet be moved ;* for that which is moved, leaveth the place where it is, and therefore will be in another place, and consequently will be moved still. Thirdly, that *whatsoever is moved, is not in one place during any time, how little soever that time be ;* for by the definition of rest, that which is in one place during any time, is at rest.

There is a certain sophism against motion, which seems to spring from the not understanding of this last proposition. For they say, that, *if any body be moved, it is moved either in the place where it is, or in the place where it is not ; both which are false ; and therefore nothing is moved.* But the falsity lies in the major proposition; for that which is moved, is neither moved in the place where it is, nor in the place where is not; but

from the place where it is, to the place where it is
not. Indeed it cannot be denied but that whatsoever is moved, is moved somewhere, that is, within some space; but then the place of that body is not that whole space, but a part of it, as is said above in the seventh article. From what is above demonstrated, namely, that whatsoever is moved, has also been moved, and will be moved, this also may be collected, that there can be no conception of motion, without conceiving past and future time.

12. Though there be no body which has not some magnitude, yet if, when any body is moved, *A point, a line, superficies, and solid, what they are.* the magnitude of it be not at all considered, the way it makes is called a *line*, or one single dimension; and the space, through which it passeth, is called *length;* and the body itself, a *point;* in which sense the earth is called a *point*, and the way of its yearly revolution, the *ecliptic line.* But if a body, which is moved, be considered as *long*, and be supposed to be so moved, as that all the several parts of it be understood to make several lines, then the way of every part of that body is called *breadth*, and the space which is made is called *superficies*, consisting of two dimensions, one whereof to every several part of the other is applied whole. Again, if a body be considered as having *superficies*, and be understood to be so moved, that all the several parts of it describe several lines, then the way of every part of that body is called *thickness* or *depth*, and the space which is made is called *solid*, consisting of three dimensions, any two whereof are applied whole to every several part of the third.

But if a body be considered as *solid*, then it is not possible that all the several parts of it should describe several lines; for what way soever it be moved, the way of the following part will fall into the way of the part before it, so that the same solid will still be made which the foremost superficies would have made by itself. And therefore there can be no other dimension in any body, as it is a body, than the three which I have now described; though, as it shall be shewed hereafter, *velocity*, which is motion according to *length*, may, by being applied to all the parts of a *solid*, make a magnitude of motion, consisting of four dimensions; as the goodness of gold, computed in all the parts of it, makes the price and value thereof.

Equal, great, greater, and less, in bodies and magnitudes, what they are.

13. *Bodies*, how many soever they be, that can fill every one the place of every one, are said to be *equal* every one to every other. Now, one body may fill the same place which another body filleth, though it be not of the same figure with that other body, if so be that it may be understood to be reducible to the same figure, either by flexion or transposition of the parts. And *one body is greater than another body, when a part of that is equal to all this; and less, when all that is equal to a part of this.* Also, *magnitudes* are *equal*, or *greater*, or *lesser*, than one another, for the same consideration, namely, when the bodies, of which they are the magnitudes, are either *equal*, or *greater*, or *less*, &c.

One and the same body has always one and the same magnitude.

14. One and the same body is always of one and the same magnitude. For seeing a body and the magnitude and place thereof cannot be comprehended in the mind otherwise than as they

are coincident, if any body be understood to be at rest, that is, to remain in the same place during some time, and the magnitude thereof be in one part of that time greater, and in another part less, that body's place, which is one and the same, will be coincident sometimes with greater, sometimes with less magnitude, that is, the same place will be greater and less than itself, which is impossible. But there would be no need at all of demonstrating a thing that is in itself so manifest, if there were not some, whose opinion concerning bodies and their magnitudes is, that a body may exist separated from its magnitude, and have greater or less magnitude bestowed upon it, making use of this principle for the explication of the nature of *rarum* and *densum*.

15. Motion, in as much as a certain length may in a certain time be transmitted by it, is called VELOCITY or *swiftness :* &c. For though *swift* be very often understood with relation to *slower* or *less swift*, as great is in respect of less, yet nevertheless, as magnitude is by philosophers taken absolutely for extension, so also *velocity* or *swiftness* may be put absolutely for motion according to length.

16. Many motions are said to be made in equal times, when every one of them begins and ends together with some other motion, or if it had begun together, would also have ended together with the same. For time, which is a phantasm of motion, cannot be reckoned but by some exposed motion; as in dials by the motion of the sun or of the hand ; and if two or more motions begin and end with this motion, they are said to be made in

equal times; from whence also it is easy to under-
stand what it is to be moved in greater or longer
time, and in less time or not so long ; namely,
that that is longer moved, which beginning with
another, ends later; or ending together, began
sooner.

Equal, greater,
and less, in ve-
locity, what.

17. Motions are said to be equally swift, when
equal lengths are transmitted in equal times ; and
greater swiftness is that, wherein greater length is
passed in equal time, or equal length in less time.
Also that swiftness by which equal lengths are
passed in equal parts of time, is called *uniform*
swiftness or motion ; and of motions *not uniform*,
such as become swifter or slower by equal in-
creasings or decreasings in equal parts of time, are
said to be *accelerated* or *retarded uniformly*.

Equal, greater,
and less, in
motion, what.

18. But motion is said to be greater, less, and
equal, not only in regard of the length which is
transmitted in a certain time, that is, in regard of
swiftness only, but of swiftness applied to every
smallest particle of magnitude ; for when any
body is moved, every part of it is also moved ; and
supposing the parts to be halves, the motions of
those halves have their swiftness equal to one
another, and severally equal to that of the whole ;
but the motion of the whole is equal to those two
motions, either of which is of equal swiftness with
it ; and therefore it is one thing for two motions
to be equal to one another, and another thing for
them to be equally swift. And this is manifest in
two horses that draw abreast, where the motion of
both the horses together is of equal swiftness with
the motion of either of them singly; but the
motion of both is greater than the motion of one

of them, namely, double. Wherefore *motions are* PART II.
said to be simply equal to one another, when the 8.
swiftness of one, computed in every part of its
magnitude, is equal to the swiftness of the other
computed also in every part of its magnitude:
and greater than one another, when the swiftness
of one computed as above, is greater than the
swiftness of the other so computed; and less,
when less. Besides, the magnitude of motion
computed in this manner is that which is commonly
called FORCE.

19. *Whatsoever is at rest, will always be at* That which
rest, unless there be some other body besides it, is at rest will always be at
which, by endeavouring to get into its place by rest, except it be moved by
motion, suffers it no longer to remain at rest. some external thing.
For suppose that some finite body exist and be at
rest, and that all space besides be empty; if now
this body begin to be moved, it will certainly be
moved some way; seeing therefore there was
nothing in that body which did not dispose it to
rest, the reason why it is moved this way is in
something out of it; and in like manner, if it had
been moved any other way, the reason of motion
that way had also been in something out of it; but
seeing it was supposed that nothing is out of it,
the reason of its motion one way would be the
same with the reason of its motion every other
way, wherefore it would be moved alike all ways
at once; which is impossible.

In like manner, *whatsoever is moved, will* That which is
always be moved, except there be some other body moved will always be moved,
besides it, which causeth it to rest. For if we unless it be hindered by some
suppose nothing to be without it, there will be no external thing.
reason why it should rest now, rather than at

I 2

another time ; wherefore its motion would cease in every particle of time alike; which is not intelligible.

20. When we say a living creature, a tree, or any other specified body is *generated* or *destroyed,* it is not to be so understood as if there were made a body of that which is not-body, or not a body of a body, but of a living creature not a living creature, of a tree not a tree, &c. that is, that those accidents for which we call one thing a living creature, another thing a tree, and another by some other name, are generated and destroyed ; and that therefore the same names are not to be given to them now, which were given them before. But that magnitude for which we give to any thing the name of body is neither generated nor destroyed. For though we may feign in our mind that a point may swell to a huge bulk, and that this may again contract itself to a point; that is, though we may imagine something to arise where before was nothing, and nothing to be there where before was something, yet we cannot comprehend in our mind how this may possibly be done in nature. And therefore philosophers, who tie themselves to natural reason, suppose that a body can neither be generated nor destroyed, but only that it may appear otherwise than it did to us, that is, under different *species*, and consequently be called by other and other names; so that that which is now called man, may at another time have the name of not-man ; but that which is once called body, can never be called not-body. But it is manifest, that all other accidents besides magnitude or extension may be generated and destroyed ;

as when a white thing is made black, the whiteness PART II.
that was in it perisheth, and the blackness that 8.
was not in it is now generated; and therefore
bodies, and the accidents under which they appear
diversely, have this difference, that bodies are
things, and not generated; accidents are generated,
and not things.

21. And therefore, when any thing appears An accident
cannot depart
otherwise than it did by reason of other and other from its subject
accidents, it is not to be thought that an accident
goes out of one subject into another, (for they are
not, as I said above, in their subjects as a part in
the whole, or as a contained thing in that which
contains it, or as a master of a family in his house,)
but that one accident perisheth, and another is
generated. For example, when the hand, being
moved, moves the pen, motion does not go out of
the hand into the pen; for so the writing might be
continued though the hand stood still; but a new
motion is generated in the pen, and is the pen's
motion.

22. And therefore also it is improper to say, an Nor be moved.
accident is moved; as when, instead of saying,
figure is an accident of a body carried away, we
say, *a body carries away its figure.*

23. Now that accident for which we give a Essence, form,
and matter,
certain name to any body, or the accident which what they are.
denominates its subject, is commonly called the
ESSENCE thereof; as rationality is the essence of
a man; whiteness, of any white thing, and exten-
sion the essence of a body. And the same essence,
in as much as it is generated, is called the FORM.
Again, a body, in respect of any accident, is called
the SUBJECT, and in respect of the form it is
called the MATTER.

Also, the production or perishing of any accident makes its subject be said *to be changed;* only the production or perishing of form makes it be said it is *generated* or *destroyed;* but in all generation and mutation, the name of *matter* still remains. For a table made of wood is not only wooden, but wood; and a statue of brass is brass as well as brazen; though Aristotle, in his *Metaphysics*, says, that whatsoever is made of any thing ought not to be called ἐκεῖνο, but ἐκείνινον; as that which is made of wood, not ξύλον, but ξύλινον, that is, not wood, but wooden.

First matter, what.

24. And as for that matter which is common to all things, and which philosophers, following Aristotle, usually call *materia prima*, that is, *first matter*, it is not any body distinct from all other bodies, nor is it one of them. What then is it ? A mere name ; yet a name which is not of vain use ; for it signifies a conception of body without the consideration of any form or other accident except only magnitude or extension, and aptness to receive form and other accident. So that whensoever we have use of the name *body in general*, if we use that of *materia prima*, we do well. For as when a man not knowing which was first, water or ice, would find out which of the two were the matter of both, he would be fain to suppose some third matter which were neither of these two ; so he that would find out what is the matter of all things, ought to suppose such as is not the matter of anything that exists. Wherefore *materia prima* is nothing ; and therefore they do not attribute to it either form or any other accident besides quantity ; whereas all singular things have their forms and accidents certain.

Materia prima, therefore, is body in general,
that is, body considered universally, not as having neither form nor any accident, but in which no form nor any other accident but quantity are at all considered, that is, they are not drawn into argumentation.

25. From what has been said, those axioms may be demonstrated, which are assumed by Euclid in the beginning of his first element, about the equality and inequality of magnitudes ; of which, omitting the rest, I will here demonstrate only this one, *the whole is greater than any part thereof;* to the end that the reader may know that those axioms are not indemonstrable, and therefore not principles of demonstration ; and from hence learn to be wary how he admits any thing for a principle, which is not at least as evident as these are.   *Greater* is defined to be that, whose part is equal to the whole of another.   Now if we suppose any whole to be A, and a part of it to be B ; seeing the whole B is equal to itself, and the same B is a part of A ; therefore a part of A will be equal to the whole B.   Wherefore, by the definition above, A is greater than B; which was to be proved.

CHAPTER IX.

OF CAUSE AND EFFECT.

1. Action and passion, what they are.—2. Action and passion mediate and immediate. — 3. Cause simply taken. Cause without which no effect follows, or cause necessary by supposition. — 4. Cause efficient and material. — 5. An entire cause is always sufficient to produce its effect. At the same instant that the cause is entire, the effect is produced. Every effect has a necessary cause.—6. The generation of effects is continual. What is the beginning in causation.—7. No cause of motion but in a body contiguous and moved.—8. The same agents and patients, if alike disposed, produce like effects though at different times. — 9. All mutation is motion. 10. Contingent accidents, what they are.

PART II.
9.
Action and passion, what they are.

1. A BODY is said to work upon or *act*, that is to say, *do* something to another body, when it either generates or destroys some accident in it : and the body in which an accident is generated or destroyed is said to *suffer*, that is, to have something *done* to it by another body ; as when one body by putting forwards another body generates motion in it, it is called the AGENT ; and the body in which motion is so generated, is called the PATIENT ; so fire that warms the hand is the *agent*, and the hand, which is warmed, is the *patient*. That accident, which is generated in the patient, is called the EFFECT.

Action and passion, mediate, and immediate.

2. When an agent and patient are contiguous to one another, their action and passion are then said to be *immediate*, otherwise, *mediate ;* and when another body, lying betwixt the agent and patient, is contiguous to them both, it is then itself both an

agent and a patient; an agent in respect of the body next after it, upon which it works, and a patient in respect of the body next before it, from which it suffers. Also, if many bodies be so ordered that every two which are next to one another be contiguous, then all those that are betwixt the first and the last are both agents and patients, and the first is an agent only, and the last a patient only.

3. An agent is understood to *produce* its deter- mined or certain effect in the patient, according to some certain accident or accidents, with which both it and the patient are affected ; that is to say, the agent hath its effect precisely such, not because it is a body, but because such a body, or so moved. For otherwise all agents, seeing they are all bodies alike, would produce like effects in all patients. And therefore the fire, for example, does not warm, because it is a body, but because it is hot ; nor does one body put forward another body because it is a body, but because it is moved into the place of that other body. The cause, therefore, of all effects consists in certain accidents both in the agents and in the patients ; which when they are all present, the effect is produced ; but if any one of them be wanting, it is not produced; and that accident either of the agent or patient, without which the effect cannot be produced, is called *causa sine qua non*, or *cause necessary by supposition,* as also the *cause requisite for the production of the effect.* But a CAUSE simply, or *an entire cause, is the aggregate of all the accidents both of the agents how many soever they be, and of the patient, put together ; which when they*

are all supposed to be present, it cannot be under-stood but that the effect is produced at the same instant; and if any one of them be wanting, it cannot be understood but that the effect is not produced.

Cause efficient, and material.

4. The aggregate of accidents in the agent or agents, requisite for the production of the effect, the effect being produced, is called the *efficient cause* thereof; and the aggregate of accidents in the patient, the effect being produced, is usually called the *material cause;* I say the effect being produced; for where there is no effect, there can be no cause; for nothing can be called a cause, where there is nothing that can be called an effect. But the efficient and material causes are both but partial causes, or parts of that cause, which in the next precedent article I called an entire cause. And from hence it is manifest, that the effect we expect, though the agents be not defective on their part, may nevertheless be frustrated by a defect in the patient; and when the patient is sufficient, by a defect in the agents.

An entire cause is always suf-ficient to pro-duce its effect.

5. An entire cause is always sufficient for the production of its effect, if the effect be at all possible. For let any effect whatsoever be pro-pounded to be produced; if the same be produced, it is manifest that the cause which produced it was a sufficient cause; but if it be not produced, and yet be possible, it is evident that something was wanting either in some agent, or in the patient, without which it could not be produced; that is, that some accident was wanting which was requi-site for its production; and therefore, that cause was not *entire,* which is contrary to what was supposed.

It follows also from hence, that in whatsoever
instant the cause is entire, in the same instant the
effect is produced. For if it be not produced, At the same instant that the
something is still wanting, which is requisite for cause is entire, the effect is produced.
the production of it ; and therefore the cause was
not entire, as was supposed.

And seeing a necessary cause is defined to be Every effect
that, which being supposed, the effect cannot but has a necessary cause.
follow ; this also may be collected, that whatsoever
effect is produced at any time, the same is produced
by a necessary cause. For whatsoever is produced,
in as much as it is produced, had an entire cause,
that is, had all those things, which being supposed,
it cannot be understood but that the effect fol-
lows ; that is, it had a necessary cause. And in the
same manner it may be shewn, that whatsoever
effects are hereafter to be produced, shall have a
necessary cause ; so that all the effects that have
been, or shall be produced, have their necessity in
things antecedent.

6. And from this, that whensoever the cause is The genera-
entire, the effect is produced in the same instant, tion of effects is continual.
it is manifest that causation and the production What is the beginning in
of effects consist in a certain continual progress ; causation.
so that as there is a continual mutation in the
agent or agents, by the working of other agents
upon them, so also the patient, upon which they
work, is continually altered and changed. For
example : as the heat of the fire increases more
and more, so also the effects thereof, namely, the
heat of such bodies as are next to it, and again, of
such other bodies as are next to them, increase
more and more accordingly ; which is already no
little argument that all mutation consists in motion

only; the truth whereof shall be further demonstrated in the ninth article. But in this progress of causation, that is, of action and passion, if any man comprehend in his imagination a part thereof, and divide the same into parts, the first part or beginning of it cannot be considered otherwise than as action or cause; for, if it should be considered as effect or passion, then it would be necessary to consider something before it, for its cause or action; which cannot be, for nothing can be before the beginning. And in like manner, the last part is considered only as effect; for it cannot be called cause, if nothing follow it; but after the last, nothing follows. And from hence it is, that in all action the beginning and cause are taken for the same thing. But every one of the intermediate parts are both action and passion, and cause and effect, according as they are compared with the antecedent or subsequent part.

No cause of motion but in a body contiguous and moved.

7. There can be no cause of motion, except in a body contiguous and moved. For let there be any two bodies which are not contiguous, and betwixt which the intermediate space is empty, or, if filled, filled with another body which is at rest; and let one of the propounded bodies be supposed to be at rest; I say it shall always be at rest. For if it shall be moved, the cause of that motion, by the 8th chapter, article 19, will be some external body; and, therefore, if between it and that external body there be nothing but empty space, then whatsoever the disposition be of that external body or of the patient itself, yet if it be supposed to be now at rest, we may conceive it will continue so till it be touched by some other body.

But seeing cause, by the definition, is the aggregate of all such accidents, which being supposed to be present, it cannot be conceived but that the effect will follow, those accidents, which are either in external bodies, or in the patient itself, cannot be the cause of future motion. And in like manner, seeing we may conceive that whatsoever is at rest will still be at rest, though it be touched by some other body, except that other body be moved; therefore in a contiguous body, which is at rest, there can be no cause of motion. Wherefore there is no cause of motion in any body, except it be contiguous and moved.

The same reason may serve to prove that whatsoever is moved, will always be moved on in the same way and with the same velocity, except it be hindered by some other contiguous and moved body; and consequently that no bodies, either when they are at rest, or when there is an interposition of vacuum, can generate or extinguish or lessen motion in other bodies. There is one that has written that things moved are more resisted by things at rest, than by things contrarily moved; for this reason, that he conceived motion not to be so contrary to motion as rest. That which deceived him was, that the words *rest* and *motion* are but contradictory names; whereas motion, indeed, is not resisted by rest, but by contrary motion.

8. But if a body work upon another body at one time, and afterwards the same body work upon the same body at another time, so that both the agent and patient, and all their parts, be in all things as they were; and there be no difference, except only in time, that is, that one action be former, the

The same agents and patients, if alike disposed, produce like effects, though at different times.

PART II. other later in time; it is manifest of itself, that the
9. effects will be equal and like, as not differing in
anything besides time.   And as effects themselves
proceed from their causes, so the diversity of them
depends upon the diversity of their causes also.

All mutation   9. This being true, it is necessary that mutation
is motion.   can be nothing else but motion of the parts of that
body which is changed.   For first, we do not say
anything is changed, but that which appears to our
senses otherwise than it appeared formerly.   Se-
condly, both those appearances are effects pro-
duced in the sentient; and, therefore, if they be
different, it is necessary, by the preceding article,
that either some part of the agent, which was for-
merly at rest, is now moved, and so the mutation
consists in this motion; or some part, which was
formerly moved, is now otherwise moved, and so
also the mutation consists in this new motion; or
which, being formerly moved, is now at rest,
which, as I have shewn above, cannot come to
pass without motion; and so again, mutation is
motion; or lastly, it happens in some of these
manners to the patient, or some of its parts; so
that mutation, howsoever it be made, will consist
in the motion of the parts, either of the body
which is perceived, or of the sentient body, or of
both.   Mutation therefore is motion, namely, of
the parts either of the agent or of the patient;
which was to be demonstrated.   And to this it is
consequent, that rest cannot be the cause of any-
thing, nor can any action proceed from it; seeing
neither motion nor mutation can be caused by it.

Contingent   10. Accidents, in respect of other accidents
accidents.   which precede them, or are before them in time.

and upon which they do not depend as upon their **PART II.** causes, are called *contingent* accidents; I say, in respect of those accidents by which they are not generated; for, in respect of their causes, all things come to pass with equal necessity; for otherwise they would have no causes at all; which, of things generated, is not intelligible.

CHAPTER X.

OF POWER AND ACT.

1. Power and cause are the same thing.—2. An act is produced at the same instant in which the power is plenary.—3. Active and passive power are parts only of plenary power.—4. An act, when said to be possible.—5. An act necessary and contingent, what.—6. Active power consists in motion.—7. Cause, formal and final, what they are.

1. CORRESPONDENT to *cause* and *effect*, are POWER and ACT; nay, those and these are the same things; though, for divers considerations, they have divers names. For whensoever any agent has all those accidents which are necessarily requisite for the production of some effect in the patient, then we say that agent has *power* to produce that effect, if it be applied to a patient. But, as I have shewn in the precedent chapter, those accidents constitute the efficient cause; and therefore the same accidents, which constitute the efficient cause, constitute also the *power* of the agent. Wherefore the *power of the agent* and the *efficient cause* are the same thing. But they are considered with this difference, that *cause* is

Power and cause are the same thing.

so called in respect of the effect already produced, and power in respect of the same effect to be produced hereafter ; so that *cause* respects the past, *power* the future time.   Also, the *power of the agent* is that which is commonly called *active power*.

In like manner, whensoever any patient has all those accidents which it is requisite it should have, for the production of some effect in it, we say it is in the *power* of that patient to produce that effect, if it be applied to a fitting agent.   But those accidents, as is defined in the precedent chapter, constitute the material cause ; and therefore the *power of the patient*, commonly called *passive power*, and *material cause*, are the same thing ; but with this different consideration, that in cause the past time, and in power the future, is respected. Wherefore the power of the agent and patient together, which may be called entire or *plenary power*, is the same thing with *entire cause ;* for they both consist in the sum or aggregate of all the accidents, as well in the agent as in the patient, which are requisite for the production of the effect. Lastly, as the accident produced is, in respect of the cause, called an effect, so in respect of the power, it is called an *act*.

An act is produced at the same instant in which the power is plenary.
2. As therefore the effect is produced in the same instant in which the cause is entire, so also every act that may be produced, is produced in the same instant in which the power is plenary.   And as there can be no effect but from a sufficient and necessary cause, so also no act can be produced but by sufficient power, or that power by which it could not but be produced.

3. And as it is manifest, as I have shewn, that the efficient and material causes are severally and by themselves parts only of an entire cause, and cannot produce any effect but by being joined together, so also power, active and passive, are parts only of plenary and entire power; nor, except they be joined, can any act proceed from them; and therefore these powers, as I said in the first article, are but conditional, namely, *the agent has power, if it be applied to a patient; and the patient has power, if it be applied to an agent;* otherwise neither of them have power, nor can the accidents, which are in them severally, be properly called powers; nor any action be said to be possible for the power of the agent alone or of the patient alone.

4. For that is an impossible act, for the production of which there is no power plenary. For seeing plenary power is that in which all things concur, which are requisite for the production of an act, if the power shall never be plenary, there will always be wanting some of those things, without which the act cannot be produced; wherefore that act shall never be produced; that is, that act is IMPOSSIBLE: and every act, which is not impossible, is POSSIBLE. Every act, therefore, which is possible, shall at some time be produced; for if it shall never be produced, then those things shall never concur which are requisite for the production of it; wherefore that act is *impossible,* by the definition; which is contrary to what was supposed.

5. A *necessary act* is that, the production whereof it is impossible to hinder; and therefore

PART II.
10.

An act neces-
sary and con-
tingent, what.

every act, that shall be produced, shall necessarily be produced; for, that it shall not be produced, is impossible; because, as is already demonstrated, every possible act shall at some time be produced; nay, this proposition, *what shall be, shall be*, is as necessary a proposition as this, *a man is a man*.

But here, perhaps, some man may ask whether those future things, which are commonly called *contingents*, are necessary. I say, therefore, that generally all contingents have their necessary causes, as is shewn in the preceding chapter; but are called contingents in respect of other events, upon which they do not depend; as the rain, which shall be tomorrow, shall be necessary, that is, from necessary causes; but we think and say it happens by chance, because we do not yet perceive the causes thereof, though they exist now; for men commonly call that *casual* or *contingent*, whereof they do not perceive the necessary cause; and in the same manner they used to speak of things past, when not knowing whether a thing be done or no, they say it is possible it never was done.

Wherefore, all propositions concerning future things, contingent or not contingent, as this, *it will rain tomorrow*, or this, *tomorrow the sun will rise*, are either necessarily true, or necessarily false; but we call them contingent, because we do not yet know whether they be true or false; whereas their verity depends not upon our knowledge, but upon the foregoing of their causes. But there are some, who though they confess this whole proposition, *tomorrow it will either rain, or not rain*, to be true, yet they will not acknowledge the parts of it, as, *tomorrow it will rain*, or, *tomorrow*

it will not rain, to be either of them true by itself; because they say neither this nor that is true *determinately.* But what is this *determinately true,* but true upon our knowledge, or evidently true? And therefore they say no more but that it is not yet known whether it be true or no; but they say it more obscurely, and darken the evidence of the truth with the same words, with which they endeavour to hide their own ignorance.

6. In the 9th article of the preceding chapter, I have shewn that the efficient cause of all motion and mutation consists in the motion of the agent, or agents; and in the first article of this chapter, that the power of the agent is the same thing with the efficient cause. From whence it may be understood, that all active power consists in motion also; and that power is not a certain accident, which differs from all acts, but is, indeed, an act, namely, motion, which is therefore called power, because another act shall be produced by it afterwards. For example, if of three bodies the first put forward the second, and this the third, the motion of the second, in respect of the first which produceth it, is the act of the second body; but, in respect of the third, it is the active power of the same second body.

7. The writers of metaphysics reckon up two other causes besides the *efficient* and *material,* namely, the ESSENCE, which some call the *formal cause,* and the END, or *final cause;* both which are nevertheless efficient causes. For when it is said the essence of a thing is the cause thereof, *as to be rational is the cause of man,* it is not intelligible; for it is all one, as if it were said, *to be a*

K 2

man is the cause of man; which is not well said.
And yet the knowledge of the *essence* of anything,
is the cause of the knowledge of the thing itself;
for, if I first know that a thing is *rational,* I know
from thence, that the same is *man;* but this is no
other than an efficient cause. A *final cause* has no
place but in such things as have sense and will;
and this also I shall prove hereafter to be an effi-
cient cause.

CHAPTER XI.

OF IDENTITY AND DIFFERENCE.

1. What it is for one thing to differ from another.—2. To differ
in number, magnitude, species, and genus, what.—3. What is
relation, proportion, and relatives.—4. Proportionals, what.—
5. The proportion of magnitudes to one another, wherein it
consists.—6. Relation is no new accident, but one of those
that were in the relative before the relation or comparison was
made. Also the causes of accidents in the correlatives, are the
cause of relation.—7. Of the beginning of individuation.

What it is
for one thing
to differ from
another.
1. HITHERTO I have spoken of body simply, and
accidents common to all bodies, as *magnitude,
motion, rest, action, passion, power, possible, &c.;*
and I should now descend to those accidents by
which one body is distinguished from another, but
that it is first to be declared what it is to be *dis-
tinct* and *not distinct,* namely, what are the SAME
and DIFFERENT; for this also is common to all
bodies, that they may be distinguished and differ-
enced from one another. Now, two bodies are
said to *differ* from one another, when something
may be said of one of them, which cannot be said
of the other at the same time.

2. And, first of all, it is manifest that no two PART II. bodies are the *same ;* for seeing they are two, they are in two places at the same time ; as that, which is To differ in number, the *same,* is at the same time in one and the same magnitude, species, and place. All bodies therefore differ from one another genus, what. in *number,* namely, as one and another ; so that the *same* and *different in number,* are names opposed to one another by contradiction.

In *magnitude* bodies differ when one is greater than another, as *a cubit long,* and *two cubits long,* of *two pound weight,* and of *three pound weight.* And to these, *equals* are opposed.

Bodies, which differ more than in magnitude, are called *unlike ;* and those, which differ only in magnitude, *like.* Also, of unlike bodies, some are said to differ in the *species,* others in the *genus ;* in the *species,* when their difference is perceived by one and the same sense, as *white* and *black ;* and in the *genus,* when their difference is not perceived but by divers senses, as *white* and *hot.*

3. And the *likeness,* or *unlikeness, equality,* or What is *inequality* of one body to another, is called their relation, proportion, RELATION ; and the bodies themselves *relatives* or and relatives. *correlatives ; Aristotle* calls them τὰ πρὸς τι ; the first whereof is usually named the *antecedent,* and the second the *consequent ;* and the *relation* of the antecedent to the consequent, according to magnitude, namely, the equality, the excess or defect thereof, is called the PROPORTION of the antecedent to the consequent ; so that *proportion* is nothing but the equality or inequality of the magnitude of the antecedent compared to the magnitude of the consequent by their difference only, or compared also with their difference. For ex-

ample, the *proportion* of three to two consists only in this, that three *exceeds* two by unity ; and the proportion of two to five in this, that two, compared with five, is *deficient* of it by three, either simply, or compared with the numbers different ; and therefore in the proportion of unequals, the proportion of the less to the greater, is called DE-FECT ; and that of the greater to the less, EXCESS.

Proportion-als, what.

4. Besides, of unequals, some are more, some less, and some equally unequal; so that there is *proportion of proportions*, as well as of *magnitudes ;* namely, where two unequals have relation to two other unequals ; as, when the inequality which is between 2 and 3, is compared with the inequality which is between 4 and 5. In which comparison there are always four magnitudes ; or, which is all one, if there be but three, the middle-most is twice numbered ; and if the proportion of the first to the second, be equal to the proportion of the third to the fourth, then the four are said to be *proportionals ;* otherwise they are not proportionals.

The proportion of magnitudes to one another, wherein it consists.

5. The proportion of the antecedent to the consequent consists in their difference, not only simply taken, but also as compared with one of the relatives ; that is, either in that part of the greater, by which it exceeds the less, or in the remainder, after the less is taken out of the greater ; as the proportion of two to five consists in the three by which five exceeds two, not in three simply only, but also as compared with five or two. For though there be the same difference between two and five, which is between nine and twelve, namely three, yet there is not the same inequality ;

and therefore the proportion of two to five is not in all relation the same with that of nine to twelve, but only in that which is called arithmetical.

6. But we must not so think of relation, as if it were an accident differing from all the other accidents of the relative; but one of them, namely, that by which the comparison is made. For example, the likeness of one *white* to another *white*, or its unlikeness to *black*, is the same accident with its *whiteness;* and *equality* and *inequality*, the same accident with the *magnitude* of the thing compared, though under another name : for that which is called *white* or *great,* when it is not compared with something else, the same when it is compared, is called *like* or *unlike, equal* or *unequal.* And from this it follows that the causes of the accidents, which are in relatives, are the causes also of *likeness, unlikeness, equality* and *inequality;* namely, that he, that makes two unequal bodies, makes also their inequality ; and he, that makes a rule and an action, makes also, if the action be congruous to the rule, their congruity; if incongruous, their incongruity. And thus much concerning *comparison* of one body with another.

Relation is no new accident, but one of those that were in the relative, before the relation or comparison was made. Also the causes of accidents in correlatives are the cause of relation.

7. But the same body may at different times be compared with itself. And from hence springs a great controversy among philosophers about the *beginning of individuation,* namely, in what sense it may be conceived that a body is at one time the same, at another time not the same it was formerly. For example, whether a man grown old be the same man he was whilst he was young, or another man ; or whether a city be in different ages the same, or another city. Some place *individuity* in

Of the beginning of individuation.

PART II.
11.
Of the begin-
ning of indi-
viduation.

the unity of *matter ;* others, in the unity of *form ;* and one says it consists in the unity of the *aggre-gate of all the accidents together.* For *matter,* it is pleaded that a lump of wax, whether it be spherical or cubical, is the same wax, because the same matter. For *form,* that when a man is grown from an infant to be an old man, though his matter be changed, yet he is still the same numerical man ; for that *identity,* which cannot be attributed to the matter, ought probably to be ascribed to the form. For the *aggregate of accidents,* no instance can be made ; but because, when any new accident is generated, a new name is commonly imposed on the thing, therefore he, that assigned this cause of *individuity,* thought the thing itself also was become another thing. According to the first opinion, he that sins, and he that is punished, should not be the same man, by reason of the per-petual flux and change of man's body ; nor should the city, which makes laws in one age and abro-gates them in another, be the same city ; which were to confound all civil rights. According to the second opinion, two bodies existing both at once, would be one and the same numerical body. For if, for example, that ship of Theseus, concern-ing the difference whereof made by continual re-paration in taking out the old planks and putting in new, the sophisters of Athens were wont to dis-pute, were, after all the planks were changed, the same numerical ship it was at the beginning ; and if some man had kept the old planks as they were taken out, and by putting them afterwards together in the same order, had again made a ship of them, this, without doubt, had also been the same nume-

rical ship with that which was at the beginning;
and so there would have been two ships numerically
the same, which is absurd.   But, according to the
third opinion, nothing would be the same it
was ; so that a man standing would not be the same
he was sitting ; nor the water, which is in the vessel,
the same with that which is poured out of it.
Wherefore the beginning of *individuation* is not
always to be taken either from matter alone, or
from form alone.

But we must consider by what name anything
is called, when we inquire concerning the *identity*
of it. For it is one thing to ask concerning Socrates,
whether he be the same man, and another to ask
whether he be the same body ; for his body, when
he is old, cannot be the same it was when he was
an infant, by reason of the difference of magnitude ;
for one body has always one and the same magni-
tude ; yet, nevertheless, he may be the same man.
And therefore, whensoever the name, by which it
is asked whether a thing be the same it was, is
given it for the matter only, then, if the matter be
the same, the thing also is *individually* the same ;
as the water, which was in the sea, is the same
which is afterwards in the cloud ; and any body is
the same, whether the parts of it be put together,
or dispersed ; or whether it be congealed, or dis-
solved.   Also, if the name be given for such form
as is the beginning of motion, then, as long as that
motion remains, it will be the same *individual*
thing ; as that man will be always the same, whose
actions and thoughts proceed all from the same
beginning of motion, namely, that which was in
his generation ; and that will be the same river

Of the begin-
ning of indi-
viduation.

which flows from one and the same fountain,
whether the same water, or other water, or some-
thing else than water, flow from thence; and that
the same city, whose acts proceed continually from
the same institution, whether the men be the same
or no. Lastly, if the name be given for some
accident, then the *identity* of the thing will depend
upon the matter; for, by the taking away and
supplying of matter, the accidents that were, are
destroyed, and other new ones are generated,
which cannot be the same numerically; so that a
ship, which signifies matter so figured, will be the
same as long as the matter remains the same; but
if no part of the matter be the same, then it is
numerically another ship; and if part of the matter
remain and part be changed, then the ship will
be partly the same, and partly not the same.

CHAPTER XII.

OF QUANTITY.

1. The definition of quantity.—2. The exposition of quantity,
what it is.—3. How line, superficies, and solid, are exposed.
4. How time is exposed.—5. How number is exposed.—6. How
velocity is exposed.—7. How weight is exposed.—8. How the
proportion of magnitudes is exposed.—9. How the proportion
of times and velocities is exposed.

Definition
of quantity.

1. WHAT and how manifold *dimension* is, has
been said in the 8th chapter, namely, that there are
three dimensions, line or length, superficies, and
solid; every one of which, if it be determined, that
is, if the limits of it be made known, is commonly
called *quantity;* for by *quantity* all {men under-

stand that which is signified by that word, by which answer is made to the question, *How much is it?* Whensoever, therefore, it is asked, for example, *How long is the journey?* it is not answered indefinitely, *length;* nor, when it is asked, *How big is the field?* is it answered indefinitely, *superficies;* nor, if a man ask, *How great is the bulk?* indefinitely, *solid:* but it is answered determinately, the journey is a hundred miles; the field is a hundred acres; the bulk is a hundred cubical feet; or at least in some such manner, that the magnitude of the thing enquired after may by certain limits be comprehended in the mind. QUANTITY, therefore, cannot otherwise be defined, than to be *a dimension determined,* or *a dimension, whose limits are set out, either by their place, or by some comparison.*

2. And *quantity* is determined two ways; one, by the sense, when some sensible object is set before it; as when a line, a superficies or solid, of a foot or cubit, marked out in some matter, is objected to the eyes; which way of determining, is called *exposition,* and the quantity so known is called *exposed quantity;* the other by memory, that is, by comparison with some exposed quantity. In the first manner, when it is asked of what quantity a thing is, it is answered, of such quantity as you see exposed. In the second manner, answer cannot be made but by comparison with some exposed quantity; for if it be asked, how long is the way? the answer is, so many thousand paces; that is, by comparing the way with a pace, or some other measure, determined and known by exposition; or the quantity of it is to some other quan-

tity known by exposition, as the diameter of a
square is to the side of the same, or by some
other the like means. But it is to be understood,
that the quantity exposed must be some standing
or permanent thing, such as is marked out in
consistent or durable matter; or at least something
which is revocable to sense; for otherwise no com-
parison can be made by it. Seeing, therefore, by
what has been said in the next preceding chapter,
comparison of one magnitude with another is the
same thing with proportion; it is manifest, that
quantity determined in the second manner is
nothing else but the proportion of a dimension not
exposed to another which is exposed; that is, the
comparison of the equality or inequality thereof
with an exposed quantity.

3. *Lines, superficies,* and *solids,* are exposed,
first, by *motion,* in such manner as in the 8th
chapter I have said they are generated; but so as
that the marks of such motion be permanent; as
when they are designed upon some matter, as a
line upon paper; or graven in some durable
matter. Secondly, by *apposition;* as when one
line or length is applied to another line or length,
one breadth to another breadth, and one thickness
to another thickness; which is as much as to
describe a line by points, a superficies by lines,
and a solid by superficies; saving that by points
in this place are to be understood very short
lines; and, by superficies, very thin solids.
Thirdly, lines and superficies may be exposed by
section, namely, a line may be made by cutting
an exposed superficies; and a superficies, by the
cutting of an exposed solid.

4. *Time* is exposed, not only by the exposition
of a line, but also of some moveable thing, which
is moved uniformly upon that line, or at least is
supposed so to be moved. For, seeing time is an
idea of motion, in which we consider former and
latter, that is succession, it is not sufficient for the
exposition of time that a line be described; but
we must also have in our mind an imagination of
some moveable thing passing over that line; and
the motion of it must be uniform, that time may
be divided and compounded as often as there shall
be need. And, therefore, when philosophers, in
their demonstrations, draw a line, and say, *Let
that line be time*, it is to be understood as if they
said, *Let the conception of uniform motion upon
that line, be time.* For though the circles in dials
be lines, yet they are not of themselves sufficient
to note time by, except also there be, or be sup-
posed to be, a motion of the shadow or the hand.

5. *Number* is exposed, either by the exposition
of points, or of the names of number, *one, two,
three, &c.;* and those points must not be conti-
guous, so as that they cannot be distinguished by
notes, but they must be so placed that they may
be *discerned* one from another; for, from this it
is, that number is called *discreet quantity*,
whereas all quantity, which is designed by motion,
is called *continual quantity.* But that number
may be exposed by the names of number, it is
necessary that they be recited by heart and in
order, as one, two, three, &c.; for by saying one,
one, one, and so forward, we know not what
number we are at beyond two or three; which
also appear to us in this manner, not as number,
but as figure.

6. For the exposition of *velocity*, which, by the definition thereof, is a motion which, in a certain time, passeth over a certain space, it is requisite, not only that time be exposed, but that there be also exposed that space which is transmitted by the body, whose velocity we would determine ; and that a body be understood to be moved in that space also ; so that there must be exposed two lines, upon one of which uniform motion must be understood to be made, that the time may be determined ; and, upon the other, the velocity is to be computed. As if we would expose the velocity of the body A, we draw two lines A B and C D, and place a body in C also ; which done, we say the velocity of the body A is so great, that it passeth over the line A B in the same time in which the body C passeth over the line C D with uniform motion.

7. *Weight* is exposed by any heavy body, of what matter soever, so it be always alike heavy.

8. The *proportion* of two magnitudes is then exposed, when the magnitudes themselves are exposed, namely, the proportion of equality, when the magnitudes are equal ; and of inequality, when they are unequal. For seeing, by the 5th article of the preceding chapter, the proportion of two unequal magnitudes consists in their difference, compared with either of them ; and when two unequal magnitudes are exposed, their difference is also exposed ; it follows, that when magnitudes, which have proportion to one another, are exposed, their proportion also is exposed with them ; and, in like manner, the proportion of equals,

which consists in this, that there is no difference
of magnitude betwixt them, is exposed at the
same time when the equal magnitudes themselves
are exposed. For example, if the exposed lines
A B and C D be equal, the propor-
tion of equality is exposed in them;
and if the exposed lines, E F and E G
be unequal, the proportion which
E F has to E G, and that which E G
has to E F are also exposed in them; for not only
the lines themselves, but also their difference, G F,
is exposed. The proportion of unequals is quan-
tity; for the difference, G F, in which it consists,
is quantity. But the proportion of equality is not
quantity; because, between equals, there is no
difference; nor is one equality greater than another,
as one inequality is greater than another inequality.

9. The proportion of two *times*, or of two *uni-* How the pro-
form velocities, is then exposed, when two lines portion of
are exposed by which two bodies are understood velocities
to be moved uniformly; and therefore the same is exposed.
two lines serve to exhibit both their own propor-
tion, and that of the times and velocities, accord-
ing as they are considered to be exposed for the
magnitudes themselves, or for the times or velo-
cities. For let the two lines A and B be ex-
posed; their proportion therefore (by the
last foregoing article) is exposed; and if
they be considered as drawn with equal
and uniform velocity, then, seeing their times are
greater, or equal, or less, according as the same
spaces are transmitted in greater, or equal, or
less time, the lines A and B will exhibit the
equality or inequality, that is, the proportion

of the times. To conclude, if the same lines, A and B, be considered as drawn in the same time, then, seeing their velocities are greater, or equal, or less, according as they pass over in the same time longer, or equal, or shorter lines, the same lines, A and B, will exhibit the equality, or inequality, that is, the proportion of their velocities.

CHAPTER XIII.

OF ANALOGISM, OR THE SAME PROPORTION.

1, 2, 3, 4. The nature and definition of proportion, arithmetical and geometrical.—5. The definition, and some properties of the same arithmetical proportion.—6, 7. The definition and transmutations of analogism, or the same geometrical proportion.—8, 9. The definitions of hyperlogism and hypologism, that is, of greater and less proportion, and their transmutations.—10, 11, 12. Comparison of analogical quantities, according to magnitude.—13, 14, 15. Composition of proportions. 16, 17, 18, 19, 20, 21, 22, 23, 24, 25. The definition and properties of continual proportion.—26, 27, 28, 29. Comparison of arithmetical and geometrical proportions.

[Note, that in this chapter the sign + signifies that the quantities betwixt which it is put, are added together; and this sign — the remainder after the latter quantity is taken out of the former. So that A+B is equal to both A and B together; and where you see A—B, there A is the whole, B the part taken out of it, and A—B the remainder. Also, two letters, set together without any sign, signify, unless they belong to a figure, that one of the quantities is multiplied by the other; as A B signifies the product of A multiplied by B.]

The nature and definition of proportion, arithmetical & geometrical.
1. GREAT and little are not intelligible, but by comparison. Now that, to which they are compared, is something exposed; that is, some magnitude either perceived by sense, or so defined by words, that it may be comprehended by the mind. Also that, to which any magnitude is compared, is either

greater or less, or equal to it. And therefore pro-
portion (which, as I have shewn, is the estimation
or comprehension of magnitudes by comparison,)
is threefold, namely, proportion of *equality*, that
is, of equal to equal ; or of *excess*, which is of the
greater to the less ; or of *defect*, which is the pro-
portion of the less to the greater.

Again, every one of these proportions is two-
fold ; for if it be asked concerning any magnitude
given, how great it is, the answer may be made
by comparing it two ways; first, by saying it is
greater or less than another magnitude, by so
much ; as seven is less than ten, by three unities ;
and this is called *arithmetical proportion*. Se-
condly, by saying it is greater or less than another
magnitude, by such a part or parts thereof ; as
seven is less than ten, by three tenth parts of the
same ten. And though this proportion be not
always explicable by number, yet it is a deter-
minate proportion, and of a different kind from
the former, and called *geometrical proportion*,
and most commonly *proportion simply*.

2. Proportion, whether it be arithmetical or
geometrical, cannot be exposed but in two magni-
tudes, (of which the former is commonly called the
antecedent, and the latter the *consequent* of the
proportion) as I have shewn in the 8th article of
the preceding chapter. And, therefore, if two
proportions be to be compared, there must be four
magnitudes exposed, namely, two antecedents and
two consequents ; for though it happen sometimes
that the consequent of the former proportion be
the same with the antecedent of the latter, yet in
that double comparison it must of necessity be

PART II.
13.

The nature
and defini-
tion of pro-
portion, &c.

twice numbered ; so that there will be always four terms.

3. Of two proportions, whether they be arithmetical or geometrical, when the magnitudes compared in both (which Euclid, in the fifth definition of his sixth book, calls the *quantities of proportions,*) are equal, then one of the proportions cannot be either greater or less than the other ; for one equality is neither greater nor less than another equality.   But of two proportions of inequality, whether they be proportions of excess or of defect, one of them may be either greater or less than the other, or they may both be equal ; for though there be propounded two magnitudes that are unequal to one another, yet there may be other two more unequal, and other two equally unequal, and other two less unequal than the two which were propounded.   And from hence it may be understood, that the proportions of excess and defect are quantity, being capable of more and less ; but the proportion of equality is not quantity, because not capable neither of *more*, nor of *less*.   And therefore proportions of inequality may be added together, or subtracted from one another, or be multiplied or divided by one another, or by number ; but proportions of equality not so.

4. Two equal proportions are commonly called *the same proportion ;* and, it is said, that the proportion of the first antecedent to the first consequent is the *same* with that of the second antecedent to the second consequent.   And when four magnitudes are thus to one another in geometrical proportion, they are called *proportionals ;* and by some, more briefly, *analogism*.   And *greater*

proportion is the proportion of a greater ante-
cedent to the same consequent, or of the same
antecedent to a less consequent; and when the
proportion of the first antecedent to the first con-
sequent is greater than that of the second ante-
cedent to the second consequent, the four magni-
tudes, which are so to one another, may be called
hyperlogism.

Less proportion is the proportion of a less ante-
cedent to the same consequent, or of the same
antecedent to a greater consequent; and when the
proportion of the first antecedent to the first conse-
quent is less than that of the second to the second,
the four magnitudes may be called *hypologism.*

5. One arithmetical proportion is the *same* with
another arithmetical proportion, when one of the
antecedents exceeds its consequent, or is exceeded
by it, as much as the other antecedent exceeds its
consequent, or is exceeded by it. And therefore,
in four magnitudes, arithmetically proportional,
the sum of the extremes is equal to the sum of the
means. For if A. B :: C. D be arithmetically pro-
portional, and the difference on both sides be the
same excess, or the same defect, E, then B + C (if
A be greater than B) will be equal to A — E + C;
and A + D will be equal to A + C — E; but A — E + C
and A + C — E are equal. Or if A be less than B,
then B + C will be equal to A + E + C; and A + D
will be equal to A + C + E; but A + E + C and A + C
+ E are equal.

The definition and some properties of the same arithmetical proportion.

Also, if there be never so many magnitudes,
arithmetically proportional, the sum of them all
will be equal to the product of half the number of
the terms multiplied by the sum of the extremes.

For if A. B : : C. D : : E. F be arithmetically pro-
portional, the couples A + F, B + E, C + D will be
equal to one another ; and their sum will be equal
to A + F, multiplied by the number of their combi-
nations, that is, by half the number of the terms.

If, of four unequal magnitudes, any two, together
taken, be equal to the other two together taken,
then the greatest and the least of them will be in
the same combination. Let the unequal magni-
tudes be A, B, C, D ; and let A + B be equal to
C + D ; and let A be the greatest of them all ; I say
B will be the least. For, if it may be, let any of
the rest, as D, be the least. Seeing therefore A
is greater than C, and B than D, A + B will be
greater than C + D ; which is contrary to what was
supposed.

If there be any four magnitudes, the sum of the
greatest and least, the sum of the means, the
difference of the two greatest, and the difference
of the two least, will be arithmetically propor-
tional. For, let there be four magnitudes, whereof
A is the greatest, D the least, and B and C the
means ; I say A + D. B + C : : A—B. C—D are
arithmetically proportional. For the difference
between the first antecedent and its consequent is
this, A + D—B—C ; and the difference between
the second antecedent and its consequent this,
A—B—C + D ; but these two differences are equal ;
and therefore, by this 5th article, A + D. B + C : :
A—B. C—D are arithmetically proportional.

If, of four magnitudes, two be equal to the other
two, they will be in reciprocal arithmetical pro-
portion. For let A + B be equal to C + D, I say
A. C : : D. B are arithmetically proportional. For

if they be not, let A. C : : D. E (supposing E to be greater or less than B) be arithmetically proportional, and then A + E will be equal to C + D ; wherefore A + B and C + D are not equal ; which is contrary to what was supposed.

6. One geometrical proportion is the same with another geometrical proportion ; when the same cause, producing equal effects in equal times, determines both the proportions.

If a point uniformly moved describe two lines, either with the same, or different velocity, all the parts of them which are contemporary, that is, which are described in the same time, will be two to two, in geometrical proportion, whether the antecedents be taken in the same line, or not. For, from the point A (in the 10th figure at the end of the 14th chapter) let the two lines, A D, A G, be described with uniform motion ; and let there be taken in them two parts A B, A E, and again, two other parts, A C, A F ; in such manner, that A B, A E, be contemporary, and likewise A C, A F contemporary. I say first (taking the antecedents A B, A C in the line A D, and the conquents A E, A F in the line A G) that A B. A C : : A E. A F are proportionals. For seeing (by the 8th chap. and the 15th art.) velocity is motion considered as determined by a certain length or line, in a certain time transmitted by it, the quantity of the line A B will be determined by the velocity and time by which the same A B is described ; and for the same reason, the quantity of the line A C will be determined by the velocity and time, by which the same A C is described ; and therefore the proportion of A B to A C, whe-

PART II.
13.

The definition
and transmuta-
tions of analo-
gism, &c.

ther it be proportion of equality, or of excess or defect, is determined by the velocities and times by which A B, A C are described ; but seeing the motion of the point A upon A B and A C is uniform, they are both described with equal velocity ; and therefore whether one of them have to the other the proportion of majority or of minority, the sole cause of that proportion is the difference of their times ; and by the same reason it is evident, that the proportion of A E to A F is determined by the difference of their times only. Seeing therefore A B, A E, as also A C, A F are contemporary, the difference of the times in which A B and A C are described, is the same with that in which A E and A F are described. Wherefore the proportion of A B to A C, and the proportion of A E to A F are both determined by the same cause. But the cause, which so determines the proportion of both, works equally in equal times, for it is uniform motion ; and therefore (by the last precedent definition) the proportion of A B to A C is the same with that of A E to A F; and consequently A B. A C : : A E. A F are proportionals ; which is the first.

Secondly, (taking the antecedents in different lines) I say, A B. A E : : A C. A F are proportionals ; for seeing A B, A E are described in the same time, the difference of the velocities in which they are described is the sole cause of the proportion they have to one another. And the same may be said of the proportion of A C to A F. But seeing both the lines A D and A G are passed over by uniform motion, the difference of the velocities in which A B, A E are described, will be the same

with the difference of the velocities, in which A C,
A F are described. Wherefore the cause which
determines the proportion of A B to A E, is the
same with that which determines the proportion of
A C to A F; and therefore A B. A E :: A C. A F,
are proportionals ; which remained to be proved.

Coroll. i. If four magnitudes be in geometrical
proportion, they will also be proportionals by *permutation*, that is, by transposing the middle terms.
For I have shown, that not only A B. A C :: A E.
A F, but also that, by *permutation*, A B. A E ::
A C. A F are proportionals.

Coroll. ii. If there be four proportionals, they
will also be proportionals by *inversion* or *conversion*, that is, by turning the antecedents into consequents. For if in the last *analogism*, I had for
A B, A C, put by inversion A C, A B, and in like
manner converted A E, A F into A F, A E, yet the
same demonstration had served. For as well A C,
A B, as A B, A C are of equal velocity; and A C,
A F, as well as A F, A C are contemporary.

Coroll. iii. If proportionals be added to proportionals, or taken from them, the aggregates, or
remainders, will be proportionals. For contemporaries, whether they be added to contemporaries,
or taken from them, make the aggregates or remainders contemporary, though the addition or
subtraction be of all the terms, or of the antecedents alone, or of the consequents alone.

Coroll. iv. If both the antecedents of four proportionals, or both the consequents, or all the
terms, be multiplied or divided by the same number or quantity, the products or quotients will be
proportionals. For the multiplication and division

PART II.
13.

The definition
and transmuta-
tions of analo-
gism, &c.

of proportionals, is the same with the addition and subtraction of them.

Coroll. v. If there be four proportionals, they will also be proportionals by *composition*, that is, by compounding an antecedent of the antecedent and consequent put together, and by taking for consequent either the consequent singly, or the antecedent singly. For this composition is nothing but addition of proportionals, namely, of consequents to their own antecedents, which by supposition are proportionals.

Coroll. vi. In like manner, if the antecedent singly, or consequent singly, be put for antecedent, and the consequent be made of both put together, these also will be proportionals. For it is the *inversion of proportion by composition,*

Coroll. vii. If there be four proportionals, they will also be proportionals by division, that is, by taking the remainder after the consequent is subtracted from the antecedent, or the difference between the antecedent and consequent for antecedent, and either the whole or the subtracted for consequent; as if A. B :: C. D be proportionals, they will by division be A—B. B :: C—D. D, and A—B. A : : C—D. C; and when the consequent is greater than the antecedent, B—A. A : : D—C. C, and B--A. B :: D—C. D. For in all these divisions, proportionals are, by the very supposition of the analogism A. B : : C. D, taken from A and B, and from C and D.

Coroll. viii. If there be four proportionals, they will also be proportionals by the *conversion of proportion*, that is, by inverting the divided proportion, or by taking the whole for antecedent, and the difference or remainder for consequent.

As, if A. B : : C. D be proportionals, then A. A—B ::C. C—D, as also B. A—B : : D. C—D will be proportionals. For seeing these inverted be proportionals, they are also themselves proportionals.

Coroll. IX. If there be two analogisms which have their quantities equal, the second to the second, and the fourth to the fourth, then either the sum or difference of the first quantities will be to the second, as the sum or difference of the third quantities is to the fourth. Let A. B : : C. D and E. B :: F. D be analogisms ; I say A + E. B : : C + F.D are proportionals. For the said analogisms will by permutation be A. C : : B. D, and E. F : : B. D ; and therefore A. C : : E. F will be proportionals, for they have both the proportion of B. to D common. Wherefore, if in the permutation of the first analogism, there be added E and F to A and C, which E and F are proportional to A and C, then (by the third coroll.) A + E. B : : C + F. D will be proportionals ; which was to be proved.

Also in the same manner it may be shown, that A --E. B : : C—F. D are proportionals.

7. If there be two analogisms, where four antecedents make an analogism, their consequents also shall make an analogism ; as also the sums of their antecedents will be proportional to the sums of their consequents. For if A. B : : C. D and E. F :: G. H be two analogisms, and A. E : : C. G be proportionals, then by permutation A. C : : E. G, and E. G : : F. H, and A. C : : B. D will be proportionals ; wherefore B. D : : E.G, that is, B.D : : F.H, and by permutation B. F : : D. H are proportionals; which is the first. Secondly, I say A + E. B + F : : C + G. D + H will be proportionals. For seeing

A. E : : C. G are proportionals, A + E. E : : C + G. G will also by composition be proportionals, and by permutation A + E. C + G : : E. G will be proportionals; wherefore, also A + E. C + G : : F. H will be proportionals. Again, seeing, as is shown above, B. F : : D. H are proportionals, B + F. F : : D + H. H will also by composition be proportionals; and by permutation B + F. D + H : : F. H will also be proportionals; wherefore A + E. C + G : : B + F. D + H are proportionals ; which remained to be proved.

Coroll. By the same reason, if there be never so many analogisms, and the antecedents be proportional to the antecedents, it may be demonstrated also that the consequents will be proportional to the consequents, as also the sum of the antecedents to the sum of the consequents.

The definition of hyperlogism and hypologism, that is, of greater and less proportion, and their transmutations.

8. In an hyperlogism, that is, where the proportion of the first antecedent to its consequent is greater than the proportion of the second antecedent to its consequent, the permutation of the proportionals, and the addition of proportionals to proportionals, and substraction of them from one another, as also their composition and division, and their multiplication and division by the same number, produce always an hyperlogism. For suppose A. B : : C. D and A. C : : E. F be analogisms, A + E. B : : C + F. D will also be an analogism ; but A + E. B : : C. D will be an hyperlogism ; wherefore by permutation, A + E. C : : B. D is an hyperlogism, because A. B : : C. D is an analogism. Secondly, if to the hyperlogism A + E. B : : C. D the proportionals G and H be added, A + E + G. B : : C + H. D will be an hyperlogism, by reason A + E + G. B : : C + F + H. D is an analogism. Also, if G and

H be taken away, $A+E$—G. B : : C—H. D will be
an hyperlogism ; for $A+E$—G. B : : $C+F$—H. D
is an analogism. Thirdly, by composition $A+E$
$+B$. B : : $C+D$. D will be an hyperlogism, because
$A+E+B$. B : : $C+F+D$. D is an analogism, and so
it will be in all the varieties of composition.
Fourthly, by division, $A+E$—B. B : : C—D. D will
by an hyperlogism, by reason A E—B. B : : $C+F$
—D. D is an analogism. Also $A+E$—B. $A+E$: :
C—D. C is an hyperlogism ; for $A+E$—B. $A+E$: :
$C+F$—D. C is an analogism. Fifthly, by multipli-
cation $4 A+4 E$. B : : 4 C. D is an hyperlogism, be-
cause $4 A$. B : : 4 C. D is an analogism ; and by
division $\frac{1}{4} A+\frac{1}{4}E$. B : : $\frac{1}{4}$ C. D is an hyperlogism,
because $\frac{1}{4} A$. B : : $\frac{1}{4}$ C. D is an analogism.

9. But if $A+E$. B : : C. D be an hyperlogism,
then by inversion B. $A+E$: : D. C will be an hy-
pologism, because B. A : : D. C being an analo-
gism, the first consequent will be too great. Also,
by conversion of proportion, $A+E$. $A+E$—B : : C.
C—D is an hypologism, because the inversion of
it, namely $A+E$—B. $A+E$: : C—D. C is an hyper-
logism, as I have shown but now. So also B. $A+$
E—B : : D. C—D is an hypologism, because, as I
have newly shown, the inversion of it, namely
$A+E$—B. B : : C—D. D is an hyperlogism. Note
that this hypologism $A+E$. $A+E$—B : : C. C—D is
commonly thus expressed ; if the proportion of
the whole, $(A+E)$ to that which is taken out of it
(B), be greater than the proportion of the whole
(C) to that which is taken out of it (D), then the
proportion of the whole $(A+E)$ to the remainder
$(A+E$—B) will be less than the proportion of the
whole (C) to the remainder (C—D).

PART II.
13,

Comparison
of analogical
quantities,
according
to magnitude.

10. If there be four proportionals, the difference of the two first, to the difference of the two last, will be as the first antecedent is to the second antecedent, or as the first consequent to the second consequent. For if A. B : : C. D be proportionals, then by division A—B. B : : C—D. D will be proportionals; and by permutation A—B. C—D : : B. D ; that is, the differences are proportional to the consequents, and therefore they are so also to the antecedents.

11. Of four proportionals, if the first be greater than the second, the third also shall be greater than the fourth. For seeing the first is greater than the second, the proportion of the first to the second is the proportion of excess; but the proportion of the third to the fourth is the same with that of the first to the second ; and therefore also the proportion of the third to the fourth is the proportion of excess ; wherefore the third is greater than the fourth. In the same manner it may be proved, that whensoever the first is less than the second, the third also is less than the fourth ; and when those are equal, that these also are equal.

12. If there be four proportionals whatsoever, A. B : : C. D, and the first and third be multiplied by any one number, as by 2 ; and again the second and fourth be multiplied by any one number, as by 3 ; and the product of the first 2 A, be greater than the product of the second 3 B ; the product also of the third 2 C, will be greater than the product of the fourth 3 D. But if the product of the first be less than the product of the second, then the product of the third will be less than that of the fourth. And lastly, if the products of the first

and second be equal, the products of the third and fourth shall also be equal. Now this theorem is all one with Euclid's definition of *the same proportion*; and it may be demonstrated thus. Seeing A. B : : C. D are proportionals, by permutation also (art. 6, coroll. I.) A. C : : B. D will be proportionals ; wherefore (by coroll. IV. art. 6) 2 A. 2 C : : 3 B. 3 D will be proportionals ; and again, by permutation, 2 A. 3 B : : 2 C. 3 D will be proportionals ; and therefore, by the last article, if 2 A be greater than 3 B, then 2 C will be greater than 3 D ; if less, less ; and if equal, equal ; which was to be demonstrated.

13. If any three magnitudes be propounded, or three things whatsoever that have any proportion one to another, as three numbers, three times, three degrees, &c. ; the proportions of the first to the second, and of the second to the third, together taken, are equal to the proportion of the first to the third. Let there be three lines, for any proportion may be reduced to the proportion of lines, A B, A C, A D ; and in the first place, let the proportion as well of the first A B to the second A C,

$$\underset{\text{_____}}{\text{A} \qquad \text{B} \qquad \text{C} \qquad \text{D}}$$

as of the second A C to the third A D, be the proportion of defect, or of less to greater ; I say the proportions together taken of A B to A C, and of A C to A D, are equal to the proportion of A B to A D. Suppose the point A to be moved over the whole line A D with uniform motion ; then the proportions as well of A B to A C, as of A C to A D, are determined by the difference of the times in which they are described ; that is, A B has to A C such proportion as is determined by the different times of their description ; and A C to A D such propor-

tion as is determined by their times. But the proportion of A B to A D is such as is determined by the difference of the times in which A B and A D are described; and the difference of the times in which A B and A C are described, together with the difference of the times in which A C and A D are described, is the same with the difference of the times in which A B and A D are described. And therefore, the same cause which determines the two proportions of A B to A C and of A C to A D, determines also the proportion of A B to A D. Wherefore, by the definition of the same proportion, delivered above in the 6th article, the proportion of A B to A C together with the proportion of A C to A D, is the same with the proportion of A B to A D.

In the second place, let A D be the first, A C the second, and A B the third, and let their proportion be the proportion of excess, or the greater to less; then, as before, the proportions of A D to A C, and of A C to A B, and of A D to A B, will be determined by the difference of their times; which in the description of A D and A C, and of A C and A B together taken, is the same with the difference of the times in the description of A D and A B. Wherefore the proportion of A D to A B is equal to the two proportions of A D to A C and of A C to A B.

In the last place. If one of the proportions, namely of A D to A B, be the proportion of excess, and another of them, as of A B to A C be the proportion of defect, thus also the proportion of A D to A C will be equal to the two proportions together taken of A D to A B, and of A B to A C. For the difference of the times in which A D and A B

are described, is excess of time; for there goes more time to the description of A D than of A B; and the difference of the times in which A B and A C are described, is defect of time, for less time goes to the description of A B than of A C; but this excess and defect being added together, make D B—B C, which is equal to D C, by which the first A D exceeds the third A C; and therefore the proportions of the first A D to the second A B, and of the second A B to the third A C, are determined by the same cause which determines the proportion of the first A D to the third A C. Wherefore, if any three magnitudes, &c.

Coroll. i. If there be never so many magnitudes having proportion to one another, the proportion of the first to the last is compounded of the proportions of the first to the second, of the second to the third, and so on till you come to the last; or, the proportion of the first to the last is the same with the sum of all the intermediate proportions. For any number of magnitudes having proportion to one another, as A, B, C, D, E being propounded, the proportion of A to E, as is newly shown, is compounded of the proportions of A to D and of D to E; and again, the proportion of A to D, of the proportions of A to C, and of C to D; and lastly, the proportion of A to C, of the proportions of A to B, and of B to C.

Coroll. ii. From hence it may be understood how any two proportions may be compounded. For if the proportions of A to B, and of C to D, be propounded to be added together, let B have to something else, as to E, the same proportion which C has to D, and let them be set in this order,

A, B, E ; for so the proportion of A to E will evidently be the sum of the two proportions of A to B, and of B to E, that is, of C to D. Or let it be as D to C, so A to something else, as to E, and let them be ordered thus, E, A, B ; for the proportion of E to B will be compounded of the proportions of E to A, that is, of C to D, and of A to B. Also, it may be understood how one proportion may be taken out of another. For if the proportion of C to D be to be subtracted out of the proportion of A to B, let it be as C to D, so A to something else, as E, and setting them in this order, A, E, B, and taking away the proportion of A to E, that is, of C to D, there will remain the proportion of E to B.

Coroll. III. If there be two orders of magnitudes which have proportion to one another, and the several proportions of the first order be the same and equal in number with the proportions of the second order ; then, whether the proportions in both orders be successively answerable to one another, which is called *ordinate proportion,* or not successively answerable, which is called *perturbed proportion,* the first and the last in both will be proportionals. For the proportion of the first to the last is equal to all the intermediate proportions ; which being in both orders the same, and equal in number, the aggregates of those proportions will also be equal to one another ; but to their aggregates, the proportions of the first to the last are equal ; and therefore the proportion of the first to the last in one order, is the same with the proportion of the first to the last in the other order. Wherefore the first and the last in both are proportionals.

14. If any two quantities be made of the mutual multiplication of many quantities, which have proportion to one another, and the efficient quantities on both sides be equal in number, the proportion of the products will be compounded of the several proportions, which the efficient quantities have to one another.

First, let the two products be A B and C D, whereof one is made of the multiplication of A into B, and the other of the multiplication of C into D. I say the proportion of A B to C D is compounded of the proportions of the efficient A to the efficient C, and of the efficient B to the efficient D. For let A B, C B and C D be set in order; and as B is to D, so let C be to another quantity as E; and let A, C, E be set also in order. Then (by coroll. IV. of the 6th art.) it will be as A B the first quantity to C B

A B.	A.
C B.	C.
C D.	E.

the second quantity in the first order, so A to C in the second order; and again, as C B to C D in the first order, so B to D, that is, by construction, so C to E in the second order; and therefore (by the last corollary) A B. C D : : A. E will be proportionals. But the proportion of A to E is compounded of the proportions of A to C, and of B to D; wherefore also the proportion of A B to C D is compounded of the same.

Secondly, let the two products be A B F, and C D G, each of them made of three efficients, the first of A, B and F, and the second of C, D and G; I say, the proportion of A B F to C D G is compounded of the proportions of A to C, of B to D, and of F to G. For let them be set in order as

before; and as B is to D, so let C be to another quantity E; and again, as F is to G, so let E be to another, H; and let the first order stand thus, ABF, CBF, CDF and CDG; and the second order thus, A, C, E, H. Then the proportion of A B F to C B F in the first order, will be as A to C in

A B F.	A.
C B F.	C.
C D F.	E.
C D G.	H.

the second; and the proportion of C B F to C D F in the first order, as B to D, that is, as C to E (by construction) in the second order; and the proportion of C D F to C D G in the first, as F to G, that is, as E to H (by construction) in the second order; and therefore A B F. C D G : : A. H will be proportionals. But the proportion of A to H is compounded of the proportions of A to C, B to D, and F to G. Wherefore the proportion of the product A B F to C D G is also compounded of the same. And this operation serves, how many soever the efficients be that make the quantities given.

From hence ariseth another way of compounding many proportions into one, namely, that which is supposed in the 5th definition of the 6th book of Euclid; which is, by multiplying all the antecedents of the proportions into one another, and in like manner all the consequents into one another. And from hence also it is evident, in the first place, that the cause why parallelograms, which are made by the duction of two straight lines into one another, and all solids which are equal to figures so made, have their proportions compounded of the proportions of the efficients; and in the second place, why the multiplication of two or more fractions into one another is the same thing

with the composition of the proportions of their
several numerators to their several denominators.
For example, if these fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ be to be
multiplied into one another, the numerators 1, 2, 3,
are first to be multiplied into one another, which
make 6 ; and next the denominators 2, 3, 4, which
make 24 ; and these two products make the frac-
tion $\frac{6}{24}$. In like manner, if the proportions of 1
to 2, of 2 to 3, and of 3 to 4, be to be compounded,
by working as I have shown above, the same pro-
portion of 6 to 24 will be produced.

15. If any proportion be compounded with itself
inverted, the compound will be the proportion of
equality. For let any proportion be given, as of
A to B, and let the inverse of it be that of C to D ;
and as C to D, so let B be to another quantity ;
for thus they will be compounded (by the second
coroll. of the 12th art.) Now seeing the propor-
tion of C to D is the inverse of the proportion of
A to B, it will be as C to D, so B to A ; and there-
fore if they be placed in order, A, B, A, the propor-
tion compounded of the proportions of A to B, and
of C to D, will be the proportion of A to A, that
is, the proportion of equality. And from hence
the cause is evident why two equal products have
their efficients reciprocally proportional. For, for
the making of two products equal, the proportions
of their efficients must be such, as being com-
pounded may make the proportion of equality,
which cannot be except one be the inverse of the
other ; for if betwixt A and A any other quantity,
as C, be interposed, their order will be A,C,A, and
the later proportion of C to A will be the inverse
of the former proportion of A to C.

16. A proportion is said to be multiplied by a number, when it is so often taken as there be unities in that number ; and if the proportion be of the greater to the less, then shall also the quantity of the proportion be increased by the multiplication ; but when the proportion is of the less to the greater, then as the number increaseth, the quantity of the proportion diminisheth ; as in these three numbers, 4, 2, 1, the proportion of 4 to 1 is not only the duplicate of 4 to 2, but also twice as great ; but inverting the order of those numbers thus, 1, 2, 4, the proportion of 1 to 2 is greater than that of 1 to 4 ; and therefore though the proportion of 1 to 4 be the duplicate of 1 to 2, yet it is not twice so great as that of 1 to 2, but contrarily the half of it.   In like manner, a proportion is said to be divided, when between two quantities are interposed one or more means in continual proportion, and then the proportion of the first to the second is said to be subduplicate of that of the first to the third, and subtriplicate of that of the first to the fourth, &c.

This mixture of proportions, where some are proportions of excess, others of defect, as in a merchant's account of debtor and creditor, is not so easily reckoned as some think ; but maketh the composition of proportions sometimes to be addition, sometimes substraction ; which soundeth absurdly to such as have always by composition understood addition, and by diminution substraction.   Therefore to make this account a little clearer, we are to consider (that which is commonly assumed, and truly) that if there be never so many quantities, the proportion of the first to

the last is compounded of the proportions of the
first to the second, and of the second to the third,
and so on to the last, without regarding their
equality, excess, or defect; so that if two propor-
tions, one of inequality, the other of equality, be
added together, the proportion is not thereby made
greater nor less; as for example, if the proportions
of A to B and of B to B be compounded, the pro-
portion of the first to the second is as much as the
sum of both, because proportion of equality, being
not quantity, neither augmenteth quantity nor
lesseneth it. But if there be three quantities,
A, B, C, unequal, and the first be the greatest, the
last least, then the proportion of B to C is an ad-
dition to that of A to B, and makes it greater;
and on the contrary, if A be the least, and C the
greatest quantity, then doth the addition of the
proportion of B to C make the compounded pro-
portion of A to C less than the proportion of A to
B, that is, the whole less than the part. The com-
position therefore of proportions is not in this case
the augmentation of them, but the diminution;
for the same quantity (Euclid v. 8) compared with
two other quantities, hath a greater proportion to
the lesser of them than to the greater. Likewise,
when the proportions compounded are one of
excess, the other of defect, if the first be of excess,
as in these numbers, 8, 6, 9, the proportion com-
pounded, namely, of 8 to 9, is less than the pro-
portion of one of the parts of it, namely, of 8
to 6; but if the proportion of the first to the
second be of defect, and that of the second to
the third be of excess, as in these numbers, 6, 8, 4,
then shall the proportion of the first to the third

PART II.
13.

The definition
and properties
of continual
proportion.

be greater than that of the first to the second, as 6 hath a greater proportion to 4 than to 8; the reason whereof is manifestly this, that the less any quantity is deficient of another, or the more one exceedeth another, the proportion of it to that other is the greater.

Suppose now three quantities in continual proportion, A B 4, A C 6, A D 9. Because therefore A D is greater than A C, but not greater than A D, the proportion of A D to A C will be (by Euclid, v. 8) greater than that of A D to A D; and likewise, because the proportions of A D to A C, and of A C to A B are the same, the proportions of A D to A C and of A C to A B, being both proportions of excess, make the whole proportion of A D to A B, or of 9 to 4, not only the duplicate of A D to A C, that is, of 9 to 6, but also the double, or twice so great. On the other side, because the proportion of A D to A D, or 9 to 9, being proportion of equality, is no quantity, and yet greater than that of AC to AD, or 6 to 9, it will be as 0—9 to 0—6, so A C to A D, and again, as 0—9 to 0--6, so 0—6 to 0--4; but 0 –4, 0—6, 0—9 are in continual proportion; and because 0—4 is greater than 0—6, the proportion of 0—4 to 0—6 will be double to the proportion of 0—4 to 0—9, double I say, and yet not duplicate, but subduplicate.

If any be unsatisfied with this ratiocination, let him first consider that (by Euclid v. 8) the proportion of A B to A C is greater than that of A B to AD, wheresoever D be placed in the line A C prolonged; and the further off the point

D is from C, so much the greater is the proportion
of A B to A C than that of A B to A D. There is
therefore some point (which suppose be E) in such
distance from C, as that the proportion of A B to
A C will be twice as great as that of A B to A E.
That considered, let him determine the length of
the line A E, and demonstrate, if he can, that A E
is greater or less than A D.

By the same method, if there be more quantities
than three, as A, B, C, D, in continual proportion,
and A be the least, it may be made appear that
the proportion of A to B is triple magnitude,
though subtriple in multitude, to the proportion of
A to D.

17. If there be never so many quantities, the
number whereof is odd, and their order such, that
from the middlemost quantity both ways they
proceed in continual proportion, the proportion of
the two which are next on either side to the mid-
dlemost is subduplicate to the proportion of the
two which are next to these on both sides, and
subtriplicate of the proportion of the two which
are yet one place more remote, &c. For let the
magnitudes be C, B, A, D, E, and let A, B, C, as
also A, D, E be in continual proportion; I say
the proportion of D to B is subduplicate of the
proportion of E to C. For the proportion of D to
B is compounded of the proportions of D to A, and
of A to B once taken; but the proportion of E to
C is compounded of the same twice taken; and
therefore the proportion of D to B is subduplicate
of the proportion of E to C. And in the same
manner, if there were three terms on either side,
it might be demonstrated that the proportion of

The definition
and properties
of continual
proportion.

D to B would be subtriplicate of that of the extremes, &c.

18. If there be never so many continual proportionals, as the first, second, third, &c. their differences will be proportional to them. For the second, third, &c. are severally consequents of the preceding, and antecedents of the following proportion. But (by art. x.) the difference of the first antecedent and consequent, to difference of the second antecedent and consequent, is as the first antecedent to the second antecedent, that is, as the first term to the second, or as the second to the third, &c. in continual proportionals.

19. If there be three continual proportionals, the sum of the extremes, together with the mean twice taken, the sum of the mean and either of the extremes, and the same extreme, are continual proportionals. For let A. B. C be continual proportionals. Seeing, therefore, A. B : : B. C are proportionals, by composition also A + B. B : : B+C. C will be proportionals; and by permutation A + B. B + C : : B. C will also be proportionals; and again, by composition A + 2 B + C. B + C : : B + C. C; which was to be proved.

20. In four continual proportionals, the greatest and the least put together is a greater quantity than the other two put together. Let A. B : : C. D be continual proportionals; whereof let the greatest be A, and the least be D; I say A + D is greater than B+C. For by art. 10, A—B. C—D : : A. C are proportionals; and therefore A—B is, by art. 11, greater than C—D. Add B on both sides, and A will be greater than C + B—D. And again, add D on both sides, and A + D will be greater than B + C; which was to be proved.

21. If there be four proportionals, the extremes multiplied into one another, and the means multiplied into one another, will make equal products. Let A. B : : C. D be proportionals; I say A D is equal to B C. For the proportion of A D to B C is compounded, by art. 13, of the proportions of A to B, and D to C, that is, its inverse B to A ; and therefore, by art. 14, this compounded proportion is the proportion of equality ; and therefore also, the proportion of A D to B C is the proportion of equality. Wherefore they are equal.

22. If there be four quantities, and the proportion of the first to the second be duplicate of the proportion of the third to the fourth, the product of the extremes to the product of the means, will be as the third to the fourth. Let the four quantities be A, B, C and D ; and let the proportion of A to B be duplicate of the proportion of C to D, I say A D, that is, the product of A into D is to B C, that is, to the product of the means, as C to D. For seeing the proportion of A to B is duplicate of the proportion of C to D, if it be as C to D, so D to another, E, then A. B : : C. E will be proportionals ; for the proportion of A to B is by supposition duplicate of the proportion of C to D ; and C to E duplicate also of that of C to D by the definition, art. 15. Wherefore, by the last article, A E or A into E is equal to B C or B into C ; but, by coroll. IV. art. 6, A D is to A E as D to E, that is, as C to D ; and therefore A D is to B C, which as I have shown is equal to A E, as C to D ; which was to be proved.

Moreover, if the proportion of the first A to the second B be triplicate of the proportion of

PART II.
13.

The definition
and properties
of continual
proportion.

the third C to the fourth D, the product of the extremes to the product of the means will be duplicate of the proportion of the third to the fourth. For if it be as C to D so D to E, and again, as D to E so E to another, F, then the proportion of C to F will be triplicate of the proportion of C to D ; and consequently, A. B : : C. F will be proportionals, and A F equal to B C. But as A D to A F, so is D to F; and therefore, also, as A D to B C, so D to F, that is, so C to E ; but the proportion of C to E is duplicate of the proportion of C to D ; wherefore, also, the proportion of A D to B C is duplicate of that of C to D, as was propounded.

23. If there be four proportionals, and a mean be interposed betwixt the first and second, and another betwixt the third and fourth, the first of these means will be to the second, as the first of the proportionals is to the third, or as the second of them is to the fourth. For let A. B : : C. D be proportionals, and let E be a mean betwixt A and B, and F a mean betwixt C and D ; I say A. C : : E. F are proportionals. For the proportion of A to E is subduplicate of the proportion of A to B, or of C to D. Also, the proportion of C to F is subduplicate of that of C to D ; and therefore A. E : : C. F are proportionals ; and by permutation A. C : : E. F are also proportionals ; which was to be proved.

24. Any thing is said to be divided into extreme and mean proportion, when the whole and the parts are in continual proportion. As for example, when A + B. A. B are continual proportionals ; or when the straight line A C is so divided in B, that

A C. A B. B C are in continual proportion. And if the same line A C be again divided in D, so as that A C. C D. A D be continual proportionals; then also

A B C

D

A C. A B. A D will be continual proportionals; and in like manner, though in contrary order, C A. CD. C B will be continual proportionals; which cannot happen in any line otherwise divided.

25. If there be three continual proportionals, and again, three other continual proportions, which have the same middle term, their extremes will be in reciprocal proportion. For let A. B. C and D. B. E be continual proportionals, I say A. D :: E. C shall be proportionals. For the proportion of A to D is compounded of the proportions of A to B, and of B to D; and the proportion of E to C is compounded of those of E to B, that is, of B to D, and of B to C, that is, of A to B. Wherefore, by equality, A. D :: E. C are proportionals.

26. If any two unequal quantities be made extremes, and there be interposed betwixt them any number of means in geometrical proportion, and the same number of means in arithmetical proportion, the several means in geometrical proportion will be less than the several means in arithmetical proportion. For betwixt A the lesser, and E the greater extreme, let there be interposed three means, B, C, D, in geometrical proportion, and as many more, F, G, H, in arithmetical proportion; I say B will be less than F, C than G, and D than H. For first, the difference between A and F is the same with that between F and G, and with that between G and H, by the definition of arithme-

tical proportion ; and therefore, the difference of
the proportionals which stand next to one another,
to the difference of the extremes, is, when there is
but one mean, half their difference ; when two, a
third part of it ; when three, a quarter, &c. ; so that
in this example it is a quarter. But the difference
between D and E, by art. 17, is more than a
quarter of the difference be-
tween the extremes, because
the proportion is geometrical,
and therefore the difference
between A and D is less than
three quarters of the same
difference of the extremes. In
like manner, if the difference

A	A
B	F
C	G
D	H
E	E

between A and D be understood to be divided
into three equal parts, it may be proved, that the
difference between A and C is less than two quar-
ters of the difference of the extremes A and E.
And lastly, if the difference between A and C be
divided into two equal parts, that the difference
between A and B is less than a quarter of the
difference of the extremes A and E.

From the consideration hereof, it is manifest,
that B, that is A together with something else
which is less than a fourth part of the difference of
the extremes A and E, is less than F, that is, than
the same A with something else which is equal to
the said fourth part.   Also, that C, that is A with
something else which is less than two fourth parts
of the said difference, is less than G, that is, than
A together with the said two-fourths.   And lastly,
that D, which exceeds A by less than three-fourths
of the said difference, is less than H, which ex-

ceeds the same A by three entire fourths of the
said difference. And in the same manner it would
be if there were four means, saving that instead
of fourths of the difference of the extremes we are
to take fifth parts ; and so on.

27. LEMMA. If a quantity being given, first one
quantity be both added to it and subtracted from
it, and then another greater or less, the propor-
tion of the remainder to the aggregate, is greater
where the less quantity is added and substracted,
than where the greater quantity is added and sub-
stracted. Let B be added to and substracted from
the quantity A ; so that A—B be the remainder,
and A+B the aggregate ; and again, let C, a
greater quantity than B, be added to and sub-
stracted from the same A, so that A—C be the
remainder and A+C the aggregate ; I say A—B.
A+B :: A—C. A+C will be an hyperlogism. For
A—B. A :: A—C. A is an hyperlogism of a greater
antecedent to the same consequent ; and therefore
A—B. A+B :: A—C. A+C is a much greater hy-
perlogism, being made of a greater antecedent to
a less consequent.

28. If unequal parts be taken from two equal
quantities, and betwixt the whole and the part of
each there be interposed two means, one in geome-
trical, the other in arithmetical proportion ; the
difference betwixt the two means will be greatest,
where the difference betwixt the whole and its part
is greatest. For let A B and A B be two equal quan-
tities, from which let two unequal parts be taken,
namely, A E the less, and A F the greater ; and
betwixt A B and A E let A G be a mean in geo-
metrical proportion, and A H a mean in arithme-

Comparison of arithmetical and geometrical proportions.

tical proportion. Also betwixt A B and A F let A I be a mean in geometrical proportion, and A K a mean in arithmetical proportion; I say H G is greater than K I.

$$\text{A} \quad\quad \text{E} \quad\quad \text{G} \quad\quad \text{H} \quad\quad \text{B}$$
$$\text{A} \quad\quad \text{F} \quad\quad \text{I} \quad\quad \text{K} \quad\quad \text{B}$$

For in the first place we have this analogism · · . .   A B. A G :: B G. G E, by article 18.

Then by composition we have this . . . .   A B + A G. A B :: B G+G E that is, B E. B G.

And by taking the halves of the antecedents this third . $\frac{1}{2}$ A B+$\frac{1}{2}$ A G. A B :: $\frac{1}{2}$ B G + $\frac{1}{2}$ G E, that is, B H. B G.

And by conversion a fourth .   A B. $\frac{1}{2}$ A B +$\frac{1}{2}$ A G :: B G. B H.

And by division this fifth   . $\frac{1}{2}$ A B —$\frac{1}{2}$ A G. $\frac{1}{2}$ A B+$\frac{1}{2}$ A G :: H G. B H.

And by doubling the first antecedent and the first consequent . . . .   AB—AG.AB + AG::HG.BH.

Also by the same method may be found out this analogism   A B—A I. A B+A I :: K I. B K.

Now seeing the proportion of A B to A E is greater than that of A B to A F, the proportion of A B to A G, which is half the greater proportion, is greater than the proportion of A B to A I the half of the less proportion ; and therefore A I is greater than A G. Wherefore the proportion of A B—A G to A B+A G, by the precedent lemma, will be greater than the proportion of A B—A I to A B+A I ; and therefore also the proportion of H G to B H will be greater than that of K I to B K, and much greater than the proportion of K I to B H, which is greater than B K ; for B H is the half of B E, as B K is the half of B F, which, by

supposition, is less than B E. Wherefore H G is greater than K I ; which was to be proved.

Coroll. It is manifest from hence, that if any quantity be supposed to be divided into equal parts infinite in number, the difference between the arithmetical and geometrical means will be infinitely little, that is, none at all. And upon this foundation, chiefly, the art of making those numbers, which are called Logarithms, seems to have been built.

29. If any number of quantities be propounded, whether they be unequal, or equal to one another ; and there be another quantity, which multiplied by the number of the propounded quantities, is equal to them all ; that other quantity is a mean in arithmetical proportion to all those propounded quantities.

CHAP. XIV.

OF STRAIT AND CROOKED, ANGLE AND FIGURE.

PART II.
14.

The definition and properties of a strait line.

1. BETWEEN two points given, the shortest line is that, whose extreme points cannot be drawn farther asunder without altering the quantity, that is, without altering the proportion of that line to any other line given. For the magnitude of a line is computed by the greatest distance which may be

between its extreme points ; so that any one line, whether it be extended or bowed, has always one and the same length, because it can have but one greatest distance between its extreme points.

And seeing the action, by which a strait line is made crooked, or contrarily a crooked line is made strait, is nothing but the bringing of its extreme points nearer to one another, or the setting of them further asunder, a *crooked line* may rightly be defined to be *that, whose extreme points may be understood to be drawn further asunder* ; and a *strait line* to be *that, whose extreme points cannot be drawn further asunder ; and comparatively, a more crooked, to be that line whose extreme points are nearer to one another than those of the other, supposing both the lines to be of equal length.* Now, howsoever a line be bowed, it makes always a *sinus* or cavity, some-times on one side, sometimes on another ; so that the same crooked line may either have its whole cavity on one side only, or it may have it part on one side and part on the other side. Which being well understood, it will be easy to understand the following comparisons of strait and crooked lines.

First, if a strait and a crooked line have their extreme points common, the crooked line is longer than the strait line. For if the extreme points of the crooked line be drawn out to their greatest distance, it will be made a strait line, of which that, which was a strait line from the beginning, will be but a part ; and therefore the strait line was shorter than the crooked line, which had the same extreme points. And for the same reason,

if two crooked lines have their extreme points common, and both of them have all their cavity on one and the same side, the outermost of the two will be the longest line.

Secondly, a strait line and a perpetually crooked line cannot be coincident, no, not in the least part. For if they should, then not only some strait line would have its extreme points common with some crooked line, but also they would, by reason of their coincidence, be equal to one another; which, as I have newly shown, cannot be.

Thirdly, between two points given, there can be understood but one strait line; because there cannot be more than one least interval or length between the same points. For if there may be two, they will either be coincident, and so both of them will be one strait line; or if they be not coincident, then the application of one to the other by extension will make the extended line have its extreme points at greater distance than the other; and consequently, it was crooked from the beginning.

Fourthly, from this last it follows, that two strait lines cannot include a superficies. For if they have both their extreme points common, they are coincident; and if they have but one or neither of them common, then at one or both ends the extreme points will be disjoined, and include no superficies, but leave all open and undetermined.

Fifthly, every part of a strait line is a strait line. For seeing every part of a strait line is the least that can be drawn between its own extreme points, if all the parts should not constitute a strait

line, they would altogether be longer than the PART II. whole line. 14.

2. A *plane* or *a plane superficies, is that which* The definition *is described by a strait line so moved, that all* of a plane su- *the several points thereof describe several strait* perficies. *lines.* A strait line, therefore, is necessarily all of it in the same plane which it describes. Also the strait lines, which are made by the points that describe a plane, are all of them in the same plane. Moreover, if any line whatsoever be moved in a plane, the lines, which are described by it, are all of them in the same plane.

All other superficies, which are not plane, are crooked, that is, are either concave or convex. And the same comparisons, which were made of strait and crooked lines, may also be made of plane and crooked superficies.

For, first, if a plane and crooked superficies be terminated with the same lines, the crooked superficies is greater than the plane superficies. For if the lines, of which the crooked superficies consists, be extended, they will be found to be longer than those of which the plane superficies consists, which cannot be extended, because they are strait.

Secondly, two superficies, whereof the one is plane, and the other continually crooked, cannot be coincident, no, not in the least part. For if they were coincident, they would be equal; nay, the same superficies would be both plane and crooked, which is impossible.

Thirdly, within the same terminating lines there can be no more than one plane superficies; because there can be but one least superficies within the same.

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Fourthly, no number of plane superficies can include a solid, unless more than two of them end in a common vertex. For if two planes have both the same terminating lines, they are coincident, that is, they are but one superficies; and if their terminating lines be not the same, they leave one or more sides open.

Fifthly, every part of a plane superficies is a plane superficies. For seeing the whole plane superficies is the least of all those, that have the same terminating lines; and also every part of the same superficies is the least of all those, that are terminated with the same lines; if every part should not constitute a plane superficies, all the parts put together would not be equal to the whole.

Several sorts of crooked lines.

3. Of straitness, whether it be in lines or in superficies, there is but one kind; but of crookedness there are many kinds; for of crooked magnitudes, some are congruous, that is, are coincident when they are applied to one other; others are incongruous. Again, some are ὁμοιομερεῖς or uniform, that is, have their parts, howsoever taken, congruous to one another; others are ἀνομοιομερεῖς or of several forms. Moreover, of such as are crooked, some are continually crooked, others have parts which are not crooked.

Definition and properties of a circular line.

4. If a strait line be moved in a plane, in such manner, that while one end of it stands still, the whole line be carried round about till it come again into the same place from whence it was first moved, it will describe a plane superficies, which will be terminated every way by that crooked line, which is made by that end of the strait line which

was carried round.   Now this superficies is called
a CIRCLE ; and of this circle, the unmoved point is
the *centre* ; the crooked line which terminates it,
the *perimeter* ; and every part of that crooked
line, a *circumference* or *arch* ; the strait line, which
generated the *circle,* is the *semidiameter* or *ra-
dius* ; and any strait line, which passeth through
the centre and is terminated on both sides in the
circumference, is called the *diameter.*   Moreover,
every point of the radius, which describes the
circle, describes in the same time its own peri-
meter, terminating its own circle, which is said to
be *concentric* to all the other circles, because this
and all those have one common centre.

Wherefore in every circle, all strait lines from
the centre to the circumference are equal.   For
they are all coincident with the radius which
generates the circle.

Also the diameter divides both the perimeter
and the circle itself into two equal parts.   For if
those two parts be applied to one another, and the
semiperimeters be coincident, then, seeing they
have one common diameter, they will be equal ;
and the semicircles will be equal also ; for these
also will be coincident.   But if the semiperimeters
be not coincident, then some one strait line, which
passes through the centre, which centre is in the
diameter, will be cut by them in two points.
Wherefore, seeing all the strait lines from the
centre to the circumference are equal, a part of
the same strait line will be equal to the whole ;
which is impossible.

For the same reason the perimeter of a circle

will be uniform, that is, any one part of it will be coincident with any other equal part of the same.

The proper-
ties of a strait
line taken in
a plane.

5. From hence may be collected this property of a strait line, namely, that it is all contained in that plane which contains both its extreme points. For seeing both its extreme points are in the plane, that strait line, which describes the plane, will pass through them both ; and if one of them be made a centre, and at the distance between both a circumference be described, whose radius is the strait line which describes the plane, that circumference will pass through the other point. Wherefore between the two propounded points, there is one strait line, by the definition of a circle, contained wholly in the propounded plane ; and therefore if another strait line might be drawn between the same points, and yet not be contained in the same plane, it would follow, that between two points two strait lines may be drawn ; which has been demonstrated to be impossible.

It may also be collected, that if two planes cut one another, their common section will be a strait line. For the two extreme points of the inter-section are in both the intersecting planes ; and between those points a strait line may be drawn ; but a strait line between any two points is in the same plane, in which the points are ; and seeing these are in both the planes, the strait line which connects them will also be in both the same planes, and therefore it is the common section of both. And every other line, that can be drawn between those points, will be either coincident with that line, that is, it will be the same line ; or it will not

be coincident, and then it will be in neither, or
but in one of those planes.

As a strait line may be understood to be
moved round about whilst one end thereof remains
fixed, as the centre; so in like manner it is easy to
understand, that a plane may be circumduced
about a strait line, whilst the strait line remains
still in one and the same place, as the *axis* of that
motion. Now from hence it is manifest, that any
three points are in some one plane. For as any
two points, if they be connected by a strait line,
are understood to be in the same plane in which
the strait line is; so, if that plane be circumduced
about the same strait line, it will in its revolution
take in any third point, howsoever it be situate;
and then the three points will be all in that plane;
and consequently the three strait lines which con-
nect those points, will also be in the same plane.

6. Two lines are said to *touch* one another,
which being both drawn to one and the same
point, will not cut one another, though they be
produced, produced, I say, in the same manner in
which they were generated. And therefore if two
strait lines touch one another in any one point,
they will be contiguous through their whole length.
Also two lines continually crooked will do the
same, if they be congruous and be applied to one
another according to their congruity; otherwise,
if they be incongruously applied, they will, as all
other crooked lines, touch one another, where they
touch, but in one point only. Which is manifest
from this, that there can be no congruity between
a strait line and a line that is continually crooked;
for otherwise the same line might be both strait

and crooked. Besides, when a strait line touches
a crooked line, if the strait line be never so little
moved about upon the point of contact, it will cut
the crooked line; for seeing it touches it but in
one point, if it incline any way, it will do more
than touch it; that is, it will either be congruous
to it, or it will cut it; but it cannot be congruous
to it; and therefore it will cut it.

The definition of an angle, and the kinds thereof.   7. An angle, according to the most general
acceptation of the word, may be thus defined;
*when two lines, or many superficies, concur in one
sole point, and diverge every where else, the
quantity of that divergence is an* ANGLE. And an
angle is of two sorts; for, first, it may be made
by the concurrence of lines, and then it is a *super-
ficial angle;* or by the concurrence of superficies,
and then it is called a *solid angle.*

Again, from the two ways by which two lines
may diverge from one another, superficial angles
are divided into two kinds. For two strait lines,
which are applied to one another, and are con-
tiguous in their whole length, may be separated or
pulled open in such manner, that their concur-
rence in one point will still remain; and this
separation or opening may be either by circular
motion, the centre whereof is their point of con-
currence, and the lines will still retain their strait-
ness, the quantity of which separation or divergence
is an *angle* simply so called; or they may be
separated by continual flexion or curvation in every
imaginable point; and the quantity of this sepa-
ration is that, which is called an *angle of con-
tingence.*

Besides, of superficial angles simply so called,

those, which are in a plane superficies, are plane;
and those, which are not plane, are denominated
from the superficies in which they are.

Lastly, those are *strait-lined angles*, which are
made by strait lines ; as those which are made by
crooked lines are *crooked-lined ;* and those which
are made both of strait and crooked lines, are
mixed angles.

8. Two arches intercepted between two radii of
concentric circles, have the same proportion to one
another, which their whole perimeters have to one
another.   For let the point A (in the first figure)
be the centre of the two circles B C D and E F G,
in which the radii A E B and A F C intercept the
arches B C and E F ; I say the proportion of the
arch B C to the arch E F is the same with that of
the perimeter BCD to the perimeter E F G.   For
if the radius A F C be understood to be moved
about the centre A with circular and uniform
motion, that is, with equal swiftness everywhere,
the point C will in a certain time describe the
perimeter B C D, and in a part of that time the
arch B C ; and because the velocities are equal by
which both the arch and the whole perimeter are
described, the proportion of the magnitude of the
perimeter B C D to the magnitude of the arch BC
is determined by nothing but the difference of the
times in which the perimeter and the arch are
described.   But both the perimeters are described
in one and the same time, and both the arches in
one and the same time ; and therefore the propor-
tions of the perimeter B C D to the arch B C, and
of the perimeter E F G to the arch E F, are
both determined by the same cause.   Wherefore

B C D. B C :: E F G. E F are proportionals (by the 6th art. of the last chapter), and by permutation B C D. EFG :: B C. E F will also be proportionals; which was to be demonstrated.

9. Nothing is contributed towards the quantity of an angle, neither by the length, nor by the equality, nor by the inequality of the lines which comprehend it. For the lines A B and A C comprehend the same angle which is comprehended by the lines A E and A F, or A B and A F. Nor is an angle either increased or diminished by the absolute quantity of the arch, which subtends the same; for both the greater arch B C and the lesser arch E F are subtended to the same angle. But the quantity of an angle is estimated by the quantity of the subtending arch compared with the quantity of the whole perimeter. And therefore the quantity of an angle simply so called may be thus defined : *the quantity of an angle is an arch or circumference of a circle, determined by its proportion to the whole perimeter*. So that when an arch is intercepted between two strait lines drawn from the centre, look how great a portion that arch is of the whole perimeter, so great is the angle. From whence it may be understood, that when the lines which contain an angle are strait lines, the quantity of that angle may be taken at any distance from the centre. But if one or both of the containing lines be crooked, then the quantity of the angle is to be taken in the least distance from the centre, or from their concurrence ; for the least distance is to be considered as a strait line, seeing no crooked line can be imagined so little, but that there may be a less strait line. And

although the least strait line cannot be given, because the least given line may still be divided, yet we may come to a part so small, as is not at all considerable; which we call a point. And this point may be understood to be in a strait line which touches a crooked line; for an angle is generated by separating, by circular motion, one strait line from another which touches it, as has been said above in the 7th article. Wherefore an angle, which two crooked lines make, is the same with that which is made by two strait lines which touch them.

10. From hence it follows, that *vertical angles*, such as are ABC, DBF in the second figure, are equal to one another. For if, from the two semi-perimeters DAC, FDA, which are equal to one another, the common arch DA be taken away, the remaining arches AC, DF will be equal to one another.

Another distinction of angles is into *right* and *oblique*. *A right angle is that, whose quantity is the fourth part of the perimeter.* And the lines, which make a right angle, are said to be *perpendicular* to one another. Also, of oblique angles, that which is greater than a right, is called an *obtuse angle;* and that which is less, an *acute angle*. From whence it follows, that all the angles that can possibly be made at one and the same point, together taken, are equal to four right angles; because the quantities of them all put together make the whole perimeter. Also, that all the angles, which are made on one side of a strait line, from any one point taken in the same, are equal to two right angles; for if that point be

made the centre, that strait line will be the diameter of a circle, by whose circumference the quantity of an angle is determined; and that diameter will divide the perimeter into two equal parts.

Of strait lines from the centre of a circle to a tangent of the same.

11. If a tangent be made the diameter of a circle, whose centre is the point of contact, a strait line drawn from the centre of the former circle to the centre of the latter circle, will make two angles with the tangent, that is, with the diameter of the latter circle, equal to two right angles, by the last article. And because, by the 6th article, the tangent has on both sides equal inclination to the circle, each of them will be a right angle; as also the semidiameter will be perpendicular to the same tangent. Moreover, the semidiameter, inasmuch as it is the semidiameter, is the least strait line which can be drawn from the centre to the tangent; and every other strait line, that reaches the tangent, will pass out of the circle, and will therefore be greater than the semidiameter. In like manner, of all the strait lines, which may be drawn from the centre to the tangent, that is the greatest which makes the greatest angle with the perpendicular; which will be manifest, if about the same centre another circle be described, whose semidiameter is a strait line taken nearer to the perpendicular, and there be drawn a perpendicular, that is, a tangent, to the same.

From whence it is also manifest, that if two strait lines, which make equal angles on either side of the perpendicular, be produced to the tangent, they will be equal.

PART II.
14.

The general
definition of
parallels;
the proper-
ties of strait
parallels.

12. There is in Euclid a definition of strait-lined parallels ; but I do not find that parallels in general are anywhere defined ; and therefore for an universal definition of them, I say that *any two lines whatsoever, strait or crooked, as also any two superficies, are* PARALLEL ; *when two equal strait lines, wheresoever they fall upon them, make always equal angles with each of them.*

From which definition it follows ; first, that any two strait lines, not inclined opposite ways, falling upon two other strait lines, which are parallel, and intercepting equal parts in both of them, are themselves also equal and parallel. As if A B and C D (in the third figure), inclined both the same way, fall upon the parallels A C and B D, and A C and B D be equal, A B and C D will also be equal and parallel. For the perpendiculars B E and D F being drawn, the right angles E B D and F D H will be equal. Wherefore, seeing E F and B D are parallel, the angles E B A and F D C will be equal. Now if D C be not equal to B A, let any other strait line equal to B A be drawn from the point D ; which, seeing it cannot fall upon the point C, let it fall upon G. Wherefore A G will be either greater or less than B D ; and therefore the angles E B A and F D C are not. equal, as was supposed. Wherefore A B and C D are equal; which is the first.

Again, because they make equal angles with the perpendiculars B E and D F ; therefore the angle C D H will be equal to the angle A B D, and, by the definition of parallels, A B and C D will be parallel; which is the second.

That plane, which is included both ways within parallel lines, is called a PARALLELOGRAM.

Coroll. I. From this last it follows, that the angles A B D and C D H are equal, that is, that a strait line, as B H, falling upon two parallels, as A B and C D, makes the internal angle A B D equal to the external and opposite angle C D H.

Coroll. II. And from hence again it follows, that a strait line falling upon two parallels, makes the alternate angles equal, that is, the angle A G F, in the fourth figure, equal to the angle G F D. For seeing G F D is equal to the external opposite angle E G B, it will be also equal to its vertical angle A G F, which is alternate to G F D.

Coroll. III. That the internal angles on the same side of the line F G are equal to two right angles. For the angles at F, namely, G F C and G F D, are equal to two right angles. But G F D is equal to its alternate angle A G F. Wherefore both the angles G F C and A G F, which are internal on the same side of the line F G, are equal to two right angles.

Coroll. IV. That the three angles of a strait-lined plain triangle are equal to two right angles; and any side being produced, the external angle will be equal to the two opposite internal angles. For if there be drawn by the vertex of the plain triangle A B C (fig. 5) a parallel to any of the sides, as to A B, the angles A and B will be equal to their alternate angles E and F, and the angle C is common. But, by the 10th article, the three angles E, C and F, are equal to two right angles; and therefore the three angles of the triangle are equal to the same; which is the first. Again, the two angles B and D are equal to two right angles, by the 10th article. Wherefore taking

away B, there will remain the angles A and C, PART II.
equal to the angle D ; which is the second. 14.

Coroll. v. If the angles A and B be equal, the General definition of parallels, &c.
sides A C and C B will also be equal, because A B
and E F are parallel ; and, on the contrary, if the
sides A C and C B be equal, the angles A and B
will also be equal. For if they be not equal, let
the angles B and G be equal. Wherefore, seeing
G B and E F are parallels, and the angles G and B
equal, the sides G C and C B will also be equal ;
and because C B and A C are equal by supposi-
tion, C G and C A will also be equal ; which cannot
be, by the 11th article.

Coroll. vi. From hence it is manifest, that if
two radii of a circle be connected by a strait line,
the angles they make with that connecting line
will be equal to one another ; and if there be
added that segment of the circle, which is sub-
tended by the same line which connects the radii,
then the angles, which those radii make with the
circumference, will also be equal to one another.
For a strait line, which subtends any arch, makes
equal angles with the same ; because, if the arch
and the subtense be divided in the middle, the two
halves of the segment will be congruous to one
another, by reason of the uniformity both of the
circumference of the circle, and of the strait line.

13. Perimeters of circles are to one another, as The circumfe-
their semidiameters are. For let there be any two rences of cir-
cles are to one
circles, as, in the first figure, B C D the greater, another as their
diameters are.
and E F G the lesser, having their common centre
at A ; and let their semidiameters be A C and A E.
I say, A C has the same proportion to A E, which
the perimeter B C D has to the perimeter E F G.

For the magnitude of the semidiameters A C and A E is determined by the distance of the points C and E from the centre A; and the same distances are acquired by the uniform motion of a point from A to C, in such manner, that in equal times the distances acquired be equal. But the perimeters B C D and E F G are also determined by the same distances of the points C and E from the centre A; and therefore the perimeters B C D and E F G, as well as the semidiameters A C and A E, have their magnitudes determined by the same cause, which cause makes, in equal times, equal spaces. Wherefore, by the 13th chapter and 6th article, the perimeters of circles and their semidiameters are proportionals; which was to be proved.

In triangles strait lines parallel to the bases are to one another, as the parts of the sides which they cut off from the vertex.

14. If two strait lines, which constitute an angle, be cut by strait-lined parallels, the intercepted parallels will be to one another, as the parts which they cut off from the vertex. Let the strait lines A B and A C, in the 6th figure, make an angle at A, and be cut by the two strait-lined parallels B C and D E, so that the parts cut off from the vertex in either of those lines, as in A B, may be A B and A D. I say, the parallels B C and D E are to one another, as the parts A B and A D. For let A B be divided into any number of equal parts, as into A F, F D, D B; and by the points F and D, let F G and D E be drawn parallel to the base B C, and cut A C in G and E; and again, by the points G and E, let other strait lines be drawn parallel to A B, and cut B C in H and I. If now the point A be understood to be moved uniformly over A B, and in the same time B be moved to C, and all the

points F, D, and B be moved uniformly and with equal swiftness over F G, D E, and B C ; then shall B pass over B H, equal to F G, in the same time that A passes over A F ; and A F and F G will be to one another, as their velocities are ; and when A is in F, D will be in K ; when A is in D, D will be in E ; and in what manner the point A passes by the points F, D, and B, in the same manner the point B will pass by the points H, I, and C ; and the strait lines F G, D K, K E, B H, H I, and I C, are equal, by reason of their parallelism ; and therefore, as the velocity in A B is to the velocity in B C, so is A D to D E ; but as the velocity in A B is to the velocity in B C, so is A B to B C ; that is to say, all the parallels will be severally to all the parts cut off from the vertex, as A F is to F G. Wherefore, A F. G F : : A D. D E : : A B. B C are proportionals.

The subtenses of equal angles in different circles, as the strait lines B C and F E (in fig. 1), are to one another as the arches which they subtend. For (by art. 8) the arches of equal angles are to one another as their perimeters are ; and (by art. 13) the perimeters as their semidiameters ; but the subtenses B C and F E are parallel to one another by reason of the equality of the angles which they make with the semidiameters ; and therefore the same subtenses, by the last precedent article, will be proportional to the semidiameters, that is, to the perimeters, that is, to the arches which they subtend.

15. If in a circle any number of equal subtenses be placed immediately after one another, and strait lines be drawn from the extreme point of the first

PART II.
14.

By what frac-
tion of a strait
line the circum-
ference of a cir-
cle is made.
subtense to the extreme points of all the rest, the
first subtense being produced will make with the
second subtense an external angle double to that,
which is made by the same first subtense, and a
tangent to the circle touching it in the extreme
points thereof; and if a strait line which subtends
two of those arches be produced, it will make an
external angle with the third subtense, triple to
the angle which is made by the tangent with the
first subtense ; and so continually.   For with the
radius A B (in fig. 7) let a circle be described, and
in it let any number of equal subtenses, B C, C D,
and D E, be placed ; also let B D and B E be drawn ;
and by producing B C, B D and B E to any dis-
tance in G, H and I, let them make angles with
the subtenses which succeed one another, namely,
the external angles G C D, and H D E.   Lastly, let
the tangent K B be drawn, making with the first
subtense the angle K B C.   I say the angle G C D
is double to the angle K B C, and the angle H D E
triple to the same angle K B C.   For if A C be
drawn cutting B D in M, and from the point C
there be drawn L C perpendicular to the same A C,
then C L and M D will be parallel, by reason of
the right angles at C and M ; and therefore the
alterne angles L C D and B D C will be equal : as
also the angles B D C and C B D will be equal,
because of the equality of the strait lines B C and
C D.   Wherefore the angle G C D is double to
either of the angles C B D or C D B ; and there-
fore also the angle G C D is double to the angle
L C D, that is, to the angle K B C.   Again, C D is
parallel to B E, by reason of the equality of the
angles C B E and D E B, and of the strait lines

C B and D E ; and therefore the angles G C D and G B E are equal ; and consequently G B E, as also D E B is double to the angle K B C. But the external angle H D E is equal to the two internal D E B and D B E ; and therefore the angle H D E is triple to the angle K B C, &c. ; which was to be proved.

Coroll. i. From hence it is manifest, that the angles K B C and C B D, as also, that all the angles that are comprehended by two strait lines meeting in the circumference of a circle and insisting upon equal arches, are equal to one another.

Coroll. ii. If the tangent B K be moved in the circumference with uniform motion about the centre B, it will in equal times cut off equal arches ; and will pass over the whole perimeter in the same time in which itself describes a semiperimeter about the centre B.

Coroll. iii. From hence also we may understand, what it is that determines the bending or curvation of a strait line into the circumference of a circle ; namely, that it is fraction continually increasing in the same manner, as numbers, from one upwards, increase by the continual addition of unity. For the indefinite strait line K B being broken in B according to any angle, as that of K B C, and again in C according to a double angle, and in D according to an angle which is triple, and in E according to an angle which is quadruple to the first angle, and so continually, there will be described a figure which will indeed be rectilineal, if the broken parts be considered as having magnitude ; but if they be understood to be the least that can be, that is, as so many points, then

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the figure described will not be rectilineal, but a circle, whose circumference will be the broken line.

Coroll. IV. From what has been said in this present article, it may also be demonstrated, that an angle in the centre is double to an angle in the circumference of the same circle, if the intercepted arches be equal. For seeing that strait line, by whose motion an angle is determined, passes over equal arches in equal times, as well from the centre as from the circumference; and while that, which is from the circumference, is passing over half its own perimeter, it passes in the same time over the whole perimeter of that which is from the centre, the arches, which it cuts off in the perimeter whose centre is A, will be double to those, which it makes in its own semiperimeter, whose centre is B. But in equal circles, as arches are to one another, so also are angles.

It may also be demonstrated, that the external angle made by a subtense produced and the next equal subtense is equal to an angle from the centre insisting upon the same arch; as in the last diagram, the angle G C D is equal to the angle C A D; for the external angle G C D is double to the angle C B D; and the angle C A D insisting upon the same arch C D is also double to the same angle C B D or K B C.

That an angle of contingence is quantity, but of a different kind from that of an angle simply so called; and that it can neither add nor
16. An angle of contingence, if it be compared with an angle simply so called, how little soever, has such proportion to it as a point has to a line; that is, no proportion at all, nor any quantity. For first, an angle of contingence is made by continual flexion; so that in the generation of it there is no

circular motion at all, in which consists the nature
of an angle simply so called ; and therefore it can-
not be compared with it according to quantity.
Secondly, seeing the external angle made by a
subtense produced and the next subtense is equal
to an angle from the centre insisting upon the
same arch, as in the last figure the angle G C D is
equal to the angle C A D, the angle of contingence
will be equal to that angle from the centre, which
is made by A B and the same A B ; for no part of
a tangent can subtend any arch ; but as the point
of contact is to be taken for the subtense, so the
angle of contingence is to be accounted for the
external angle, and equal to that angle whose arch
is the same point B.

Now, seeing an angle in general is defined to be
the opening or divergence of two lines, which con-
cur in one sole point ; and seeing one opening is
greater than another, it cannot be denied, but that
by the very generation of it, an angle of contin-
gence is quantity ; for wheresoever there is greater
and less, there is also quantity ; but this quantity
consists in greater and less flexion ; for how much
the greater a circle is, so much the nearer comes
the circumference of it to the nature of a strait
line ; for the circumference of a circle being made
by the curvation of a strait line, the less that strait
line is, the greater is the curvation ; and therefore,
when one strait line is a tangent to many circles,
the angle of contingence, which it makes with a
less circle, is greater than that which it makes
with a greater circle.

Nothing therefore is added to or taken from an
angle simply so called, by the addition to it or

taking from it of never so many angles of contin-
gence. And as an angle of one sort can never be
equal to an angle of the other sort, so they cannot
be either greater or less than one another.

From whence it follows, that an angle of a seg-
ment, that is, the angle, which any strait line
makes with any arch, is equal to the angle which
is made by the same strait line, and another which
touches the circle in the point of their concur-
rence ; as in the last figure, the angle which is
made between G B and B K is equal to that which
is made between G B and the arch B C.

That the incli-
nation of planes
is angle simply
so called.

17. An angle, which is made by two planes, is
commonly called the inclination of those planes ;
and because planes have equal inclination in all
their parts, instead of their inclination an angle is
taken, which is made by two strait lines, one of
which is in one, the other in the other of those
planes, but both perpendicular to the common
section.

A solid angle
what it is.

18. A solid angle may be conceived two ways.
First, for the aggregate of all the angles, which are
made by the motion of a strait line, while one ex-
treme point thereof remaining fixed, it is carried
about any plain figure, in which the fixed point of
the strait line is not contained. And in this sense,
it seems to be understood by Euclid. Now it is
manifest, that the quantity of a solid angle so con-
ceived is no other, than the aggregate of all the
angles in a superficies so described, that is, in the
superficies of a pyramidal solid. Secondly, when
a pyramis or cone has its vertex in the centre of a
sphere, a solid angle may be understood to be the
proportion of a spherical superficies subtending

that vertex to the whole superficies of the sphere. In which sense, solid angles are to one another as the spherical bases of solids, which have their vertex in the centre of the same sphere.

19. All the ways, by which two lines respect one another, or all the variety of their position, may be comprehended under four kinds; for any two lines whatsoever are either parallels, or being produced, if need be, or moved one of them to the other parallelly to itself, they make an angle; or else, by the like production and motion, they touch one another; or lastly, they are *asymptotes*. The nature of parallels, angles, and tangents, has been already declared. It remains that I speak briefly of the nature of asymptotes.

Asymptosy depends upon this, that quantity is infinitely divisible. And from hence it follows, that any line being given, and a body supposed to be moved from one extreme thereof towards the other, it is possible, by taking degrees of velocity always less and less, in such proportion as the parts of the line are made less by continual division, that the same body may be always moved forwards in that line, and yet never reach the end of it. For it is manifest, that if any strait line, as A F, (in the 8th figure) be cut anywhere in B, and again B F be cut in C, and C F in D, and D F in E, and so eternally, and there be drawn from the point F, the strait line F F at any angle A F F; and lastly, if the strait lines A F, B F, C F, D F, E F, &c., having the same proportion to one another with the segments of the line A F, be set in order and parallel to the same A F, the crooked line A B C D E, and the strait line F F, will be *asymptotes*, that is, they

will always come nearer and nearer together, but never touch one another. Now, because any line may be cut eternally according to the proportions which the segments have to one another, therefore the divers kinds of asymptotes are infinite in number, and not necessary to be further spoken of in this place. In the nature of asymptotes in general there is no more, than that they come still nearer and nearer, but never touch. But in special in the asymptosy of hyperbolic lines, it is understood they should approach to a distance less than any given quantity.

Situation, by what it is determined.

20. SITUATION *is the relation of one place to another;* and where there are many places, their situation is determined by four things; by *their distances from one another;* by *several distances from a place assigned*; by *the order of strait lines drawn from a place assigned to the places of them all;* and by *the angles which are made by the lines so drawn.* For if their distances, order, and angles, be given, that is, be certainly known, their several places will also be so certainly known, as that they can be no other.

What is like situation; what is figure; and what are like figures.

21. Points, how many soever they be, have *like situation* with an equal number of other points, when all the strait lines, that are drawn from some one point to all these, have severally the same proportion to those, that are drawn in the same order and at equal angles from some one point to all those. For let there be any number of points as A, B, and C, (in the 9th figure) to which from some one point D let the strait lines D A, D B, and D C be drawn; and let there be an equal number of other points, as E, F, and G, and from some

point H let the strait lines H E, H F, and H G be
drawn, so that the angles A D B and B D C be
severally and in the same order equal to the angles
E H F and F H G, and the strait lines D A, D B,
and D C proportional to the strait lines H E, H F,
and H G ; I say, the three points A, B, and C, have
like situation with the three points E, F, and G, or
are placed alike. For if H E be understood to be
laid upon D A, so that the point H be in D, the
point F will be in the strait line D B, by reason of
the equality of the angles A D B and E H F ; and
the point G will be in the strait line D C, by reason
of the equality of the angles B D C and F H G ;
and the strait lines A B and E F, as also B C and
F G, will be parallel, because A D. E H : : B D.
F H : : C D. G H are proportionals by construction ;
and therefore the distances between the points A
and B, and the points B and C, will be propor-
tional to the distances between the points E and F,
and the points F and G. Wherefore, in the situa-
tion of the points A, B, and C, and the situation
of the points E, F and G, the angles in the same
order are equal ; so that their situations differ in
nothing but the inequality of their distances from
one another, and of their distances from the points
D and H. Now, in both the orders of points, those
inequalities are equal ; for A B. B C : : E F. F G,
which are their distances from one another, as
also D A. D B. D C : : H E. H F. H G, which are
their distances from the assumed points D and
H, are proportionals. Their difference, therefore,
consists solely in the magnitude of their distances.
But, by the definition of *like*, (chapter I. article 2)
those things, which differ only in magnitude, are
like. Wherefore the points A, B, and C, have to

PART II.
14.

What is like
situation ; what
is figure ; &c.

What is like
situation; what
is figure, &c.

one another like situation with the points E, F, and G, or are placed alike; which was to be proved.

FIGURE *is quantity, determined by the situation or placing of all its extreme points.* Now I call those points extreme, which are contiguous to the place which is without the figure. In lines therefore and superficies, all points may be called extreme; but in solids only those which are in the superficies that includes them.

Like figures are those, whose extreme points in one of them are all placed like all the extreme points in the other; for such figures differ in nothing but magnitude.

And like figures are *alike placed,* when in both of them the homologal strait lines, that is, the strait lines which connect the points which answer one another, are parallel, and have their proportional sides inclined the same way.

And seeing every strait line is like every other strait line, and every plane like every other plane, when nothing but planeness is considered; if the lines, which include planes, or the superficies, which include solids, have their proportions known, it will not be hard to know whether any figure be like or unlike to another propounded figure.

And thus much concerning the *first grounds of philosophy.* The next place belongs to *geometry;* in which the quantities of figures are sought out from the proportions of lines and angles. Wherefore it is necessary for him, that would study geometry, to know first what is the nature of quantity, proportion, angle and figure. Having therefore explained these in the three last chapters, I thought fit to add them to this part; and so pass to the next.

PART III.

PROPORTIONS OF MOTIONS
AND MAGNITUDES.

CHAPTER XV.

OF THE NATURE, PROPERTIES, AND DIVERS
CONSIDERATIONS OF MOTION AND
ENDEAVOUR.

1. The next things in order to be treated of are MOTION and MAGNITUDE, which are the most common accidents of all bodies. This place therefore most properly belongs to the elements of geometry. But because this part of philosophy, having been improved by the best wits of all ages, has afforded greater plenty of matter than can well

PART III.

15.

Repetition of some principles of the doctrine of motion formerly set down.

be thrust together within the narrow limits of this discourse, I thought fit to admonish the reader, that before he proceed further, he take into his hands the works of Euclid, Archimedes, Apollonius, and other as well ancient as modern writers. For to what end is it, to do over again that which is already done? The little therefore that I shall say concerning geometry in some of the following chapters, shall be such only as is new, and conducing to natural philosophy.

I have already delivered some of the principles of this doctrine in the eighth and ninth chapters; which I shall briefly put together here, that the reader in going on may have their light nearer at hand.

First, therefore, in chap. VIII. art. 10, *motion* is defined to be *the continual privation of one place, and acquisition of another*.

Secondly, it is there shown, that *whatsoever is moved is moved in time*.

Thirdly, in the same chapter, art. 11, I have defined *rest to be when a body remains for some time in one place*.

Fourthly, it is there shown, that *whatsoever is moved is not in any determined place ;* as also that the same *has been moved, is still moved, and will yet be moved ;* so that in every part of that space, in which motion is made, we may consider three times, namely, the *past*, the *present*, and the *future time*.

Fifthly, in art. 15 of the same chapter, I have defined *velocity* or *swiftness to be motion considered as power, namely, that power by which a body moved may in a certain time transmit a*

certain length; which also may more briefly be enunciated thus, *velocity is the quantity of motion determined by time and line.*

Sixthly, in the same chapter, art. 16, I have shown that *motion is the measure of time.*

Seventhly, in the same chapter, art. 17, I have defined motions to be equally swift, when in equal times equal lengths are transmitted by them.

Eighthly, in art. 18 of the same chapter, *motions are defined to be equal, when the swiftness of one moved body, computed in every part of its magnitude, is equal to the swiftness of another, computed also in every part of its magnitude.* From whence it is to be noted, that *motions equal to one another, and motions equally swift,* do not signify the same thing; for when two horses draw abreast, the motion of both is greater than the motion of either of them singly; but the swiftness of both together is but equal to that of either.

Ninthly, in art. 19 of the same chapter, I have shown, that *whatsoever is at rest will always be at rest, unless there be some other body besides it, which by getting into its place suffers it no longer to remain at rest.* And that *whatsoever is moved, will always be moved, unless there be some other body besides it, which hinders its motion.*

Tenthly, in chap. IX. art. 7, I have demonstrated, that *when any body is moved which was formerly at rest, the immediate efficient cause of that motion is in some other moved and contiguous body.*

Eleventhly, I have shown in the same place, that *whatsoever is moved, will always be moved in the same way, and with the same swiftness, if it be*

PART III.
15.

not hindered by some other moved and contiguous body.

Other principles added to them.

2. To which principles I shall here add those that follow. First, I define ENDEAVOUR *to be motion made in less space and time than can be given;* that is, *less than can be determined or assigned by exposition or number* ; that is, *motion made through the length of a point, and in an instant or point of time.* For the explaining of which definition it must be remembered, that by a point is not to be understood that which has no quantity, or which cannot by any means be divided ; for there is no such thing in nature ; but that, whose quantity is not at all considered, that is, whereof neither quantity nor any part is computed in demonstration ; so that a point is not to be taken for an indivisible, but for an undivided thing ; as also an instant is to be taken for an undivided, and not for an indivisible time.

In like manner, endeavour is to be conceived as motion ; but so as that neither the quantity of the time in which, nor of the line in which it is made, may in demonstration be at all brought into comparison with the quantity of that time, or of that line of which it is a part. And yet, as a point may be compared with a point, so one endeavour may be compared with another endeavour, and one may be found to be greater or less than another. For if the vertical points of two angles be compared, they will be equal or unequal in the same proportion which the angles themselves have to one another. Or if a strait line cut many circumferences of concentric circles, the inequality of the points of intersection will be in the same propor-

tion which the perimeters have to one another. And in the same manner, if two motions begin and end both together, their endeavours will be equal or unequal, according to the proportion of their velocities ; as we see a bullet of lead descend with greater endeavour than a ball of wool.

Secondly, I define IMPETUS, *or quickness of motion, to be the swiftness or velocity of the body moved, but considered in the several points of that time in which it is moved. In which sense* impetus *is nothing else but the quantity or velocity of endeavour. But considered with the whole time, it is the whole velocity of the body moved taken together throughout all the time, and equal to the product of a line representing the time, multiplied into a line representing the arith- metically mean* impetus *or quickness.* Which arithmetical mean, what it is, is defined in the 29th article of chapter XIII.

And because in equal times the ways that are passed are as the velocities, and the *impetus* is the velocity they go withal, reckoned in all the several points of the times, it followeth that during any time whatsoever, howsoever the *impetus* be in- creased or decreased, the length of the way passed over shall be increased or decreased in the same proportion ; and the same line shall represent both the way of the body moved, and the several *impetus* or degrees of swiftness wherewith the way is passed over.

And if the body moved be not a point, but a strait line moved so as that every point thereof make a several strait line, the plane described by its motion, whether uniform, accelerated, or re-

tarded, shall be greater or less, the time being the same, in the same proportion with that of the *impetus* reckoned in one motion to the *impetus* reckoned in the other.   For the reason is the same in parallelograms and their sides.

For the same cause also, if the body moved be a plane, the solid described shall be still greater or less in the proportions of the several *impetus* or quicknesses reckoned through one line, to the several *impetus* reckoned through another.

This understood, let A B C D, (in figure 1, chap. xvii.) be a parallelogram ; in which suppose the side A B to be moved parallelly to the opposite side C D, decreasing all the way till it vanish in the point C, and so describing the figure A B E F C ; the point B, as A B decreaseth, will therefore describe the line B E F C ; and suppose the time of this motion designed by the line C D ; and in the same time C D, suppose the side A C to be moved parallel and uniformly to B D.   From the point O taken at adventure in the line C D, draw O R parallel to B D, cutting the line B E F C in E, and the side A B in R.   And again, from the point Q taken also at adventure in the line C D, draw Q S parallel to B D, cutting the line B E F C in F, and the side A B in S ; and draw E G and F H parallel to C D, cutting A C in G and H.   Lastly, suppose the same construction done in all the points possible of the line B E F C.   I say, that as the proportions of the swiftness wherewith Q F, O E, D B, and all the rest supposed to be drawn parallel to D B and terminated in the line B E F C, are to the proportions of their several times designed by the several parallels H F, G E, A B, and all the

rest supposed to be drawn parallel to the line of time C D and terminated in the line B E F C, the aggregate to the aggregate, so is the area or plane D B E F C to the area or plane A C F E B. For as A B decreasing continually by the line B E F C vanisheth in the time C D into the point C, so in the same time the line D C continually decreasing vanisheth by the same line C F E B into the point B ; and the point D describeth in that decreasing motion the line D B equal to the line A C described by the point A in the decreasing motion of A B ; and their swiftnesses are therefore equal. Again, because in the time G E the point O describeth the line O E, and in the same time the point S describeth the line S E, the line O E shall be to the line S E, as the swiftness wherewith O E is described to the swiftness wherewith S E is described. In like manner, because in the same time H F the point Q describeth the line Q F, and the point R the line R F, it shall be as the swiftness by which Q F is described to the swiftness by which R F is described, so the line itself Q F to the line itself R F ; and so in all the lines that can possibly be drawn parallel to B D in the points where they cut the line B E F C. But all the parallels to B D, as S E, R F, A C, and the rest that can possibly be drawn from the line A B to the line B E F C, make the area of the plane A B E F C; and all the parallels to the same B D, as Q F, O E, D B and the rest drawn to the points where they cut the same line B E F C, make the area of the plane B E F C D. As therefore the aggregate of the swiftnesses wherewith the plane B E F C D is described, is to the aggregate of the swiftnesses wherewith

PART III.
15.

Other principles added to them.

the plane A C F E B is described, so is the plane itself B E F C D to the plane itself A C F E B. But the aggregate of the times represented by the parallels A B, G E, H F and the rest, maketh also the area A C F E B. And therefore, as the aggregate of all the lines Q F, O E, D B and all the rest of the lines parallel to B D and terminated in the line B E F C, is to the aggregate of all the lines H F, G E, A B and all the rest of the lines parallel to C D and terminated in the same line B E F C; that is, as the aggregate of the lines of swiftness to the aggregate of the lines of time, or as the whole swiftness in the parallels to D B to the whole time in the parallels to C D, so is the plane B E F C D to the plane A C F E B. And the proportions of Q F to F H, and of O E to E G, and of D B to B A, and so of all the rest taken together, are the proportions of the plane D B E F C to the plane A B E F C. But the lines Q F, O E, D B and the rest are the lines that design the swiftness ; and the lines H F, G E, A B and the rest are the lines that design the times of the motions ; and therefore the proportion of the plane D B E F C to the plane A B E F C is the proportion of all the velocities taken together to all the times taken together. Wherefore, as the proportions of the swiftnesses, &c. ; which was to be demonstrated.

The same holds also in the diminution of the circles, whereof the lines of time are the semidiameters, as may easily be conceived by imagining the whole plane A B C D turned round upon the axis B D ; for the line B E F C will be everywhere in the superficies so made, and the lines H F, G E, A B, which are here parallelograms, will be there

cylinders, the diameters of whose bases are the lines H F, G E, A B, &c. and the altitude a point, that is to say, a quantity less than any quantity that can possibly be named ; and the lines Q F, O E, D B, &c. small solids whose lengths and breadths are less than any quantity that can be named.

But this is to be noted, that unless the proportion of the sum of the swiftnesses to the proportion of the sum of the times be determined, the proportion of the figure D B E F C to the figure A B E F C cannot be determined.

Thirdly, I define RESISTANCE *to be the endeavour of one moved body either wholly or in part contrary to the endeavour of another moved body, which toucheth the same.* I say, wholly contrary, when the endeavour of two bodies proceeds in the same strait line from the opposite extremes, and contrary in part, when two bodies have their endeavour in two lines, which, proceeding from the extreme points of a strait line, meet without the same.

Fourthly, that I may define what it is to PRESS, I say, that *of two moved bodies one presses the other, when with its endeavour it makes either all or part of the other body to go out of its place.*

Fifthly, *a body, which is pressed and not wholly removed, is said to* RESTORE *itself, when, the pressing body being taken away, the parts which were moved do, by reason of the internal constitution of the pressed body, return every one into its own place.* And this we may observe in springs, in blown bladders, and in many other bodies, whose parts yield more or less to the endeavour which the pressing body makes at the

P 2

first arrival; but afterwards, when the pressing body is removed, they do, by some force within them, *restore* themselves, and give their whole body the same figure it had before.

Sixthly, I define FORCE *to be the* impetus *or quickness of motion multiplied either into itself, or into the magnitude of the movent, by means whereof the said movent works more or less upon the body that resists it.*

3. Having premised thus much, I shall now demonstrate, first, that if a point moved come to touch another point which is at rest, how little soever the impetus or quickness of its motion be, it shall move that other point. For if by that impetus it do not at all move it out of its place, neither shall it move it with double the same impetus. For nothing doubled is still nothing; and for the same reason it shall never move it with that impetus, how many times soever it be multiplied, because nothing, however it be multiplied, will for ever be nothing. Wherefore, when a point is at rest, if it do not yield to the least impetus, it will yield to none ; and consequently it will be impossible that that, which is at rest, should ever be moved.

Secondly, that when a point moved, how little soever the impetus thereof be, falls upon a point of any body at rest, how hard soever that body be, it will at the first touch make it yield a little. For if it do not yield to the impetus which is in that point, neither will it yield to the impetus of never so many points, which have all their impetus severally equal to the impetus of that point. For seeing all those points together work equally, if any one

of them have no effect, the aggregate of them all
together shall have no effect as many times told as there are points in the whole body, that is, still no effect at all ; and by consequent there would be some bodies so hard that it would be impossible to break them ; that is, a finite hardness, or a finite force, would not yield to that which is infinite ; which is absurd.

Coroll. It is therefore manifest, that rest does nothing at all, nor is of any efficacy ; and that nothing but motion gives motion to such things as be at rest, and takes it from things moved.

Thirdly, that cessation in the movent does not cause cessation in that which was moved by it. For (by number 11 of art. 1 of this chapter) whatsoever is moved perseveres in the same way and with the same swiftness, as long as it is not hindered by something that is moved against it. Now it is manifest, that cessation is not contrary motion ; and therefore it follows that the standing still of the movent does not make it necessary that the thing moved should also stand still.

Coroll. They are therefore deceived, that reckon the taking away of the impediment or resistance for one of the causes of motion.

4. Motion is brought into account for divers respects; first, as in a body *undivided*, that is, considered as a point ; or, as in a *divided* body. In an undivided body, when we suppose the way, by which the motion is made, to be a line ; and in a divided body, when we compute the motion of the several parts of that body, as of parts.

Secondly, from the diversity of the regulation of motion, it is in body, considered as undivided;

sometimes *uniform* and sometimes *multiform*. *Uniform* is that by which equal lines are always transmitted in equal times ; and *multiform*, when in one time more, in another time less space is transmitted. Again, of multiform motions, there are some in which the degrees of acceleration and retardation proceed in the same proportions, which the spaces transmitted have, whether duplicate, or triplicate, or by whatsoever number multiplied ; and others in which it is otherwise.

Thirdly, from the number of the movents ; that is, one motion is made by one movent only, and another by the concourse of many movents.

Fourthly, from the position of that line in which a body is moved, in respect of some other line ; and from hence one motion is called *perpendicular*, another *oblique*, another *parallel*.

Fifthly, from the position of the movent in respect of the moved body ; from whence one motion is *pulsion* or driving, another *traction* or drawing. *Pulsion*, when the movent makes the moved body go before it ; and *traction*, when it makes it follow. Again, there are two sorts of *pulsion ;* one, when the motions of the movent and moved body begin both together, which may be called *trusion* or *thrusting* and *vection ;* the other, when the movent is first moved, and afterwards the moved body, which motion is called *percussion* or *stroke*.

Sixthly, motion is considered sometimes from the effect only which the movent works in the moved body, which is usually called *moment*. Now *moment is the excess of motion which the movent has above the motion or endeavour of the resisting body.*

Seventhly, it may be considered from the diversity of the *medium;* as one motion may be made in *vacuity* or *empty place;* another in a *fluid*; another in a *consistent medium,* that is, a *medium* whose parts are by some power so *consistent* and *cohering,* that no part of the same will yield to the movent, unless the whole yield also.

Eighthly, when a moved body is considered as having parts, there arises another distinction of motion into *simple* and *compound.* *Simple,* when all the several parts describe several equal lines; *compounded,* when the lines described are unequal.

5. All endeavour tends towards that part, that is to say, in that way which is determined by the motion of the movent, if the movent be but one; or, if there be many movents, in that way which their concourse determines. For example, if a moved body have direct motion, its first endeavour will be in a strait line; if it have circular motion, its first endeavour will be in the circumference of a circle. *The way by which the first endeavour of bodies moved tendeth.*

6. And whatsoever the line be, in which a body has its motion from the concourse of two movents, as soon as in any point thereof the force of one of the movents ceases, there immediately the former endeavour of that body will be changed into an endeavour in the line of the other movent. *In motion, which is made by concourse, one of the movents ceasing, the endeavour is made by the way by which the rest tend.*

Wherefore, when any body is carried on by the concourse of two winds, one of those winds ceasing, the endeavour and motion of that body will be in that line, in which it would have been carried by that wind alone which blows still. And in the describing of a circle, where that which is moved has its motion determined by a movent in a

tangent, and by the radius which keeps it in a certain distance from the centre, if the retention of the radius cease, that endeavour, which was in the circumference of the circle, will now be in the tangent, that is, in a strait line. For, seeing endeavour is computed in a less part of the circumference than can be given, that is, in a point, the way by which a body is moved in the circumference is compounded of innumerable strait lines, of which every one is less than can be given; which are therefore called points. Wherefore when any body, which is moved in the circumference of a circle, is freed from the retention of the radius, it will proceed in one of those strait lines, that is, in a tangent.

All endeavour is propagated in infinitum.   7. All endeavour, whether strong or weak, is propagated to infinite distance; for it is motion. If therefore the first endeavour of a body be made in space which is empty, it will always proceed with the same velocity; for it cannot be supposed that it can receive any resistance at all from empty space; and therefore, (by art. 7, chap. IX) it will always proceed in the same way and with the same swiftness. And if its endeavour be in space which is filled, yet, seeing endeavour is motion, that which stands next in its way shall be removed, and endeavour further, and again remove that which stands next, and so infinitely. Wherefore the propagation of endeavour, from one part of full space to another, proceeds infinitely. Besides, it reaches in any instant to any distance, how great soever. For in the same instant in which the first part of the full *medium* removes that which is next it, the second also removes that part which is next

to it ; and therefore all endeavour, whether it be in empty or in full space, proceeds not only to any distance, how great soever, but also in any time, how little soever, that is, in an instant. Nor makes it any matter, that endeavour, by proceeding, grows weaker and weaker, till at last it can no longer be perceived by sense ; for motion may be insensible ; and I do not here examine things by sense and experience, but by reason.

8. When two movents are of equal magnitude, How much greater the velocity or magnitude is of a movent, so much the greater is the efficacy there-of upon any other body in its way. the swifter of them works with greater force than the slower, upon a body that resists their motion. Also, if two movents have equal velocity, the greater of them works with more force than the less. For where the magnitude is equal, the movent of greater velocity makes the greater impression upon that body upon which it falls ; and where the velocity is equal, the movent of greater magnitude falling upon the same point, or an equal part of another body, loses less of its velocity, because the resisting body works only upon that part of the movent which it touches, and therefore abates the impetus of that part only ; whereas in the mean time the parts, which are not touched, proceed, and retain their whole force, till they also come to be touched ; and their force has some effect. Wherefore, for example, in batteries a longer than a shorter piece of timber of the same thickness and velocity, and a thicker than a slenderer piece of the same length and velocity, work a greater effect upon the wall.

CHAPTER XVI.

OF MOTION ACCELERATED AND UNIFORM, AND
OF MOTION BY CONCOURSE.

1. The velocity of any body, in what time soever it be computed, is that which is made of the multiplication of the impetus, or quickness of its motion into the time.—2-5. In all motion, the lengths which are passed through are to one another, as the products made by the impetus multiplied into the time.—6. If two bodies be moved with uniform motion through two lengths, the proportion of those lengths to one another will be compounded of the proportions of time to time, and impetus to impetus, directly taken.—7. If two bodies pass through two lengths with uniform motion, the proportion of their times to one another will be compounded of the proportions of length to length, and impetus to impetus reciprocally taken; also the proportion of their impetus to one another will be compounded of the proportions of length to length, and time to time reciprocally taken.—8. If a body be carried on with uniform motion by two movents together, which meet in an angle, the line by which it passes will be a strait line, subtending the complement of that angle to two right angles.—9, &c. If a body be carried by two movents together, one of them being moved with uniform, the other with accelerated motion, and the proportion of their lengths to their times being explicable in numbers, how to find out what line that body describes.

PART III.
16.

The velocity of any body, in what time so-ever it be computed, is that which is made of the multiplication of the impetus or quickness

1. THE velocity of any body, in whatsoever time it be moved, has its quantity determined by the sum of all the several quicknesses or impetus, which it hath in the several points of the time of the body's motion. For seeing velocity, (by the definition of it, chap. VIII, art. 15) is that power by which a body can in a certain time pass through a certain length; and quickness of motion or impetus, (by

chap. xv, art. 2, num. 2) is velocity taken in one
point of time only, all the impetus, together taken
in all the points of time, will be the same thing
with the mean impetus multiplied into the whole
time, or which is all one, will be the velocity of the
whole motion.

PART III.
16.
of its motion
into the time.

Coroll. If the impetus be the same in every
point, any strait line representing it may be taken
for the measure of time : and the quicknesses or
impetus applied ordinately to any strait line
making an angle with it, and representing the way
of the body's motion, will design a parallelogram
which shall represent the velocity of the whole
motion.   But if the impetus or quickness of mo-
tion begin from rest and increase uniformly, that
is, in the same proportion continually with the
times which are passed, the whole velocity of the
motion shall be represented by a triangle, one side
whereof is the whole time, and the other the
greatest impetus acquired in that time ; or else by
a parallelogram, one of whose sides is the whole
time of motion, and the other, half the greatest
impetus ; or lastly, by a parallelogram having for
one side a mean proportional between the whole
time and the half of that time, and for the other
side the half of the greatest impetus.   For both
these parallelograms are equal to one another, and
severally equal to the triangle which is made of
the whole line of time, and of the greatest ac-
quired impetus ; as is demonstrated in the ele-
ments of geometry.

2. In all uniform motions the lengths which are
transmitted are to one another, as the product of
the mean impetus multiplied into its time, to the

In all motion,
the lengths
which are pass-
ed through are

product of the mean impetus multiplied also into its time.

For let A B (in fig. 1) be the time, and A C the impetus by which any body passes with uniform motion through the length D E; and in any part of the time A B, as in the time A F, let another body be moved with uniform motion, first, with the same impetus A C. This body, therefore, in the time A F with the impetus A C will pass through the length A F. Seeing, therefore, when bodies are moved in the same time, and with the same velocity and impetus in every part of their motion, the proportion of one length transmitted to another length transmitted, is the same with that of time to time, it followeth, that the length transmitted in the time A B with the impetus A C will be to the length transmitted in the time A F with the same impetus A C, as A B itself is to A F, that is, as the parallelogram A I is to the parallelogram A H, that is, as the product of the time A B into the mean impetus A C is to the product of the time A F into the same impetus A C. Again, let it be supposed that a body be moved in the time A F, not with the same but with some other uniform impetus, as A L. Seeing therefore, one of the bodies has in all the parts of its motion the impetus A C, and the other in like manner the impetus A L, the length transmitted by the body moved with the impetus A C will be to the length transmitted by the body moved with the impetus A L, as A C itself is to A L, that is, as the parallelogram A H is to the parallelogram F L. Wherefore, by ordinate proportion it will be, as the parallelogram A I to the parallelogram F L, that is, as the pro-

duct of the mean impetus into the time is to the
product of the mean impetus into the time, so the
length transmitted in the time A B with the impe-
tus A C, to the length transmitted in the time A F
with the impetus A L; which was to be demon-
strated.

Coroll. Seeing, therefore, in uniform motion, as
has been shown, the lengths transmitted are to
one another as the parallelograms which are made
by the multiplication of the mean impetus into the
times, that is, by reason of the equality of the im-
petus all the way, as the times themselves, it will
also be, by permutation, as time to length, so time
to length; and in general, to this place are appli-
cable all the properties and transmutations of ana-
logisms, which I have set down and demonstrated
in chapter XIII.

3. In motion begun from rest and uniformly
accelerated, that is, where the impetus increaseth
continually according to the proportion of the
times, it will also be, as one product made by the
mean impetus multiplied into the time, to another
product made likewise by the mean impetus multi-
plied into the time, so the length transmitted in
the one time to the length transmitted in the other
time.

For let A B (in fig. 1) represent a time; in the
beginning of which time A, let the impetus be as
the point A; but as the time goes on, so let the
impetus increase uniformly, till in the last point of
that time A B, namely in B, the impetus acquired
be B I. Again, let A F represent another time, in
whose beginning A, let the impetus be as the point
itself A; but as the time proceeds, so let the im-

petus increase uniformly, till in the last point F of
the time A F the impetus acquired be F K ; and
let D E be the length passed through in the time
A B with impetus uniformly increased.   I say, the
length D E is to the length transmitted in the time
A F, as the time A B multiplied into the mean of
the impetus increasing through the time A B, is to
the time A F multiplied into the mean of the im-
petus increasing through the time A F.

For seeing the triangle A B I is the whole velo-
city of the body moved in the time A B, till the
impetus acquired be B I ; and the triangle A F K
the whole velocity of the body moved in the time
A F with impetus increasing till there be acquired
the impetus F K ; the length D E to the length
acquired in the time A F with impetus increasing
from rest in A till there be acquired the impetus
F K, will be as the triangle A B I to the triangle
A F K, that is, if the triangles A B I and A F K be
like, in duplicate proportion of the time A B to the
time A F ; but if unlike, in the proportion com-
pounded of the proportions of A B to A F and of
B I to F K.   Wherefore, as A B I is to A F K, so
let D E be to D P ; for so, the length transmitted
in the time A B with impetus increasing to B I,
will be to the length transmitted in the time A F
with impetus increasing to F K, as the triangle
A B I is to the triangle A F K ; but the triangle
A B I is made by the multiplication of the time
A B into the mean of the impetus increasing to
B I ; and the triangle A F K is made by the multi-
plication of the time A F into the mean of the
impetus increasing to F K ; and therefore the
length D E which is transmitted in the time A B

with impetus increasing to B I, to the length D P
which is transmitted in the time A F with impetus
increasing to F K, is as the product which is made
of the time A B multiplied into its mean impetus,
to the product of the time A F multiplied also into
its mean impetus ; which was to be proved.

Coroll. I. In motion uniformly accelerated, the
proportion of the lengths transmitted to that of
their times, is compounded of the proportions of
their times to their times, and impetus to impetus.

Coroll. II. In motion uniformly accelerated, the
lengths transmitted in equal times, taken in conti-
nual succession from the beginning of motion, are
as the differences of square numbers beginning
from unity, namely, as 3, 5, 7, &c.   For if in the
first time the length transmitted be as 1, in the
first and second times the length transmitted will
be as 4, which is the square of 2, and in the three
first times it will be as 9, which is the square of 3,
and in the four first times as 16, and so on.   Now
the differences of these squares are 3, 5, 7, &c.

Coroll. III. In motion uniformly accelerated from
rest, the length transmitted is to another length
transmitted uniformly in the same time, but with
such impetus as was acquired by the accelerated
motion in the last point of that time, as a triangle
to a parallelogram, which have their altitude and
base common.   For seeing the length D E (in fig. 1)
is passed through with velocity as the triangle
A B I, it is necessary that for the passing through
of a length which is double to D E, the velocity be
as the parallelogram A I ; for the parallelogram A I
is double to the triangle A B I.

4. In motion, which beginning from rest is so ac-

lerated, that the impetus thereof increases conti-
nually in proportion duplicate to the proportion of
the times in which it is made, a length transmitted
in one time will be to a length transmitted in ano-
ther time, as the product made by the mean impetus
multiplied into the time of one of those motions, to
the product of the mean impetus multiplied into
the time of the other motion.

For let A B (in fig. 2) represent a time, in whose
first instant A let the impetus be as the point A ;
but as the time proceeds, so let the impetus in-
crease continually in duplicate proportion to that
of the times, till in the last point of time B the
impetus acquired be B I ; then taking the point F
anywhere in the time A B, let the impetus F K
acquired in the time A F be ordinately applied to
that point F.   Seeing therefore the proportion of
F K to B I is supposed to be duplicate to that of
A F to A B, the proportion of A F to A B will be
subduplicate to that of F K to B I ; and that of
A B to A F will be (by chap. XIII. art 16) dupli-
cate to that of B I to F K ; and consequently the
point K will be in a parabolical line, whose dia-
meter is A B and base B I ; and for the same
reason, to what point soever of the time A B the
impetus acquired in that time be ordinately ap-
plied, the strait line designing that impetus will be
in the same parabolical line A K I.   Wherefore the
mean impetus multiplied into the whole time A B
will be the parabola A K I B, equal to the paralle-
logram A M, which parallelogram has for one side
the line of time A B and for the other the line of
the impetus A L, which is two-thirds of the im-
petus B I ; for every parabola is equal to two-

thirds of that parallelogram with which it has its
altitude and base common.   Wherefore the whole
velocity in A B will be the parallelogram A M, as
being made by the multiplication of the impetus
A L into the time A B.   And in like manner, if
F N be taken, which is two-thirds of the impetus
F K, and the parallelogram F O be completed, F O
will be the whole velocity in the time A F, as being
made by the uniform impetus A O or F N multi-
plied into the time A F.   Let now the length
transmitted in the time A B and with the velocity
A M be the strait line D E; and lastly, let the
length transmitted in the time A F with the velo-
city A N be D P; I say that as A M is to A N, or as
the parabola A K I B to the parabola A K F, so is
D E to D P.   For as A M is to F L, that is, as A B
is to A F, so let D E be to D G.   Now the propor-
tion of A M to A N is compounded of the propor-
tions of A M to F L, and of F L to A N.   But as
A M to F L, so by construction is D E to D G;
and as F L is to A N (seeing the time in both is the
same, namely, A F), so is the length D G to the
length D P; for lengths transmitted in the same
time are to one another as their velocities are.
Wherefore by ordinate proportion, as A M is to
A N, that is, as the mean impetus A L multiplied
into its time A B, is to the mean impetus A O
multiplied into A F, so is D E to D P; which was
to be proved.

Coroll. 1. Lengths transmitted with motion so
accelerated, that the impetus increase continually
in duplicate proportion to that of their times, if
the base represent the impetus, are in triplicate
proportion of their impetus acquired in the last

point of their times. For as the length D E is to the length D P, so is the parallelogram A M to the parallelogram A N, and so the parabola A K I B to the parabola A K F. But the proportion of the parabola A K I B to the parabola A K F is triplicate to the proportion which the base B I has to the base F K. Wherefore also the proportion of D E to D P is triplicate to that of B I to F K.

Coroll. II. Lengths transmitted in equal times succeeding one another from the beginning, by motion so accelerated, that the proportion of the impetus be duplicate to the proportion of the times, are to one another as the differences of cubic numbers beginning at unity, that is as 7, 19, 37, &c. For if in the first time the length transmitted be as 1, the length at the end of the second time will be as 8, at the end of the third time as 27, and at the end of the fourth time as 64, &c.; which are cubic numbers, whose differences are 7, 19, 37, &c.

Coroll. III. In motion so accelerated, as that the length transmitted be always to the length transmitted in duplicate proportion to their times, the length uniformly transmitted in the whole time, and with impetus all the way equal to that which is last acquired, is as a parabola to a parallelogram of the same altitude and base, that is, as 2 to 3. For the parabola A K I B is the impetus increasing in the time A B ; and the parallelogram A I is the greatest uniform impetus multiplied into the same time A B. Wherefore the lengths transmitted will be as a parabola to a parallelogram, &c., that is, as 2 to 3.

5. If I should proceed to the explication of such motions as are made by impetus increasing in pro-

portion triplicate, quadruplicate, quintuplicate, &c., to that of their times, it would be a labour infinite and unnecessary. For by the same method by which I have computed such lengths, as are transmitted with impetus increasing in single and duplicate proportion, any man may compute such as are transmitted with impetus increasing in triplicate, quadruplicate, or what other proportion he pleases.

In making which computation he shall find, that where the impetus increase in proportion triplicate to that of the times, there the whole velocity will be designed by the first parabolaster (of which see the next chapter); and the lengths transmitted will be in proportion quadruplicate to that of the times. And in like manner, where the impetus increase in quadruplicate proportion to that of the times, that there the whole velocity will be designed by the second parabolaster, and the lengths transmitted will be in quintuplicate proportion to that of the times; and so on continually.

6. If two bodies with uniform motion transmit two lengths, each with its own impetus and time, the proportion of the lengths transmitted will be compounded of the proportions of time to time, and impetus to impetus, directly taken.

Let two bodies be moved uniformly (as in fig. 3), one in the time A B with the impetus A C, the other in the time A D with the impetus A E. I say the lengths transmitted have their proportion to one another compounded of the proportions of A B to A D, and of A C to A E. For let any length whatsoever, as Z, be transmitted by one of the bodies in the time A B with the impetus A C; and any other length, as X, be transmitted by the

other body in the time A D with the impetus A E ; and let the parallelograms A F and A G be completed. Seeing now Z is to X (by art. 2) as the impetus A C multiplied into the time A B is to the impetus A E multiplied into the time A D, that is, as A F to A G ; the proportion of Z to X will be compounded of the same proportions, of which the proportion of A F to A G is compounded ; but the proportion of A F to A G is compounded of the proportions of the side A B to the side A D, and of the side A C to the side A E (as is evident by the Elements of Euclid), that is, of the proportions of the time A B to the time A D, and of the impetus A C to the impetus A E. Wherefore also the proportion of Z to X is compounded of the same proportions of the time A B to the time A D, and of the impetus A C to the impetus A E ; which was to be demonstrated.

Coroll. i. When two bodies are moved with uniform motion, if the times and impetus be in reciprocal proportion, the lengths transmitted shall be equal. For if it were as A B to A D (in the same fig. 3) so reciprocally A E to A C, the proportion of A F to A G would be compounded of the proportions of A B to A D, and of A C to A E, that is, of the proportions of A B to A D, and of A D to A B. Wherefore, A F would be to A G as A B to A B, that is, equal ; and so the two products made by the multiplication of impetus into time would be equal ; and by consequent, Z would be equal to X.

Coroll. ii. If two bodies be moved in the same time, but with different impetus, the lengths transmitted will be as impetus to impetus. For if the

time of both of them be A D, and their different impetus be A E and A C, the proportion of A G to D C will be compounded of the proportions of A E to A C and of A D to A D, that is, of the proportions of A E to A C and of A C to A C; and so the proportion of A G to D C, that is, the proportion of length to length, will be as A E to A C, that is, as that of impetus to impetus. In like manner, if two bodies be moved uniformly, and both of them with the same impetus, but in different times, the proportion of the lengths transmitted by them will be as that of their times. For if they have both the same impetus A C, and their different times be A B and A D, the proportion of A F to D C will be compounded of the proportions of A B to A D and of A C to A C; that is, of the proportions of A B to A D and of A D to A D; and therefore the proportion of A F to D C, that is, of length to length, will be the same with that of A B to A D, which is the proportion of time to time.

7. If two bodies pass through two lengths with uniform motion, the proportion of the times in which they are moved will be compounded of the proportions of length to length and impetus to impetus reciprocally taken.

For let any two lengths be given, as (in the same fig. 3) Z and X, and let one of them be transmitted with the impetus A C, the other with the impetus A E. I say the proportion of the times in which they are transmitted, will be compounded of the proportions of Z to X, and of A E, which is the impetus with which X is transmitted, to A C, the impetus with which Z is transmitted. For seeing A F is the product of the impetus A C multiplied

If two bodies pass through two lengths with uniform motion, the proportion of their times to one another, will be compounded of the proportions of length to length, and impetus to impetus reciprocally taken; also the proportion of their impetus to one another, will be compounded of the proportions of length to length, and

into the time A B, the time of motion through Z
will be a line, which is made by the application of
the parallelogram A F to the strait line A C, which
line is A B; and therefore A B is the time of
motion through Z.   In like manner, seeing A G is
the product of the impetus A E multiplied into the
time A D, the time of motion through X will be a
line which is made by the application of A G to the
strait line A D; but A D is the time of motion
through X.   Now the proportion of A B to A D
is compounded of the proportions of the parallelo-
gram A F to the parallelogram A G, and of the
impetus A E to the impetus A C; which may be
demonstrated thus.   Put the parallelograms in
order A F, A G, D C, and it will be manifest that
the proportion of A F to D C is compounded of the
proportions of A F to A G and of A G to D C; but
A F is to D C as A B to A D; wherefore also the
proportion of A B to A D is compounded of the
proportions of A F to A G and of A G to D C.
And because the length Z is to the length X as
A F is to A G, and the impetus A E to the impetus
A C as A G to D C, therefore the proportion of
A B to A D will be compounded of the proportions
of the length Z to the length X, and of the impetus
A E to the impetus A C; which was to be demon-
strated.

In the same manner it may be proved, that in
two uniform motions the proportion of the impetus
is compounded of the proportions of length to
length and of time to time reciprocally taken.

For if we suppose A C (in the same fig. 3) to be
the time, and A B the impetus with which the
length Z is passed through; and A E to be the

time, and A D the impetus with which the length X
is passed through, the demonstration will proceed
as in the last article.

8. If a body be carried by two movents toge-
ther, which move with strait and uniform motion,
and concur in any given angle, the line by which
that body passes will be a strait line.

If a body be
carried on with
uniform motion
by two mo-
vents together,
which meet in
an angle, the
line by which it
passes will be a
strait line, sub-
tending the
complement of
that angle to 2
right angles.

Let the movent A B (in fig. 4) have strait and
uniform motion, and be moved till it come into the
place C D; and let another movent A C, having
likewise strait and uniform motion, and making
with the movent A B any given angle C A B, be
understood to be moved in the same time to D B;
and let the body be placed in the point of their
concourse, A.   I say the line which that body de-
scribes with its motion is a strait line.   For let the
parallelogram A B D C be completed, and its dia-
gonal A D be drawn; and in the strait line A B
let any point E be taken; and from it let E F be
drawn parallel to the strait lines A C and B D,
cutting A D in G; and through the point G let H I
be drawn parallel to the strait lines A B and C D;
and lastly, let the measure of the time be A C.
Seeing therefore both the motions are made in the
same time, when A B is in C D, the body also
will be in C D; and in like manner, when A C is
in B D, the body will be in B D.   But A B is in
C D at the same time when A C is in B D; and
therefore the body will be in C D and B D at the
same time; wherefore it will be in the common
point D.   Again, seeing the motion from A C to
B D is uniform, that is, the spaces transmitted by
it are in proportion to one another as the times
in which they are transmitted, when A C is in E F,

the proportion of A B to A E will be the same with that of E F to E G, that is, of the time A C to the time A H. Wherefore A B will be in H I in the same time in which A C is in E F, so that the body will at the same time be in E F and H I, and therefore in their common point G. And in the same manner it will be, wheresoever the point E be taken between A and B. Wherefore the body will always be in the diagonal A D; which was to be demonstrated.

Coroll. From hence it is manifest, that the body will be carried through the same strait line A D, though the motion be not uniform, provided it have like acceleration; for the proportion of A B to A E will always be the same with that of A C to A H.

If a body be carried by two movents toge-ther, which meet in any given angle, and are moved, the one uniformly, the other with motion uniformly accelerated from rest, that is, that the proportion of their impetus be as that of their times, that is, that the proportion of their lengths be duplicate to that of the lines of their times, till the line of greatest impetus acquired by accelera-tion be equal to that of the line of time of the uni-form motion; the line in which the body is carried will be the crooked line of a semiparabola, whose base is the impetus last acquired, and vertex the point of rest.

If a body be carried by two movents toge- ther, one of them being moved with uniform, the other with ac- celerated mo- tion, and the proportion of their lengths to their times be- ing explicable in numbers, how to find out what line that body describes.

Let the straight line A B (in fig. 5) be under-stood to be moved with uniform motion to C D; and let another movent in the strait line A C be supposed to be moved in the same time to B D, but with motion uniformly accelerated, that is,

with such motion, that the proportion of the spaces which are transmitted be always duplicate to that of the times, till the impetus acquired be B D equal to the strait line A C ; and let the semiparabola A G D B be described. I say that by the concourse of those two movents, the body will be carried through the semiparabolical crooked line A G D.   For let the parallelogram A B D C be completed ; and from the point E, taken anywhere in the strait line A B, let E F be drawn parallel to A C and cutting the crooked line in G ; and lastly, through the point G let H I be drawn parallel to the strait lines A B and C D.   Seeing therefore the proportion of A B to A E is by supposition duplicate to the proportion of E F to E G, that is, of the time A C to the time A H, at the same time when A C is in E F, A B will be in H I ; and therefore the moved body will be in the common point G.   And so it will always be, in what part soever of A B the point E be taken. Wherefore the moved body will always be found in the parabolical line A G D ; which was to be demonstrated.

10. If a body be carried by two movents together, which meet in any given angle, and are moved the one uniformly, the other with impetus increasing from rest, till it be equal to that of the uniform motion, and with such acceleration, that the proportion of the lengths transmitted be every where triplicate to that of the times in which they are transmitted ; the line, in which that body is moved, will be the crooked line of the first semiparabolaster of two means, whose base is the impetus last acquired.

Let the strait line A B (in the 6th figure) be moved

uniformly to C D ; and let another movent A C be moved at the same time to B D with motion so accelerated, that the proportion of the lengths transmitted be everywhere triplicate to the proportion of their times ; and let the impetus acquired in the end of that motion be B D, equal to the strait line A C ; and lastly, let A G D be the crooked line of the first semiparabolaster of two means. I say, that by the concourse of the two movents together, the body will be always in that crooked line A G D.   For let the parallelogram A B D C be completed ; and from the point E, taken anywhere in the strait line A B, let E F be drawn parallel to A C, and cutting the crooked line in G ; and through the point G let H I be drawn parallel to the strait lines A B and C D. Seeing therefore the proportion of A B to A E is, by supposition, triplicate to the proportion of E F to E G, that is, of the time A C to the time A H, at the same time when A C is in E F, A B will be in H I ; and therefore the moved body will be in the common point G. And so it will always be, in what part soever of A B the point E be taken ; and by consequent, the body will always be in the crooked line A G D ; which was to be demonstrated.

11. By the same method it may be shown, what line it is that is made by the motion of a body carried by the concourse of any two movents, which are moved one of them uniformly, the other with acceleration, but in such proportions of spaces and times as are explicable by numbers, as *duplicate, triplicate,* &c., or such as may be designed by any broken number whatsoever. For which this is the rule. Let the two numbers of the length

and time be added together ; and let their sum be
the denominator of a fraction, whose numerator
must be the number of the length. Seek this frac-
tion in the table of the third article of the xviith
chapter ; and the line sought will be that, which
denominates the three-sided figure noted on the
left hand ; and the kind of it will be that, which is
numbered above over the fraction.   For example,
let there be a concourse of two movents, whereof
one is moved uniformly, the other with motion so
accelerated, that the spaces are to the times as 5
to 3.   Let a fraction be made whose denominator
is the sum of 5 and 3, and the numerator 5, namely
the fraction $\frac{5}{8}$.   Seek in the table, and you will
find $\frac{5}{8}$ to be the third in that row, which belongs
to the three-sided figure of four means. Wherefore
the line of motion made by the concourse of two
such movents, as are last of all described, will be
the crooked line of the third parabolaster of four
means.

12. If motion be made by the concourse of two
movents, whereof one is moved uniformly, the
other beginning from rest in the angle of concourse
with any acceleration whatsoever ; the movent,
which is moved uniformly, shall put forward the
moved body in the several parallel spaces, less
than if both the movents had uniform motion ; and
still less and less, as the motion of the other
movent is more and more accelerated.

Let the body be placed in A, (in the 7th figure)
and be moved by two movents, by one with uni-
form motion from the strait line A B to the strait
line C D parallel to it ; and by the other with any
acceleration, from the strait line A C to the strait

PART III.
16.

If a body be
carried, &c.

line B D parallel to it; and in the parallelogram
A B D C let a space be taken between any two pa-
rallels E F and G H. I say, that whilst the movent
A C passes through the latitude which is between
E F and G H, the body is less moved forwards from
A B towards C D, than it would have been, if the
motion from A C to B D had been uniform.

For suppose that whilst the body is made to
descend to the parallel E F by the power of the
movent from A C towards B D, the same body in
the same time is moved forwards to any point F
in the line E F, by the power of the movent from
A B towards C D; and let the strait line A F be
drawn and produced indeterminately, cutting G H
in H. Seeing therefore, it is as A E to A G, so E F
to G H; if A C should descend towards B D with
uniform motion, the body in the time G H, (for I
make A C and its parallels the measure of time,)
would be found in the point H. But because A C
is supposed to be moved towards B D with motion
continually accelerated, that is, in greater propor-
tion of space to space, than of time to time, in the
time G H the body will be in some parallel beyond
it, as between G H and B D. Suppose now that in
the end of the time G H it be in the parallel I K,
and in I K let I L be taken equal to G H. When
therefore the body is in the parallel I K, it will be
in the point L. Wherefore when it was in the
parallel G H, it was in some point between G and
H, as in the point M; but if both the motions had
been uniform, it had been in the point H; and
therefore whilst the movent A C passes over the
latitude which is between E F and G H, the body is
less moved forwards from A B towards C D, than

it would have been, if both the motions had been uniform; which was to be demonstrated.

13. Any length being given, which is passed through in a given time with uniform motion, to find out what length shall be passed through in the same time with motion uniformly accelerated, that is, with such motion that the proportion of the lengths passed through be continually duplicate to that of their times, and that the line of the impetus last acquired be equal to the line of the whole time of the motion.

Let A B (in the 8th figure) be a length, transmitted with uniform motion in the time A C; and let it be required to find another length, which shall be transmitted in the same time with motion uniformly accelerated, so that the line of the impetus last acquired be equal to the strait line A C.

Let the parallelogram A B D C be completed; and let B D be divided in the middle at E; and between B E and B D let B F be a mean proportional; and let A F be drawn and produced till it meet with C D produced in G; and lastly, let the parallelogram A C G H be completed. I say, A H is the length required.

For as duplicate proportion is to single proportion, so let A H be to A I, that is, let A I be the half of A H; and let I K be drawn parallel to the strait line A C, and cutting the diagonal A D in K, and the strait line A G in L. Seeing therefore A I is the half of A H, I L will also be the half of B D, that is, equal to B E; and I K equal to B F; for B D, that is, G H, B F, and B E, that is, I L, being continual proportionals, A H, A B and A I will also be continual proportionals. But as A B is to

A I, that is, as A H is to A B, so is B D to I K, and so also is G H, that is, B D to B F; and therefore B F and I K are equal. Now the proportion of A H to A I is duplicate to the proportion of A B to A I, that is, to that of B D to I K, or of G H to I K. Wherefore the point K will be in a parabola, whose diameter is A H, and base G H, which G H is equal to A C. The body therefore proceeding from rest in A, with motion uniformly accelerated in the time A C, when it has passed through the length A H, will acquire the impetus G H equal to the time A C, that is, such impetus, as that with it the body will pass through the length A C in the time A C. Wherefore any length being given, &c., which was propounded to be done.

14. Any length being given, which in a given time is transmitted with uniform motion, to find out what length shall be transmitted in the same time with motion so accelerated, that the lengths transmitted be continually in triplicate proportion to that of their times, and the line of the impetus last of all acquired be equal to the line of time given.

Let the given length A B (in the 9th figure) be transmitted with uniform motion in the time A C; and let it be required to find what length shall be transmitted in the same time with motion so accelerated, that the lengths transmitted be continually in triplicate proportion to that of their times, and the impetus last acquired be equal to the time given.

Let the parallelogram A B D C be completed; and let B D be so divided in E, that B E be a third part of the whole B D; and let B F be a mean pro-

portional between B D and B E; and let A F be drawn and produced till it meet the strait line C D in G; and lastly, let the parallelogram A C G H be completed. I say, A H is the length required.

For as triplicate proportion is to single proportion, so let A H be to another line, A I, that is, make A I a third part of the whole A H; and let I K be drawn parallel to the strait line A C, cutting the diagonal A D in K, and the strait line A G in L; then, as A B is to A I, so let A I be to another, A N; and from the point N let N Q be drawn parallel to A C, cutting A G, A D, and F K produced in P, M, and O; and last of all, let F O and L M be drawn, which will be equal and parallel to the strait lines B N and I N. By this construction, the lengths transmitted A H, A B, A I, and A N, will be continual proportionals; and, in like manner, the times G H, B F, I L and N P, that is, N Q, N O, N M and N P, will be continual proportionals, and in the same proportion with A H, A B, A I and A N. Wherefore the proportion of A H to A N is the same with that of B D, that is, of N Q to N P; and the proportion of N Q to N P triplicate to that of N Q to N O, that is, triplicate to that of B D to I K; wherefore also the length A H is to the length A N in triplicate proportion to that of the time B D, to the time I K; and therefore the crooked line of the first three-sided figure of two means whose diameter is A H, and base G H equal to A C, shall pass through the point O; and consequently, A H shall be transmitted in the time A C, and shall have its last acquired impetus G H equal to A C, and the proportions of the lengths acquired in any of the times triplicate to the pro-

portions of the times themselves.   Wherefore A H is the length required to be found out.

By the same method, if a length be given which is transmitted with uniform motion in any given time, another length may be found out which shall be transmitted in the same time with motion so accelerated, that the lengths transmitted shall be to the times in which they are transmitted, in proportion quadruplicate, quintuplicate, and so on infinitely.   For if B D be divided in E, so that B D be to B E as 4 to 1 ; and there be taken between B D and B E a mean proportional F B; and as A H is to A B, so A B be made to a third, and again so that third to a fourth, and that fourth to a fifth, A N, so that the proportion of A H to A N be quadruplicate to that of A H to A B, and the parallelogram N B F O be completed, the crooked line of the first three-sided figure of three means will pass through the point O ; and consequently, the body moved will acquire the impetus G H equal to A C in the time A C.   And so of the rest.

15. Also, if the proportion of the lengths transmitted be to that of their times, as any number to any number, the same method serves for the finding out of the length transmitted with such impetus, and in such time.

For let A C (in the 10th figure) be the time in which a body is transmitted with uniform motion from A to B; and the parallelogram A B D C being completed, let it be required to find out a length in which that body may be moved in the same time A C from A, with motion so accelerated, that the proportion of the lengths transmitted to that of the times be continually as 3 to 2.

Let B D be so divided in E, that B D be to B E

as 3 to 2; and between B D and B E let B F be a mean proportional; and let A F be drawn and pro- duced till it meet with C D produced in G; and making A M a mean proportional between A H and A B, let it be as A M to A B, so A B to A I; and so the proportion of A H to A I will be to that of A H to A B as 3 to 2; for of the proportions, of which that of A H to A M is one, that of A H to A B is two, and that of A H to A I is three; and consequently, as 3 to 2 to that of G H to B F, and (F K being drawn parallel to B I and cutting A D in K) so likewise to that of G H or B D to I K. Wherefore the proportion of the length A H to A I is to the proportion of the time B D to I K as 3 to 2; and therefore if in the time A C the body be moved with accelerated motion, as was pro- pounded, till it acquire the impetus H G equal to A C, the length transmitted in the same time will be A H.

16. But if the proportion of the lengths to that of the times had been as 4 to 3, there should then have been taken two mean proportionals between A H and A B, and their proportion should have been continued one term further, so that A H to A B might have three of the same proportions, of which A H to A I has four; and all things else should have been done as is already shown. Now the way how to interpose any number of means between two lines given, is not yet found out. Nevertheless this may stand for a general rule; *if there be a time given, and a length be transmitted in that time with uniform motion; as for example, if the time be* A C, *and the length* A B, *the strait line* A G, *which determines the length* C G *or* A H,

transmitted in the same time A C *with any acce-*
lerated motion, shall so cut B D *in* F, *that* B F
shall be a mean proportional between B D *and*
B E, B E *being so taken in* B D, *that the propor-*
tion of length to length be everywhere to the pro-
portion of time to time, as the whole B D *is to its*
part B E.

17. If in a given time two lengths be trans-
mitted, one with uniform motion, the other with
motion accelerated in any proportion of the lengths
to the times ; and again, in part of the same time,
parts of the same lengths be transmitted with the
same motions, the whole length will exceed the
other length in the same proportion in which one
part exceeds the other part.

For example, let A B (in the 8th figure) be a
length transmitted in the time A C, with uniform
motion ; and let A H be another length transmitted
in the same time with motion uniformly accele-
rated, so that the impetus last acquired be G H
equal to A C ; and in A H let any part A I be taken,
and transmitted in part of the time A C with uni-
form motion ; and let another part A B be taken
and transmitted in the same part of the time A C
with motion uniformly accelerated ; I say, that as
A H is to A B, so will A B be to A I.

Let B D be drawn parallel and equal to H G,
and divided in the midst at E, and between B D and
B E let a mean proportional be taken as B F ;
and the strait line A G, by the demonstration of
art. 13, shall pass through F. And dividing A H
in the midst at I, A B shall be a mean proportional
between A H and A I. Again, because A I and A B
are described by the same motions, if I K be

drawn parallel and equal to B F or A M, and
divided in the midst at N, and between I K and
I N be taken the mean proportional I L, the strait
line A F will, by the demonstration of the same
art. 13, pass through L.   And dividing A B in the
midst at O, the line A I will be a mean proportional
between A B and A O.   Where A B is divided in
I and O, in like manner as A H is divided in B and
I; and as A H to A B, so is A B to A I.   Which
was to be proved.

Coroll. Also as A H to A B, so is H B to B I;
and so also B I to I O.

And as this, where one of the motions is uni-
formly accelerated, is proved out of the demonstra-
tion of art. 13; so, when the accelerations are
in double proportion to the times, the same may be
proved by the demonstration of art. 14; and by
the same method in all other accelerations, whose
proportions to the times are explicable in numbers.

18. If two sides, which contain an angle in any
parallelogram, be moved in the same time to the
sides opposite to them, one of them with uniform
motion, the other with motion uniformly accele-
rated; that side, which is moved uniformly, will
affect as much with its concourse through the
whole length transmitted, as it would do if the
other motion were also uniform, and the length
transmitted by it in the same time were a mean
proportional between the whole and the half.

Let the side A B of the parallelogram A B D C,
(in the 11th figure) be understood to be moved with
uniform motion till it be coincident with C D; and
let the time of that motion be A C or B D.   Also
in the same time let the side A C be understood to

R 2

be moved with motion uniformly accelerated, till it be coincident with B D; then dividing A B in the middle in E, let A F be made a mean proportional between A B and A E; and drawing F G parallel to A C, let the side A C be understood to be moved in the same time A C with uniform motion till it be coincident with F G. I say, the whole A B confers as much to the velocity of the body placed in A, when the motion of A C is uniformly accelerated till it comes to B D, as the part A F confers to the same, when the side A C is moved uniformly and in the same time to F G.

For seeing A F is a mean proportional between the whole A B and its half A E, B D will (by the 13th article) be the last impetus acquired by A C, with motion uniformly accelerated till it come to the same B D; and consequently, the strait line F B will be the excess, by which the length, transmitted by A C with motion uniformly accelerated, will exceed the length transmitted by the same A C in the same time with uniform motion, and with impetus every where equal to B D. Wherefore, if the whole A B be moved uniformly to C D in the same time in which A C is moved uniformly to F G, the part F B, seeing it concurs not at all with the motion of the side A C which is supposed to be moved only to F G, will confer nothing to its motion. Again, supposing the side A C to be moved to B D with motion uniformly accelerated, the side A B with its uniform motion to C D will less put forwards the body when it is accelerated in all the parallels, than when it is not at all accelerated; and by how much the greater the acceleration is, by so much the less it will put it for-

wards, as is shown in the 12th article. When
therefore A C is in F G with accelerated motion,
the body will not be in the side C D at the point G,
but at the point D ; so that G D will be the excess,
by which the length transmitted with accelerated
motion to B D exceeds the length transmitted with
uniform motion to F G ; so that the body by its
acceleration avoids the action of the part A F, and
comes to the side C D in the time A C, and makes
the length C D, which is equal to the length A B.
Wherefore uniform motion from A B to C D in the
time A C, works no more in the whole length A B
upon the body uniformly accelerated from A C to
B D, than if A C were moved in the same time
with uniform motion to F G ; the difference con-
sisting only in this, that when A B works upon the
body uniformly moved from A C to F G, that, by
which the accelerated motion exceeds the uniform
motion, is altogether in F B or G D ; but when the
same A B works upon the body accelerated, that,
by which the accelerated motion exceeds the uni-
form motion, is dispersed through the whole length
A B or C D, yet, so that if it were collected and
put together, it would be equal to the same F B or
G D. Wherefore, if two sides which contain an
angle, &c. ; which was to be demonstrated.

19. If two transmitted lengths have to their
times any other proportion explicable by number,
and the side A B be so divided in E, that A B be
to A E in the same proportion which the lengths
transmitted have to the times in which they are
transmitted, and between A B and A E there be
taken a mean proportional A F ; it may be shown
by the same method, that the side, which is moved

PART III.
16.

with uniform motion, works as much with its con-
course through the whole length A B, as it would
do if the other motion were also uniform, and the
length transmitted in the same time A C were that
mean proportional A F.

And thus much concerning motion by concourse.

CHAP. XVII.

OF FIGURES DEFICIENT.

1. Definitions of a deficient figure; of a complete figure; of the complement of a deficient figure; and of proportions which are proportional and commensurable to one another.— 2. The proportion of a deficient figure to its complement.—3. The proportions of deficient figures to the parallelograms in which they are described, set forth in a table.—4. The description and production of the same figures.—5. The drawing of tangents to them.—6. In what proportion the same figures exceed a strait-lined triangle of the same altitude and base.— 7. A table of solid deficient figures described in a cylinder.—8. In what proportion the same figures exceed a cone of the same altitude and base.—9. How a plain deficient figure may be described in a parallelogram, so that it be to a triangle of the same base and altitude, as another deficient figure, plain or solid, twice taken, is to the same deficient figure, together with the complete figure in which it is described.—10. The transferring of certain properties of deficient figures described in a parallelogram to the proportions of the spaces transmitted with several degrees of velocity.—11. Of deficient figures described in a circle.—12. The proposition demonstrated in art. 2 confirmed from the elements of philosophy.—13. An unusual way of reasoning concerning the equality between the superficies of a portion of a sphere and a circle.—14. How from the description of deficient figures in a parallelogram, any number of mean proportionals may be found out between two given strait lines.

Definition of a deficient figure. 1. I CALL those *deficient figures* which may be understood to be generated by the uniform motion

of some quantity, which decreases continually, till
at last it have no magnitude at all.

And I call that a *complete figure*, answering to
a deficient figure, which is generated with the
same motion and in the same time, by a quantity
which retains always its whole magnitude.

The *complement* of a deficient figure is that which
being added to the deficient figure makes it com-
plete.

Four proportions are said to be *proportional*,
when the first of them is to the second as the third
is to the fourth.   For example, if the first propor-
tion be duplicate to the second, and again, the
third be duplicate to the fourth, those proportions
are said to be *proportional*.

And *commensurable* proportions are those, which
are to one another as number to number.   As
when to a proportion given, one proportion is
duplicate, another triplicate, the duplicate propor-
tion will be to the triplicate proportion as 2 to 3 ;
but to the given proportion it will be as 2 to 1 ;
and therefore I call those three proportions *com-
mensurable*.

2. A deficient figure, which is made by a quantity
continually decreasing to nothing by proportions
everywhere proportional and commensurable, is to
its complement, as the proportion of the whole
altitude to an altitude diminished in any time is
to the proportion of the whole quantity, which
describes the figure, to the same quantity dimi-
nished in the same time.

Let the quantity A B (in fig. 1), by its motion
through the altitude A C, describe the complete
figure A D ; and again, let the same quantity, by

PART III.
17.

The proportion
of a deficient
figure to its
complement.
decreasing continually to nothing in C, describe
the deficient figure A B E F C, whose complement
will be the figure B D C F E.   Now let A B be
supposed to be moved till it lie in G K, so that the
altitude diminished be G C, and A B diminished
be G E ; and let the proportion of the whole alti-
tude A C to the diminished altitude G C, be, for
example, triplicate to the proportion of the whole
quantity A B or G K to the diminished quantity
G E.   And in like manner, let H I be taken equal
to G E, and let it be diminished to H F ; and let
the proportion of G C to H C be triplicate to that
of H I to H F ; and let the same be done in as
many parts of the strait line A C as is possible ;
and a line be drawn through the points B, E, F
and C.   I say the deficient figure A B E F C is to
its complement B D C F E as 3 to 1, or as the pro-
portion of A C to G C is to the proportion of A B,
that is, of G K to G E.

For (by art. 2, chapter x v.) the proportion of
the complement B E F C D to the deficient figure
A B E F C is all the proportions of D B to B A,
O E to E G, Q F to F H, and of all the lines
parallel to D B terminated in the line B E F C, to
all the parallels to A B terminated in the same
points of the line B E F C.   And seeing the pro-
portions of D B to O E, and of D B to Q F &c.
are everywhere triplicate of the proportions of A B
to G E, and of A B to H F &c. the proportions of
H F to A B, and of G E to A B &c. (by art. 16,
chap. XIII.), are triplicate of the proportions of
Q F to D B, and of O E to D B &c. and therefore
the deficient figure A B E F C, which is the aggre-

PART III.
17.

The proportion
of a deficient
figure to its
complement.

gate of all the lines H F, G E, A B, &c. is triple to the complement B E F C D made of all the lines Q F, O E, D B, &c.; which was to be proved.

It follows from hence, that the same complement B E F C D is $\frac{1}{4}$ of the whole parallelogram. And by the same method may be calculated in all other deficient figures, generated as above declared, the proportion of the parallelogram to either of its parts; as that when the parallels increase from a point in the same proportion, the parallelogram will be divided into two equal triangles; when one increase is double to the other, it will be divided into a semiparabola and its complement, or into 2 and 1.

The same construction standing, the same conclusion may otherwise be demonstrated thus.

Let the strait line C B be drawn cutting G K in L, and through L let M N be drawn parallel to the strait line A C; wherefore the parallelograms G M and L D will be equal. Then let L K be divided into three equal parts, so that it may be to one of those parts in the same proportion which the proportion of A C to G C, or of G K to G L, hath to the proportion of G K to G E. Therefore LK will be to one of those three parts as the arithmetical proportion between G K and G L is to the arithmetical proportion between G K and the same G K wanting the third part of L K; and K E will be somewhat greater than a third of L K. Seeing now the altitude A G or M L is, by reason of the continual decrease, to be supposed less than any quantity that can be given; L K, which is intercepted between the diagonal B C and the side B D,

The proportion
of a deficient
figure to its
complement.

will be also less than any quantity that can be given; and consequently, if G be put so near to A in g, as that the difference between Cg and C A be less than any quantity that can be assigned, the difference also between Cl (removing L to l) and CB, will be less than any quantity that can be assigned; and the line g l being drawn and produced to the line B D in k, cutting the crooked line in e, the proportion of Gk to Gl will still be triplicate to the proportion of Gk to Ge, and the difference between k and e, the third part of k l, will be less than any quantity that can be given; and therefore the parallelogram e D will differ from a third part of the parallelogram Ae by a less difference than any quantity that can be assigned. Again, let H I be drawn parallel and equal to G E, cutting C B in P, the crooked line in F, and O E in I, and the proportion of Cg to C H will be triplicate to the proportion of H F to H P, and I F will be greater than the third part of P I. But again, setting H in h so near to g, as that the difference between Ch and Cg may be but as a point, the point P will also in p be so near to l, as that the difference between Cp and Cl will be but as a point; and drawing h p till it meet with B D in i, cutting the crooked line in f, and having drawn e o parallel to B D, cutting D C in o, the parallelogram f o will differ less from the third part of the parallelogram g f, than by any quantity that can be given. And so it will be in all other spaces generated in the same manner. Wherefore the differences of the arithmetical and geometrical means, which are but as so many points B, e, f, &c.

(seeing the whole figure is made up of so many indivisible spaces) will constitute a certain line, such as is the line B E F C, which will divide the complete figure A D into two parts, whereof one, namely, A B E F C, which I call a deficient figure, is triple to the other, namely, B D C F E, which I call the complement thereof. And whereas the proportion of the altitudes to one another is in this case everywhere triplicate to that of the decreasing quantities to one another; in the same manner, if the proportion of the altitudes had been everywhere quadruplicate to that of the decreasing quantities, it might have been demonstrated that the deficient figure had been quadruple to its complement; and so in any other proportion. Wherefore, a deficient figure, which is made, &c. which was to be demonstrated.

The same rule holdeth also in the diminution of the bases of cylinders, as is demonstrated in the second article of chapter xv.

3. By this proposition, the magnitudes of all deficient figures, when the proportions by which their bases decrease continually are proportional to those by which their altitudes decrease, may be compared with the magnitudes of their complements; and consequently, with the magnitudes of their complete figures. And they will be found to be, as I have set them down in the following tables; in which I compare a parallelogram with three-sided figures; and first, with a strait-lined triangle, made by the base of the parallelogram continually decreasing in such manner, that the altitudes be always in proportion to one another

The proportion of deficient figures to the parallelograms in which they are described, set forth in a table

as the bases are, and so the triangle will be equal
to its complement; or the proportions of the alti-
tudes and bases will be as 1 to 1, and then the
triangle will be half the parallelogram.   Secondly,
with that three-sided figure which is made by the
continual decreasing of the bases in subduplicate
proportion to that of the altitudes; and so the
deficient figure will be double to its complement,
and to the parallelogram as 2 to 3.   Then, with
that where the proportion of the altitudes is tripli-
cate to that of the bases; and then the deficient
figure will be triple to its complement, and to the
parallelogram as 3 to 4.   Also the proportion of
the altitudes to that of the bases may be as 3 to 2;
and then the deficient figure will be to its comple-
ment as 3 to 2, and to the parallelogram as 3 to 5;
and so forwards, according as more mean propor-
tionals are taken, or as the proportions are more
multiplied, as may be seen in the following table.
For example, if the bases decrease so, that the
proportion of the altitudes to that of the bases be
always as 5 to 2, and it be demanded what pro-
portion the figure made has to the parallelogram,
which is supposed to be unity; then, seeing that
where the proportion is taken five times, there
must be four means; look in the table amongst
the three-sided figures of four means, and seeing
the proportion was as 5 to 2, look in the upper-
most row for the number 2, and descending in
the second column till you meet with that three-
sided figure, you will find $\frac{5}{7}$; which shows that the
deficient figure is to the parallelogram as $\frac{5}{7}$ to 1,
or as 5 to 7.

	1	2	3	4	5	6	7
Parallelogram	1	:	:	:	:	:	:
Strait-sided triangle	$\frac{1}{2}$	:	:	:	:	:	:
Three-sided figure of 1 mean .	$\frac{2}{3}$	:	:	:	:	:	:
Three-sided figure of 2 means .	$\frac{3}{4}$	$\frac{3}{5}$	:	:	:	:	:
Three-sided figure of 3 means .	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$	:	:	:	:
Three-sided figure of 4 means .	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{5}{9}$	:	:	:
Three-sided figure of 5 means .	$\frac{6}{7}$	$\frac{6}{8}$	$\frac{6}{9}$	$\frac{6}{10}$	$\frac{6}{11}$	:	:
Three-sided figure of 6 means .	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$	$\frac{7}{11}$	$\frac{7}{12}$	$\frac{7}{13}$	:
Three-sided figure of 7 means .	$\frac{8}{9}$	$\frac{8}{10}$	$\frac{8}{11}$	$\frac{8}{12}$	$\frac{8}{13}$	$\frac{8}{14}$	$\frac{9}{15}$

4. Now for the better understanding of the nature of these three-sided figures, I will show how they may be described by points; and first, those which are in the first column of the table. Any parallelogram being described, as A B C D (in figure 2) let the diagonal B D be drawn; and the strait-lined triangle B C D will be half the parallelogram; then let any number of lines, as E F, be drawn parallel to the side B C, and cutting the diagonal B D in G; and let it be everywhere, as E F to E G, so E G to another, E H; and through all the points H let the line B H H D be drawn; and the figure B H H D C will be that which I call a three-sided figure of one mean, because in three proportionals, as E F, E G and E H, there is but one mean, namely, E G; and this three-sided figure will be $\frac{2}{3}$ of the parallelogram, and is called a *parabola*.   Again, let it be as E G to E H, so E H to another, E I, and let the line B I I D be drawn, making the three-sided figure B I I D C; and this will be $\frac{3}{4}$ of the parallelogram, and is by many called a *cubic parabola*.   In like manner, if the

Description & production of the same figures.

PART III.
17.

Description &
production of
the same
figures.

proportions be further continued in E F, there will be made the rest of the three-sided figures of the first column ; which I thus demonstrate. Let there be drawn strait lines, as H K and G L, parallel to the base D C. Seeing therefore the proportion of E F to E H is duplicate to that of E F to E G, or of B C to B L, that is, of C D to L G, or of K M (producing K H to A D in M) to K H, the proportion of B C to B K will be duplicate to that of K M to K H ; but as B C is to B K, so is D C or K M to K N, and therefore the proportion of K M to K N is duplicate to that of K M to K H ; and so it will be wheresoever the parallel K M be placed. Wherefore the figure B H H D C is double to its complement B H H D A, and consequently ⅔ of the whole parallelogram. In the same manner, if through I be drawn O P I Q parallel and equal to C D, it may be demonstrated that the proportion of O Q to O P, that is, of B C to B O, is triplicate that of O Q to O I, and therefore that the figure B I I D C is triple to its complement B I I D A, and consequently ¾ of the whole parallelogram, &c.

Secondly, such three-sided figures as are in any of the transverse rows, may be thus described. Let A B C D (in fig. 3) be a parallelogram, whose diagonal is B D. I would describe in it such figures, as in the preceding table I call three-sided figures of three means. Parallel to D C, I draw E F as often as is necessary, cutting B D in G ; and between E F and E G, I take three proportionals E H, E I and E K. If now there be drawn lines through all the points H, I and K, that through all the points H will make the figure B H D C, which is the first of those three-sided figures ; and that

through all the points I, will make the figure
B I D C, which is the second ; and that which is
drawn through all the points K, will make the
figure B K D C the third of those three-sided
figures. The first of these, seeing the proportion
of E F to E G is quadruplicate of that E F to E H,
will be to its complement as 4 to 1, and to the
parallelogram as 4 to 5. The second, seeing the
proportion of E F to E G is to that of E F to E I as
4 to 2, will be double to its complement, and $\frac{4}{6}$ or
$\frac{2}{3}$ of the parallelogram. The third, seeing the pro-
proportion of E F to E G is that of E F to E K as
4 to 3, will be to its complement as 4 to 3, and to
the parallelogram as 4 to 7.

Any of these figures being described may be
produced at pleasure, thus ; let A B C D (in fig. 4)
be a parallelogram, and in it let the figure B K D C
be described, namely, the third three-sided figure
of three means. Let B D be produced indefinitely
to E, and let E F be made parallel to the base D C,
cutting A D produced in G, and B C produced in
F ; and in G E let the point H be so taken, that the
proportion of F E to F G may be quadruplicate to
that of F E to F H, which may be done by making
F H the greatest of three proportionals between
F E and F G ; the crooked line B K D produced,
will pass through the point H. For if the strait
line B H be drawn, cutting C D in I, and H L be
drawn parallel to G D, and meeting C D produced
in L ; it will be as F E to F G, so C L to C I, that
is, in quadruplicate proportion to that of F E to
F H, or of C D to C I. Wherefore if the line B K D
be produced according to its generation, it will
fall upon the point H.

5. A strait line may be drawn so as to touch the crooked line of the said figure in any point, in this manner. Let it be required to draw a tangent to the line B K D H (in fig. 4) in the point D. Let the points B and D be connected, and drawing D A equal and parallel to B C, let B and A be connected; and because this figure is by construction the third of three means, let there be taken in A B three points, so, that by them the same A B be divided into four equal parts; of which take three, namely, A M, so that A B may be to A M, as the figure B K D C is to its complement. I say, the strait line M D will touch the figure in the point given D. For let there be drawn anywhere between A B and D C a parallel, as R Q, cutting the strait line B D, the crooked line B K D, the strait line M D, and the strait line A D, in the points P, K, O and Q. R K will therefore, by construction, be the least of three means in geometrical proportion between R Q and R P. Wherefore (by coroll. of art. 28, chapter XIII.) R K will be less than R O; and therefore M D will fall without the figure. Now if M D be produced to N, F N will be the greatest of three means in arithmetical proportion between F E and F G; and F H will be the greatest of three means in geometrical proportion between the same F E and F G. Wherefore (by the same coroll. of art. 28, chapter XIII.) F H will be less than F N; and therefore D N will fall without the figure, and the strait line M N will touch the same figure only in the point D.

6. The proportion of a deficient figure to its complement being known, it may also be known

what proportion a strait-lined triangle has to the excess of the deficient figure above the same triangle; and these proportions I have set down in the following table; where if you seek, for example, how much the fourth three-sided figure of five means exceeds a triangle of the same altitude and base, you will find in the concourse of the fourth column with the three-sided figures of five means $\frac{2}{10}$; by which is signified, that that three-sided figure exceeds the triangle by two-tenths or by one-fifth part of the same triangle.

PART III. 17. — a strait-lined triangle of the same altitude and base.

		1	2	3	4	5	6	7
	The triangle	1	:	:	:	:	:	:
The excess of	A three-sided fig. of 1 mean .	$\frac{1}{3}$	:	:	:	:	:	:
	A three-sided fig. of 2 means	$\frac{2}{4}$	$\frac{1}{5}$	:	:	:	:	:
	A three-sided fig. of 3 means	$\frac{3}{5}$	$\frac{2}{6}$	$\frac{1}{7}$	:	:	:	:
	A three-sided fig. of 4 means	$\frac{4}{6}$	$\frac{3}{7}$	$\frac{2}{8}$	$\frac{1}{9}$	:	:	:
	A three-sided fig. of 5 means	$\frac{5}{7}$	$\frac{4}{8}$	$\frac{3}{9}$	$\frac{2}{10}$	$\frac{1}{11}$	:	:
	A three-sided fig. of 6 means	$\frac{6}{8}$	$\frac{5}{9}$	$\frac{4}{10}$	$\frac{3}{11}$	$\frac{2}{12}$	$\frac{1}{13}$	:
	A three-sided fig. of 7 means	$\frac{7}{9}$	$\frac{6}{10}$	$\frac{5}{11}$	$\frac{4}{12}$	$\frac{3}{13}$	$\frac{2}{14}$	$\frac{1}{15}$

7. In the next table are set down the proportion of a cone and the solids of the said three-sided figures, namely, the proportions between them and a cylinder. As for example, in the concourse of the second column with the three-sided figures of four means, you have $\frac{5}{9}$; which gives you to understand, that the solid of the second three-sided figure of four means is to the cylinder as $\frac{5}{9}$ to 1, or as 5 to 9.

A table of solid deficient figures described in a cylinder.

PART III.
17.

The solids of		1	2	3	4	5	6	7
	A cylinder	1	:	:	:	:	:	:
	A cone	$\frac{1}{3}$	:	:	:	:	:	:
	A three-sided fig. of 1 mean	$\frac{2}{4}$		:	:	:	:	:
	A three-sided fig. of 2 means	$\frac{3}{5}$	$\frac{3}{7}$		:	:	:	:
	A three-sided fig. of 3 means	$\frac{4}{6}$	$\frac{4}{8}$	$\frac{4}{10}$		:	:	:
	A three-sided fig. of 4 means	$\frac{5}{7}$	$\frac{5}{9}$	$\frac{5}{11}$	$\frac{5}{13}$		:	:
	A three-sided fig. of 5 means	$\frac{6}{8}$	$\frac{6}{10}$	$\frac{6}{12}$	$\frac{6}{14}$	$\frac{6}{16}$		:
	A three-sided fig. of 6 means	$\frac{7}{9}$	$\frac{7}{11}$	$\frac{7}{13}$	$\frac{7}{15}$	$\frac{7}{17}$	$\frac{7}{19}$	
	A three-sided fig. of 7 means	$\frac{8}{10}$	$\frac{8}{12}$	$\frac{8}{14}$	$\frac{8}{16}$	$\frac{8}{18}$	$\frac{8}{20}$	$\frac{8}{22}$

In what proportion the same figures exceed a cone of the same altitude and base.

8. Lastly, the excess of the solids of the said three-sided figures above a cone of the same altitude and base, are set down in the table which follows :

The excesses of the solids of these three-sided figures above a cone.		1	2	3	4	5	6	7
	The Cone	1	:	:	:	:	:	:
	Of the solid of a three-sided figure of 1 mean	$\frac{0}{12}$	:	:	:	:	:	:
	Ditto ditto, 2 means	$\frac{12}{15}$	$\frac{6}{21}$	:	:	:	:	:
	Ditto ditto, 3 means	$\frac{18}{18}$	$\frac{12}{24}$	$\frac{6}{30}$	:	:	:	:
	Ditto ditto, 4 means	$\frac{24}{21}$	$\frac{18}{27}$	$\frac{12}{33}$	$\frac{6}{30}$	:	:	:
	Ditto ditto, 5 means	$\frac{30}{24}$	$\frac{24}{30}$	$\frac{18}{36}$	$\frac{12}{42}$	$\frac{6}{48}$	:	:
	Ditto ditto, 6 means	$\frac{36}{27}$	$\frac{30}{33}$	$\frac{24}{39}$	$\frac{18}{45}$	$\frac{12}{51}$	$\frac{6}{57}$	:
	Ditto ditto, 7 means	$\frac{42}{30}$	$\frac{36}{36}$	$\frac{30}{42}$	$\frac{24}{48}$	$\frac{18}{54}$	$\frac{12}{60}$	$\frac{6}{66}$

How a plain deficient figure may be described in a parallelogram, so that it be to a triangle of the same base and altitude, as another deficient figure, plain or solid, twice taken, is to the same deficient

9. If any of these deficient figures, of which I have now spoken, as A B C D (in the 5th figure) be inscribed within the complete figure B E, having A D C E for its complement; and there be made upon C B produced the triangle A B I; and there be the parallelogram A B I K be completed; and there be drawn parallel to the strait line C I, any number of lines, as M F, cutting every one of them the

crooked line of the deficient figure in D, and the
strait lines A C, A B and A I in H, G, and L; and
as G F is to G D, so G L be made to another, G N; figure, together
and through all the points N there be drawn the with the com-
line A N I : there will be a deficient figure A N I B, plete figure, in
whose complement will be A N I K.   I say, the which it is de-
figure A N I B is to the triangle A B I, as the de- scribed.
ficient figure A B C D twice taken is to the same
deficient figure together with the complete figure
B E.

For as the proportion of A B to A G, that is, of
G M to G L, is to the proportion of G M to G N,
so is the magnitude of the figure A N I B to that
of its complement A N I K, by the second article
of this chapter.

But, by the same article, as the proportion of
A B to A G, that is, of G M to G L, is to the pro-
portion of G F to G D, that is, by construction, of
G L to G N, so is the figure A B C D to its comple-
ment A D C E.

And by composition, as the proportion of G M
to G L, together with that of G L to G N, is to the
proportion of G M to G L, so is the complete figure
B E to the deficient figure A B C D.

And by conversion, as the proportion of G M to
G L is to both the proportions of G M to G L and
of G L to G N, that is, to the proportion of G M to
G N, which is the proportion compounded of both,
so is the deficient figure A B C D to the complete
figure B E.

But it was, as the proportion of G M to G L to
that of G M to G N, so the figure A N I B to its
complement A N I K.   And therefore, A B C D. B E
: : A N I B. A N I K are proportionals.   And by com-

position, A B C D + B E. A B C D : : B K. A N I B are proportionals.

And by doubling the consequents, A B C D + B E. 2 A B C D : : B K. 2 A N I B are proportionals.

And by taking the halves of the third and the fourth, A B C D + B E. 2 A B C D : : A B I. A N I B are also proportionals; which was to be proved.

The transferring of certain properties of deficient figures described in a parallelogram to the proportions of spaces transmitted with several degrees of velocity.

10. From what has been said of deficient figures described in a parallelogram, may be found out what proportions spaces, transmitted with accelerated motion in determined times, have to the times themselves, according as the moved body is accelerated in the several times with one or more degrees of velocity.

For let the parallelogram A B C D, in the 6th figure, and in it the three-sided figure D E B C be described; and let F G be drawn anywhere parallel to the base, cutting the diagonal B D in H, and the crooked line B E D in E; and let the proportion of B C to B F be, for example, triplicate to that of F G to F E; whereupon the figure D E B C will be triple to its complement B E D A; and in like manner, I E being drawn parallel to B C, the three-sided figure E K B F will be triple to its complement B K E I. Wherefore the parts of the deficient figure cut off from the vertex by strait lines parallel to the base, namely, D E B C and E K B F, will be to one another as the parallelograms A C and I F; that is, in proportion compounded of the proportions of the altitudes and bases. Seeing therefore the proportion of the altitude B C to the altitude B F is triplicate to the proportion of the base D C to the base F E, the figure D E B C to the

figure E K B F will be quadruplicate to the propor-
tion of the same D C to F E. And by the same
method, may be found out what proportion any of The transfer-
ring of certain
the said three-sided figures has to any part of the properties
of deficient
same, cut off from the vertex by a strait line pa- figures, &c.
rallel to the base.

Now as the said figures are understood to be
described by the continual decreasing of the base,
as of C D, for example, till it end in a point, as in
B ; so also they may be understood to be described
by the continual increasing of a point, as of B, till
it acquire any magnitude, as that of C D.

Suppose now the figure B E D C to be described
by the increasing of the point B to the magnitude
C D. Seeing therefore the proportion of B C to
B F is triplicate to that of C D to F E, the propor-
tion of F E to C D will, by conversion, as I shall
presently demonstrate, be triplicate to that B F to
B C. Wherefore if the strait line B C be taken for
the measure of the time in which the point B is
moved, the figure E K B F will represent the sum
of all the increasing velocities in the time B F ; and
the figure D E B C will in like manner represent
the sum of all the increasing velocities in the time
B C. Seeing therefore the proportion of the figure
E K B F to the figure D E B C is compounded of
the proportions of altitude to altitude, and base to
base ; and seeing the proportion of F E to C D is
triplicate to that of B F to B C ; the proportion of
the figure E K B F to the figure D E B C will be
quadruplicate to that of B F to B C ; that is, the
proportion of the sum of the velocities in the time
B F, to the sum of the velocities in the time B C,
will be quadruplicate to the proportion of B F to

PART III.
17.
The transfer-
ring of certain
properties
of deficient
figures, &c.

B C.   Wherefore if a body be moved from B with velocity so increasing, that the velocity acquired in the time B F be to the velocity acquired in the time B C in triplicate proportion to that of the times themselves B F to B C, and the body be carried to F in the time B F; the same body in the time B C will be carried through a line equal to the fifth proportional from B F in the continual proportion of B F to B C.   And by the same manner of working, we may determine what spaces are transmitted by velocities increasing according to any other proportions.

It remains that I demonstrate the proportion of F E to C D to be triplicate to that of B F to B C. Seeing therefore the proportion of C D, that is, of F G to F E is subtriplicate to that of B C to B F; the proportion of F G to F E will also be subtripli- cate to that of F G to F H. Wherefore the propor- tion of F G to F H is triplicate to that of F G, that is, of C D to F E.   But in four continual propor- tionals, of which the least is the first, the propor- tion of the first to the fourth, (by the 16th article of chapter XIII), is subtriplicate to the proportion of the third to the same fourth.   Wherefore the proportion of F H to G F is subtriplicate to that of F E to C D; and therefore the proportion of F E to C D is triplicate to that of F H to F G, that is, of B F to B C; which was to be proved.

It may from hence be collected, that when the velocity of a body is increased in the same propor- tion with that of the times, the degrees of velocity above one another proceed as numbers do in im- mediate succession from unity, namely, as 1, 2, 3, 4, &c.   And when the velocity is increased in pro-

portion duplicate to that of the times, the degrees proceed as numbers from unity, skipping one, as 1, 3, 5, 7, &c. Lastly, when the proportions of the velocities are triplicate to those of the times, the progression of the degrees is as that of numbers from unity, skipping two in every place, as 1, 4, 7, 10, &c., and so of other proportions. For geometrical proportionals, when they are taken in every point, are the same with arithmetical proportionals.

11. Moreover, it is to be noted that as in quantities, which are made by any magnitudes decreasing, the proportions of the figures to one another are as the proportions of the altitudes to those of the bases; so also it is in those, which are made with motion decreasing, which motion is nothing else but that power by which the described figures are greater or less. And therefore in the description of *Archimedes' spiral,* which is done by the continual diminution of the semidiameter of a circle in the same proportion in which the circumference is diminished, the space, which is contained within the semidiameter and the spiral line, is a third part of the whole circle. For the semidiameters of circles, inasmuch as circles are understood to be made up of the aggregate of them, are so many sectors; and therefore in the description of a spiral, the sector which describes it is diminished in duplicate proportions to the diminutions of the circumference of the circle in which it is inscribed; so that the complement of the spiral, that is, that space in the circle which is without the spiral line, is double to the space within the spiral line. In the same manner, if

there be taken a mean proportional everywhere between the semidiameter of the circle, which contains the spiral, and that part of the semidiameter which is within the same, there will be made another figure, which will be half the circle. And to conclude, this rule serves for all such spaces as may be described by a line or superficies decreasing either in magnitude or power; so that if the proportions, in which they decrease, be commensurable to the proportions of the times in which they decrease, the magnitudes of the figures they describe will be known.

The proposition demonstrated in art. 2 confirmed from the elements of philosophy.

12. The truth of that proposition, which I demonstrated in art. 2, which is the foundation of all that has been said concerning deficient figures, may be derived from the elements of philosophy, as having its original in this; *that all equality and inequality between two effects, that is, all proportion, proceeds from, and is determined by, the equal and unequal causes of those effects, or from the proportion which the causes, concurring to one effect, have to the causes which concur to the producing of the other effect;* and that therefore the proportions of quantities are the same with the proportions of their causes. Seeing, therefore, two deficient figures, of which one is the complement of the other, are made, one by motion decreasing in a certain time and proportion, the other by the loss of motion in the same time; the causes, which make and determine the quantities of both the figures, so that they can be no other than they are, differ only in this, that the proportions by which the quantity which generates the figure proceeds in describing of the same, that

is, the proportions of the remainders of all the
times and altitudes, may be other proportions than
those by which the same generating quantity de-
creases in making the complement of that figure,
that is, the proportions of the quantity which gene-
rates the figure continually diminished. Wherefore,
as the proportion of the times in which motion is
lost, is to that of the decreasing quantities by
which the deficient figure is generated, so will the
defect or complement be to the figure itself which
is generated.

13. There are also other quantities which are An unusual
determinable from the knowledge of their causes, way of reason-
namely, from the comparison of the motions by the equality
which they are made; and that more easily than superficies
from the common elements of geometry. For of a sphere
example, that the superficies of any portion of a and a circle.
sphere is equal to that circle, whose radius is a
strait line drawn from the pole of the portion to
the circumference of its base, I may demonstrate
in this manner.   Let B A C (in fig. 7) be a portion
of a sphere, whose axis is A E, and whose base is
B C; and let A B be the strait line drawn from
the pole A to the base in B; and let A D, equal to
A B, touch the great circle B A C in the pole A.
It is to be proved that the circle made by the
radius A D is equal to the superficies of the portion
B A C.   Let the plain A E B D be understood to
make a revolution about the axis A E; and it is
manifest that by the strait line A D a circle will be
described; and by the arch A B the superficies of
a portion of a sphere; and lastly, by the subtense
A B the superficies of a right cone.   Now seeing
both the strait line A B and the arch A B make

one and the same revolution, and both of them have the same extreme points A and B, the cause why the spherical superficies, which is made by the arch, is greater than the conical superficies, which is made by the subtense, is, that A B the arch is greater than A B the subtense; and the cause why it is greater consists in this, that although they be both drawn from A to B, yet the subtense is drawn strait, but the arch angularly, namely, according to that angle which the arch makes with the subtense, which angle is equal to the angle D A B (for an angle of contingence adds nothing to an angle of a segment, as has been shown in chapter xiv, article 16.)   Wherefore the magnitude of the angle D A B is the cause why the superficies of the portion, described by the arch A B, is greater than the superficies of the right cone described by the subtense A B.

Again, the cause why the circle described by the tangent A D is greater than the superficies of the right cone described by the subtense A B (notwithstanding that the tangent and the subtense are equal, and both moved round in the same time) is this, that A D stands at right angles to the axis, but A B obliquely ; which obliquity consists in the same angle D A B.   Seeing therefore the quantity of the angle D A B is that which makes the excess both of the superficies of the portion, and of the circle made by the radius A D, above the superficies of the right cone described by the subtense A B ; it follows, that both the superficies of the portion and that of the circle do equally exceed the superficies of the cone. Wherefore the circle made by A D or A B, and

the spherical superficies made by the arch A B, are equal to one another; which was to be proved.

14. If these deficient figures, which I have de-- scribed in a parallelogram, were capable of exact description, then any number of mean propor- tionals might be found out between two strait lines given. For example, in the parallelogram A B C D, (in figure 8) let the three-sided figure of two means be described (which many call a *cubical parabola*); and let R and S be two given strait lines; between which, if it be required to find two mean propor- tionals, it may be done thus. Let it be as R to S, so B C to B F; and let F E be drawn parallel to B A, and cut the crooked line in E; then through E let G H be drawn parallel and equal to the strait line A D, and cut the diagonal B D in I; for thus we have G I the greatest of two means between G H and G E, as appears by the description of the figure in article 4. Wherefore, if it be as G H to G I, so R to another line, T, that T will be the greatest of two means between R and S. And therefore if it it be again as R to T, so T to ano- ther line, X, that will be done which was required.

In the same manner, four mean proportionals may be found out, by the description of a three- sided figure of four means; and so any other num- ber of means, &c.

CHAPTER XVIII.

OF THE EQUATION OF STRAIT LINES WITH THE CROOKED LINES OF PARABOLAS AND OTHER FIGURES MADE IN IMITATION OF PARABOLAS.

1. To find the strait line equal to the crooked line of a semiparabola.—2. To find a strait line equal to the crooked line of the first semiparabolaster, or to the crooked line of any other of the deficient figures of the table of the 3d article of the precedent chapter.

PART III.
18.

To find a strait line equal to the crooked line of a semiparabola.

1. A PARABOLA being given, to find a strait line equal to the crooked line of the semiparabola.

Let the parabolical line given be A B C (in figure 1), and the diameter found be A D, and the base drawn D C; and the parallelogram A D C E being completed, draw the strait line A C. Then dividing A D into two equal parts in F, draw F H equal and parallel to D C, cutting A C in K, and the parabolical line in O; and between F H and F O take a mean proportional F P, and draw A O, A P and P C. I say that the two lines A P and P C, taken together as one line, are equal to the parabolical line A B O C.

For the line A B O C being a parabolical line, is generated by the concourse of two motions, one uniform from A to E, the other in the same time uniformly accelerated from rest in A to D. And because the motion from A to E is uniform, A E may represent the times of both those motions from the beginning to the end. Let therefore A E be the time; and consequently the lines ordi-

nately applied in the semiparabola will design the parts of time wherein the body, that describeth the line A B O C, is in every point of the same ; so that as at the end of the time A E or D C it is in C, so at the end of the time F O it will be in O. And because the velocity in A D is increased uniformly, that is, in the same proportion with the time, the same lines ordinately applied in the semiparabola will design also the continual augmentation of the impetus, till it be at the greatest, designed by the base D C.   Therefore supposing uniform motion in the line A F, in the time F K the body in A by the concourse of the two uniform motions in A F and F K will be moved uniformly in the line A K ; and K O will be the increase of the impetus or swiftness gained in the time F K ; and the line A O will be uniformly described by the concourse of the two uniform motions in A F and F O in the time F O.   From O draw O L parallel to E C, cutting A C in L ; and draw L N parallel to D C, cutting E C in N, and the parabolical line in M ; and produce it on the other side to A D in I ; and I N, I M and I L will be, by the construction of a parabola, in continual proportion, and equal to the three lines F H, F P and F O ; and a strait line parallel to E C passing through M will fall on P ; and therefore O P will be the increase of impetus gained in the time F O or I L.   Lastly, produce P M to C D in Q ; and Q C or M N or P H will be the increase of impetus proportional to the time F P or I M or D Q.   Suppose now uniform motion from H to C in the time P H. Seeing therefore in the time F P with uniform motion and the impetus increased in proportion to the times, is described

the straight line A P ; and in the rest of the time and impetus, namely, P H, is described the line C P uniformly ; it followeth that the whole line A P C is described with the whole impetus, and in the same time wherewith is described the parabolical line A B C ; and therefore the line A P C, made of the two strait lines A P and P C, is equal to the parabolical line A B C ; which was to be proved.

2. To find a strait line equal to the crooked line of the first semiparabolaster.

Let A B C be the crooked line of the first semiparabolaster ; A D the diameter ; D C the base ; and let the parallelogram completed be A D C E, whose diagonal is A C. Divide the diameter into two equal parts in F, and draw F H equal and parallel to D C, cutting A C in K, the crooked line in O, and E C in H. Then draw O L parallel to E C, cutting A C in L ; and draw L N parallel to the base D C, cutting the crooked line in M, and the strait line E C in N ; and produce it on the other side to A D in I. Lastly, through the point M draw P M Q parallel and equal to H C, cutting F H in P ; and join C P, A P and A O. I say, the two strait lines A P and P C are equal to the crooked line A B O C.

For the line A B O C, being the crooked line of the first semiparabolaster, is generated by the concourse of two motions, one uniform from A to E, the other in the same time accelerated from rest in A to D, so as that the impetus increaseth in proportion perpetually triplicate to that of the increase of the time, or which is all one, the lengths transmitted are in proportion triplicate to

that of the times of their transmission; for as the
impetus or quicknesses increase, so the lengths
transmitted increase also. And because the mo-
tion from A to E is uniform, the line A E may
serve to represent the time, and consequently the
lines, ordinately drawn in the semiparabolaster,
will design the parts of time wherein the body,
beginning from rest in A, describeth by its
motion the crooked line A B O C. And because
D C, which represents the greatest acquired im-
petus, is equal to A E, the same ordinate lines will
represent the several augmentations of the impetus
increasing from rest in A. Therefore, supposing
uniform motion from A to F, in the time F K there
will be described, by the concourse of the two
uniform motions A F and F K, the line A K uni-
formly, and K O will be the increase of impetus in
the time F K; and by the concourse of the two
uniform motions in A F and F O will be described
the line A O uniformly. Through the point L
draw the strait line L M N parallel to D C, cutting
the strait line A D in I, the crooked line A B C in
M, and the strait line E C in N; and through the
point M the strait line P M Q parallel and equal to
H C, cutting D C in Q and F H in P. By the
concourse therefore of the two uniform motions in
A F and F P in the time F P will be uniformly
described the strait line A P; and L M or O P
will be the increase of impetus to be added for the
time F O. And because the proportion of I N to
I L is triplicate to the proportion of I N to I M,
the proportion of F H to F O will also be tripli-
cate to the proportion of F H to F P; and the
proportional impetus gained in the time F P is P H.

So that F H being equal to D C, which designed
the whole impetus acquired by the acceleration,
there is no more increase of impetus to be com-
puted.   Now in the time P H suppose an uniform
motion from H to C; and by the two uniform mo-
tions in C H and H P will be described uniformly
the strait line P C. Seeing therefore the two strait
lines A P and P C are described in the time A E
with the same increase of impetus, wherewith the
crooked line A B O C is described in the same
time A E, that is, seeing the line A P C and the
line A B O C are transmitted by the same body in
the same time and with equal velocities, the lines
themselves are equal; which was to be demon-
strated.

By the same method (if any of the semipara-
bolasters in the table of art. 3 of the precedent
chapter be exhibited) may be found a strait line
equal to the crooked line thereof, namely, by
dividing the diameter into two equal parts, and
proceeding as before.   Yet no man hitherto hath
compared any crooked with any strait line, though
many geometricians of every age have endeavoured
it.   But the cause, why they have not done it,
may be this, that there being in Euclid no defini-
tion of equality, nor any mark by which to judge
of it besides congruity (which is the 8th axiom of
the first Book of his Elements) a thing of no use
at all in the comparing of strait and crooked; and
others after Euclid (except Archimedes and Apol-
lonius, and in our time Bonaventura) thinking the
industry of the ancients had reached to all that
was to be done in geometry, thought also, that
all that could be propounded was either to be

deduced from what they had written, or else that it was not at all to be done : it was therefore disputed by some of those ancients themselves, whether there might be any equality at all between crooked and strait lines ; which question Archimedes, who assumed that some strait line was equal to the circumference of a circle, seems to have despised, as he had reason. And there is a late writer that granteth that between a strait and a crooked line there is equality; but now, says he, since the fall of Adam, without the special assistance of Divine Grace it is not to be found.

PART III.
18.
To find a strait line, &c.

CHAPTER XIX.

OF ANGLES OF INCIDENCE AND REFLECTION,
EQUAL BY SUPPOSITION.

1. If two strait lines falling upon another strait line be parallel, the lines reflected from them shall also be parallel.—2. If two strait lines drawn from one point fall upon another strait line, the lines reflected from them, if they be drawn out the other way, will meet in an angle equal to the angle made by the lines of incidence.—3. If two strait parallel lines, drawn not oppositely, but from the same parts, fall upon the circumference of a circle, the lines reflected from them, if produced they meet within the circle, will make an angle double to that which is made by two strait lines drawn from the centre to the points of incidence.—4. If two strait lines drawn from the same point without a circle fall upon the circumference, and the lines reflected from them being produced meet within the circle, they will make an angle equal to twice that angle, which is made by two strait lines drawn from the centre to the points of incidence, together with the angle which the incident lines themselves make.—5. If two strait lines drawn from one point

fall upon the concave circumference of a circle, and the angle they make be less than twice the angle at the centre, the lines reflected from them and meeting within the circle will make an angle, which being added to the angle of the incident lines will be equal to twice the angle at the centre.—6. If through any one point two unequal chords be drawn cutting one another, and the centre of the circle be not placed between them, and the lines reflected from them concur wheresoever, there cannot through the point, through which the two former lines were drawn, be drawn any other strait line whose reflected line shall pass through the common point of the two former lines reflected.—7. In equal chords the same is not true. 8. Two points being given in the circumference of a circle, to draw two strait lines to them, so that their reflected lines may contain any angle given.—9. If a strait line falling upon the circumference of a circle be produced till it reach the semi-diameter, and that part of it, which is intercepted between the circumference and the semidiameter, be equal to that part of the semidiameter which is between the point of concourse and the centre, the reflected line will be parallel to the semi-diameter.—10. If from a point within a circle, two strait lines be drawn to the circumference, and their reflected lines meet in the circumference of the same circle, the angle made by the reflected lines will be a third part of the angle made by the incident lines.

PART III.
19.

Angles of incidence and reflection.

WHETHER a body falling upon the superficies of another body and being reflected from it, do make equal angles at that superficies, it belongs not to this place to dispute, being a knowledge which depends upon the natural causes of reflection; of which hitherto nothing has been said, but shall be spoken of hereafter.

In this place, therefore, let it be supposed that the angle of incidence is equal to the angle of reflection; that our present search may be applied, not to the finding out of the causes, but some consequences of the same.

I call an *angle of incidence*, that which is made

between a strait line and another line, strait or
crooked, upon which it falls, and which I call the
line reflecting ; and an *angle of reflection* equal
to it, that which is made at the same point between
the strait line which is reflected and the line
reflecting.

1. If two strait lines, which fall upon another
strait line, be parallel, their reflected lines shall be
also parallel.

Let the two strait lines A B and C D (in fig. 1),
which fall upon the strait line E F, at the points
B and D, be parallel; and let the lines reflected
from them be B G and D H.   I say, B G and D H
are also parallel.

For the angles A B E and C D E are equal by
reason of the parallelism of A B and C D; and the
angles G B F and H D F are equal to them by sup-
position; for the lines B G and D H are reflected
from the lines A B and C D.   Wherefore B G and
D H are parallel.

If two strait lines falling upon another strait line be parallel, the lines reflected from them shall also be parallel.

2. If two strait lines drawn from the same
point fall upon another strait line, the lines re-
flected from them, if they be drawn out the other
way, will meet in an angle equal to the angle of the
incident lines.

From the point A (in fig. 2) let the two strait
lines A B and A D be drawn ; and let them fall
upon the strait line E K at the points B and D ;
and let the lines B I and D G be reflected from
them.   I say, I B and G D do converge, and that if
they be produced on the other side of the line E K,
they shall meet, as in F ; and that the angle B F D
shall be equal to the angle B A D.

If two strait lines drawn from one point fall upon another strait line, the lines reflected from them, if they be drawn out the other way, will meet in an angle equal to the angle made by the lines of incidence.

For the angle of reflection I B K is equal to the

T 2

angle of incidence A B E; and to the angle I B K
its vertical angle E B F is equal; and therefore
the angle A B E is equal to the angle E B F.
Again, the angle A D E is equal to the angle of
reflection G D K, that is, to its vertical angle
E D F; and therefore the two angles A B D and
A D B of the triangle A B D are one by one equal
to the two angles F B D and F D B of the triangle
F B D; wherefore also the third angle B A D is
equal to the third angle B F D; which was to be
proved.

Coroll. i. If the strait line A F be drawn, it will
be perpendicular to the strait line E K. For both
the angles at E will be equal, by reason of the
equality of the two angles A B E and F B E, and
of the two sides A B and F B.

Coroll. ii. If upon any point between B and D
there fall a strait line, as AC, whose reflected line is
CH, this also produced beyond C, will fall upon F;
which is evident by the demonstration above.

If two strait
parallel lines,
drawn not op-
positely, but
from the same
parts, fall upon
the circumfe-
rence of a cir-
cle, the lines
reflected from
them, if pro-
duced they
meet within
the circle, will
make an angle
double to that
which is made
by two strait
lines drawn
from the centre
to the points of
incidence.

3. If from two points taken without a circle,
two strait parallel lines, drawn not oppositely, but
from the same parts, fall upon the circumference;
the lines reflected from them, if produced they
meet within the circle, will make an angle double
to that which is made by two strait lines drawn
from the centre to the points of incidence.

Let the two strait parallels A B and D C (in
fig. 3) fall upon the circumference B C at the
points B and C; and let the centre of the circle be
E; and let A B reflected be B F, and D C reflected
be C G; and let the lines F B and G C produced
meet within the circle in H; and let E B and E C

be connected. I say the angle F H G is double to the angle B E C.

For seeing A B and D C are parallels, and E B cuts A B in B, the same E B produced will cut D C somewhere ; let it cut it in D ; and let D C be produced howsoever to I, and let the intersection of D C and B F be at K.   The angle therefore I C H, being external to the triangle C K H, will be equal to the two opposite angles C K H and C H K.   Again, I C E being external to the triangle C D E, is equal to the two angles at D and E. Wherefore the angle I C H, being double to the angle I C E, is equal to the angles at D and E twice taken ; and therefore the two angles C K H and C H K are equal to the two angles at D and E twice taken.   But the angle C K H is equal to the angles D and A B D, that is, D twice taken ; for A B and D C being parallels, the altern angles D and A B D are equal.   Wherefore C H K, that is the angle F H G is also equal to the angle at E twice taken ; which was to be proved.

Coroll. If from two points taken within a circle two strait parallels fall upon the circumference, the lines reflected from them shall meet in an angle, double to that which is made by two strait lines drawn from the centre to the points of incidence.   For the parallels A B and I C falling upon the points B and C, are reflected in the lines B H and C H, and make the angle at H double to the angle at E, as was but now demonstrated.

4. If two strait lines drawn from the same point without a circle fall upon the circumference, and the lines reflected from them being produced meet within the circle, they will make an angle equal to

twice that angle, which is made by two strait lines drawn from the centre to the points of incidence, together with the angle which the incident lines themselves make.

Let the two strait lines A B and A C (in fig. 4) be drawn from the point A to the circumference of the circle, whose centre is D; and let the lines reflected from them be B E and C G, and, being produced, make within the circle the angle H; also let the two strait lines D B and D C be drawn from the centre D to the points of incidence B and C. I say, the angle H is equal to twice the angle at D together with the angle at A.

For let A C be produced howsoever to I. Therefore the angle I C H, which is external to the triangle C K H, will be equal to the two angles C K H and C H K. Again, the angle I C D, which is external to the triangle C L D, will be equal to the two angles C L D and C D L. But the angle I C H is double to the angle I C D, and is therefore equal to the angles C L D and C D L twice taken. Wherefore the angles C K H and C H K are equal to the angles C L D and C D L twice taken. But the angle C L D, being external to the triangle A L B, is equal to the two angles L A B and L B A; and consequently C L D twice taken is equal to L A B and L B A twice taken. Wherefore C K H and C H K are equal to the angle C D L together with L A B and L B A twice taken. Also the angle C K H is equal to the angle L A B once and A B K, that is, L B A twice taken. Wherefore the angle C H K is equal to the remaining angle C D L, that is, to the angle at D, twice taken, and

the angle L A B, that is, the angle at A, once taken ; which was to be proved.

Coroll. If two strait converging lines, as I C and M B, fall upon the concave circumference of a circle, their reflected lines, as C H and B H, will meet in the angle H, equal to twice the angle D, together with the angle at A made by the incident lines produced. Or, if the incident lines be H B and I C, whose reflected lines C H and B M meet in the point N, the angle C N B will be equal to twice the angle D, together with the angle C K H made by the lines of incidence. For the angle C N B is equal to the angle H, that is, to twice the angle D, together with the two angles A, and N B H, that is, K B A. But the angles K B A and A are equal to the angle C K H. Wherefore the angle C N B is equal to twice the angle D, together with the angle C K H made by the lines of incidence I C and H B produced to K.

5. If two strait lines drawn from one point fall upon the concave circumference of a circle, and the angle they make be less than twice the angle at the centre, the lines reflected from them and meeting within the circle will make an angle, which being added to the angle of the incident lines, will be equal to twice the angle at the centre.

Let the two lines A B and A C (in fig. 5), drawn from the point A, fall upon the concave circumference of the circle whose centre is D ; and let their reflected lines B E and C E meet in the point E ; also let the angle A be less than twice the angle D. I say, the angles A and E together taken are equal to twice the angle D.

For let the strait lines A B and E C cut the

If two strait
lines drawn
from one, &c.

strait lines D C and D B in the points G and H ; and the angle B H C will be equal to the two angles E B H and E ; also the same angle B H C will be equal to the two angles D and D C H ; and in like manner the angle B G C will be equal to the two angles A C D and A, and the same angle B G C will be also equal to the two angles D B G and D. Wherefore the four angles E B H, E, A C D and A, are equal to the four angles D, D C H, D B G and D. If, therefore, equals be taken away on both sides, namely, on one side A C D and E B H, and on the other side D C H and D B G, (for the angle E B H is equal to the angle D B G, and the angle A C D equal to the angle D C H), the remainders on both sides will be equal, namely, on one side the angles A and E, and on the other the angle D twice taken. Wherefore the angles A and E are equal to twice the angle D.

Coroll. If the angle A be greater than twice the angle D, their reflected lines will diverge. For, by the corollary of the third proposition, if the angle A be equal to twice the angle D, the reflected lines B E and C E will be parallel ; and if it be less, they will concur, as has now been demonstrated. And therefore, if it be greater, the reflected lines B E and C E will diverge, and consequently, if they be produced the other way, they will concur and make an angle equal to the excess of the angle A above twice the angle D ; as is evident by art. 4.

If through any
one point two
unequal chords
be drawn cut-
ting one ano-
ther, and the
centre of the
circle be not

6. If through any one point two unequal chords be drawn cutting one another, either within the circle, or, if they be produced, without it, and the centre of the circle be not placed between them, and the lines reflected from them concur where-

soever; there cannot, through the point through
which the former lines were drawn, be drawn
another strait line, whose reflected line shall pass placed between
them, and the
through the point where the two former reflected lines reflected
from them con-
lines concur. cur whereso-
ever, there
Let any two unequal chords, as B K and C H cannot through
the point,
(in fig. 6), be drawn through the point A in the through which
circle B C; and let their reflected lines B D and the two former
lines were
C E meet in F; and let the centre not be between drawn, be
A B and A C; and from the point A let any other drawn any
other strait line
strait line, as A G, be drawn to the circumference whose reflected
line shall pass
between B and C. I say, G N, which passes through the
common point
through the point F, where the reflected lines B D of the two for-
and C E meet, will not be the reflected line of A G. mer lines re-
flected.

For let the arch B L be taken equal to the arch
B G, and the strait line B M equal to the strait
line B A; and L M being drawn, let it be produced
to the circumference in O. Seeing therefore B A
and B M are equal, and the arch B L equal to the
arch B G, and the angle M B L equal to the angle
A B G, A G and M L will also be equal, and, pro-
ducing G A to the circumference in I, the whole
lines L O and G I will in like manner be equal.
But L O is greater than G F N, as shall presently
be demonstrated; and therefore also G I is greater
than G N. Wherefore the angles N G C and I G B
are not equal. Wherefore the line G F N is not
reflected from the line of incidence A G, and con-
sequently no other strait line, besides A B and
A C, which is drawn through the point A, and
falls upon the circumference B C, can be reflected
to the point F; which was to be demonstrated.

It remains that I prove L O to be greater than
G N; which I shall do in this manner. L O and

G N cut one another in P ; and P L is greater than P G. Seeing now L P. P G : : P N. P O are proportionals, therefore the two extremes L P and P O together taken, that is L O, are greater than P G and P N together taken, that is, G N; which remained to be proved.

In equal chords the same is not true.

7. But if two equal chords be drawn through one point within a circle, and the lines reflected from them meet in another point, then another strait line may be drawn between them through the former point, whose reflected line shall pass through the latter point.

Let the two equal chords B C and E D (in the 7th figure) cut one another in the point A within the circle B C D; and let their reflected lines C H and D I meet in the point F. Then dividing the arch C D equally in G, let the two chords G K and G L be drawn through the points A and F. I say, G L will be the line reflected from the chord K G. For the four chords B C, C H, E D and D I are by supposition all equal to one another; and therefore the arch B C H is equal to the arch E D I ; as also the angle B C H to the angle E D I ; and the angle A M C to its verticle angle F M D ; and the strait line D M to the strait line G M ; and, in like manner, the strait line A C to the strait line F D ; and the chords C G and G D being drawn, will also be equal ; and also the angles F D G and A C G, in the equal segments G D I and G C B. Wherefore the strait lines F G and A G are equal ; and, therefore, the angle F G D is equal to the angle A G C, that is, the angle of incidence equal to the angle of reflection. Wherefore the line G L is reflected from the incident line C G ; which was to be proved.

Coroll. By the very sight of the figure it is mani- fest, that if G be not the middle point between C and D, the reflected line G L will not pass through the point F.

8. Two points in the circumference of a circle being given to draw two strait lines to them, so as that their reflected lines may be parallel, or contain any angle given.

In the circumference of the circle, whose centre is A, (in the 8th figure) let the two points B and C be given ; and let it be required to draw to them from two points taken without the circle two incident lines, so that their reflected lines may, first, be parallel.

Let A B and A C be drawn ; as also any incident line D C, with its reflected line C F ; and let the angle E C D be made double to the angle A ; and let H B be drawn parallel to E C, and produced till it meet with D C produced in I. Lastly, producing A B indefinitely to K, let G B be drawn so that the angle G B K may be equal to the angle H B K, and then G B will be the reflected line of the incident line H B. I say, D C and H B are two incident lines, whose reflected lines C F and B G are parallel.

For seeing the angle E C D is double to the angle B A C, the angle H I C is also, by reason of the parallels E C and H I, double to the same B A C ; therefore also F C and G B, namely, the lines reflected from the incident lines D C and H B, are parallel. Wherefore the first thing required is done.

Secondly, let it be required to draw to the points B and C two strait lines of incidence, so that the

PART III.
19.

Two points be-
ing given in the
circumference
of a circle, &c.
lines reflected from them may contain the given angle Z.

To the angle E C D made at the point C, let there be added on one side the angle D C L equal to half Z, and on the other side the angle E C M equal to the angle D C L; and let the strait line B N be drawn parallel to the strait line C M; and let the angle K B O be made equal to the angle N B K; which being done, B O will be the line of reflection from the line of incidence N B. Lastly, from the incident line L C, let the reflected line C O be drawn, cutting B O at O, and making the angle C O B. I say, the angle C O B is equal to the angle Z.

Let N B be produced till it meet with the strait line L C produced in P. Seeing, therefore, the angle L C M is, by construction, equal to twice the angle B A C, together with the angle Z; the angle N P L, which is equal to L C M by reason of the parallels N P and M C, will also be equal to twice the same angle B A C, together with the angle Z. And seeing the two strait lines O C and O B fall from the point O upon the points C and B; and their reflected lines L C and N B meet in the point P; the angle N P L will be equal to twice the angle B A C together with the angle C O B. But I have already proved the angle N P L to be equal to twice the angle B A C together with the angle Z. Therefore the angle C O B is equal to the angle Z; wherefore, two points in the circumference of a circle being given, I have drawn, &c.; which was to be done.

But if it be required to draw the incident lines from a point within the circle, so that the lines reflected from them may contain an angle equal to

the angle Z, the same method is to be used, saving
that in this case the angle Z is not to be added to twice the angle B A C, but to be taken from it.

9. If a strait line, falling upon the circumference of a circle, be produced till it reach the semidiameter, and that part of it which is intercepted between the circumference and the semidiameter be equal to that part of the semidiameter which is between the point of concourse and the centre, the reflected line will be parallel to the semidiameter.

If a strait line falling upon the circumference of a circle be produced till it reach the semidiameter, and that part of it, which is intercepted between the circumference and the semidiameter, be equal to that part of the semidiameter which is between the point of concourse and the centre, the reflected line will be parallel to the semidiameter.

Let any line A B (in the 9th figure) be the semidiameter of the circle whose centre is A ; and upon the circumference B D let the strait line C D fall, and be produced till it cut A B in E, so that E D and E A may be equal ; and from the incident line C D let the line D F be reflected.   I say, A B and D F will be parallel.

Let A G be drawn through the point D.   Seeing, therefore, E D and E A are equal, the angles E D A and E A D will also be equal.   But the angles F D G and E D A are equal ; for each of them is half the angle E D H or F D C.   Wherefore the angles F D G and E A D are equal ; and consequently D F and A B are parallel ; which was to be proved.

Coroll. If E A be greater then E D, then D F and A B being produced will concur ; but if E A be less than E D, then B A and D H being produced will concur.

10. If from a point within a circle two strait lines be drawn to the circumference, and their reflected lines meet in the circumference of the same circle, the angle made by the lines of reflection will be a third part of the angle made by the lines of incidence.

If from a point within a circle two strait lines be drawn to the circumference, and their reflected lines meet in the circumference,

cumference of the same circle, the angle made by the reflected lines will be a third part of the angle made by the incident lines.

From the point B (in the 10th figure) taken within the circle whose centre is A, let the two strait lines B C and B D be drawn to the circumference; and let their reflected lines C E and D E meet in the circumference of the same circle at the point E. I say, the angle C E D will be a third part of the angle C B D.

Let A C and A D be drawn. Seeing, therefore, the angles C E D and C B D together taken are equal to twice the angle C A D (as has been demonstrated in the 5th article); and the angle C A D twice taken is quadruple to the angle C E D; the angles C E D and C B D together taken will also be equal to the angle C E D four times taken; and therefore if the angle C E D be taken away on both sides, there will remain the angle C B D on one side, equal to the angle C E D thrice taken on the other side; which was to be demonstrated.

Coroll. Therefore a point being given within a circle, there may be drawn two lines from it to the circumference, so as their reflected lines may meet in the circumference. For it is but trisecting the angle C B D, which how it may be done shall be shown in the following chapter.

CHAPTER XX.

OF THE DIMENSION OF A CIRCLE, AND THE
DIVISION OF ANGLES OR ARCHES.

1. The dimension of a circle never determined in numbers by
Archimedes and others.—2. The first attempt for the finding out
of the dimension of a circle by lines.—3. The second attempt
for the finding out of the dimension of a circle from the
consideration of the nature of crookedness.—4. The third
attempt; and some things propounded to be further searched
into.—5. The equation of the spiral of Archimedes with a
strait line.—6. Of the analysis of geometricians by the powers
of lines.

1. In the comparing of an arch of a circle with a strait line, many and great geometricians, even from the most ancient times, have exercised their wits; and more had done the same, if they had not seen their pains, though undertaken for the common good, if not brought to perfection, vilified by those that envy the praises of other men. Amongst those ancient writers whose works are come to our hands, Archimedes was the first that brought the length of the perimeter of a circle within the limits of numbers very little differing from the truth; demonstrating the same to be less than three diameters and a seventh part, but greater than three diameters and ten seventy-one parts of the diameter. So that supposing the radius to consist of 10,000,000 equal parts, the arch of a quadrant will be between 15,714,285 and 15,704,225 of the same parts. In our times, Ludovicus Van Cullen and Willebrordus Snellius,

PART III.
20.

The dimension
of a circle never
determined in
numbers by
Archimedes
and others.

The dimension of a circle never determined in numbers by Archimedes and others.

with joint endeavour, have come yet nearer to the truth ; and pronounced from true principles, that the arch of a quadrant, putting, as before, 10,000,000 for radius, differs not one whole unity from the number 15,707,963 ; which, if they had exhibited their arithmetical operations, and no man had discovered any error in that long work of theirs, had been demonstrated by them. This is the furthest progress that has been made by the way of numbers ; and they that have proceeded thus far deserve the praise of industry. Nevertheless, if we consider the benefit, which is the scope at which all speculation should aim, the improvement they have made has been little or none. For any ordinary man may much sooner and more accurately find a strait line equal to the perimeter of a circle, and consequently square the circle, by winding a small thread about a given cylinder, than any geometrician shall do the same by dividing the radius into 10,000,000 equal parts. But though the length of the circumference were exactly set out, either by numbers, or mechanically, or only by chance, yet this would contribute no help at all towards the section of angles, unless happily these two problems, *to divide a given angle according to any proportion assigned*, and *to find a strait line equal to the arch of a circle*, were reciprocal, and followed one another. Seeing therefore the benefit proceeding from the knowledge of the length of the arch of a quadrant consists in this, that we may thereby divide an angle according to any proportion, either accurately, or at least accurately enough for common use ; and seeing this cannot be done by arithmetic, I

thought fit to attempt the same by geometry,
and in this chapter to make trial whether it might not be performed by the drawing of strait and circular lines.

2. Let the square A B C D (in the first figure) be described; and with the radii A B, B C, and D C, the three arches B D, C A, and A C; of which let the two B D and C A cut one another in E, and the two B D and A C in F. The diagonals therefore B D and A C being drawn will cut one another in the centre of the square G, and the two arches B D and C A in two equal parts in H and Y; and the arch B H D will be trisected in F and E. Through the centre G let the two strait lines K G L and M G N be drawn parallel and equal to the sides of the square A B and A D, cutting the four sides of the same square in the points K, L, M, and N; which being done, K L will pass through F, and M N through E. Then let O P be drawn parallel and equal to the side B C, cutting the arch B F D in F, and the sides A B and D C in O and P. Therefore O F will be the sine of the arch B F, which is an arch of 30 degrees; and the same O F will be equal to half the radius. Lastly, dividing the arch B F in the middle in Q, let R Q, the sine of the arch B Q, be drawn and produced to S, so that Q S be equal to R Q, and consequently R S be equal to the chord of the arch B F; and let F S be drawn and produced to T in the side B C. I say, the strait line B T is equal to the arch B F; and consequently that B V, the triple of B T, is equal to the arch of the quadrant B F E D.

Let T F be produced till it meet the side B A produced in X; and dividing O F in the middle

The first
attempt for
the finding
out of the
dimension of a
circle by lines.

in Z, let Q Z be drawn and produced till it meet with the side B A produced.   Seeing therefore the strait lines R S and O F are parallel, and divided in the midst in Q and Z, Q Z produced will fall upon X, and X Z Q produced to the side B C will cut B T in the midst in α.

Upon the strait line F Z, the fourth part of the radius A B, let the equilateral triangle a Z F be constituted ; and upon the centre a, with the radius a Z, let the arch Z F be drawn ; which arch Z F will therefore be equal to the arch Q F, the half of the arch B F.   Again, let the strait line Z O be cut in the midst in b, and the strait line b O in the midst in c ; and let the bisection be continued in this manner till the last part O c be the least that can possibly be taken ; and upon it, and all the rest of the parts equal to it into which the strait line O F may be cut, let so many equilateral triangles be understood to be constituted ; of which let the last be d O c.   If, therefore, upon the centre d, with the radius d O, be drawn the arch O c, and upon the rest of the equal parts of the strait line O F be drawn in like manner so many equal arches, all those arches together taken will be equal to the whole arch B F, and the half of them, namely, those that are comprehended between O and Z, or between Z and F, will be equal to the arch B Q or Q F, and in sum, what part soever the strait line O c be of the strait line O F, the same part will the arch O c be of the arch B F, though both the arch and the chord be infinitely bisected.   Now seeing the arch O c is more crooked than that part of the arch B F which is equal to it ; and seeing also

that the more the strait line X c is produced, the
more it diverges from the strait line X O, if the
points O and c be understood to be moved for-
wards with strait motion in X O and X c, the
arch O c will thereby be extended by little and
little, till at the last it come somewhere to have
the same crookedness with that part of the arch B F
which is equal to it. In like manner, if the strait
line X b be drawn, and the point b be understood
to be moved forwards at the same time, the arch
$c\,b$ will also by little and little be extended, till
its crookedness come to be equal to the crooked-
ness of that part of the arch B F which is equal
to it. And the same will happen in all those
small equal arches which are described upon so
many equal parts of the strait line O F. It is also
manifest, that by strait motion in X O and X Z
all those small arches will lie in the arch B F,
in the points B, Q and F. And though the same
small equal arches should not be coincident with
the equal parts of the arch B F in all the other
points thereof, yet certainly they will constitute
two crooked lines, not only equal to the two
arches B Q and Q F, and equally crooked, but
also having their cavity towards the same parts ;
which how it should be, unless all those small
arches should be coincident with the arch B F in
all its points, is not imaginable. They are there-
fore coincident, and all the strait lines drawn
from X, and passing through the points of division
of the strait line O F, will also divide the arch
B F into the same proportions into which O F is
divided.

Now seeing X b cuts off from the point B the

PART III.
20.

The first
attempt for
the finding
out of the
dimension of a
circle by lines.

fourth part of the arch B F, let that fourth part be B e ; and let the sine thereof, $f\,e$, be produced to F T in g, for so $f\,e$ will be the fourth part of the strait line $f\,g$, because as O b is to O F, so is $f\,e$ to $f\,g$. But B T is greater than $f\,g$; and therefore the same B T is greater than four sines of the fourth part of the arch B F. And in like manner, if the arch B F be subdivided into any number of equal parts whatsoever, it may be proved that the strait line B T is greater than the sine of one of those small arches, so many times taken as there be parts made of the whole arch B F. Wherefore the strait line B T is not less than the arch B F. But neither can it be greater, because if any strait line whatsoever, less than B T, be drawn below B T, parallel to it, and terminated in the strait lines X B and X T, it would cut the arch B F; and so the sine of some one of the parts of the arch B F, taken so often as that small arch is found in the whole arch B F, would be greater than so many of the same arches ; which is absurd. Wherefore the strait line B T is equal to the arch B F ; and the strait line B V equal to the arch of the quadrant B F D ; and B V four times taken, equal to the perimeter of the circle described with the radius A B. Also the arch B F and the strait line B T are everywhere divided into the same proportions ; and consequently any given angle, whether greater or less than B A F, may be divided into any proportion given.

But the strait line B V, though its magnitude fall within the terms assigned by Archimedes, is found, if computed by the canon of signs, to be somewhat greater than that which is exhibited by

PART III.
20.

The first
attempt for
the finding
out of the
dimension of a
circle by lines.

the Rudolphine numbers. Nevertheless, if in the place of B T, another strait line, though never so little less, be substituted, the division of angles is immediately lost, as may by any man be demonstrated by this very scheme.

Howsoever, if any man think this my strait line B V to be too great, yet, seeing the arch and all the parallels are everywhere so exactly divided, and B V comes so near to the truth, I desire he would search out the reason, why, granting B V to be precisely true, the arches cut off should not be equal.

But some man may yet ask the reason why the strait lines, drawn from X through the equal parts of the arch B F, should cut off in the tangent B V so many strait lines equal to them, seeing the connected straight line X V passes not through the point D, but cuts the strait line A D produced in *l;* and consequently require some determination of this problem. Concerning which, I will say what I think to be the reason, namely, that whilst the magnitude of the arch doth not exceed the magnitude of the radius, that is, the magnitude of the tangent B C, both the arch and the tangent are cut alike by the strait lines drawn from X ; otherwise not. For A V being connected, cutting the arch B H D in I, if X C being drawn should cut the same arch in the same point I, it would be as true that the arch B I is equal to the radius B C, as it is true that the arch B F is equal to the strait line B T; and drawing X K it would cut the arch B I in the midst in *i;* also drawing A *i* and producing it to the tangent B C in *k,* the strait line B *k* will be the

tangent of the arch B *i*, (which arch is equal to half the radius) and the same strait line B *k* will be equal to the strait line *k* I. I say all this is true, if the preceding demonstration be true ; and consequently the proportional section of the arch and its tangent proceeds hitherto. But it is manifest by the golden rule, that taking B *k* double to B T, the line X *k* shall not cut off the arch B E, which is double to the arch B F, but a much greater. For the magnitude of the straight lines X M, X B, and M E, being known (in numbers), the magnitude of the strait line cut off in the tangent by the strait line X E produced to the tangent, may also be known ; and it will be found to be less than B *k*; Wherefore the strait line X *k* being drawn, will cut off a part of the arch of the quadrant greater than the arch B E. But I shall speak more fully in the next article concerning the magnitude of the arch B I.

And let this be the first attempt for the finding out of the dimension of a circle by the section of the arch B F.

The second attempt for the finding out of the dimension of a circle from the consideration of the nature of crookedness.

3. I shall now attempt the same by arguments drawn from the nature of the crookedness of the circle itself; but I shall first set down some premises necessary for this speculation ; and

First, if a strait line be bowed into an arch of a circle equal to it, as when a stretched thread, which toucheth a right cylinder, is so bowed in every point, that it be everywhere coincident with the perimeter of the base of the cylinder, the flexion of that line will be equal, in all its points ; and consequently the crookedness of the arch of a

circle is everywhere uniform ; which needs no other demonstration than this, that the perimeter of a circle is an uniform line.

Secondly, and consequently : if two unequal arches of the same circle be made by the bowing of two strait lines equal to them, the flexion of the longer line, whilst it is bowed into the greater arch, is greater than the flexion of the shorter line, whilst it is bowed into the lesser arch, according to the proportion of the arches themselves ; and consequently, the crookedness of the greater arch is to the crookedness of the lesser arch, as the greater arch is to the lesser arch.

Thirdly : if two unequal circles and a strait line touch one another in the same point, the crookedness of any arch taken in the lesser circle, will be greater than the crookedness of an arch equal to it taken in the greater circle, in reciprocal proportion to that of the radii with which the circles are described ; or, which is all one, any strait line being drawn from the point of contact till it cut both the circumferences, as the part of that strait line cut off by the circumference of the greater circle to that part which is cut off by the circumference of the lesser circle.

For let A B and A C (in the second figure) be two circles, touching one another, and the strait line A D in the point A ; and let their centres be E and F ; and let it be supposed, that as A E is to A F, so is the arch A B to the arch A H. I say the crookedness of the arch A C is to the crookedness of the arch A H, as A E is to A F. For let the strait line A D be supposed to be equal to the arch A B, and the strait line A G to the arch A C ; and

let A D, for example, be double to A G. There-
fore, by reason of the likeness of the arches A B
and A C, the strait line A B will be double to the
strait line A C, and the radius A E double to the ra-
dius A F, and the arch A B double to the arch A H.
And because the strait line A D is so bowed to be
coincident with the arch A B equal to it, as the
strait line A G is bowed to be coincident with the
arch A C equal also to it, the flexion of the strait
line A G into the crooked line A C will be equal to
the flexion of the strait line A D into the crooked
line A B. But the flexion of the strait line A D
into the crooked line A B is double to the flexion
of the strait line A G into the crooked line A H ;
and therefore the flexion of the strait line A G into
the crooked line A C is double to the flexion of
the same strait line A G into the crooked line
A H. Wherefore, as the arch A B is to the arch
A C or A H ; or as the radius A E is to the radius
A F ; or as the chord A B is to the chord A C ; so
reciprocally is the flexion or uniform crookedness
of the arch A C, to the flexion or uniform crooked-
ness of the arch A H, namely, here double. And
this may by the same method be demonstrated in
circles whose perimeters are to one another triple,
quadruple, or in whatsoever given proportion. The
crookedness therefore of two equal arches taken in
several circles are in proportion reciprocal to that
of their radii, or like arches, or like chords ; which
was to be demonstrated.

Let the square A B C D be again described (in
the third figure), and in it the quadrants A B D,
B C A and D A C ; and dividing each side of the
square A B C D in the midst in E, F, G and H, let

E G and F H be connected, which will cut one an-
other in the centre of the square at I, and divide
the arch of the quadrant A B D into three equal
parts in K and L.   Also the diagonals A C and
B D being drawn will cut one another in I, and
divide the arches B K D and C L A into two equal
parts in M and N.   Then with the radius B F let
the arch F E be drawn, cutting the diagonal B D
in O ; and dividing the arch B M in the midst in P,
let the strait line E a equal to the chord B P be set
off from the point E in the arch E F, and let the
arch $a\,b$ be taken equal to the arch O a, and let
B a and B b be drawn and produced to the arch
A N in c and d; and lastly, let the strait line A d
be drawn.   I say the strait line A d is equal to the
arch A N or B M.

I have proved in the preceding article, that the
arch E O is twice as crooked as the arch B P, that
is to say, that the arch E O is so much more
crooked than the arch B P, as the arch B P is more
crooked than the strait line E a.   The crookedness
therefore of the chord E a, of the arch B P, and of
the arch E O, are as 0, 1, 2.   Also the difference
between the arches E O and E O, the difference
between the arches E O and E a, and the difference
between the arches E O and E b, are as 0, 1, 2.   So
also the difference between the arches A N and
A N, the difference between the arches A N and
A c, and the difference between the arches A N
and A d, are as 0, 1, 2 ; and the strait line A c is
double to the chord B P or E a, and the strait line
A d double to the chord E b.

Again, let the strait line B F be divided in the
midst in Q, and the arch B P in the midst in R ;

PART III.
20.

The second at-
tempt for the
finding, &c.

and describing the quadrant B Q S (whose arch
Q S is a fourth part of the arch of the quadrant
B M D, as the arch B R is a fourth part of the arch
B M, which is the arch of the semiquadrant A B M)
let the chord S e equal to the chord B R be set off
from the point S in the arch S Q ; and let B e be
drawn and produced to the arch A N in f; which
being done, the strait line Af will be quadruple to
the chord B R or S e. And seeing the crooked-
ness of the arch S e, or of the arch A c, is double
to the crookedness of the arch B R, the excess of
the crookedness of the arch Af above the crook-
edness of the arch A c will be subduple to the
excess of the crookedness of the arch A c above the
crookedness of the arch A N ; and therefore the
arch N c will be double to the arch cf. Wherefore
the arch $c\,d$ is divided in the midst in f, and the
arch Nf is $\frac{3}{4}$ of the arch N d. And in like manner
if the arch B R be bisected in V, and the strait
line B Q in X, and the quadrant B X Y be de-
scribed, and the strait line Y g equal to the chord
B V be set off from the point Y in the arch Y X,
it may be demonstrated that the strait line B g
being drawn and produced to the arch A N, will
cut the arch $f\,d$ into two equal parts, and that a
strait line drawn from A to the point of that sec-
tion, will be equal to eight chords of the arch B V,
and so on perpetually ; and consequently, that the
strait line A d is equal to so many equal chords of
equal parts of the arch B M, as may be made by
infinite bisections. Wherefore the strait line A d
is equal to the arch B M or A N, that is, to half
the arch of the quadrant A B D or B C A.

Coroll. An arch being given not greater than

the arch of a quadrant (for being made greater, it comes again towards the radius B A produced, from which it receded before) if a strait line double to the chord of half the given arch be adapted from the beginning of the arch, and by how much the arch that is subtended by it is greater than the given arch, by so much a greater arch be subtended by another strait line, this strait line shall be equal to the first given arch.

Supposing the strait line BV (in fig. 1) be equal to the arch of the quadrant B H D, and A V be connected cutting the arch B H D in I, it may be asked what proportion the arch B I has to the arch I D. Let therefore the arch A Y be divided in the midst in o, and in the strait line A D let A p be taken equal, and A q double to the drawn chord A o. Then upon the centre A, with the radius A q, let an arch of a circle be drawn cutting the arch A Y in r, and let the arch Y r be doubled at t; which being done, the drawn strait line A t (by what has been last demonstrated) will be equal to the arch A Y. Again, upon the centre A with the radius A t let the arch $t u$ be drawn cutting A D in u; and the strait line A u will be equal to the arch A Y. From the point u let the strait line $u s$ be drawn equal and parallel to the strait line A B, cutting M N in x, and bisected by M N in the same point x. Therefore the strait line A x being drawn and produced till it meet with B C produced in V, it will cut off B V double to B s, that is, equal to the arch B H D. Now let the point, where the strait line A V cuts the arch B H D, be I; and let the arch D I be divided in the midst in y; and in the strait line D C, let D z

PART III.
20.

The second at-
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finding, &c.
be taken equal, and D δ double to the drawn chord
D y; and upon the centre D with the radius D δ
let an arch of a circle be drawn cutting the arch
B H D in the point n; and let the arch $n\,m$ be
taken equal to the arch I n; which being done,
the strait line D m will (by the last foregoing
corollary) be equal to the arch D I. If now the
strait lines D m and C V be equal, the arch B I
will be equal to the radius A B or B C; and con-
sequently X C being drawn, will pass through the
point I.   Moreover, if the semicircle B H D ϵ being
completed, the strait lines ϵ I and B I be drawn,
making a right angle (in the semicircle) at I, and
the arch B I be divided in the midst at i, it will
follow that A i being connected will be parallel to
the strait line ϵ I, and being produced to B C in k,
will cut off the strait line B k equal to the strait
line k I, and equal also to the strait line A γ cut
off in A D by the strait line ϵ I.   All which is
manifest, supposing the arch B I and the radius
B C to be equal.

But that the arch B I and the radius B C are
precisely equal, cannot (how true soever it be) be
demonstrated, unless that be first proved which is
contained in art. 1, namely, that the strait lines
drawn from X through the equal parts of O F
(produced to a certain length) cut off so many
parts also in the tangent B C severally equal to
the several arches cut off; which they do most
exactly as far as B C in the tangent, and B I in the
arch B E; insomuch that no inequality between
the arch B I and the radius B C can be discovered
either by the hand or by ratiocination.   It is
therefore to be further enquired, whether the

strait line A V cut the arch of the quadrant in I
in the same proportion as the point C divides the
strait line B V, which is equal to the arch of the
quadrant. But however this be, it has been de-
monstrated that the strait line B V is equal to the
arch B H D.

4. I shall now attempt the same dimension of a
circle another way, assuming the two following
lemmas.

Lemma I. If to the arch of a quadrant, and the
radius, there be taken in continual proportion a
third line Z; then the arch of the semiquadrant,
half the chord of the quadrant, and Z, will also be
in continual proportion.

For seeing the radius is a mean proportional
between the chord of a quadrant and its semi-
chord, and the same radius a mean proportional
between the arch of the quadrant and Z, the
square of the radius will be equal as well to the
rectangle made of the chord and semichord of the
quadrant, as to the rectangle made of the arch of
the quadrant and Z; and these two rectangles
will be equal to one another. Wherefore, as the
arch of a quadrant is to its chord, so reciprocally
is half the chord of the quadrant to Z. But as the
arch of the quadrant is to its chord, so is half the
arch of the quadrant to half the chord of the
quadrant. Wherefore, as half the arch of the
quadrant is to half the chord of the quadrant (or
to the sine of 45 degrees), so is half the chord of
the quadrant to Z; which was to be proved.

Lemma II. The radius, the arch of the semi-
quadrant, the sine of 45 degrees, and the semi-
radius, are proportional.

For seeing the sine of 45 degrees is a mean proportional between the radius and the semi-radius; and the same sine of 45 degrees is also a mean proportional (by the precedent lemma) between the arch of 45 degrees and Z; the square of the sine of 45 degrees will be equal as well to the rectangle made of the radius and semiradius, as to the rectangle made of the arch of 45 degrees and Z. Wherefore, as the radius is to the arch of 45 degrees, so reciprocally is Z to the semiradius; which was to be demonstrated.

Let now A B C D (in fig. 4) be a square; and with the radii A B, B C and D A, let the three quadrants A B D, B C A and D A C, be described; and let the strait lines E F and G H, drawn parallel to the sides B C and A B, divide the square A B C D into four equal squares. They will therefore cut the arch of the quadrant A B D into three equal parts in I and K, and the arch of the quadrant B C A into three equal parts in K and L. Also let the diagonals A C and B D be drawn, cutting the arches B I D and A L C in M and N. Then upon the centre H with the radius H F equal to half the chord of the arch B M D, or to the sine of 45 degrees, let the arch F O be drawn cutting the arch C K in O; and let A O be drawn and produced till it meet with B C produced in P; also let it cut the arch B M D in Q, and the strait line D C in R. If now the strait line H Q be equal to the strait line D R, and being produced to D C in S, cut off D S equal to half the strait line B P; I say then the strait line B P will be equal to the arch B M D.

For seeing P B A and A D R are like triangles,

it will be as P B to the radius B A or A D, so A D
to D R ; and therefore as well P B, A D and D R,
as P B, A D (or A Q) and Q H are in continual
proportion ; and producing H O to D C in T, DT
will be equal to the sine of 45 degrees, as shall by
and by be demonstrated.   Now D S, D T and D R
are in continual proportion by the first lemma ;
and by the second lemma D C. D S : : D R. D F are
proportionals.   And thus it will be, whether B P
be equal or not equal to the arch of the quadrant
B M D.   But if they be equal, it will then be, as
that part of the arch B M D which is equal to the
radius, is to the remainder of the same arch B M D;
so A Q to H Q, or so B C to C P.   And then will
B P and the arch B M D be equal.   But it is not
demonstrated that the strait lines H Q and D R
are equal ; though if from the point B there be
drawn (by the construction of fig. 1) a strait line
equal to the arch B M D, then D R to H Q, and
also the half of the strait line B P to D S, will
always be so equal, that no inequality can be dis-
covered between them.   I will therefore leave this
to be further searched into.   For though it be
almost out of doubt, that the strait line B P and
the arch B M D are equal, yet that may not be
received without demonstration ; and means of
demonstration the circular line admitteth none
that is not grounded upon the nature of flexion, or
of angles.   But by that way I have already exhi-
bited a strait line equal to the arch of a quadrant
in the first and second aggression.

It remains that I prove D T to be equal to the
sine of 45 degrees.

In B A produced let A V be taken equal to the sine of 45 degrees ; and drawing and producing V H, it will cut the arch of the quadrant C N A in the midst in N, and the same arch again in O, and the strait line D C in T, so that D T will be equal to the sine of 45 degrees, or to the strait line A V ; also the strait line V H will be equal to the strait line H I, or the sine of 60 degrees.

For the square of A V is equal to two squares of the semiradius ; and consequently the square of V H is equal to three squares of the semiradius. But H I is a mean proportional between the semiradius and three semiradii ; and, therefore, the square of H I is equal to three squares of the semiradius. Wherefore H I is equal to H V. But because A D is cut in the midst in H, therefore V H and H T are equal ; and, therefore, also D T is equal to the sine of 45 degrees. In the radius B A let B X be taken equal to the sine of 45 degrees ; for so V X will be equal to the radius ; and it will be as V A to A H the semiradius, so V X the radius to X N the sine of 45 degrees. Wherefore V H produced passes through N. Lastly, upon the centre V with the radius V A let the arch of a circle be drawn cutting V H in Y ; which being done, V Y will be equal to H O (for H O is, by construction, equal to the sine of 45 degrees) and Y H will be equal to O T ; and, therefore, V T passes through O. All which was to be demonstrated.

I will here add certain problems, of which if any analyst can make the construction, he will thereby be able to judge clearly of what I have now said concerning the dimension of a circle. Now

these problems are nothing else (at least to sense)
but certain symptoms accompanying the construc-
tion of the first and third figure of this chapter.

Describing, therefore, again, the square A B C D
(in fig. 5) and the three quadrants A B D, B C A
and D A C, let the diagonals A C and B D be drawn,
cutting the arches B H D and C I A in the middle
in H and I ; and the strait lines E F and G L, di-
viding the square A B C D into four equal squares,
and trisecting the arches B H D and C I A, namely,
B H D in K and M, and C I A in M and O.   Then
dividing the arch B K in the midst in P, let Q P
the sine of the arch B P, be drawn and produced to
R, so that Q R be double to Q P ; and, connecting
K R, let it be produced one way to B C in S, and
the other way to B A produced in T. Also let B V
be made triple to B S, and consequently, (by the
second article of this chapter) equal to the arch
B D.   This construction is the same with that of
the first figure, which I thought fit to renew dis-
charged of all lines but such as are necessary for my
present purpose.

In the first place, therefore, if A V be drawn,
cutting the arch B H D in X, and the side D C in
Z, I desire some analyst would, if he can, give a
reason why the strait lines T E and T C should cut
the arch B D, the one in Y, the other in X, so as
to make the arch B Y equal to the arch Y X ; or if
they be not equal, that he would determine their
difference.

Secondly, if in the side D A, the strait line D a
be taken equal to D Z, and V a be drawn ; why
V a and V B should be equal ; or if they be not
equal, what is the difference.

Thirdly, drawing $Z b$ parallel and equal to the side C B, cutting the arch B H D in c, and drawing the strait line A c, and producing it to B V in d; why A d should be equal and parallel to the strait line a V, and consequently equal also to the arch B D.

Fourthly, drawing e K the sine of the arch B K, and taking (in e A produced) ef equal to the diagonal A C, and connecting f C; why f C should pass through a (which point being given, the length of the arch B H D is also given) and c; and why fe and fc should be equal; or if not, why unequal.

Fifthly, drawing fZ, I desire he would show, why it is equal to B V, or to the arch B D; or if they be not equal, what is their difference.

Sixthly, granting fZ to be equal to the arch B D, I desire he would determine whether it fall all without the arch B C A, or cut the same, or touch it, and in what point.

Seventhly, the semicircle B D g being completed, why g I being drawn and produced, should pass through X, by which point X the length of the arch B D is determined. And the same g I being yet further produced to D C in h, why A d, which is equal to the arch B D, should pass through that point h.

Eighthly, upon the centre of the square A B C D, which let be k, the arch of the quadrant E i L being drawn, cutting e K produced in i, why the drawn strait line i X should be parallel to the side C D.

Ninthly, in the sides B A and B C taking $g l$ and B m severally equal to half B V, or to the arch B H, and drawing mn parallel and equal to the

side B A, cutting the arch B D in *o*, why the strait line which connects V *l* should pass through the point *o*.

Tenthly, I would know of him why the strait line which connects *a* H should be equal to B *m* ; or if not, how much it differs from it.

The analyst that can solve these problems without knowing first the length of the arch B D, or using any other known method than that which proceeds by perpetual bisection of an angle, or is drawn from the consideration of the nature of flexion, shall do more than ordinary geometry is able to perform.   But if the dimension of a circle cannot be found by any other method, then I have either found it, or it is not at all to be found.

From the known length of the arch of a quadrant, and from the proportional division of the arch and of the tangent B C, may be deduced the section of an angle into any given proportion ; as also the squaring of the circle, the squaring of a given sector, and many the like propositions, which it is not necessary here to demonstrate.   I will, therefore, only exhibit a strait line equal to the spiral of Archimides, and so dismiss this speculation.

5. The length of the perimeter of a circle being found, that strait line is also found, which touches a spiral at the end of its first conversion. For upon the centre A (in fig. 6) let the circle B C D E be described ; and in it let Archimedes' spiral A F G H B be drawn, beginning at A and ending at B. Through the centre A let the strait line C E be drawn, cutting the diameter B D at right angles ; and let it be produced to I, so that A I be equal to the perimeter B C D E B.   Therefore I B being drawn will touch

The equation of
the spiral of Ar-
chimedes with
a strait line.

The equation
of the spiral of
Archimedes
with á strait line

the spiral A F G H B in B; which is demonstrated by Archimedes in his book *De Spiralibus*.

And for a strait line equal to the given spiral A F G H B, it may be found thus.

Let the strait line A I, which is equal to the perimeter B C D E, be bisected in K; and taking K L equal to the radius A B, let the rectangle I L be completed. Let M L be understood to be the axis, and K L the base of a parabola, and let M K be the crooked line thereof. Now if the point M be conceived to be so moved by the concourse of two movents, the one from I M to K L with velocity encreasing continually in the same proportion with the times, the other from M L to I K uniformly, that both those motions begin together in M and end in K; Galilæus has demonstrated that by such motion of the point M, the crooked line of a parabola will be described. Again, if the point A be conceived to be moved uniformly in the strait line A B, and in the same time to be carried round upon the centre A by the circular motion of all the points between A and B; Archimedes has demonstrated that by such motion will be described a spiral line. And seeing the circles of all these motions are concentric in A ; and the interior circle is always less than the exterior in the proportion of the times in which A B is passed over with uniform motion ; the velocity also of the circular motion of the point A will continually increase proportionally to the times. And thus far the generations of the parabolical line M K, and of the spiral line A F G H B, are like. But the uniform motion in A B concurring with circular motion in the perimeters of all the concentric circles, describes that

circle, whose centre is A, and perimeter B C D E;
and, therefore, that circle is (by the coroll. of art.
1, chap. xvi) the aggregate of all the velocities to-
gether taken of the point A whilst it describes the
spiral A F G H B.   Also the rectangle I K L M is
the aggregate of all the velocities together taken
of the point M, whilst it describes the crooked line
M K.   And, therefore the whole velocity by which
the parabolical line M K is described, is to the
whole velocity with which the spiral line A F G H B
is described in the same time, as the rectangle
I K L M is to the circle B C D E, that is to the
triangle A I B.   But because A I is bisected in K,
and the strait lines I M and A B are equal, there-
fore the rectangle I K L M and the triangle A I B
are also equal.   Wherefore the spiral line AFGHB,
and the parabolical line M K, being described with
equal velocity and in equal times, are equal to one
another.   Now, in the first article of chap. xviii, a
strait line' is found out equal to any parabolical
line.   Wherefore also a strait line is found out equal
to a given spiral line of the first revolution described
by Archimedes ; which was to be done.

    6. In the sixth chapter, which is of *Method*,
that which I should there have spoken of the ana-
lytics of geometricians I thought fit to defer, be-
cause I could not there have been understood, as
not having then so much as named *lines, superfi-
cies, solids, equal* and *unequal, &c.*   Wherefore I
will in this place set down my thoughts concern-
ing it.

    Analysis is continual reasoning from the defini-
tions of the terms of a proposition we suppose
true, and again from the definitions of the terms of

Of the analysis of geometricians by the powers of lines.

those definitions, and so on, till we come to some things known, the composition whereof is the demonstration of the truth or falsity of the first supposition ; and this composition or demonstration is that we call *Synthesis.* *Analytica*, therefore, is that art, by which our reason proceeds from something supposed, to principles, that is, to prime propositions, or to such as are known by these, till we have so many known propositions as are sufficient for the demonstration of the truth or falsity of the thing supposed. *Synthetica* is the art itself of demonstration. Synthesis, therefore, and analysis, differ in nothing, but in proceeding forwards or backwards ; and *Logistica* comprehends both. So that in the analysis or synthesis of any question, that is to say, of any problem, the terms of all the propositions ought to be convertible ; or if they be enunciated hypothetically, the truth of the consequent ought not only to follow out of the truth of its antecedent, but contrarily also the truth of the antecedent must necessarily be inferred from the truth of the consequent. For otherwise, when by resolution we are arrived at principles, we cannot by composition return directly back to the thing sought for. For those terms which are the first in analysis, will be the last in synthesis ; as for example, when in *resolving*, we say, these two rectangles are equal, and therefore their sides are reciprocally proportional, we must necessarily in *compounding* say, the sides of these rectangles are reciprocally proportional, and therefore the rectangles themselves are equal ; which we could not say, unless *rectangles have their sides recipro-*

cally proportional, and *rectangles are equal*,
were terms convertible.

Now in every analysis, that which is sought is the proportion of two quantities; by which proportion, a figure being described, the quantity sought for may be exposed to sense. And this *exposition* is the end and solution of the question, or the construction of the problem.

And seeing analysis is reasoning from something supposed, till we come to principles, that is, to definitions, or to theorems formerly known; and seeing the same reasoning tends in the last place to some equation, we can therefore make no end of resolving, till we come at last to the causes themselves of equality and inequality, or to theorems formerly demonstrated from those causes; and so have a sufficient number of those theorems for the demonstration of the thing sought for.

And seeing also, that the end of the analytics is either the construction of such a problem as is possible, or the detection of the impossibility thereof; whensoever the problem may be solved, the analyst must not stay, till he come to those things which contain the efficient cause of that whereof he is to make construction. But he must of necessity stay, when he comes to prime propositions; and these are definitions. These definitions therefore must contain the efficient cause of his construction; I say of his construction, not of the conclusion which he demonstrates; for the cause of the conclusion is contained in the premised propositions; that is to say, the truth of the proposition he proves is drawn from the propositions which prove the same.

PART III.
20.

Of the analysis of geometricians by the powers of lines.

But the cause of his construction is in the things themselves, and consists in motion, or in the concourse of motions. Wherefore those propositions, in which analysis ends, are definitions, but such as signify in what manner the construction or generation of the thing proceeds. For otherwise, when he goes back by synthesis to the proof of his problem, he will come to no demonstration at all; there being no true demonstration but such as is scientifical; and no demonstration is scientifical, but that which proceeds from the knowledge of the causes from which the construction of the problem is drawn. To collect therefore what has been said into few words; ANALYSIS *is ratiocination from the supposed construction or generation of a thing to the efficient cause or coefficient causes of that which is constructed or generated. And* SYN-THESIS *is ratiocination from the first causes of the construction, continued through all the middle causes till we come to the thing itself which is constructed or generated.*

But because there are many means by which the same thing may be generated, or the same problem be constructed, therefore neither do all geometricians, nor doth the same geometrician always, use one and the same method. For, if to a certain quantity given, it be required to construct another quantity equal, there may be some that will inquire whether this may not be done by means of some motion. For there are quantities, whose equality and inequality may be argued from motion and time, as well as from congruence; and there is motion, by which two quantities, whether lines or superficies, though one of them be crooked, the

PART III.
20.
Of the analysis
of geometri-
cians by the
powers of lines.

other strait, may be made congruous or coincident. And this method Archimedes made use of in his book *De Spiralibus*. Also the equality or inequality of two quantities may be found out and demonstrated from the consideration of weight, as the same Archimedes did in his quadrature of the parabola. Besides, equality and inequality are found out often by the division of the two quantities into parts which are considered as indivisable; as Cavallerius Bonaventura has done in our time, and Archimedes often.   Lastly, the same is performed by the consideration of the powers of lines, or the roots of those powers, and by the multiplication, division, addition, and subtraction, as also by the extraction of the roots of those powers, or by finding where strait lines of the same proportion terminate.   For example, when any number of strait lines, how many soever, are drawn from a strait line and pass all through the same point, look what proportion they have, and if their parts continued from the point retain everywhere the same proportion, they shall all terminate in a strait line.   And the same happens if the point be taken between two circles. So that the places of all their points of termination make either strait lines, or circumferences of circles, and are called *plane places*.   So also when strait parallel lines are applied to one strait line, if the parts of the strait line to which they are applied be to one another in proportion duplicate to that of the contiguous applied lines, they will all terminate in a conical section; which section, being the place of their termination, is called a *solid place*, because it serves for the finding out of the quantity of any

PART III.
20.

Of the analysis
of geometri-
cians by the
powers of lines.

equation which consists of three dimensions. There are therefore three ways of finding out the cause of equality or inequality between two given quantities; namely, first, by the computation of *motions ;* for by equal motion, and equal time, equal spaces are described; and ponderation is motion. Secondly, by *indivisibles :* because all the parts together taken are equal to the whole. And thirdly, by the *powers :* for when they are equal, their roots also are equal; and contrarily, the powers are equal, when their roots are equal. But if the question be much complicated, there cannot by any of these ways be constituted a certain rule, from the supposition of which of the unknown quantities the analysis may best begin ; nor out of the variety of equations, that at first appear, which we were best to choose ; but the success will depend upon dexterity, upon formerly acquired science, and many times upon fortune.

For no man can ever be a good analyst without being first a good geometrician ; nor do the rules of analysis make a geometrician, as synthesis doth; which begins at the very elements, and proceeds by a logical use of the same. For the true teaching of geometry is by synthesis, according to Euclid's method ; and he that hath Euclid for his master, may be a geometrician without Vieta, though Vieta was a most admirable geometrician ; but he that has Vieta for his master, not so, without Euclid.

And as for that part of analysis which works by the powers, though it be esteemed by some geometricians, not the chiefest, to be the best way of solving all problems, yet it is a thing of no great extent ; it being all contained in the doctrine of

rectangles, and rectangled solids. So that although
they come to an equation which determines the
quantity sought, yet they cannot sometimes by
art exhibit that quantity in a plane, but in some
conic section; that is, as geometricians say, not
geometrically, but mechanically. Now such pro-
blems as these, they call *solid*; and when they
cannot exhibit the quantity sought for with the
help of a conic section, they call it a *lineary* pro-
blem. And therefore in the quantities of angles,
and of the arches of circles, there is no use at all
of the analytics which proceed by the powers; so
that the ancients pronounced it impossible to ex-
hibit in a plane the division of angles, except
bisection, and the bisection of the bisected parts,
otherwise than mechanically. For Pappus, (before
the 31st proposition of his fourth book) distin-
guishing and defining the several kinds of pro-
blems, says that "some are *plane*, others *solid*,
and others *lineary*. Those, therefore, which may
be solved by strait lines and the circumferences of
circles, (that is, which may be described with the
rule and compass, without any other instrument),
are fitly called *plane*; for the lines, by which
such problems are found out, have their generation
in a plane. But those which are solved by the
using of some one or more conic sections in their
construction, are called *solid*, because their con-
struction cannot be made without using the super-
ficies of solid figures, namely, of cones. There
remains the third kind, which is called *lineary*,
because other lines besides those already mentioned
are made use of in their construction, &c." And a

Of the analysis of geometri- cians by the powers of lines.

little after he says, " of this kind are the *spiral* lines, the *quadratrices*, the *conchoeides*, and the *cissoeides*. And geometricians think it no small fault, when for the finding out of a plane problem any man makes use of conics, or new lines." Now he ranks the trisection of an angle among solid problems, and the quinquesection among lineary. But what! are the ancient geometricians to be blamed, who made use of the quadratrix for the finding out of a strait line equal to the arch of a circle? And Pappus himself, was he faulty, when he found out the trisection of an angle by the help of an hyperbole? Or am I in the wrong, who think I have found out the construction of both these problems by the rule and compass only? Neither they, nor I. For the ancients made use of this analysis which proceeds by the powers; and with them it was a fault to do that by a more remote power, which might be done by a nearer ; as being an argument that they did not sufficiently understand the nature of the thing.

The virtue of this kind of analysis consists in the changing and turning and tossing of rectangles and analogisms; and the skill of analysts is mere logic, by which they are able methodically to find out whatsoever lies hid either in the subject or predicate of the conclusion sought for.   But this doth not properly belong to algebra, or the analytics specious, symbolical, or cossick ; which are, as I may say, the *brachygraphy* of the analytics, and an art neither of teaching nor learning geometry, but of registering with brevity and celerity the inventions of geometricians.   For though it be easy to discourse

by symbols of very remote propositions; yet
whether such discourse deserve to be thought very
profitable, when it is made without any ideas of
the things themselves, I know not.

CHAPTER XXI.

OF CIRCULAR MOTION.

1. In simple motion, every strait line taken in the body moved is so carried, that it is always parallel to the places in which it formerly was.—2. If circular motion be made about a resting centre, and in that circle there be an epicycle, whose revolution is made the contrary way, in such manner that in equal times it make equal angles, every strait line taken in that epicycle will be so carried, that it will always be parallel to the places in which it formerly was.—3. The properties of simple motion.—4. If a fluid be moved with simple circular motion, all the points taken in it will describe their circles in times proportional to the distances from the centre.—5. Simple motion dissipates heterogeneous and congregates homogeneous bodies.—6. If a circle made by a movent moved with simple motion be commensurable to another circle made by a point which is carried about by the same movent, all the points of both the circles will at some time return to the same situation. 7. If a sphere have simple motion, its motion will more dissipate heterogeneous bodies by how much it is more remote from the poles.—8. If the simple circular motion of a fluid body be hindered by a body which is not fluid, the fluid body will spread itself upon the superficies of that body.—9. Circular motion about a fixed centre casteth off by the tangent such things as lie upon the circumference and stick not to it. 10. Such things, as are moved with simple circular motion, beget simple circular motion.—11. If that which is so moved have one side hard and the other side fluid, its motion will not be perfectly circular.

1. I HAVE already defined *simple motion* to be
that, in which the several points taken in a moved

In simple
motion, every
strait line
taken in the
body moved
is so carried,
that it is always
parallel to
the places
in which it
formerly was.

body do in several equal times describe several equal arches. And therefore in simple circular motion it is necessary that every strait line taken in the moved body be always carried parallel to itself; which I thus demonstrate.

First, let A B (in the first figure) be any strait line taken in any solid body; and let A D be any arch drawn upon any centre C and radius C A. Let the point B be understood to describe towards the same parts the arch B E, like and equal to the arch A D. Now in the same time in which the point A transmits the arch A D, the point B, which by reason of its simple motion is supposed to be carried with a velocity equal to that of A, will transmit the arch B E; and at the end of the same time the whole A B will be in D E; and therefore A B and D E are equal. And seeing the arches AD and BE are like and equal, their subtending strait lines AD and BE will also be equal; and therefore the four-sided figure A B D E will be a parallelogram. Wherefore A B is carried parallel to itself. And the same may be proved by the same method, if any other strait line be taken in the same moved body in which the strait line A B was taken. So that all strait lines, taken in a body moved with simple circular motion, will be carried parallel to themselves.

Coroll. I. It is manifest that the same will also happen in any body which hath simple motion, though not circular. For all the points of any strait line whatsoever will describe lines, though not circular, yet equal; so that though the crooked lines A D and B E were not arches of circles, but of parabolas, ellipses, or of any other figures,

yet both they, and their subtenses, and the strait
lines which join them, would be equal and parallel.

Coroll. II. It is also manifest, that the radii of the equal circles A D and B E, or the axis of a sphere, will be so carried, as to be always parallel to the places in which they formerly were. For the strait line B F drawn to the centre of the arch B E being equal to the radius A C, will also be equal to the strait line F E or C D ; and the angle B F E will be equal to the angle A C D. Now the intersection of the strait lines C A and B E being at G, the angle C G E (seeing B E and A D are parallel) will be equal to the angle D A C. But the angle E B F is equal to the same angle D A C ; and therefore the angles C G E and E B F are also equal. Wherefore A C and B F are parallel ; which was to be demonstrated.

2. Let there be a circle given (in the second figure) whose centre is A, and radius A B ; and upon the centre B and any radius B C let the epicycle C D E be described. Let the centre B be understood to be carried about the centre A, and the whole epicycle with it till it be coincident with the circle F G H, whose centre is I ; and let B A I be any angle given. But in the time that the centre B is moved to I, let the epicycle C D E have a contrary revolution upon its own centre, namely from E by D to C, according to the same proportions ; that is, in such manner, that in both the circles, equal angles be made in equal times. I say E C, the axis of the epicycle, will be always carried parallel to itself. Let the angle F I G be made equal to the angle B A I ; I F and A B will

therefore be parallel; and how much the axis A G has departed from its former place A C (the measure of which progression is the angle C A G, or C B D, which I suppose equal to it) so much in the same time has the axis I G, the same with B C, departed from its own former situation. Wherefore, in what time B C comes to I G by the motion from B to I upon the centre A, in the same time G will come to F by the contrary motion of the epicycle; that is, it will be turned backwards to F, and I G will lie in I F. But the angles F I G and G A C are equal; and therefore A C, that is, B C, and I F, (that is the axis, though in different places) will be parallel. Wherefore, the axis of the epicycle E D C will be carried always parallel to itself; which was to be proved.

Coroll. From hence it is manifest, that those two annual motions which Copernicus ascribes to the earth, are reducible to this one circular simple motion, by which all the points of the moved body are carried always with equal velocity, that is, in equal times they make equal revolutions uniformly.

This, as it is the most simple, so it is the most frequent of all circular motions; being the same which is used by all men when they turn anything round with their arms, as they do in grinding or sifting. For all the points of the thing moved describe lines which are like and equal to one another. So that if a man had a ruler, in which many pens' points of equal length were fastened, he might with this one motion write many lines at once.

3. Having shown what simple motion is, I will
here also set down some properties of the same.

First, when a body is moved with simple motion
in a fluid medium which hath no vacuity, it changes
the situation of all the parts of the fluid ambient
which resist its motion; I say there are no parts
so small of the fluid ambient, how far soever it be
continued, but do change their situation in such
manner, as that they leave their places continually
to other small parts that come into the same.

For (in the same second figure) let any body,
as K L M N, be understood to be moved with
simple circular motion; and let the circle, which
every point thereof describes, have any deter-
mined quantity, suppose that of the same K L M N.
Wherefore the centre A and every other point,
and consequently the moved body itself, will be
carried sometimes towards the side where is K,
and sometimes towards the other side where
is M. When therefore it is carried to K, the
parts of the fluid medium on that side will go
back; and, supposing all space to be full, others
on the other side will succeed. And so it will be
when the body is carried to the side M, and to N,
and every way. Now when the nearest parts of
the fluid medium go back, it is necessary that the
parts next to those nearest parts go back also;
and supposing still all space to be full, other parts
will come into their places with succession perpe-
tual and infinite. Wherefore all, even the least
parts of the fluid medium, change their places, &c.
Which was to be proved.

It is evident from hence, that simple motion,
whether circular or not circular, of bodies which

PART III.
21.

make perpetual returns to their former places, hath greater or less force to dissipate the parts of resisting bodies, as it is more or less swift; and as the lines described have greater or less magnitude. Now the greatest velocity that can be, may be understood to be in the least circuit, and the least in the greatest; and may be so supposed, when there is need.

If a fluid be moved with simple circular motion, all the points taken in it will describe their circles in times proportional to the distances from the centre.

4. Secondly, supposing the same simple motion in the air, water, or other fluid medium; the parts of the medium, which adhere to the moved body, will be carried about with the same motion and velocity, so that in what time soever any point of the movent finishes its circle, in the same time every part of the medium, which adheres to the movent, shall also describe such a part of its circle, as is equal to the whole circle of the movent; I say, it shall describe a part, and not the whole circle, because all its parts receive their motion from an interior concentric movent, and of concentric circles the exterior are always greater than the interior; nor can the motion imprinted by any movent be of greater velocity than that of the movent itself. From whence it follows, that the more remote parts of the fluid ambient shall finish their circles in times, which have to one another the same proportion with their distances from the movent. For every point of the fluid ambient, as long as it toucheth the body which carries it about, is carried about with it, and would make the same circle, but that it is left behind so much as the exterior circle exceeds the interior. So that if we suppose some thing, which is not fluid, to float in that part of the fluid ambient which is

nearest to the movent, it will together with the movent be carried about. Now that part of the fluid ambient, which is not the nearest but almost the nearest, receiving its degree of velocity from the nearest, which degree cannot be greater than it was in the giver, doth therefore in the same time make a circular line, not a whole circle, yet equal to the whole circle of the nearest. Therefore in the same time that the movent describes its circle, that which doth not touch it shall not describe its circle; yet it shall describe such a part of it, as is equal to the whole circle of the movent. And after the same manner, the more remote parts of the ambient will describe in the same time such parts of their circles, as shall be severally equal to the whole circle of the movent; and, by consequent, they shall finish their whole circles in times proportional to their distances from the movent; which was to be proved.

5. Thirdly, the same simple motion of a body placed in a fluid medium, congregates or gathers into one place such things as naturally float in that medium, if they be homogeneous; and if they be heterogeneous, it separates and dissipates them. But if such things as be heterogeneous do not float, but settle, then the same motion stirs and mingles them disorderly together. For seeing bodies, which are unlike to one another, that is, heterogeneous bodies, are not unlike in that they are bodies; for bodies, as bodies, have no difference; but only from some special cause, that is, from some internal motion, or motions of their smallest parts (for I have shown in chap. IX, art. 9, that all mutation is such motion), it remains that

Simple motion dissipates heterogeneous and congregates homogeneous bodies.

PART III.
21.

Simple motion dissipates heterogeneous, &c.

heterogeneous bodies have their unlikeness or difference from one another from their internal or specifical motions. Now bodies which have such difference receive unlike and different motions from the same external common movent ; and therefore they will not be moved together, that is to say, they will be dissipated. And being dissipated they will necessarily at some time or other meet with bodies like themselves, and be moved alike and together with them ; and afterwards meeting with more bodies like themselves, they will unite and become greater bodies. Wherefore homogeneous bodies are congregated, and heterogeneous dissipated by simple motion in a medium where they naturally float. Again, such as being in a fluid medium do not float, but sink, if the motion of the fluid medium be strong enough, will be stirred up and carried away by that motion, and consequently they will be hindered from returning to that place to which they sink naturally, and in which only they would unite, and out of which they are promiscuously carried ; that is, they are disorderly mingled.

Now this motion, by which homogeneous bodies are congregated and heterogeneous are scattered, is that which is commonly called *fermentation,* from the Latin *fervere ;* as the Greeks have their Ζύμη, which signifies the same, from Ζέω *ferveo.* For seething makes all the parts of the water change their places ; and the parts of any thing, that is thrown into it, will go several ways according to their several natures. And yet all *fervour* or seething is not caused by fire ; for new wine and many other things have also their fer-

mentation and fervour, to which fire contributes little, and sometimes nothing. But when in fermentation we find heat, it is made by the fermentation.

6. Fourthly, in what time soever the movent, whose centre is A (in fig. 2) moved in K L N, shall, by any number of revolutions, that is, when the perimeters B I and K L N be commensurable, have described a line equal to the circle which passes through the points B and I; in the same time all the points of the floating body, whose centre is B, shall return to have the same situation in respect of the movent, from which they departed. For seeing it is as the distance B A, that is, as the radius of the circle which passes through B I is to the perimeter itself B I, so the radius of the circle K L N is to the perimeter K L N; and seeing the velocities of the points B and K are equal, the time also of the revolution in I B to the time of one revolution in K L N, will be as the perimeter B I to the perimeter K L N; and therefore so many revolutions in K L N, as together taken are equal to the perimeter B I, will be finished in the same time in which the whole perimeter B I is finished; and therefore also the points L, N, F and H, or any of the rest, will in the same time return to the same situation from which they departed; and this may be demonstrated, whatsoever be the points considered. Wherefore all the points shall in that time return to the same situation; which was to be proved.

From hence it follows, that if the perimeters B I and L K N be not commensurable, then all the

If a circle made by a movent moved with simple motion be commensurable to another circle made by a point which is carried about by the same movent, all the points of both the circles will at some time return to the same situation.

points will never return to have the same situation or configuration in respect of one another.

7. In simple motion, if the body moved be of a spherical figure, it hath less force towards its poles than towards its middle to dissipate heterogeneous, or to congregate homogeneous bodies.

Let there be a sphere (as in the third figure) whose centre is A and diameter B C; and let it be conceived to be moved with simple circular motion; of which motion let the axis be the strait line D E, cutting the diameter B C at right angles in A. Let now the circle, which is described by any point B of the sphere, have B F for its diameter; and taking F G equal to B C, and dividing it in the middle in H, the centre of the sphere A will, when half a revolution is finished, lie in H. And seeing H F and A B are equal, a circle described upon the centre H with the radius H F or H G, will be equal to the circle whose centre is A and radius A B. And if the same motion be continued, the point B will at the end of another half revolution return to the place from whence it began to be moved; and therefore at the end of half a revolution, the point B will be carried to F, and the whole hemisphere D B E into that hemisphere in which are the points L, K and F. Wherefore that part of the fluid medium, which is contiguous to the point F, will in the same time go back the length of the strait line B F; and in the return of the point F to B, that is, of G to C, the fluid medium will go back as much in a strait line from the point C. And this is the effect of simple motion in the middle of the sphere, where the distance from the poles is greatest. Let now the point I be taken in the same sphere nearer to

the pole E, and through it let the strait line I K be drawn parallel to the strait line B F, cutting the arch F L in K, and the axis H L in M ; then connecting H K, upon H F let the perpendicular K N be drawn. In the same time therefore that B comes to F the point I will come to K, B F and I K being equal and described with the same velocity. Now the motion in I K to the fluid medium upon which it works, namely, to that part of the medium which is contiguous to the point K, is oblique, whereas if it proceeded in the strait line H K it would be perpendicular ; and therefore the motion which proceeds in I K has less power than that which proceeds in H K with the same velocity. But the motions in H K and H F do equally thrust back the medium ; and therefore the part of the sphere at K moves the medium less than the part at F, namely, so much less as K N is less than H F. Wherefore also the same motion hath less power to disperse heterogeneous, and to congregate homogeneous bodies, when it is nearer, than when it is more remote from the poles ; which was to be proved.

Coroll. It is also necessary, that in planes which are perpendicular to the axis, and more remote than the pole itself from the middle of the sphere, this simple motion have no effect. For the axis D E with simple motion describes the superficies of a cylinder ; and towards the bases of the cylinder there is in this motion no endeavour at all.

8. If in a fluid medium moved about, as hath been said, with simple motion, there be conceived to float some other spherical body which is not fluid, the parts of the medium, which are stopped by that

If a simple
circular motion
of a fluid body
be hindered by
a body which is
not fluid, the

PART III.
21.

fluid body will spread itself upon the superficies of that body.

body, will endeavour to spread themselves every way upon the superficies of it. And this is manifest enough by experience, namely, by the spreading of water poured out upon a pavement. But the reason of it may be this. Seeing the sphere A (in fig. 3) is moved towards B, the medium also in which it is moved will have the same motion. But because in this motion it falls upon a body not liquid, as G, so that it cannot go on ; and seeing the small parts of the medium cannot go forwards, nor can they go directly backwards against the force of the movent; it remains, therefore, that they diffuse themselves upon the superficies of that body, as towards O and P; which was to be proved.

Circular motion about a fixed centre casteth off by the tangent such things as lie upon the circumference & stick not to it.

9. Compounded circular motion, in which all the parts of the moved body do at once describe circumferences, some greater, others less, according to the proportion of their several distances from the common centre, carries about with it such bodies, as being not fluid, adhere to the body so moved; and such as do not adhere, it casteth forwards in a strait line which is a tangent to the point from which they are cast off.

For let there be a circle whose radius is A B (in fig. 4) ; and let a body be placed in the circumference in B, which if it be fixed there, will necessarily be carried about with it, as is manifest of itself. But whilst the motion proceeds, let us suppose that body to be unfixed in B. I say, the body will continue its motion in the tangent B C. For let both the radius A B and the sphere B be conceived to consist of hard matter; and let us suppose the radius A B to be stricken in the point B

by some other body which falls upon it in the tangent D B. Now, therefore, there will be a motion made by the concourse of two things, the one, endeavour towards C in the strait line D B produced, in which the body B would proceed, if it were not retained by the radius A B; the other, the retention itself. But the retention alone causeth no endeavour towards the centre; and, therefore, the retention being taken away, which is done by the unfixing of B, there will remain but one endeavour in B, namely, that in the tangent B C. Wherefore the motion of the body B unfixed will proceed in the tangent B C; which was to be proved.

By this demonstration it is manifest, that circular motion about an unmoved axis shakes off and puts further from the centre of its motion such things as touch, but do not stick fast to its superficies; and the more, by how much the distance is greater from the poles of the circular motion; and so much the more also, by how much the things, that are shaken off, are less driven towards the centre by the fluid ambient, for other causes.

10. If in a fluid medium a spherical body be moved with simple circular motion, and in the same medium there float another sphere whose matter is not fluid, this sphere also shall be moved with simple circular motion.

Let B C D (in fig. 5) be a circle, whose centre is A, and in whose circumference there is a sphere so moved, that it describes with simple motion the the perimeter B C D. Let also E F G be another sphere of consistent matter, whose semidiameter is E H, and centre H; and with the radius A H let the circle H I be described. I say, the sphere

E F G will, by the motion of the body in B C D, be moved in the circumference H I with simple motion.

For seeing the motion in B C D (by art. 4 of this chapter) makes all the points of the fluid medium describe in the same time circular lines equal to one another, the points E, H and G of the strait line E H G will in the same time describe with equal radii equal circles. Let E B be drawn equal and parallel to the strait line A H ; and let A B be connected, which will therefore be equal and parallel to E H ; and therefore also, if upon the centre B and radius B E the arch E K be drawn equal to the arch H I, and the strait lines A I, B K and I K be drawn, B K and A I will be equal; and they will also be parallel, because the two arches E K and H I, that is, the two angles K B E and I A H are equal; and, consequently, the strait lines A B and K I, which connect them, will also be equal and parallel. Wherefore K I and E H are parallel. Seeing, therefore, E and H are carried in the same time to K and I, the whole strait line I K will be parallel to E H, from whence it departed. And, therefore, seeing the sphere E F G is supposed to be of consistent matter, so as all its points keep always the same situation, it is necessary that every other strait line, taken in the same sphere, be carried always parallel to the places in which it formerly was. Wherefore the sphere E F G is moved with simple circular motion ; which was to be demonstrated.

If that which is
so moved have
one side hard
and the other
11. If in a fluid medium, whose parts are stirred by a body moved with simple motion, there float another body, which hath its superficies either

wholly hard, or wholly fluid, the parts of this body
shall approach the centre equally on all sides ; that
is to say, the motion of the body shall be circular,
and concentric with the motion of the movent.
But if it have one side hard, and the other side
fluid, then both those motions shall not have the
same centre, nor shall the floating body be moved
in the circumference of a perfect circle.

Let a body be moved in the circumference of the
circle K L M N (in fig 2.) whose centre is A. And
let there be another body at I, whose superficies is
either all hard or all fluid. Also let the medium, in
which both these bodies are placed, be fluid. I
say, the body at I will be moved in the circle I B
about the centre A. For this has been demonstrated
in the last article.

Wherefore let the superficies of the body at I be
fluid on one side, and hard on the other. And
first, let the fluid side be towards the centre. See-
ing, therefore, the motion of the medium is such,
as that its parts do continually change their places,
(as hath been shown in art 5) ; if this change of
place be considered in those parts of the medium
which are contiguous to the fluid superficies, it must
needs be that the small parts of that superficies
enter into the places of the small parts of the me-
dium which are contiguous to them ; and the like
change of place will be made with the next conti-
guous parts towards A. And if the fluid parts of
the body at I have any degree at all of tenacity (for
there are degrees of tenacity, as in the air and
water) the whole fluid side will be lifted up a little,
but so much the less, as its parts have less tena-
city ; whereas the hard part of the superficies.

PART III.
21.
If that which is
so moved, &c.
which is contiguous to the fluid part, has no cause at all of elevation, that is to say, no endeavour towards A.

Secondly, let the hard superficies of the body at I be towards A. By reason, therefore, of the said change of place of the parts which are contiguous to it, the hard superficies must, of necessity, seeing by supposition there is no empty space, either come nearer to A, or else its smallest parts must supply the contiguous places of the medium, which otherwise would be empty. But this cannot be, by reason of the supposed hardness; and, therefore, the other must needs be, namely, that the body come nearer to A. Wherefore the body at I has greater endeavour towards the centre A, when its hard side is next it, than when it is averted from it. But the body in I, while it is moving in the circumference of the circle I B, has sometimes one side, sometimes another, turned towards the centre; and, therefore, it is sometimes nearer, sometimes further off from the centre A. Wherefore the body at I is not carried in the circumference of a perfect circle; which was to be demonstrated.

CHAPTER XXII.

OF OTHER VARIETY OF MOTION.

1. I HAVE already (chapter xv. art. 2) defined endeavour to be motion through some length, though not considered as length, but as a point. Whether, therefore, there be resistance or no resistance, the endeavour will still be the same. For simply to endeavour is to go. But when two bodies, having opposite endeavours, press one another, then the endeavour of either of them is that which we call pressure, and is mutual when their pressures are opposite.

Two kinds of mediums in which bodies are moved.

2. Bodies moved, and also the mediums in which they are moved, are of two kinds. For either they have their parts coherent in such manner, as no part of the moved body will easily yield to the movent, except the whole body yield also, and such are the things we call *hard:* or else their parts, while the whole remains unmoved, will easily yield to the movent, and these we call *fluid* or *soft* bodies. For the words *fluid, soft, tough,* and *hard,* in the same manner as *great* and *little,* are used only comparatively; and are not different kinds, but different degrees of quality.

Propagation of motion, what it is.

3. To *do,* and to *suffer,* is to move and to be moved; and nothing is moved but by that which toucheth it and is also moved, as has been formerly shown. And how great soever the distance be, we say the first movent moveth the last moved body, but mediately; namely so, as that the first moveth the second, the second the third, and so on, till the last of all be touched. When therefore one body, having opposite endeavour to another body, moveth the same, and that moveth a third, and so on, I call that action *propagation of motion.*

What motion bodies have when they press one another.

4. When two fluid bodies, which are in a free and open space, press one another, their parts will endeavour, or be moved, towards the sides; not only those parts which are there where the mutual contact is, but all the other parts. For in the first contact, the parts, which are pressed by both the endeavouring bodies, have no place either forwards or backwards in which they can be moved; and therefore they are pressed out towards the sides. And this expressure, when the forces are equal, is

in a line perpendicular to the bodies pressing. But whensoever the foremost parts of both the bodies are pressed, the hindermost also must be pressed at the same time ; for the motion of the hindermost parts cannot in an instant be stopped by the resistance of the foremost parts, but proceeds for some time ; and therefore, seeing they must have some place in which they may be moved, and that there is no place at all for them forwards, it is necessary that they be moved into the places which are towards the sides every way. And this effect follows of necessity, not only in fluid, but in consistent and hard bodies, though it be not always manifest to sense. For though from the compression of two stones we cannot with our eyes discern any swelling outwards towards the sides, as we perceive in two bodies of wax ; yet we know well enough by reason, that some tumour must needs be there, though it be but little.

5. But when the space is enclosed, and both the bodies be fluid, they will, if they be pressed together, penetrate one another, though differently, according to their different endeavours. For suppose a hollow cylinder of hard matter, well stopped at both ends, but filled first, below with some heavy fluid body, as quicksilver, and above with water or air. If now the bottom of the cylinder be turned upwards, the heaviest fluid body, which is now at the top, having the greatest endeavour downwards, and being by the hard sides of the vessel hindered from extending itself sideways, must of necessity either be received by the lighter body, that it may sink through it, or else it must open a passage through itself, by

which the lighter body may ascend. For of the two bodies, that, whose parts are most easily separated, will be the first divided ; which being done, it is not necessary that the parts of the other suffer any separation at all. And therefore when two liquors, which are enclosed in the same vessel, change their places, there is no need that their smallest parts should be mingled with one another; for a way being opened through one of them, the parts of the other need not be separated.

Now if a fluid body, which is not enclosed, press a hard body, its endeavour will indeed be towards the internal parts of that hard body ; but being excluded by the resistance of it, the parts of the fluid body will be moved every way according to the superficies of the hard body, and that equally, if the pressure be perpendicular ; for when all the parts of the cause are equal, the effects will be equal also. But if the pressure be not perpendicular, then the angles of the incidence being unequal, the expansion also will be unequal, namely, greater on that side where the angle is greater, because that motion is most direct which proceeds by the directest line.

When one body
presseth an-
other and doth
not penetrate
it, the action of
the pressing
body is perpen-
dicular to the
superficies of
the body
pressed.

6. If a body, pressing another body, do not penetrate it, it will nevertheless give to the part it presseth an endeavour to yield, and recede in a strait line perpendicular to its superficies in that point in which it is pressed.

Let A B C D (in fig. 1) be a hard body, and let another body, falling upon it in the strait line E A, with any inclination or without inclination, press it in the point A. I say the body so pressing, and not penetrating it, will give to the part A an

endeavour to yield or recede in a strait line per-
pendicular to the line A D.

For let A B be perpendicular to A D, and let
B A be produced to F.   If therefore A F be coin-
cident with A E, it is of itself manifest that the
motion in E A will make A to endeavour in the
line A B.   Let now E A be oblique to A D, and
from the point E let the strait line E C be drawn,
cutting A D at right angles in D, and let the
rectangles A B C D and A D E F be completed.   I
have shown (in the 8th article of chapter xvi)
that the body will be carried from E to A by the
concourse of two uniform motions, the one in E F
and its parallels, the other in E D and its parallels.
But the motion in E F and its parallels, whereof
D A is one, contributes nothing to the body in A
to make it endeavour or press towards B; and
therefore the whole endeavour, which the body
hath in the inclined line E A to pass or press the
strait line A D, it hath it all from the perpendicular
motion or endeavour in F A.   Wherefore the body
E, after it is in A, will have only that perpendicu-
lar endeavour which proceeds from the motion in
F A, that is, in A B; which was to be proved.

7. If a hard body falling upon or pressing an-
other body penetrate the same, its endeavour
after its first penetration will be neither in the
inclined line produced, nor in the perpendicular,
but sometimes betwixt both, sometimes without
them.

Let E A G (in the same fig. 1) be the inclined
line produced; and first, let the passage through
the medium, in which E A is, be easier than the
passage through the medium in which A G is. As

soon therefore as the body is within the medium in which is A G, it will find greater resistance to its motion in D A and its parallels, than it did whilst it was above A D ; and therefore below A D it will proceed with slower motion in the parallels of D A, than above it. Wherefore the motion which is compounded of the two motions in E F and E D will be slower below A D than above it ; and therefore also, the body will not proceed from A in E A produced, but below it. Seeing, therefore, the endeavour in A B is generated by the endeavour in F A ; if to the endeavour in F A there be added the endeavour in D A, which is not all taken away by the immersion of the point A into the lower medium, the body will not proceed from A in the perpendicular A B, but beyond it ; namely, in some strait line between A B and A G, as in the line A H.

Secondly, let the passage through the medium E A be less easy than that through A G. The motion, therefore, which is made by the concourse of the motions in E F and F B, is slower above A D than below it ; and consequently, the endeavour will not proceed from A in E A produced, but beyond it, as in A I. Wherefore, if a hard body falling, &c. ; which was to be proved.

This divergency of the strait line A H from the strait line A G is that which, the writers of optics commonly called *refraction*, which, when the passage is easier in the first than in the second medium, is made by diverging from the line of inclination towards the perpendicular ; and contrarily, when the passage is not so easy in the

first medium, by departing further from the per-
pendicular.

8. By the 6th theorem it is manifest, that the
force of the movent may be so placed, as that
the body moved by it may proceed in a way almost
directly contrary to that of the movent, as we see
in the motion of ships.

For let A B (in fig. 2) represent a ship, whose
length from the prow to the poop is A B, and let
the wind lie upon it in the strait parallel lines C B,
D E and F G; and let D E and F G be cut in E and
and G by a strait line drawn from B perpendicular
to A B; also let B E and E G be equal, and the
angle A B C any angle how small soever.   Then
between B C and B A let the strait line B I be
drawn; and let the sail be conceived to be spread
in the same line B I, and the wind to fall upon it
in the points L, M and B; from which points, per-
pendicular to B I, let B K, M Q and L P be drawn.
Lastly, let E N and G O be drawn perpendicular to
B G, and cutting B K in H and K; and let H N
and K O be made equal to one another, and seve-
rally equal to B A.   I say, the ship B A, by the
wind falling upon it in C B, D E, F G, and other
lines parallel to them, will be carried forwards
almost opposite to the wind, that is to say, in a
way almost contrary to the way of the movent.

For the wind that blows in the line C B will (as
hath been shown in art. 6.) give to the point B an en-
deavour to proceed in a strait line perpendicular to
the strait line B I, that is, in the strait line B K;
and to the points M and L an endeavour to pro-
ceed in the strait lines M Q and L P, which are
parallel to B K.   Let now the measure of the time

z 2

PART III.
22.

Motion some-
times opposite
to that of the
movent.
be B G, which is divided in the middle in E ; and
let the point B be carried to H in the time B E.
In the same time, therefore, by the wind blowing
in D M and F L, and as many other lines as may
be drawn parallel to them, the whole ship will be
applied to the strait line H N.   Also at the end of
the second time E G, it will be applied to the strait
line K O.   Wherefore the ship will always go for-
ward ; and the angle it makes with the wind will
be equal to the angle A B C, how small soever that
angle be; and the way it makes will in every time be
equal to the strait line E H.   I say, thus it would
be, if the ship might be moved with as great
celerity sideways from B A towards K O, as it may
be moved forwards in the line B A.   But this is
impossible, by reason of the resistance made by the
great quantity of water which presseth the side, much
exceeding the resistance made by the much smaller
quantity which presseth the prow of the ship ; so
that the way the ship makes sideways is scarce
sensible ; and, therefore, the point B will proceed
almost in the very line B A, making with the wind
the angle A B C, how acute soever; that is to say,
it will proceed almost in the strait line B C, that
is, in a way almost contrary to the way of the
movent ; which was to be demonstrated.

But the sail in B I must be so stretched as that .
there be left in it no bosom at all ; for otherwise
the strait lines L P, M Q and B K will not be per-
pendicular to the plane of the sail, but falling below
P, Q and K, will drive the ship backwards.   But
by making use of a small board for a sail, a little
waggon with wheels for the ship, and of a smooth
pavement for the sea, I have by experience found

this to be so true, that I could scarce oppose the board to the wind in any obliquity, though never so small, but the waggon was carried forwards by it.

By the same 6th theorem it may be found, how much a stroke, which falls obliquely, is weaker than a stroke falling perpendicularly, they being like and equal in all other respects.

Let a stroke fall upon the wall A B obliquely, as for example, in the strait line C A (in fig. 3.) Let C E be drawn parallel to A B, and D A perpendicular to the same A B and equal to C A ; and let both the velocity and time of the motion in C A be equal to the velocity and time of the motion in D A. I say, the stroke in C A will be weaker than that in D A, in the proportion of E A to D A. For producing D A howsoever to F, the endeavour of both the strokes will (by art. 6) proceed from A in the perpendicular A F. But the stroke in C A is made by the concourse of two motions in C E and E A, of which that in C E contributes nothing to the stroke in A, because C E and B A are parallels ; and, therefore, the stroke in C A is made by the motion which is in E A only. But the velocity or force of the perpendicular stroke in E A, to the velocity or force of the stroke in D A, is as E A to D A. Wherefore the oblique stroke in C A is weaker than the perpendicular stroke in D A, in the proportion of E A to D A or C A ; which was to be proved.

9. In a full medium, all endeavour proceeds as far as the medium itself reacheth; that is to say, if the medium be infinite, the endeavour will proceed infinitely.

In a full medium, motion is propagated to any distance.

For whatsoever endeavoureth is moved, and therefore whatsoever standeth in its way it maketh it yield, at least a little, namely, so far as the movent itself is moved forwards. But that which yieldeth is also moved, and consequently maketh that to yield which is in its way, and so on successively as long as the medium is full; that is to say, infinitely, if the full medium be infinite; which was to be proved.

Now although endeavour thus perpetually propagated do not always appear to the senses as motion, yet it appears as action, or as the efficient cause of some mutation. For if there be placed before our eyes some very little object, as for example, a small grain of sand, which at a certain distance is visible; it is manifest that it may be removed to such a distance as not to be any longer seen, though by its action it still work upon the organs of sight, as is manifest from that which was last proved, that all endeavour proceeds infinitely. Let it be conceived therefore to be removed from our eyes to any distance how great soever, and a sufficient number of other grains of sand of the same bigness added to it; it is evident that the aggregate of all those sands will be visible; and though none of them can be seen when it is single and severed from the rest, yet the whole heap or hill which they make will manifestly appear to the sight; which would be impossible, if some action did not proceed from each several part of the whole heap.

Dilatation & contraction what they are.

10 Between the degrees of hard and soft are those things which we call *tough*, tough being that which may be bent without being altered from

what it was; and the bending of a line is either
the adduction or diduction of the extreme parts,
that is, a motion from straitness to crookedness,
or contrarily, whilst the line remains still the same
it was; for by drawing out the extreme points of
a line to their greatest distance, the line is made
strait, which otherwise is crooked. So also the
bending of a superficies is the diduction or adduc-
tion of its extreme lines, that is, their dilatation and
contraction.

11. *Dilatation* and *contraction*, as also all *flexion*, Dilatation
supposes necessarily that the internal parts of the & contraction
body bowed do either come nearer to the external suppose mu-
tation of the
smallest parts
parts, or go further from them. For though *flexion* in respect of
their situation.
be considered only in the length of a body, yet
when that body is bowed, the line which is made
on one side will be convex, and the line on the
other side will be concave; of which the concave,
being the interior line, will, unless something be
taken from it and added to the convex line, be the
more crooked, that is, the greater of the two.
But they are equal; and, therefore, in flexion there
is an accession made from the interior to the ex-
terior parts; and, on the contrary, in tension, from
the exterior to the interior parts. And as for those
things which do not easily suffer such transposition
of their parts, they are called brittle; and the
great force they require to make them yield, makes
them also with sudden motion to leap asunder, and
break in pieces.

12. Also motion is distinguished into *pulsion* All traction
and *traction*. And pulsion, as I have already de- is pulsion.
fined it, is when that which is moved goes before
that which moveth it. But contrarily, in traction

the movent goes before that which is moved. Nevertheless, considering it with greater attention, it seemeth to be the same with pulsion.   For of two parts of a hard body, when that which is foremost drives before it the medium in which the motion is made, at the same time that which is thrust forwards thrusteth the next, and this again the next, and so on successively.   In which action, if we suppose that there is no place void, it must needs be, that by continual pulsion, namely, when that action has gone round, the movent will be behind that part, which at the first seemed not to be thrust forwards, but to be drawn ; so that now the body, which was drawn, goes before the body which gives it motion ; and its motion is no longer traction, but pulsion.

Such things as being pressed or bent restore themselves, have motion in their internal parts.

13. Such things as are removed from their places by forcible compression or extension, and, as soon as the force is taken away, do presently return and restore themselves to their former situation, have the beginning of their restitution within them-selves, namely, a certain motion in their internal parts, which was there, when, before the taking away of the force, they were compressed, or extended.   For that restitution is motion, and that which is at rest cannot be moved, but by a moved and a contiguous movent.   Nor doth the cause of their restitution proceed from the taking away of the force by which they were compressed or extended ; for the removing of impediments hath not the efficacy of a cause, as has been shown at the end of the 3rd article of chap. xv.   The cause therefore of their restitution is some motion either of the parts of the ambient, or of the parts of the

body compressed or extended. But the parts of
the ambient have no endeavour which contributes
to their compression or extension, nor to the set-
ting of them at liberty, or restitution. It remains
therefore that from the time of their compression or
extension there be left some endeavour or motion,
by which, the impediment being removed, every
part resumes its former place; that is to say, the
whole restores itself.

14. In the carriage of bodies, if that body, which *Though that*
which carries
carries another, hit upon any obstacle, or be by *another be*
any means suddenly stopped, and that which is *stopped, the*
body carried
carried be not stopped, it will go on, till its motion *will proceed.*
be by some external impediment taken away.

For I have demonstrated (chap. viii, art. 19)
that motion, unless it be hindered by some external
resistance, will be continued eternally with the
same celerity; and in the 7th article of chap. ix,
that the action of an external agent is of no effect
without contact. When therefore that, which car-
rieth another thing, is stopped, that stop doth not
presently take away the motion of that which is
carried. It will therefore proceed, till its motion
be by little and little extinguished by some external
resistance: which was to be proved; though expe-
rience alone had been sufficient to prove this.

In like manner, if that body which carrieth
another be put from rest into sudden motion, that
which is carried will not be moved forwards toge-
ther with it, but will be left behind. For the con-
tiguous part of the body carried hath almost the
same motion with the body which carries it; and
the remote parts will receive different velocities
according to their different distances from the body

that carries them ; namely, the more remote the parts are, the less will be their degrees of velocity. It is necessary, therefore, that the body, which is carried, be left accordingly more or less behind. And this also is manifest by experience, when at the starting forward of the horse the rider falleth backwards.

The effects of percussion not to be compared with those of weight.

15. In *percussion*, therefore, when one hard body is in some small part of it stricken by another with great force, it is not necessary that the whole body should yield to the stroke with the same celerity with which the stricken part yields.   For the rest of the parts receive their motion from the motion of the part stricken and yielding, which motion is less propagated every way towards the sides, than it is directly forwards.   And hence it is, that sometimes very hard bodies, which being erected can hardly be made to stand, are more easily broken than thrown down by a violent stroke ; when, nevertheless, if all their parts together were by any weak motion thrust forwards, they would easily be cast down.

16. Though the difference between *trusion* and *percussion* consist only in this, that in trusion the motion both of the movent and moved body begin both together in their very contact ; and in percussion the striking body is first moved, and afterwards the body stricken ; yet their effects are so different, that it seems scarce possible to compare their forces with one another.   I say, any effect of percussion being propounded, as for example, the stroke of a beetle of any weight assigned, by which a pile of any given length is to be driven

into earth of any tenacity given, it seems to me very hard, if not impossible, to define with what weight, or with what stroke, and in what time, the same pile may be driven to a depth assigned into the same earth. The cause of which difficulty is this, that the velocity of the percutient is to be compared with the magnitude of the ponderant. Now velocity, seeing it is computed by the length of space transmitted, is to be accounted but as one dimension; but weight is as a solid thing, being measured by the dimension of the whole body. And there is no comparison to be made of a solid body with a length, that is, with a line.

17. If the internal parts of a body be at rest, or retain the same situation with one another for any time how little soever, there cannot in those parts be generated any new motion or endeavour, whereof the efficient cause is not without the body of which they are parts. For if any small part, which is comprehended within the superficies of the whole body, be supposed to be now at rest, and by and by to be moved, that part must of necessity receive its motion from some moved and contiguous body. But by supposition, there is no such moved and contiguous part within the body. Wherefore, if there be any endeavour or motion or change of situation in the internal parts of that body, it must needs arise from some efficient cause that is without the body which contains them; which was to be proved.

Motion cannot begin first in the internal parts of a body.

18. In hard bodies, therefore, which are compressed or extended, if, that which compresseth or extendeth them being taken away, they restore

themselves to their former place or situation, it must needs be that that endeavour or motion of their internal parts, by which they were able to recover their former places or situations, was not extinguished when the force by which they were compressed or extended was taken away. Therefore, when the lath of a cross-bow bent doth, as soon as it is at liberty, restore itself, though to him, that judges by sense, both it and all its parts seem to be at rest; yet he, that judging by reason doth not account the taking away of impediment for an efficient cause, nor conceives that without an efficient cause any thing can pass from rest to motion, will conclude that the parts were already in motion before they began to restore themselves.

Action and reaction proceed in the same line.

19. *Action* and *reaction* proceed in the same line, but from opposite terms. For seeing reaction is nothing but endeavour in the patient to restore itself to that situation from which it was forced by the agent; the endeavour or motion both of the agent and patient or reagent will be propagated between the same terms; yet so, as that in action the term, *from which*, is in reaction the term *to which*. And seeing all action proceeds in this manner, not only between the opposite terms of the whole line in which it is propagated, but also in all the parts of that line, the terms *from which* and *to which*, both of the action and reaction, will be in the same line. Wherefore action and reaction proceed in the same line, &c.

Habit, what it is.

20. To what has been said of motion, I will add what I have to say concerning habit. *Habit*, therefore, is a generation of motion, not of motion

simply, but an easy conducting of the moved body in a certain and designed way.   And seeing it is attained by the weakening of such endeavours as divert its motion, therefore such endeavours are to be weakened by little and little. But this cannot be done but by the long continuance of action, or by actions often repeated; and therefore custom begets that facility, which is commonly and rightly called *habit ;* and it may be defined thus: HABIT *is motion made more easy and ready by custom ; that is to say, by perpetual endeavour, or by iterated endeavours in a way differing from that in which the motion proceeded from the beginning, and opposing such endeavours as resist.*   And to make this more perspicuous by example, we may observe, that when one that has no skill in music first puts his hand to an instrument, he cannot after the first stroke carry his hand to the place where he would make the second stroke, without taking it back by a new endeavour, and, as it were beginning again, pass from the first to the second. Nor will he be able to go on to the third place without another new endeavour ; but he will be forced to draw back his hand again, and so successively, by renewing his endeavour at every stroke ; till at the last, by doing this often, and by compounding many interrupted motions or endeavours into one equal endeavour, he be able to make his hand go readily on from stroke to stroke in that order and way which was at the first designed. Nor are habits to be observed in living creatures only, but also in bodies inanimate.   For we find that when the lath of a cross-bow is strongly bent,

Habit,
what it is.

and would if the impediment were removed return again with great force ; if it remain a long time bent, it will get such a habit, that when it is loosed and left to its own freedom, it will not only not restore itself, but will require as much force for the bringing of it back to its first posture, as it did for the bending of it at the first.

CHAP. XXIII.

OF THE CENTRE OF EQUIPONDERATION ; OF BODIES PRESSING DOWNWARDS IN STRAIT PARALLEL LINES.

1. Definitions and suppositions.— 2. Two planes of equiponderation are not parallel.—3. The centre of equiponderation is in every plane of equiponderation.—4. The moments of equal ponderants are to one another as their distances from the centre of the scale.— 5, 6. The moments of unequal ponderants have their proportion to one another compounded of the proportions of their weights and distances from the centre of the scale.—7. If two ponderants have their weights and distances from the centre of the scale in reciprocal proportion, they are equally poised ; and contrarily.—8. If the parts of any ponderant press the beams of the scale every where equally, all the parts cut off, reckoned from the centre of the scale, will have their moments in the same proportion with that of the parts of a triangle cut off from the vertex by strait lines parallel to the base.—9. The diameter of equiponderation of figures, which are deficient according to commensurable proportions of their altitudes and bases, divides the axis, so that the part taken next the vertex is to the other part of the complete figure to the deficient figure.—10. The diameter of equiponderation of the complement of the half of any of the said deficient figures, divides that line which is drawn through the vertex parallel to the base, so that the part next the vertex is to the other part as the complete figure to the

complement.—11. The centre of equiponderation of the half of any of the deficient figures in the first row of the table of art. 3, chap. XVII, may be found out by the numbers of the second row.—12. The centre of equiponderation of the half of any of the figures of the second row of the same table, may be found out by the numbers of the fourth row.—13. The centre of equiponderation of the half of any of the figures in the same table being known, the centre of the excess of the same figure above a triangle of the same altitude and base is also known.—14. The centre of equiponderation of a solid sector is in the axis so divided, that the part next the vertex be to the whole axis, wanting half the axis of the portion of the sphere, as 3 to 4.

DEFINITIONS.

I. A *scale* is a strait line, whose middle point is immovable, all the rest of its points being at liberty ; and that part of the scale, which reaches from the centre to either of the weights, is called the *beam*.

II. *Equiponderation* is when the endeavour of one body, which presses one of the beams, resists the endeavour of another body pressing the other beam, so that neither of them is moved ; and the bodies, when neither of them is moved, are said to be *equally poised*.

III. *Weight* is the aggregate of all the endeavours, by which all the points of that body, which presses the beam, tend downwards in lines parallel to one another ; and the body which presses is called the *ponderant*.

IV. *Moment* is the power which the ponderant has to move the beam, by reason of a determined situation.

V. *The plane of equiponderation* is that by which the ponderant is so divided, that the moments on both sides remain equal.

VI. *The diameter of equiponderation* is the common section of the two planes of equiponderation, and is in the strait line by which the weight is hanged.

VII. *The centre of equiponderation* is the common point of the two diameters of equiponderation.

SUPPOSITIONS.

Suppositions.   I. When two bodies are equally poised, if weight be added to one of them and not to the other, their equiponderation ceases.

II. When two ponderants of equal magnitude, and of the same species or matter, press the beam on both sides at equal distances from the centre of the scale, their moments are equal.   Also when two bodies endeavour at equal distances from the centre of the scale, if they be of equal magnitude and of the same species, their moments are equal.

Two planes of equiponderation are not parallel.

2. No two planes of equiponderation are parallel.

Let A B C D (in fig. 1) be any ponderant whatsoever; and in it let E F be a plane of equiponderation; parallel to which, let any other plane be drawn, as G H.   I say, G H is not a plane of equiponderation.   For seeing the parts A E F D and E B C F of the ponderant A B C D are equally poised; and the weight E G H F · is added to the part A E F D, and nothing is added to the part E B C F, but the weight E G H F is taken from it; therefore, by the first supposition, the parts A G H D and G B C H will not be equally poised; and consequently G H is not a plane of equiponderation.   Wherefore, no two planes of equiponderation are parallel; which was to be proved.

3. The centre of equiponderation is in every plane of equiponderation.

For if another plane of equiponderation be taken, it will not, by the last article, be parallel to the former plane; and therefore both those planes will cut one another. Now that section (by the 6th definition) is the diameter of equiponderation. Again, if another diameter of equiponderation be taken, it will cut that former diameter; and in that section (by the 7th definition) is the centre of equiponderation. Wherefore the centre of equiponderation is in that diameter which lies in the said plane of equiponderation.

4. The moment of any ponderant applied to one point of the beam, to the moment of the same or an equal ponderant applied to any other point of the beam, is as the distance of the former point from the centre of the scale, to the distance of the latter point from the same centre. Or thus, those moments are to one another, as the arches of circles which are made upon the centre of the scale through those points, in the same time. Or lastly thus, they are as the parallel bases of two triangles, which have a common angle at the centre of the scale.

Let A (in fig. 2) be the centre of the scale; and let the equal ponderants D and E press the beam A B in the points B and C; also let the strait lines B D and C E be diameters of equiponderation; and the points D and E in the ponderants D and E be their centres of equiponderation. Let A G F be drawn howsoever, cutting D B produced in F, and E C in G; and lastly, upon the common centre A, let the two arches B H and C I be described, cut-

ting A G F in H and I.   I say, the moment of the ponderant D to the moment of the ponderant E is as A B to A C, or as B H to C I, or as B F to C G. For the effect of the ponderant D, in the point B, is circular motion in the arch B H; and the effect of the ponderant E, in the point C, circular motion in the arch C I; and by reason of the equality of the ponderants D and E, these motions are to one another as the quicknesses or velocities with which the points B and C describe the arches B H and C I, that is, as the arches themselves B H and C I, or as the strait parallels B F and C G, or as the parts of the beam A B and A C; for A B. A C :: B F. C G :: B H. C I. are proportionals; and therefore the effects, that is, by the 4th definition, the moments of the equal ponderants applied to several points of the beam, are to one another as A B and A C; or as the distances of those points from the centre of the scale; or as the parallel bases of the triangles which have a common angle at A; or as the concentric arches B H and C I; which was to be demonstrated.

The moments of unequal ponderants have their proportion to one another compounded of the proportions of their weights and distances from the centre of the scale.

5. Unequal ponderants, when they are applied to several points of the beam, and hang at liberty, that is, so as the line by which they hang be the diameter of equiponderation, whatsoever be the figure of the ponderant, have their moments to one another in proportion compounded of the proportions of their distances from the centre of the scale, and of their weights.

Let A (in fig. 3) be the centre of the scale, and A B the beam; to which let the two ponderants C and D be applied at the points B and E.   I say, the proportion of the moment of the ponderant C

to the moment of the ponderant D, is compounded of the proportions of A B to A E, and of the weight C to the weight D ; or, if C and D be of the same species, of the magnitude C to the magnitude D.

Let either of them, as C, be supposed to be bigger than the other, D. If, therefore, by the addition of F, F and D together be as one body equal to C, the moment of C to the moment of F + D will be (by the last article) as B G is to E H. Now as F + D is to D, so let E H be to another E I ; and the moment of F + D, that is of C, to the moment of D, will be as B G to E I. But the proportion of B G to E I is compounded of the proportions of B G to E H, that is, of A B to A E, and of E H to E I, that is, of the weight C to the weight D. Wherefore unequal ponderants, when they are applied, &c. Which was to be proved.

6. The same figure remaining, if I K be drawn parallel to the beam A B, and cutting A G in K ; and K L be drawn parallel to B G, cutting A B in L, the distances A B and A L from the centre will be proportional to the moments of C and D. For the moment of C is B G, and the moment of D is E I, to which K L is equal. But as the distance A B from the centre is to the distance A L from the centre, so is B G, the moment of the ponderant C, to L K, or E I the moment of the ponderant D.

7. If two ponderants have their weights and distances from the centre in reciprocal proportion, and the centre of the scale be between the points to which the ponderants are applied, they will be equally poised. And contrarily, if they be equally poised, their weights and distances from the centre of the scale will be in reciprocal proportion.

A A 2

Let the centre of the scale (in the same third figure) be A, the beam A B; and let any ponderant C, having B G for its moment, be applied to the point B; also let any other ponderant D, whose moment is E I, be applied to the point E. Through the point I let I K be drawn parallel to the beam A B, cutting A G in K; also let K L be drawn parallel to B G, K L will then be the moment of the ponderant D; and by the last article, it will be as B G, the moment of the ponderant C in the point B, to L K the moment of the ponderant D in the point E, so A B to A L. On the other side of the centre of the scale, let A N be taken equal to A L; and to the point N let there be applied the ponderant O, having to the ponderant C the proportion of A B to A N. I say, the ponderants in B and N will be equally poised. For the proportion of the moment of the ponderant O, in the point N, to the moment of the ponderant C in the point B, is by the 5th article, compounded of the proportions of the weight O to the weight C, and of the distance from the centre of the scale A N or A L to the distance from the centre of the scale A B. But seeing we have supposed, that the distance A B to the distance A N is in reciprocal proportion of the weight O to the weight C, the proportion of the moment of the ponderant O, in the point N, to the moment of the ponderant C, in the point B, will be compounded of the proportions of A B to A N, and of A N to A B. Wherefore, setting in order A B, A N, A B, the moment of O to the moment of C will be as the first to the last, that is, as A B to A B. Their moments therefore are equal; and consequently the plane which passes through

A will (by the fifth definition) be a plane of equiponderation. Wherefore they will be equally poised; as was to be proved.

Now the converse of this is manifest. For if there be equiponderation and the proportion of the weights and distances be not reciprocal, then both the weights will always have the same moments, although one of them have more weight added to it or its distance changed.

Coroll. When ponderants are of the same species, and their moments be equal; their magnitudes and distances from the centre of the scale will be reciprocally proportional. For in homogeneous bodies, it is as weight to weight, so magnitude to magnitude.

8. If to the whole length of the beam there be applied a parallelogram, or a parallelopipedum, or a prisma, or a cylinder, or the superficies of a cylinder, or of a sphere, or of any portion of a sphere or prisma; the parts of any of them cut off with planes parallel to the base will have their moments in the same proportion with the parts of a triangle, which has its vertex in the centre of the scale, and for one of its sides the beam itself, which parts are cut off by planes parallel to the base.

First, let the rectangled parallelogram A B C D (in figure 4) be applied to the whole length of the beam A B; and producing C B howsoever to E, let the triangle A B E be described. Let now any part of the parallelogram, as A F, be cut off by the plane F G, parallel to the base C B; and let F G be produced to A E in the point H. 1 say, the moment of the whole A B C D to the moment of its

part A F, is as the triangle A B E to the triangle A G H, that is, in proportion duplicate to that of the distances from the centre of the scale.

For, the parallelogram A B C D being divided into equal parts, infinite in number, by strait lines drawn parallel to the base ; and supposing the moment of the strait line C B to be B E, the moment of the strait line F G will (by the 7th article) be G H ; and the moments of all the strait lines of that parallelogram will be so many strait lines in the triangle A B E drawn parallel to the base B E; all which parallels together taken are the moment of the whole parallelogram A B C D ; and the same parallels do also constitute the superficies of the triangle A B E. Wherefore the moment of the parallelogram A B C D is the triangle A B E ; and for the same reason, the moment of the parallelogram A F is the triangle A G H ; and therefore the moment of the whole parallelogram to the moment of a parallelogram which is part of the same, is as the triangle A B E to the triangle A G H, or in proportion duplicate to that of the beams to which they are applied. And what is here demonstrated in the case of a parallelogram may be understood to serve for that of a cylinder, and of a prisma, and their superficies ; as also for the superficies of a sphere, of an hemisphere, or any portion of a sphere. For the parts of the superficies of a sphere have the same proportion with that of the parts of the axis cut off by the same parallels, by which the parts of the superficies are cut off, as Archimedes has demonstrated ; and therefore when the parts of any of these figures are equal and at equal

distances from the centre of the scale, their mo-
ments also are equal, in the same manner as they
are in parallelograms.

Secondly, let the parallelogram A K I B not be
rectangled; the strait line I B will nevertheless
press the point B perpendicularly in the strait line
B E ; and the strait line L G will press the point
G perpendicularly in the strait line G H ; and all
the rest of the strait lines which are parallel to I B
will do the like. Whatsoever therefore the moment
be which is assigned to the strait line I B, as here,
for example, it is supposed to be B E, if A E be
drawn, the moment of the whole parallelogram A I
will be the triangle A B E ; and the moment of the
part A L will be the triangle A G H. Wherefore
the moment of any ponderant, which has its sides
equally applied to the beam, whether they be
applied perpendicularly or obliquely, will be always
to the moment of a part of the same in such pro-
portion as the whole triangle has to a part of the
same cut off by a plane which is parallel to the base.

9. The centre of equiponderation of any figure,
which is deficient according to commensurable
proportions of the altitude and base diminished,
and whose complete figure is either a parallelogram
or a cylinder, or a parallelopipedum, divides the
axis, so, that the part next the vertex, to the other
part, is as the complete figure to the deficient
figure.

For let C I A P E (in fig. 5) be a deficient figure,
whose axis is A B, and whose complete figure is
C D F E ; and let the axis A B be so divided in Z,
that A Z be to Z B as C D F E is to C I A P E. I

say, the centre of equiponderation of the figure C I A P E will be in the point Z.

First, that the centre of equiponderation of the figure C I A P E is somewhere in the axis A B is manifest of itself; and therefore A B is a diameter of equiponderation. Let A E be drawn, and let B E be put for the moment of the strait line C E; the triangle A B E will therefore (by the third article) be the moment of the complete figure C D F E. Let the axis A B be equally divided in L, and let G L H be drawn parallel and equal to the strait line C E, cutting the crooked line C I A P E in I and P, and the strait lines A C and A E in K and M. Moreover, let Z O be drawn parallel to the same C E; and let it be, as L G to L I, so L M to another, L N; and let the same be done in all the rest of the strait lines possible, parallel to the base; and through all the points N, let the line A N E be drawn; the three-sided figure A N E B will therefore be the moment of the figure C I A P E. Now the triangle A B E is (by the 9th article of chapter xvii) to the three-sided figure A N E B, as A B C D + A I C B is to A I C B twice taken, that is, as C D F E + C I A P E is to C I A P E twice taken. But as C I A P E is to C D F E, that is, as the weight of the deficient figure is to the weight of the complete figure, so is C I A P E twice taken to C D F E twice taken. Wherefore, setting in order C D F E + C I A P E. 2 C I A P E. 2 C D F E; the proportion of C D F E + C I A P E to C D F E twice taken will be compounded of the proportion of C D F E + C I A P E to C I A P E twice taken, that is, of the proportion

of the triangle A B E to the three-sided figure
A N E B, that is, of the moment of the complete
figure to the moment of the deficient figure, and of
the proportion of C I A P E twice taken to C D F E
twice taken, that is, to the proportion reciprocally
taken of the weight of the deficient figure to the
weight of the complete figure.

Again, seeing by supposition A Z. Z B : : C D F E.
C I A P E are proportionals ; A B. A Z : : C D F E +
C I A P E. C D F E will also, by compounding, be
proportionals.   And seeing A L is the half of A B,
A L. A Z : : C D F E + C I A P E. 2 C D F E will also
be proportionals.   But the proportion of CDFE +
C I A P E to 2 C D F E is compounded, as was but
now shown, of the proportions of moment to mo-
ment, &c., and therefore the proportion of A L to
A Z is compounded of the proportion of the mo-
ment of the complete figure CDFE to the moment
of the deficient figure C I A P E, and of the pro-
portion of the weight of the deficient figure CIAPE
to the weight of the complete figure C D F E ; but
the proportion of A L to A Z is compounded of the
proportions of A L to B Z and of B Z to A Z.   Now
the proportion of B Z to A Z is the proportion of
the weights reciprocally taken, that is to say, of the
weight C I A P E to the weight C D F E.   There-
fore the remaining proportion of A L to B Z, that
is, of L B to B Z, is the proportion of the moment
of the weight CDFE to the moment of the weight
C I A P E.   But the proportion of A L to B Z is
compounded of the proportions of A L to A Z and
of A Z to Z B ; of which proportions that of A Z to
Z B is the proportion of the weight C D F E to the
weight C I A P E.   Wherefore (by art. 5 of this

chapter) the remaining proportion of A L to A Z is
the proportion of the distances of the points Z and
L from the centre of the scale, which is Λ.   And,
therefore, (by art. 6) the weight CI A P E shall hang
from O in the strait line O Z.   So that O Z is one
diameter of equiponderation of the weight CIAPE.
But the strait line A B is the other diameter of equi-
ponderation of the same weight CI A P E.   Where-
fore (by the 7th definition) the point Z is the centre
of the same equiponderation; which point, by con-
struction, divides the axis so, that the part Λ Z,
which is the part next the vertex, is to the other
part Z B, as the complete figure C D F E is to the
deficient figure CI A P E ; which is that which was
to be demonstrated.

Coroll. I. The centre of equiponderation of any of
those plane three-sided figures, which are compared
with their complete figures in the table of art. 3,
chap. XVII, is to be found in the same table, by
taking the denominator of the fraction for the part
of the axis cut off next the vertex, and the nume-
rator for the other part next the base. For example,
if it be required to find the centre of equipondera-
tion of the second three-sided figure of four means,
there is in the concourse of the second column
with the row of three-sided figures of four means
this fraction $\frac{5}{7}$, which signifies that that figure is
to its parallelogram or complete figure as $\frac{5}{7}$ to
unity, that is, as $\frac{5}{7}$ to $\frac{7}{7}$, or as 5 to 7 ; and, there-
fore the centre of equiponderation of that figure
divides the axis, so that the part next the vertex
is to the other part as 7 to 5.

Coroll. II. The centre of equiponderation of any
of the solids of those figures, which are contained

in the table of art. 7 of the same chap. XVII, is exhibited in the same table. For example, if the centre of equiponderation of a cone be sought for, the cone will be found to be $\frac{1}{3}$ of its cylinder ; and, therefore, the centre of its equiponderation will so divide the axis, that the part next the vertex to the other part will be as 3 to 1. Also the solid of a three-sided figure of one mean, that is, a parabolical solid, seeing it is $\frac{2}{3}$, that is $\frac{1}{2}$ of its cylinder, will have its centre of equiponderation in that point, which divides the axis, so that the part towards the vertex be double to the part towards the base.

10. The diameter of equiponderation of the complement of the half of any of those figures which are contained in the table of art. 3, chap. XVII, divides that line which is drawn through the vertex parallel and equal to the base, so that the part next the vertex will be to the other part, as the complete figure to the complement.

For let A I C B (in the same fig. 5) be the half of a parabola, or of any other of those three-sided figures which are in the table of art. 3, chap. XVII, whose axis is A B, and base B C, having A D drawn from the vertex, equal and parallel to the base B C, and whose complete figure is the parallelogram A B C D. Let I Q be drawn at any distance from the side C D, but parallel to it ; and let A D be the altitude of the complement A I C D, and Q I a line ordinately applied in it. Wherefore the altitude A L in the deficient figure A I C B is equal to Q I the line ordinately applied in its complement ; and contrarily, L I the line ordinately applied in the figure A I C B is equal to the altitude

A Q in its complement; and so in all the rest of the ordinate lines and altitudes the mutation is such, that that line, which is ordinately applied in the figure, is the altitude of its complement. And, therefore, the proportion of the altitudes decreasing to that of the ordinate lines decreasing, being multiplicate according to any number in the deficient figure, is submultiplicate according to the same number in its complement. For example, if A I C B be a parabola, seeing the proportion of A B to A L is duplicate to that of B C to L I, the proportion of AD to AQ in the complement AICD, which is the same with that of B C to L I, will be subduplicate to that of C D to Q I, which is the same with that of A B to A L; and consequently, in a parabola, the complement will be to the parallelogram as 1 to 3; in a three-sided figure of two means, as 1 to 4; in a three-sided figure of three means, as 1 to 5, &c. But all the ordinate lines together in A I C D are its moment; and all the ordinate lines in AICB are its moment. Wherefore the moments of the complements of the halves of deficient figures in the table of art. 3 of chap. XVII, being compared, are as the deficient figures themselves; and, therefore, the diameter of equiponderation will divide the strait line A D in such proportion, that the part next the vertex be to the other part, as the complete figure A B C D is to the complement A I C D.

Coroll. The diameter of equiponderation of these halves may be found by the table of art. 3 of chap. XVII, in this manner. Let there be propounded any deficient figure, namely, the second three-sided figure of two means. This figure is to the com-

plete figure as $\frac{3}{5}$ to 1, that is 3 to 5. Wherefore the complement to the same complete figure is as 2 to 5 ; and, therefore, the diameter of equiponderation of this complement will cut the strait line drawn from the vertex parallel to the base, so that the part next the vertex will be to the other part as 5 to 2. And, in like manner, any other of the said three-sided figures being propounded, if the numerator of its fraction found out in the table be taken from the denominator, the strait line drawn from the vertex is to be divided, so that the part next the vertex be to the other part, as the denominator is to the remainder which that subtraction leaves.

11. The centre of equiponderation of the half of any of those crooked-lined figures, which are in the first row of the table of art. 3 of chap. XVII, is in that strait line which, being parallel to the axis, divides the base according to the numbers of the fraction next below it in the second row, so that the numerator be answerable to that part which is towards the axis.

For example, let the first figure of three means be taken, whose half is A B C D (in fig. 6), and let the rectangle A B E D be completed. The complement therefore will be B C D E. And seeing A B E D is to the figure A B C D (by the table) as 5 to 4, the same A B E D will be to the complement B C D E as 5 to 1. Wherefore, if F G be drawn parallel to the base D A, cutting the axis so that A G be to G B as 4 to 5, the centre of equiponderation of the figure A BC D will, by the precedent article, be somewhere in the same F G. Again, seeing, by the same article, the complete

figure A B E D, is to the complement B C D E as
5 to 1, therefore if B E and A D be divided in I
and H as 5 to 1, the centre of equiponderation of
the complement B C D E will be somewhere in the
strait line which connects H and I.   Let now the
strait line L K be drawn through M the centre of
the complete figure, parallel to the base ; and the
strait line N O through the same centre M, perpen-
dicular to it; and let the strait lines LK and FG cut
the strait line H I in P and Q.   Let P R be taken
quadruple to P Q ; and let R M be drawn and pro-
duced to F G in S.   R M therefore will be to M S
as 4 to 1, that is, as the figure A B C D to its com-
plement B C D E.   Wherefore, seeing M is the
centre of the complete figure A B E D, and the dis-
tances of R and S from the centre M be in propor-
tion reciprocal to that of the weight of the com-
plement BCDE to the weight of the figure ABCD,
R and S will either be the centres of equiponderation
of their own figures, or those centres will be in some
other points of the diameters of equiponderation
HI and FG. But this last is impossible. For no other
strait line can be drawn through the point M ter-
minating in the strait lines H I and F G, and retain-
ing the proportion of M R to M S, that is, of the
figure A B C D to its complement B C D E.   The
centre, therefore, of equiponderation of the figure
A B C D is in the point S.   Now, seeing P M hath
the same proportion to Q S which R P hath to R Q,
Q S will be 5 of those parts of which P M is four,
that is, of which I N is four.   But I N or P M is 2
of those parts of which E B or F G is 6 ; and, there-
fore, if it be as 4 to 5, so 2 to a fourth, that fourth

will be $2\frac{1}{2}$. Wherefore Q S is $2\frac{1}{2}$ of those parts of which F G is 6. But F Q is 1 ; and, therefore, FS is $3\frac{1}{2}$. Wherefore the remaining part GS is $2\frac{1}{2}$. So that F G is so divided in S, that the part towards the axis is in proportion to the other part, as $2\frac{1}{2}$ to $3\frac{1}{2}$, that is as 5 to 7 ; which answereth to the fraction $\frac{5}{7}$ in the second row, next under the fraction $\frac{4}{5}$ in the first row. Wherefore drawing S T parallel to the axis, the base will be divided in like manner.

By this method it is manifest, that the base of a semiparabola will be divided into 3 and 5 ; and the base of the first three-sided figure of two means, into 4 and 6 ; and of the first three-sided figure of four means, into 6 and 8. The fractions, therefore, of the second row denote the proportions, into which the bases of the figures of the first row are divided by the diameters of equiponderation. But the first row begins one place higher than the second row.

12. The centre of equiponderation of the half of any of the figures in the second row of the same table of art. 3, chap. XVII, is in a strait line parallel to the axis, and dividing the base according to the numbers of the fraction in the fourth row, two places lower, so as that the numerator be answerable to that part which is next the axis.

Let the half of the second three-sided figure of two means be taken ; and let it be A B C D (in fig. 7) ; whose complement is B C D E, and the rectangle completed A B E D. Let this rectangle be divided by the two strait lines L K and N O, cutting one another in the centre M at right angles ; and because A B E D is to A B C D as 5 to

3, let A B be divided in G, so that A G to B G be as 3 to 5 ; and let F G be drawn parallel to the base. Also because A B E D is (by art. 9) to B C D E as 5 to 2, let B E be divided in the point I, so that B I be to I E as 5 to 2 ; and let I H be drawn parallel to the axis, cutting L K and F G in P and Q. Let now P R be so taken, that it be to P Q as 3 to 2, and let R M be drawn and produced to F G in S. Seeing, therefore, R P is to P Q, that is, R M to M S, as A B C D is to its complement B C D E, and the centres of equiponderation of A B C D and B C D E are in the strait lines F G and H I, and the centre of equiponderation of them both toge-ther in the point M ; R will be the centre of the complement B C D E, and S the centre of the figure A B C D.   And seeing P M, that is I N, is to Q S, as R P is to R Q ; and I N or P M is 3 of those parts, of which B E, that is F G, is 14 ; there-fore Q S is 5 of the same parts ; and E I, that is F Q, 4 ; and F S, 9 ; and G S, 5.   Wherefore the strait line S T being drawn parallel to the axis, will divide the base A D into 5 and 9.   But the fraction $\frac{9}{5}$ is found in the fourth row of the table, two places below the fraction $\frac{2}{5}$ in the second row.

By the same method, if in the same second row there be taken the second three-sided figure of three means, the centre of equiponderation of the half of it will be found to be in a strait line parallel to the axis, dividing the base according to the numbers of the fraction $\frac{9}{10}$, two places below in the fourth row.   And the same way serves for all the rest of the figures in the second row.   In like manner, the centre of equiponderation of the third three-sided figure of three means will be found to

be in a strait line parallel to the axis, dividing the base, so that the part next the axis be to the other part as 7 to 13, &c.

Coroll. The centres of equiponderation of the halves of the said figures are known, seeing they are in the intersection of the strait lines S T and F G, which are both known.

13. The centre of equiponderation of the half of any of the figures, which (in the table of art. 3, chap. XVII) are compared with their parallelograms, being known; the centre of equiponderation of the excess of the same figure above its triangle is also known.

For example, let the semiparabola A B C D (in fig. 8) be taken, whose axis is A B ; whose complete figure is A B E D ; and whose excess above its triangle is B C D B. Its centre of equiponderation may be found out in this manner. Let F G be drawn parallel to the base, so that A F be a third part of the axis ; and let H I be drawn parallel to the axis, so that A H be a third part of the base. This being done, the centre of equiponderation of the triangle A B D will be I. Again, let K L be drawn parallel to the base, so that A K be to A B as 2 to 5 ; and M N parallel to the axis, so that A M be to A D as 3 to 8 ; and let M N terminate in the strait line K L. The centre, therefore, of equiponderation of the parabola A B C D is N ; and therefore we have the centres of equiponderation of the semiparabola A B C D, and of its part the triangle A B D. That we may now find the centre of equiponderation of the remaining part B C D B, let I N be drawn and produced to O, so that N O be triple to I N ; and

PART III.
23.

The centre of equiponderation of the half, &c.

O will be the centre sought for. For seeing the weight of A B D to the weight of B C D B is in proportion reciprocal to that of the strait line N O to the strait line I N ; and N is the centre of the whole, and I the centre of the triangle A B D ; O will be the centre of the remaining part, namely, of the figure B D C B ; which was to be found.

Coroll. The centre of equiponderation of the figure B D C B is in the concourse of two strait lines, whereof one is parallel to the base, and divides the axis, so that the part next the base be $\frac{2}{5}$ or $\frac{9}{15}$ of the whole axis; the other is parallel to the axis, and so divides the base, that the part towards the axis be $\frac{1}{2}$, or $\frac{12}{24}$ of the whole base. For drawing O P parallel to the base, it will be as I N to N O, so F K to K P, that is, so 1 to 3, or 5 to 15. But A F is $\frac{5}{15}$, or $\frac{1}{3}$ of the whole A B ; and A K is $\frac{6}{15}$, or $\frac{2}{5}$; and F K $\frac{1}{15}$; and K P $\frac{3}{15}$; and therefore A P is $\frac{9}{15}$ of the axis A B. Also A H is $\frac{1}{3}$, or $\frac{8}{24}$; and A M $\frac{3}{9}$, or $\frac{9}{24}$ of the whole base ; and therefore O Q being drawn parallel to the axis, M Q, which is triple to H M, will be $\frac{3}{24}$. Wherefore A Q is $\frac{12}{24}$, or $\frac{1}{2}$ of the base A D.

The excesses of the rest of the three-sided figures in the first row of the table of art. 3, chap. XVII, have their centres of equiponderation in two strait lines, which divide the axis and base according to those fractions, which add 4 to the numerators of the fractions of a parabola $\frac{9}{15}$ and $\frac{12}{24}$; and 6 to the denominators, in this manner :—

In a parabola,   the axis $\frac{9}{15}$, the base $\frac{12}{24}$.
In the first three-sided figure,   the axis $\frac{13}{21}$, the base $\frac{16}{30}$.
In the second three-sided figure, the axis $\frac{17}{27}$, the base $\frac{20}{36}$, &c.

And by the same method, any man, if it be

worth the pains, may find out the centres of equi-
ponderation of the excesses above their triangles
of the rest of the figures in the second and third
row, &c.

14. The centre of equiponderation of the sector The centre of equiponderation of a solid sector is in the axis so divided,
of a sphere, that is, of a figure compounded of a
right cone, whose vertex is the centre of the
sphere, and the portion of the sphere whose base that the part next the vertex
is the same with that of the cone, divides the strait be to the whole axis, wanting
line which is made of the axis of the cone and half half the axis of
the axis of the portion together taken, so that the the portion of the sphere, as
part next the vertex be triple to the other part, or 3 to 4.
to the whole strait line as 3 to 4.

For let A B C (in fig. 9) be the sector of a
sphere, whose vertex is the centre of the sphere A;
whose axis is A D ; and the circle upon B C is the
common base of the portion of the sphere and of
the cone whose vertex is A ; the axis of which
portion is E D, and the half thereof F D ; and the
axis of the cone, A E.   Lastly, let A G be $\frac{3}{4}$ of the
strait line A F.   I say, G is the centre of equipon-
deration of the sector A B C.

Let the strait line F H be drawn of any length,
making right angles with A F at F ; and drawing
the strait line A H, let the triangle AFH be made.
Then upon the same centre A let any arch I K be
drawn, cutting A D in L ; and its chord, cutting
A D in M ; and dividing M L equally in N, let N O
be drawn parallel to the strait line FH, and meet-
ing with the strait line A H in O.

Seeing now B D C is the spherical superficies of
the portion cut off with a plane passing through
B C, and cutting the axis at right angles; and
seeing F H divides E D, the axis of the portion,

PART III.
23.

Centre of equi-
ponderation of
a solid, &c.
into two equal parts in F ; the centre of equipon-
deration of the superficies B D C will be in F (by
art. 8) ; and for the same reason the centre of
equiponderation of the superficies I L K, K being
in the strait line A C, will be in N. And in like
manner, if there were drawn, between the centre
of the sphere A and the outermost spherical super-
ficies of the sector, arches infinite in number, the
centres of equiponderation of the spherical super-
ficies, in which those arches are, would be found
to be in that part of the axis, which is intercepted
between the superficies itself and a plane passing
along by the chord of the arch, and cutting the
axis in the middle at right angles.

Let it now be supposed that the moment of the
outermost spherical superficies B D C is F H. See-
ing therefore the superficies B D C is to the super-
ficies I L K in proportion duplicate to that of the
arch B D C to the arch I L K, that is, of B E to
I M, that is, of F H to N O ; let it be as F H to
N O, so N O to another N P ; and again, as N O to
N P, so N P to another N Q ; and let this be done
in all the strait lines parallel to the base F H that
that can possibly be drawn between the base and
the vertex of the triangle A F H. If then through
all the points Q there be drawn the crooked line
A Q H, the figure A F H Q A will be the comple-
ment of the first three-sided figure of two means ;
and the same will also be the moment of all the
spherical superficies, of which the solid sector
A B C D is compounded ; and by consequent, the
moment of the sector itself. Let now F H be un-
derstood to be the semidiameter of the base of a
right cone, whose side is A H, and axis A F

PART III.
23.
Centre of equi-
ponderation of
a solid, &c.

Wherefore, seeing the bases of the cones, which pass through F and N and the rest of the points of the axis, are in proportion duplicate to that of the strait lines .F H and N O, &c., the moment of all the bases together, that is, of the whole cone, will be the figure itself A F H Q A ; and therefore the centre of equiponderation of the cone A F H is the same with that of the solid sector. Wherefore, seeing A G is $\frac{3}{4}$ of the axis A F, the centre of equi-ponderation of the cone A F H is in G; and there-fore the centre of the solid sector is in G also, and divides the part A F of the axis so that A G is triple to G F; that is, A G is to A F as 3 to 4 ; which was to be demonstrated.

Note, that when the sector is a hemisphere, the axis of the cone vanisheth into that point which is the centre of the sphere; and therefore it addeth nothing to half the axis of the portion. Wherefore, if in the axis of the hemisphere there be taken from the centre $\frac{3}{4}$ of half the axis, that is, $\frac{3}{8}$ of the semidiameter of the sphere, there will be the centre of equiponderation of the hemisphere.

CHAPTER XXIV.

OF REFRACTION AND REFLECTION.

1. Definitions.—2. In perpendicular motion there is no refraction.—3. Things thrown out of a thinner into a thicker medium are so refracted that the angle refracted is greater than the angle of inclination.—4. Endeavour, which from one point tendeth every way, will be so refracted, as that the sine of the angle refracted will be to the sine of the angle of inclination, as the density of the first medium is to the density of the second medium, reciprocally taken.—5. The sine of the refracted angle in one inclination is to the sine of the refracted angle in another inclination, as the sine of the angle of that inclination is to the sine of the angle of this inclination.—6. If two lines of incidence, having equal inclination, be the one in a thinner, the other in a thicker medium, the sine of the angle of inclination will be a mean proportional between the two sines of the refracted angles.—7. If the angle of inclination be semirect, and the line of inclination be in the thicker medium, and the proportion of their densities be the same with that of the diagonal to the side of a square, and the separating superficies be plane, the refracted line will be in the separating superficies.—8. If a body be carried in a strait line upon another body, and do not penetrate the same, but be reflected from it, the angle of reflection will be equal to the angle of incidence.—9. The same happens in the generation of motion in the line of incidence.

DEFINITIONS.

I. REFRACTION is the *breaking* of that strait line, in which a body is moved or its action would proceed in one and the same medium, into two strait lines, by reason of the different natures of the two mediums.

II. The former of these is called the *line of incidence*; the latter the *refracted line*.

III. The *point of refraction* is the common point of the line of incidence, and of the refracted line.

IV. The *refracting superficies*, which also is the *separating superficies* of the two mediums, is that in which is the point of refraction.

V. The *angle refracted* is that, which the refracted line makes in the point of refraction with that line, which from the same point is drawn perpendicular to the separating superficies in a different medium.

VI. The *angle of refraction* is that which the refracted line makes with the line of incidence produced.

VII. The *angle of inclination* is that which the line of incidence makes with that line, which from the point of refraction is drawn perpendicular to the separating superficies.

VIII. The *angle of incidence* is the complement to a right angle of the angle of inclination.

And so, (in fig. 1) the refraction is made in A B F. The refracted line is B F. The line of incidence is A B. The point of incidence and of refraction is B. The refracting or separating superficies is D B E. The line of incidence produced directly is A B C. The perpendicular to the separating superficies is B H. The angle of refraction is C B F. The angle refracted is H B F. The angle of inclination is A B G or H B C. The angle of incidence is A B D.

IX. Moreover the *thinner medium* is understood to be that in which there is less resistance to motion, or to the generation of motion; and the thicker that wherein there is greater resistance.

PART III.
24.

x. And that medium in which there is equal resistance everywhere, is a *homogeneous medium*. All other mediums are *heterogeneous*.

In perpendicular motion there is no refraction.

2. If a body pass, or there be generation of motion from one medium to another of different density, in a line perpendicular to the separating superficies, there will be no refraction.

For seeing on every side of the perpendicular all things in the mediums are supposed to be like and equal, if the motion itself be supposed to be perpendicular, the inclinations also will be equal, or rather none at all; and therefore there can be no cause from which refraction may be inferred to be on one side of the perpendicular, which will not conclude the same refraction to be on the other side. Which being so, refraction on one side will destroy refraction on the other side; and consequently either the refracted line will be everywhere, which is absurd, or there will be no refracted line at all; which was to be demonstrated.

Coroll. It is manifest from hence, that the cause of refraction consisteth only in the obliquity of the line of incidence, whether the incident body penetrate both the mediums, or without penetrating, propagate motion by pressure only.

Things thrown out of a thinner into a thicker medium are so refracted that the angle refracted is greater than the angle of inclination.

3. If a body, without any change of situation of its internal parts, as a stone, be moved obliquely out of the thinner medium, and proceed penetrating the thicker medium, and the thicker medium be such, as that its internal parts being moved restore themselves to their former situation; the angle refracted will be greater than the angle of inclination,

PART III.
24.
Things thrown
out of a thinner
into a thicker
medium, &c.

For let D B E (in the same first figure) be the separating superficies of two mediums; and let a body, as a stone thrown, be understood to be moved as is supposed in the strait line A B C; and let A B be in the thinner medium, as in the air; and B C in the thicker, as in the water. I say the stone, which being thrown, is moved in the line A B, will not proceed in the line B C, but in some other line, namely, that, with which the perpendicular B H makes the refracted angle H B F greater than the angle of inclination H B C.

For seeing the stone coming from A, and falling upon B, makes that which is at B proceed towards H, and that the like is done in all the strait lines which are parallel to B H; and seeing the parts moved restore themselves by contrary motion in the same line; there will be contrary motion generated in H B, and in all the strait lines which are parallel to it. Wherefore, the motion of the stone will be made by the concourse of the motions in A G, that is, in D B, and in G B, that is, in B H, and lastly, in H B, that is, by the concourse of three motions. But by the concourse of the motions in A G and B H, the stone will be carried to C; and therefore by adding the motion in H B, it will be carried higher in some other line, as in B F, and make the angle H B F greater than the angle H B C.

And from hence may be derived the cause, why bodies which are thrown in a very oblique line, if either they be any thing flat, or be thrown with great force, will, when they fall upon the water, be cast up again from the water into the air.

For let A B (in fig. 2) be the superficies of the

water; into which, from the point C, let a stone be thrown in the strait line C A, making with the line B A produced a very little angle C A D; and producing B A indefinitely to D, let C D be drawn perpendicular to it, and A E parallel to C D. The stone therefore will be moved in C A by the concourse of two motions in C D and D A, whose velocities are as the lines themselves C D and D A. And from the motion in C D and all its parallels downwards, as soon as the stone falls upon A, there will be reaction upwards, because the water restores itself to its former situation. If now the stone be thrown with sufficient obliquity, that is, if the strait line C D be short enough, that is, if the endeavour of the stone downwards be less than the reaction of the water upwards, that is, less than the endeavour it hath from its own gravity (for that may be), the stone will by reason of the excess of the endeavour which the water hath to restore itself, above that which the stone hath downwards, be raised again above the superficies A B, and be carried higher, being reflected in a line which goes higher, as the line A G.

Endeavour, which from one point tendeth every way, will be so refracted, as that the sine of the angle refracted will be to the sine of the angle of inclination, as the density of the first medium is to the density of the second medium, reciprocally taken.

4. If from a point, whatsoever the medium be, endeavour be propagated every way into all the parts of that medium; and to the same endeavour there be obliquely opposed another medium of a different nature, that is, either thinner or thicker; that endeavour will be so refracted, that the sine of the angle refracted, to the sine of the angle of inclination, will be as the density of the first medium to the density of the second medium, reciprocally taken.

First, let a body be in the thinner medium in A

(fig. 3), and let it be understood to have endeavour every way, and consequently, that its endeavour proceed in the lines A B and A *b*; to which let B *b* the superficies of the thicker medium be obliquely opposed in B and *b*, so that A B and A *b* be equal; and let the strait line B *b* be produced both ways. From the points B and *b*, let the perpendiculars B C and *b c* be drawn; and upon the centres B and *b*, and at the equal distances B A and *b* A, let the circles A C and A *c* be described, cutting B C and *b c* in C and *c*, and the same C B and *c b* produced in D and *d*, as also A B and A *b* produced in E and *e*. Then from the point A to the strait lines B C and *b c* let the perpendiculars A F and A*f* be drawn. A F therefore will be the sine of the angle of inclination of the strait line A B, and A*f* the sine of the angle of inclination of the strait line A *h*, which two inclinations are by construction made equal. I say, as the density of the medium in which are B C and *b c* is to the density of the medium in which are B D and *b d*, so is the sine of the angle refracted, to the sine of the angle of inclination.

Let the strait line F G be drawn parallel to the strait line A B, meeting with the strait line *b* B produced in G.

Seeing therefore A F and B G are also parallels, they will be equal; and consequently, the endeavour in A F is propagated in the same time, in which the endeavour in B G would be propagated if the medium were of the same density. But because B G is in a thicker medium, that is, in a medium which resists the endeavour more than the medium in which A F is, the endeavour will be

propagated less in B G than in A F, according to the proportion which the density of the medium, in which A F is, hath to the density of the medium in which B G is. Let therefore the density of the medium, in which B G is, be to the density of the medium, in which A F is, as B G is to B H ; and let the measure of the time be the radius of the circle. Let H I be drawn parallel to B D, meeting with the circumference in I ; and from the point I let I K be drawn perpendicular to B D ; which being done, B H and I K will be equal ; and I K will be to A F, as the density of the medium in which is A F is to the density of the medium in which is I K. Seeing therefore in the time A B, which is the radius of the circle, the endeavour is propagated in A F in the thinner medium, it will be propagated in the same time, that is, in the time B I in the thicker medium from K to I. Therefore, B I is the refracted line of the line of incidence A B ; and I K is the sine of the angle refracted : and A F the sine of the angle of inclination. Wherefore, seeing I K is to A F, as the density of the medium in which is A F to the density of the medium in which is I K ; it will be as the density of the medium in which is A F or B C to the density of the medium in which is I K or B D, so the sine of the angle refracted to the sine of the angle of inclination. And by the same reason it may be shown, that as the density of the thinner medium is to the density of the thicker medium, so will K I the sine of the angle refracted be to A F the sine of the angle of inclination.

Secondly, let the body, which endeavoureth every

way, be in the thicker medium at I. If, therefore, both the mediums were of the same density, the endeavour of the body in I B would tend directly to L; and the sine of the angle of inclination L M would be equal to I K or B H. But because the density of the medium, in which is I K, to the density of the medium, in which is L M, is as B H to B G, that is, to A F, the endeavour will be propagated further in the medium in which L M is, than in the medium in which I K is, in the proportion of density to density, that is, of M L to A F. Wherefore, B A being drawn, the angle refracted will be C B A, and its sine A F. But L M is the sine of the angle of inclination; and therefore again, as the density of one medium is to the density of the different medium, so reciprocally is the sine of the angle refracted to the sine of the angle of inclination; which was to be demonstrated.

In this demonstration, I have made the separating superficies B b plane by construction. But though it were concave or convex, the theorem would nevertheless be true. For the refraction being made in the point B of the plane separating superficies, if a crooked line, as P Q, be drawn, touching the separating line in the point B; neither the refracted line B I, nor the perpendicular B D, will be altered; and the refracted angle K B I, as also its sine K I, will be still the same they were.

5. The sine of the angle refracted in one inclination is to the sine of the angle refracted in another inclination, as the sine of the angle of that inclination to the sine of the angle of this inclination.

nation, as the sine of the angle of that inclination is to the sine of the angle of this inclination.

For seeing the sine of the refracted angle is to the sine of the angle of inclination, whatsoever that inclination be, as the density of one medium to the density of the other medium ; the proportion of the sine of the refracted angle, to the sine of the angle of inclination, will be compounded of the proportions of density to density, and of the sine of the angle of one inclination to the sine of the angle of the other inclination. But the proportions of the densities in the same homogeneous body are supposed to be the same. Wherefore refracted angles in different inclinations are as the sines of the angles of those inclinations; which was to be demonstrated.

If two lines of incidence, having equal inclination, be one in a thinner the other in a thicker medium, the sine of the angle of inclination will be a mean proportional between the two sines of the re-fracted angles.

6. If two lines of incidence, having equal inclination, be the one in a thinner, the other in a thicker medium, the sine of the angle of their inclination will be a mean proportional between the two sines of their angles refracted.

For let the strait line A B (in fig. 3) have its inclination in the thinner medium, and be refracted in the thicker medium in B I ; and let E B have as much inclination in the thicker medium, and be refracted in the thinner medium in B S ; and let R S, the sine of the angle refracted, be drawn. I say, the strait lines R S, A F, and I K are in continual proportion. For it is, as the density of the thicker medium to the density of the thinner medium, so R S to A F. But it is also as the density of the same thicker medium to that of the same thinner medium, so A F to I K. Wherefore R S. A F : : A F. I K are proportionals ; that is, R S, A F, and I K are in continual proportion, and A F is the mean proportional ; which was to be proved.

7. If the angle of inclination be semirect, and the line of inclination be in the thicker medium, and the proportion of the densities be as that of a diagonal to the side of its square, and the separating superficies be plain, the refracted line will be in that separating superficies.

For in the circle A C (fig. 4) let the angle of inclination A B C be an angle of 45 degrees. Let C B be produced to the circumference in D ; and let C E, the sine of the angle E B C, be drawn, to which let B F be taken equal in the separating line B G. B C E F will therefore be a parallelogram, and F E and B C, that is F E and B G equal. Let A G be drawn, namely the diagonal of the square whose side is B G, and it will be, as A G to E F so B G to B F ; and so, by supposition, the density of the medium, in which C is, to the density of the medium in which D is; and so also the sine of the angle refracted to the sine of the angle of inclination.   Drawing therefore F D, and from D the line D H perpendicular to A B produced, D H will be the sine of the angle of inclination. And seeing the sine of the angle refracted is to the sine of the angle of inclination, as the density of the medium, in which is C, is to the density of the medium in which is D, that is, by supposition, as A G is to F E, that is as B G is to D H ; and seeing D H is the sine of the angle of inclination, B G will therefore be the sine of the angle refracted.   Wherefore B G will be the refracted line, and lye in the plain separating superficies ; which was to be demonstrated.

Coroll. It is therefore manifest, that when the inclination is greater than 45 degrees, as also

PART III.
21.

If the angle of inclination be semirect, and the line of inclination be in the thicker medium, and the proportion of their densities be the same with that of the diagonal to the side of a square, and the separating superficies be plain, the refracted line will be in the separating superficies.

when it is less, provided the density be greater, it may happen that the refraction will not enter the thinner medium at all.

If a body be carried in a strait line upon another body, and do not penetrate it,but be reflected from it, the angle of reflection will be equal to the angle of incidence.

8. If a body fall in a strait line upon another body, and do not penetrate it, but be reflected from it, the angle of reflection will be equal to the angle of incidence.

Let there be a body at A (in fig. 5), which falling with strait motion in the line A C upon another body at C, passeth no further, but is reflected; and let the angle of incidence be any angle, as A C D. Let the strait line C E be drawn, making with D C produced the angle E C F equal to the angle A C D; and let A D be drawn perpendicular to the strait line D F. Also in the same strait line D F let C G be taken equal to C D; and let the perpendicular G E be raised, cutting C E in E. This being done, the triangles A C D and E C G will be equal and like. Let C H be drawn equal and parallel to the strait line A D; and let H C be produced indefinitely to I. Lastly let E A be drawn, which will pass through H, and be parallel and equal to G D. I say the motion from A to C, in the strait line of incidence A C, will be reflected in the strait line C E.

For the motion from A to C is made by two co-efficient or concurrent motions, the one in A H parallel to D G, the other in A D perpendicular to the same D G; of which two motions that in A H works nothing upon the body A after it has been moved as far as C, because, by supposition, it doth not pass the strait line D G; whereas the endeavour in A D, that is in H C, worketh further towards I. But seeing it doth only press and not

penetrate, there will be reaction in H, which causeth motion from C towards H ; and in the meantime the motion in H E remains the same it was in A H ; and therefore the body will now be moved by the concourse of two motions in C H and H E, which are equal to the two motions it had formerly in A H and H C. Wherefore it will be carried on in C E. The angle therefore of reflection will be E C G, equal, by construction, to the angle A C D ; which was to be demonstrated.

Now when the body is considered but as a point, it is all one whether the superficies or line in which the reflection is made be strait or crooked ; for the point of incidence and reflection C is as well in the crooked line which toucheth D G in C, as in D G itself.

9. But if we suppose that not a body be moved, but some endeavour only be propagated from A to C, the demonstration will nevertheless be the same. For all endeavour is motion ; and when it hath reached the solid body in C, it presseth it, and endeavoureth further in C I. Wherefore the reaction will proceed in C H ; and the endeavour in C H concurring with the endeavour in H E, will generate the endeavour in C E, in the same manner as in the repercussion of bodies moved.

If therefore endeavour be propagated from any point to the concave superficies of a spherical body, the reflected line with the circumference of a great circle in the same sphere will make an angle equal to the angle of incidence.

For if endeavour be propagated from A (in fig. 6) to the circumference in B, and the centre of the sphere be C, and the line C B be drawn, as

also the tangent D B E; and lastly if the angle F B D be made equal to the angle A B E, the reflection will be made in the line B F, as hath been newly shown. Wherefore the angles, which the strait lines A B and F B make with the circumference, will also be equal. But it is here to be noted, that if C B be produced howsoever to G, the endeavour in the line G B C will proceed only from the perpendicular reaction in G B; and that therefore there will be no other endeavour in the point B towards the parts which are within the sphere, besides that which tends towards the centre.

And here I put an end to the third part of this discourse; in which I have considered motion and magnitude by themselves in the abstract. The fourth and last part, concerning the *phenomena of nature*, that is to say, concerning the motions and magnitudes of the bodies which are parts of the world, real and existent, is that which follows.

PART IV.

PHYSICS,

OR THE PHENOMENA OF NATURE.

CHAPTER XXV.

OF SENSE AND ANIMAL MOTION.

1. I HAVE, in the first chapter, defined philosophy
to be *knowledge of effects acquired by true ratio-
cination, from knowledge first had of their causes
and generation; and of such causes or genera-
tions as may be, from former knowledge of their
effects or appearances.* There are, therefore,
two methods of philosophy; one, from the gene-
ration of things to their possible effects; and the

PART IV.
25.

The connexion
of what hath
been said with
that which
followeth.

c c 2

other, from their effects or appearances to some
possible generation of the same. In the former
of these the truth of the first principles of our
ratiocination, namely definitions, is made and con-
stituted by ourselves, whilst we consent and agree
about the appellations of things. And this part I
have finished in the foregoing chapters; in which,
if I am not deceived, I have affirmed nothing,
saving the definitions themselves, which hath not
good coherence with the definitions I have given ;
that is to say, which is not sufficiently demonstrated
to all those, that agree with me in the use of words
and appellations ; for whose sake only I have
written the same. I now enter upon the other
part ; which is the finding out by the appearances
or effects of nature, which we know by sense, some
ways and means by which they may be, I do not
say they are, generated. The principles, therefore,
upon which the following discourse depends, are not
such as we ourselves make and pronounce in gene-
ral terms, as definitions ; but such, as being placed
in the things themselves by the Author of Nature,
are by us observed in them ; and we make use of
them in single and particular, not universal propo-
sitions. Nor do they impose upon us any necessity
of constituting theorems ; their use being only,
though not without such general propositions as
have been already demonstrated, to show us the
possibility of some production or generation. See-
ing, therefore, the science, which is here taught,
hath its principles in the appearances of nature,
and endeth in the attaining of some knowledge of
natural causes, I have given to this part the title
of PHYSICS, or the *Phenomena of Nature.* Now

such things as appear, or are shown to us by na-
ture, we call phenomena or appearances.

Of all the phenomena or appearances which are
near us, the most admirable is apparition itself,
τὸ φαίνεσθαι; namely, that some natural bodies have
in themselves the patterns almost of all things, and
others of none at all.   So that if the appearances
be the principles by which we know all other
things, we must needs acknowledge sense to be the
principle by which we know those principles, and
that all the knowledge we have is derived from it.
And as for the causes of sense, we cannot begin
our search of them from any other phenomenon
than that of sense itself.   But you will say, by what
sense shall we take notice of sense?   I answer, by
sense itself, namely, by the memory which for some
time remains in us of things sensible, though they
themselves pass away.   For he that perceives that
he hath perceived, remembers.

In the first place, therefore, the causes of our
perception, that is, the causes of those ideas and
phantasms which are perpetually generated within us
whilst we make use of our senses, are to be enquired
into ; and in what manner their generation pro-
ceeds.   To help which inquisition, we may observ
first of all, that our phantasms or ideas are not
always the same ; but that new ones appear to us,
and old ones vanish, according as we apply our
organs of sense, now to one object, now to another.
Wherefore they are generated, and perish.   And
from hence it is manifest, that they are some
change or mutation in the sentient.

2. Now that all mutation or alteration is mo-
tion or endeavour (and endeavour also is motion)

PART IV.
25.

The investiga-
tion of the na-
ture of sense,
and the defini-
tion of sense.

in the internal parts of the thing that is altered, hath been proved (in art. 9, chap. VIII) from this, that whilst even the least parts of any body remain in the same situation in respect of one another, it cannot be said that any alteration, unless perhaps that the whole body together hath been moved, hath happened to it ; but that it both appeareth and is the same it appeared and was before. Sense, therefore, in the sentient, can be nothing else but motion in some of the internal parts of the sentient ; and the parts so moved are parts of the organs of sense. For the parts of our body, by which we perceive any thing, are those we commonly call the organs of sense. And so we find what is the subject of our sense, namely, that in which are the phantasms ; and partly also we have discovered the nature of sense, namely, that it is some internal motion in the sentient.

I have shown besides (in chap. IX, art. 7) that no motion is generated but by a body contiguous and moved : from whence it is manifest, that the immediate cause of sense or perception consists in this, that the first organ of sense is touched and pressed. For when the uttermost part of the organ is pressed, it no sooner yields, but the part next within it is pressed also ; and, in this manner, the pressure or motion is propagated through all the parts of the organ to the innermost. And thus also the pressure of the uttermost part proceeds from the pressure of some more remote body, and so continually, till we come to that from which, as from its fountain, we derive the phantasm or idea that is made in us by our sense. And this, whatsoever it be, is that we commonly call *the object*,

Sense, therefore, is some internal motion in the sentient, generated by some internal motion of the parts of the object, and propagated through all the media to the innermost part of the organ. By which words I have almost defined what sense is.

Moreover, I have shown (art. 2, chap. xv) that all resistance is endeavour opposite to another endeavour, that is to say, reaction. Seeing, therefore, there is in the whole organ, by reason of its own internal natural motion, some resistance or reaction against the motion which is propagated from the object to the innermost part of the organ, there is also in the same organ an endeavour opposite to the endeavour which proceeds from the object; so that when that endeavour inwards is the last action in the act of sense, then from the reaction, how little soever the duration of it be, a phantasm or idea hath its being; which, by reason that the endeavour is now outwards, doth always appear as something situate without the organ. So that now I shall give you the whole definition of sense, as it is drawn from the explication of the causes thereof and the order of its generation, thus: SENSE *is a phantasm, made by the reaction and endeavour outwards in the organ of sense, caused by an endeavour inwards from the object, remaining for some time more or less.*

3. The *subject* of sense is the *sentient* itself, namely, some living creature; and we speak more correctly, when we say a living creature seeth, than when we say the eye seeth. The object is the thing received; and it is more accurately said, that we see the sun, than that we see the light. For light and colour, and heat and sound, and

The subject and object of sense

other qualities which are commonly called sensible, are not objects, but phantasms in the sentients. For a phantasm is the act of sense, and differs no otherwise from sense than *fieri*, that is, being a doing, differs from *factum esse*, that is, being done; which difference, in things that are done in an instant, is none at all; and a phantasm is made in an instant. For in all motion which proceeds by perpetual propagation, the first part being moved moves the second, the second the third, and so on to the last, and that to any distance, how great soever. And in what point of time the first or foremost part proceeded to the place of the second, which is thrust on, in the same point of time the last save one proceeded into the place of the last yielding part; which by reaction, in the same instant, if the reaction be strong enough, makes a phantasm; and a phantasm being made, perception is made together with it.

4. The *organs* of sense, which are in the sentient, are such parts thereof, that if they be hurt, the very generation of phantasms is thereby destroyed, though all the rest of the parts remain entire. Now these parts in the most of living creatures are found to be certain spirits and membranes, which, proceeding from the *pia mater*, involve the brain and all the nerves; also the brain itself, and the arteries which are in the brain; and such other parts, as being stirred, the heart also, which is the fountain of all sense, is stirred together with them. For whensoever the action of the object reacheth the body of the sentient, that action is by some nerve propagated to the brain; and if the nerve leading thither be

so hurt or obstructed, that the motion can be propagated no further, no sense follows. Also if the motion be intercepted between the brain and the heart by the defect of the organ by which the action is propagated, there will be no perception of the object.

5. But though all sense, as I have said, be made by reaction, nevertheless it is not necessary that every thing that reacteth should have sense. I know there have been philosophers, and those learned men, who have maintained that all bodies are endued with sense. Nor do I see how they can be refuted, if the nature of sense be placed in reaction only. And, though by the reaction of bodies inanimate a phantasm might be made, it would nevertheless cease, as soon as ever the object were removed. For unless those bodies had organs, as living creatures have, fit for the retaining of such motion as is made in them, their sense would be such, as that they should never remember the same. And therefore this hath nothing to do with that sense which is the subject of my discourse. For by sense, we commonly understand the judgment we make of objects by their phantasms; namely, by comparing and distinguishing those phantasms; which we could never do, if that motion in the organ, by which the phantasm is made, did not remain there for some time, and make the same phantasm return. Wherefore sense, as I here understand it, and which is commonly so called, hath necessarily some memory adhering to it, by which former and later phantasms may be compared together, and distinguished from one another.

Sense, therefore, properly so called, must necessarily have in it a perpetual variety of phantasms, that they may be discerned one from another. For if we should suppose a man to be made with clear eyes, and all the rest of his organs of sight well disposed, but endued with no other sense; and that he should look only upon one thing, which is always of the same colour and figure, without the least appearance of variety, he would seem to me, whatsoever others may say, to see, no more than I seem to myself to feel the bones of my own limbs by my organs of feeling; and yet those bones are always and on all sides touched by a most sensible membrane. I might perhaps say he were astonished, and looked upon it; but I should not say he saw it; it being almost all one for a man to be always sensible of one and the same thing, and not to be sensible at all of any thing.

6. And yet such is the nature of sense, that it does not permit a man to discern many things at once. For seeing the nature of sense consists in motion; as long as the organs are employed about one object, they cannot be so moved by another at the same time, as to make by both their motions one sincere phantasm of each of them at once. And therefore two several phantasms will not be made by two objects working together, but only one phantasm compounded from the action of both.

Besides, as when we divide a body, we divide its place; and when we reckon many bodies, we must necessarily reckon as many places; and contrarily, as I have shown in the seventh chapter; so what number soever we say there be of times, we

must understand the same number of motions also; and as oft as we count many motions, so oft we reckon many times.   For though the object we look upon be of divers colours, yet with those divers colours it is but one varied object, and not variety of objects.

Moreover, whilst those organs which are common to all the senses, such as are those parts of every organ which proceed in men from the root of the nerves to the heart, are vehemently stirred by a strong action from some one object, they are, by reason of the contumacy which the motion, they have already, gives them against the reception of all other motion, made the less fit to receive any other impression from whatsoever other objects, to what sense soever those objects belong. And hence it is, that an earnest studying of one object, takes away the sense of all other objects for the present.   For *study* is nothing else but a possession of the mind, that is to say, a vehement motion made by some one object in the organs of sense, which are stupid to all other motions as long as this lasteth ; according to what was said by Terence, " *Populus studio stupidus in funambulo animum occuparat.*"   For what is *stupor* but that which the Greeks call ἀναισθησία, that is, a cessation from the sense of other things ?   Wherefore at one and the same time, we cannot by sense perceive more than one single object ; as in reading, we see the letters successively one by one, and not all together, though the whole page be presented to our eye ; and though every several letter be distinctly written there, yet when we look upon the whole page at once, we read nothing.

From hence it is manifest, that every endeavour of the organ outwards, is not to be called sense, but that only, which at several times is by vehemence made stronger and more predominant than the rest ; which deprives us of the sense of other phantasms, no otherwise than the sun deprives the rest of the stars of light, not by hindering their action, but by obscuring and hiding them with his excess of brightness.

Imagination, the remains of past sense ; which also is memory. Of sleep.

7. But the motion of the organ, by which a phantasm is made, is not commonly called sense, except the object be present. And the phantasm remaining after the object is removed or past by, is called *fancy*, and in Latin *imaginatio ;* which word, because all phantasms are not images, doth not fully answer the signification of the word *fancy* in its general acceptation. Nevertheless I may use it safely enough, by understanding it for the Greek Φαντασία.

IMAGINATION therefore is nothing else but *sense decaying*, or *weakened*, by the absence of the object. But what may be the cause of this decay or weakening ? Is the motion the weaker, because the object is taken away ? If it were, then phantasms would always and necessarily be less clear in the imagination, than they are in sense ; which is not true. For in dreams, which are the imaginations of those that sleep, they are no less clear than in sense itself. But the reason why in men waking the phantasms of things past are more obscure than those of things present, is this, that their organs being at the same time moved by other present objects, those phantasms are the less predominant. Whereas in sleep, the passages

being shut up, external action doth not at all
disturb or hinder internal motion.

If this be true, the next thing to be considered, will be, whether any cause may be found out, from the supposition whereof it will follow, that the passage is shut up from the external objects of sense to the internal organ. I suppose, therefore, that by the continual action of objects, to which a reaction of the organ, and more especially of the spirits, is necessarily consequent, the organ is wearied, that is, its parts are no longer moved by the spirits without some pain ; and consequently the nerves being abandoned and grown slack, they retire to their fountain, which is the cavity either of the brain or of the heart; by which means the action which proceeded by the nerves is necessarily intercepted. For action upon a patient, that retires from it, makes but little impression at the first; and at last, when the nerves are by little and little slackened, none at all. And therefore there is no more reaction, that is, no more sense, till the organ being refreshed by rest, and by a supply of new spirits recovering strength and motion, the sentient awaketh. And thus it seems to be always, unless some other preternatural cause intervene ; as heat in the internal parts from lassitude, or from some disease stirring the spirits and other parts of the organ in some extraordinary manner.

8. Now it is not without cause, nor so casual a thing as many perhaps think it, that phantasms in this their great variety proceed from one another; and that the same phantasms sometimes bring into the mind other phantasms like themselves, and at

other times extremely unlike. For in the motion of any continued body, one part follows another by cohesion ; and therefore, whilst we turn our eyes and other organs successively to many objects, the motion which was made by every one of them remaining, the phantasms are renewed as often as any one of those motions comes to be predominant above the rest ; and they become predominant in the same order in which at any time formerly they were generated by sense. So that when by length of time very many phantasms have been generated within us by sense, then almost any thought may arise from any other thought ; insomuch that it may seem to be a thing indifferent and casual, which thought shall follow which. But for the most part this is not so uncertain a thing to waking as to sleeping men. For the thought or phantasm of the desired end brings in all the phantasms, that are means conducing to that end, and that in order backwards from the last to the first, and again forwards from the beginning to the end. But this supposes both appetite, and judgment to discern what means conduce to the end, which is gotten by experience ; and experience is store of phantasms, arising from the sense of very many things. For φαντάζεσθαι and *meminisse, fancy* and *memory*, differ only in this, that memory supposeth the time past, which fancy doth not. In memory, the phantasms we consider are as if they were worn out with time ; but in our fancy we consider them as they are ; which distinction is not of the things themselves, but of the considerations of the sentient. For there is in memory something like that which happens in looking upon things at a great

distance ; in which as the small parts of the object
are not discerned, by reason of their remoteness; so
in memory, many accidents and places and parts
of things, which were formerly perceived by sense,
are by length of time decayed and lost.

The perpetual arising of phantasms, both in
sense and imagination, is that which we commonly
call discourse of the mind, and is common to men
with other living creatures.   For he that thinketh,
compareth the phantasms that pass, that is, taketh
notice of their likeness or unlikeness to one an-
other.   And as he that observes readily the like-
nesses of things of different natures, or that are
very remote from one another, is said to have a
good fancy ; so he is said to have a good judgment,
that finds out the unlikenesses or differences of
things that are like one another.   Now this obser-
vation of differences is not perception made by a
common organ of sense, distinct from sense or
perception properly so called, but is memory of the
differences of particular phantasms remaining for
some time ; as the distinction between hot and
lucid, is nothing else but the memory both of a
heating, and of an enlightening object.

9. The phantasms of men that sleep, are *dreams*.
Concerning which we are taught by experience
these five things.   First, that for the most part
there is neither order nor coherence in them.
Secondly, that we dream of nothing but what is
compounded and made up of the phantasms of
sense past.   Thirdly, that sometimes they proceed,
as in those that are drowsy, from the interruption
of their phantasms by little and little, broken and
altered through sleepiness ; and sometimes also

Dreams,
whence they
proceed.

they begin in the midst of sleep. Fourthly, that they are clearer than the imaginations of waking men, except such as are made by sense itself, to which they are equal in clearness. Fifthly, that when we dream, we admire neither the places nor the looks of the things that appear to us. Now from what hath been said, it is not hard to show what may be the causes of these phenomena. For as for the first, seeing all order and coherence proceeds from frequent looking back to the end, that is, from consultation ; it must needs be, that seeing in sleep we lose all thought of the end, our phantasms succeed one another, not in that order which tends to any end, but as it happeneth, and in such manner, as objects present themselves to our eyes when we look indifferently upon all things before us, and see them, not because we would see them, but because we do not shut our eyes ; for then they appear to us without any order at all. The second proceeds from this, that in the silence of sense there is no new motion from the objects, and therefore no new phantasm, unless we call that new, which is compounded of old ones, as a chimera, a golden mountain, and the like. As for the third, why a dream is sometimes as it were the continuation of sense, made up of broken phantasms, as in men distempered with sickness, the reason is manifestly this, that in some of the organs sense remains, and in others it faileth. But how some phantasms may be revived, when all the exterior organs are benumbed with sleep, is not so easily shown. Nevertheless that, which hath already been said, contains the reason of this also. For whatsoever strikes the *pia mater*, reviveth

some of those phantasms that are still in motion
in the brain; and when any internal motion of the
heart reacheth that membrane, then the predomi-
nant motion in the brain makes the phantasm.
Now the motions of the heart are appetites and
aversions, of which I shall presently speak further.
And as appetites and aversions are generated by
phantasms, so reciprocally phantasms are gene‐
rated by appetites and aversions. For example,
heat in the heart proceeds from anger and fight-
ing; and again, from heat in the heart, whatsoever
be the cause of it, is generated anger and the
image of an enemy, in sleep. And as love and
beauty stir up heat in certain organs; so heat in
the same organs, from whatsoever it proceeds,
often causeth desire and the image of an unresist-
ing beauty. Lastly, cold doth in the same manner
generate fear in those that sleep, and causeth them
to dream of ghosts, and to have phantasms of
horror and danger; as fear also causeth cold in
those that wake. So reciprocal are the motions
of the heart and brain. The fourth, namely, that
the things we seem to see and feel in sleep, are as
clear as in sense itself, proceeds from two causes;
one, that having then no sense of things without
us, that internal motion which makes the phan-
tasm, in the absence of all other impressions, is
predominant; and the other, that the parts of our
phantasms which are decayed and worn out by
time, are made up with other fictitious parts. To
conclude, when we dream, we do not wonder at
strange places and the appearances of things un-
known to us, because admiration requires that the
things appearing be new and unusual, which can

happen to none but those that remember former appearances; whereas in sleep, all things appear as present.

But it is here to be observed, that certain dreams, especially such as some men have when they are between sleeping and waking, and such as happen to those that have no knowledge of the nature of dreams and are withal superstitious, were not heretofore nor are now accounted dreams.   For the apparitions men thought they saw, and the voices they thought they heard in sleep, were not believed to be phantasms, but things subsisting of themselves, and objects without those that dreamed. For to some men, as well sleeping as waking, but especially to guilty men, and in the night, and in hallowed places, fear alone, helped a little with the stories of such apparitions, hath raised in their minds terrible phantasms, which have been and are still deceitfully received for things really true, under the names of *ghosts* and *incorporeal substances*.

Of the senses, their kinds, their organs and phantasms, proper and common.

10. In most living creatures there are observed five kinds of senses, which are distinguished by their organs, and by their different kinds of phantasms ; namely, *sight, hearing, smell, taste,* and *touch ;* and these have their organs partly peculiar to each of them severally, and partly common to them all. The organ of sight is partly animate, and partly inanimate.   The inanimate parts are the three humours ; namely, the watery humour, which by the interposition of the membrane called uvea, the perforation whereof is called the apple of the eye, is contained on one side by the first concave superficies of the eye, and on the other side by the

ciliary processes, and the coat of the crystalline humour; the crystalline, which, hanging in the midst between the ciliary processes, and being almost of spherical figure, and of a thick consistence, is enclosed on all sides with its own transparent coat; and the vitreous or glassy humour, which filleth all the rest of the cavity of the eye, and is somewhat thicker then the watery humour, but thinner than the crystalline. The animate part of the organ is, first, the membrane *choroeides*, which is a part of the *pia mater*, saving that it is covered with a coat derived from the marrow of the optic nerve, which is called the *retina ;* and this *choroeides*, seeing it is part of the *pia mater*, is continued to the beginning of the *medulla spinalis* within the scull, in which all the nerves which are within the head have their roots.   Wherefore all the animal spirits that the nerves receive, enter into them there ; for it is not imaginable that they can enter into them anywhere else.   Seeing therefore sense is nothing else but the action of objects propagated to the furthest part of the organ ; and seeing also that animal spirits are nothing but vital spirits purified by the heart, and carried from it by the arteries; it follows necessarily, that the action is derived from the heart by some of the arteries to the roots of the nerves which are in the head, whether those arteries be the *plexus retiformis*, or whether they be other arteries which are inserted into the substance of the brain.   And, therefore, those arteries are the complement or the remaining part of the whole organ of sight.   And this last part is a common organ to all the senses; whereas, that which reacheth from the eye to the

roots of the nerves is proper only to sight. The proper organ of hearing is the tympanum of the ear and its own nerve; from which to the heart the organ is common. So the proper organs of smell and taste are nervous membranes, in the palate and tongue for the taste, and in the nostrils for the smell; and from the roots of those nerves to the heart all is common. Lastly, the proper organ of touch are nerves and membranes dispersed through the whole body; which membranes are derived from the root of the nerves. And all things else belonging alike to all the senses seem to be administered by the arteries, and not by the nerves.

The proper phantasm of sight is light; and under this name of light, colour also, which is nothing but perturbed light, is comprehended. Wherefore the phantasm of a lucid body is light; and of a coloured body, colour. But the object of sight, properly so called, is neither light nor colour, but the body itself which is lucid, or enlightened, or coloured. For light and colour, being phantasms of the sentient, cannot be accidents of the object. Which is manifest enough from this, that visible things appear oftentimes in places in which we know assuredly they are not, and that in different places they are of different colours, and may at one and the same time appear in divers places. Motion, rest, magnitude, and figure, are common both to the sight and touch; and the whole appearance together of figure, and light or colour, is by the Greeks commonly called εἶδος, and εἴδωλον, and ἰδέα; and by the Latins, *species* and

imago ; all which names signify no more but ap-
pearance.

The phantasm, which is made by hearing, is sound ; by smell, odour ; by taste, savour ; and by touch, hardness and softness, heat and cold, wetness, oiliness, and many more, which are easier to be distinguished by sense than words.   Smoothness, roughness, rarity, and density, refer to figure, and are therefore common both to touch and sight. And as for the objects of hearing, smell, taste, and touch, they are not sound, odour, savour, hardness, &c., but the bodies themselves from which sound, odour, savour, hardness, &c. proceed ; of the causes of which, and of the manner how they are produced, I shall speak hereafter.

But these phantasms, though they be effects in the sentient, as subject, produced by objects working upon the organs ; yet there are also other effects besides these, produced by the same objects in the same organs ; namely certain motions proceeding from sense, which are called *animal motions.*   For seeing in all sense of external things there is mutual action and reaction, that is, two endeavours opposing one another, it is manifest that the motion of both of them together will be continued every way, especially to the confines of both the bodies.   And when this happens in the internal organ, the endeavour outwards will proceed in a solid angle, which will be greater, and consequently the idea greater, than it would have been if the impression had been weaker.

11. From hence the natural cause is manifest, first, why those things seem to be greater, which, *cæteris paribus,* are seen in a greater angle :

secondly, why in a serene cold night, when the moon doth not shine, more of the fixed stars appear than at another time. For their action is less hindered by the serenity of the air, and not obscured by the greater light of the moon, which is then absent; and the cold, making the air more pressing, helpeth or strengtheneth the action of the stars upon our eyes; in so much as stars may then be seen which are seen at no other time. And this may suffice to be said in general concerning sense made by the reaction of the organ. For, as for the place of the image, the deceptions of sight, and other things of which we have experience in ourselves by sense, seeing they depend for the most part upon the fabric itself of the eye of man, I shall speak of them then when I come to speak of man.

Pleasure, pain, appetite, and aversion, what they are.
12. But there is another kind of sense, of which I will say something in this place, namely, the sense of pleasure and pain, proceeding not from the reaction of the heart outwards, but from continual action from the outermost part of the organ towards the heart. For the original of life being in the heart, that motion in the sentient, which is propagated to the heart, must necessarily make some alteration or diversion of vital motion, namely, by quickening or slackening, helping or hindering the same. Now when it helpeth, it is pleasure; and when it hindereth, it is pain, trouble, grief, &c. And as phantasms seem to be without, by reason of the endeavour outwards, so pleasure and pain, by reason of the endeavour of the organ inwards, seem to be within; namely, there where the first cause of the pleasure or pain is; as when the

pain proceeds from a wound, we think the pain
and the wound are both in the same place.

Now vital motion is the motion of the blood, perpetually circulating (as hath been shown from many infallible signs and marks by Doctor Harvey, the first observer of it) in the veins and arteries. Which motion, when it is hindered by some other motion made by the action of sensible objects, may be restored again either by bending or setting strait the parts of the body ; which is done when the spirits are carried now into these, now into other nerves, till the pain, as far as is possible, be quite taken away.   But if vital motion be helped by motion made by sense, then the parts of the organ will be disposed to guide the spirits in such manner as conduceth most to the preservation and augmentation of that motion, by the help of the nerves.   And in animal motion this is the very first endeavour, and found even in the embryo ; which while it is in the womb, moveth its limbs with voluntary motion, for the avoiding of what-soever troubleth it, or for the pursuing of what pleaseth it.   And this first endeavour, when it tends towards such things as are known by expe-rience to be pleasant, is called *appetite*, that is, an approaching ; and when it shuns what is trouble-some, *aversion*, or flying from it.   And little in-fants, at the beginning and as soon as they are born, have appetite to very few things, as also they avoid very few, by reason of their want of experi-ence and memory ; and therefore they have not so great a variety of animal motion as we see in those that are more grown.   For it is not possible, with-

PART IV.
25.

out such knowledge as is derived from sense, that is, without experience and memory, to know what will prove pleasant or hurtful ; only there is some place for conjecture from the looks or aspects of things. And hence it is, that though they do not know what may do them good or harm, yet sometimes they approach and sometimes retire from the same thing, as their doubt prompts them. But afterwards, by accustoming themselves by little and little, they come to know readily what is to be pursued and what to be avoided ; and also to have a ready use of their nerves and other organs, in the pursuing and avoiding of good and bad. Wherefore appetite and aversion are the first endeavours of animal motion.

Consequent to this first endeavour, is the impulsion into the nerves and retraction again of animal spirits, of which it is necessary there be some receptacle or place near the original of the nerves ; and this motion or endeavour is followed by a swelling and relaxation of the muscles; and lastly, these are followed by contraction and extension of the limbs, which is animal motion.

Deliberation and will, what.

13. The considerations of appetites and aversions are divers. For seeing living creatures have sometimes appetite and sometimes aversion to the same thing, as they think it will either be for their good or their hurt; while that vicissitude of appetites and aversions remains in them, they have that series of thoughts which is called *deliberation;* which lasteth as long as they have it in their power to obtain that which pleaseth, or to avoid that which displeaseth them. Appetite, therefore, and aversion are simply so called as long as they follow

not deliberation. But if deliberation have gone
before, then the last act of it, if it be appetite, is
called *will;* if aversion, *unwillingness.* So that
the same thing is called both will and appetite;
but the consideration of them, namely, before and
after deliberation, is divers. Nor is that which is
done within a man whilst he willeth any thing,
different from that which is done in other living
creatures, whilst, deliberation having preceded,
they have appetite.

Neither is the freedom of willing or not willing,
greater in man, than in other living creatures. For
where there is appetite, the entire cause of appetite
hath preceded ; and, consequently, the act of
appetite could not choose but follow, that is, hath
of necessity followed (as is shown in chapter IX,
article 5). And therefore such a liberty as is free
from necessity, is not to be found in the will either
of men or beasts. But if by liberty we understand
the faculty or power, not of willing, but of doing
what they will, then certainly that liberty is to be
allowed to both, and both may equally have it,
whensoever it is to be had.

Again, when appetite and aversion do with cele-
rity succeed one another, the whole series made by
them hath its name sometimes from one, some-
times from the other. For the same deliberation,
whilst it inclines sometimes to one, sometimes to
the other, is from appetite called *hope,* and from
aversion, *fear.* For where there is no hope, it is
not to be called fear, but *hate ;* and where no fear,
not hope, but *desire.* To conclude, all the passions,
called passions of the mind, consist of appetite and
aversion, except pure pleasure and pain, which are

a certain fruition of good or evil ; as anger is aversion from some imminent evil, but such as is joined with appetite of avoiding that evil by force. But because the passions and perturbations of the mind are innumerable, and many of them not to be discerned in any creatures besides men; I will speak of them more at large in that section which is concerning *man*. As for those objects, if there be any such, which do not at all stir the mind, we are said to contemn them.

And thus much of sense in general. In the next place I shall speak of sensible objects.

CHAPTER XXVI.

OF THE WORLD AND OF THE STARS.

1. The magnitude and duration of the world, inscrutable.—2. No place in the world empty.—3. The arguments of Lucretius for vacuum, invalid.—4. Other arguments for the establishing of vacuum, invalid.—5. Six suppositions for the salving of the phenomena of nature.—6. Possible causes of the motions annual and diurnal; and of the apparent direction, station, and retrogradation of the planets.—7. The supposition of simple motion, why likely.—8. The cause of the eccentricity of the annual motion of the earth.—9. The cause why the moon hath always one and the same face turned towards the earth. 10. The cause of the tides of the ocean.—11. The cause of the precession of the equinoxes.

The magnitude
and duration of
the world, in-
scrutable. 1. CONSEQUENT to the contemplation of sense is the contemplation of bodies, which are the efficient causes or objects of sense. Now every object is either a part of the whole world, or an aggregate of parts. The greatest of all bodies, or sensible objects, is the world itself; which we behold when

we look round about us from this point of the same
which we call the earth.   Concerning the world,
as it is one aggregate of many parts, the things
that fall under inquiry are but few ; and those we
can determine, none.   Of the whole world we may
inquire what is its magnitude, what its duration,
and how many there be, but nothing else.   For as
for place and time, that is to say, magnitude and
duration, they are only our own fancy of a body
simply so called, that is to say, of a body indefi-
nitely taken, as I have shown before in chapter VII.
All other phantasms are of bodies or objects, as
they are distinguished from one another ; as colour,
the phantasm of coloured bodies ; sound, of bodies
that move the sense of hearing, &c.   The questions
concerning the magnitude of the world are whether
it be finite or infinite, full or not full ; concerning
its duration, whether it had a beginning, or be
eternal; and concerning the number, whether there
be one or many; though as concerning the num-
ber, if it were of infinite magnitude, there could
be no controversy at all.   Also if it had a begin-
ning, then by what cause and of what matter it was
made ; and again, from whence that cause and
that matter had their being, will be new questions ;
till at last we come to one or many eternal cause
or causes.   And the determination of all these
things belongeth to him that professeth the uni-
versal doctrine of philosophy, in case as much
could be known as can be sought.   But the know-
ledge of what is infinite can never be attained by a
finite inquirer. Whatsoever we know that are men,
we learn it from our phantasms ; and of infinite,
whether magnitude or time, there is no phantasm

PART IV.
26.

The magnitude
and duration of
the world, in-
scrutable.

at all; so that it is impossible either for a man or any other creature to have any conception of infinite. And though a man may from some effect proceed to the immediate cause thereof, and from that to a more remote cause, and so ascend continually by right ratiocination from cause to cause; yet he will not be able to proceed eternally, but wearied will at last give over, without knowing whether it were possible for him to proceed to an end or not. But whether we suppose the world to be finite or infinite, no absurdity will follow. For the same things which now appear, might appear, whether the Creator had pleased it should be finite or infinite. Besides, though from this, that nothing can move itself, it may rightly be inferred that there was some first eternal movent; yet it can never be inferred, though some used to make such inference, that that movent was eternally immoveable, but rather eternally moved. For as it is true, that nothing is moved by itself; so it is true also that nothing is moved but by that which is already moved. The questions therefore about the magnitude and beginning of the world, are not to be determined by philosophers, but by those that are lawfully authorized to order the worship of God. For as Almighty God, when he had brought his people into Judæa, allowed the priests the first fruits reserved to himself; so when he had delivered up the world to the disputations of men, it was his pleasure that all opinions concerning the nature of infinite and eternal, known only to himself, should, as the first fruits of wisdom, be judged by those whose ministry he meant to use in the ordering of religion. I cannot therefore commend those that

boast they have demonstrated, by reasons drawn from natural things, that the world had a beginning. They are contemned by idiots, because they understand them not; and by the learned, because they understand them; by both deservedly. For who can commend him that demonstrates thus? "If the world be eternal, then an infinite number of days, or other measures of time, preceded the birth of Abraham. But the birth of Abraham preceded the birth of Isaac; and therefore one infinite is greater than another infinite, or one eternal than another eternal; which," he says, "is absurd." This demonstration is like his, who from this, that the number of even numbers is infinite, would conclude that there are as many even numbers as there are numbers simply, that is to say, the even numbers are as many as all the even and odd together. They, which in this manner take away eternity from the world, do they not by the same means take away eternity from the Creator of the world? From this absurdity therefore they run into another, being forced to call eternity *nunc stans*, a standing still of the present time, or an abiding now; and, which is much more absurd, to give to the infinite number of numbers the name of unity. But why should eternity be called an abiding now, rather than an abiding then? Wherefore there must either be many eternities, or *now* and *then* must signify the same. With such demonstrators as these, that speak in another language, it is impossible to enter into disputation. And the men, that reason thus absurdly, are not idiots, but, which makes the absurdity unpardonable, geometricians, and such as take upon them to be judges,

PART IV.
26.

The magnitude
and duration of
the world, in-
scrutable.

impertinent, but severe judges of other men's demonstrations. The reason is this, that as soon as they are entangled in the words *infinite* and *eternal*, of which we have in our mind no idea, but that of our own insufficiency to comprehend them, they are forced either to speak something absurd, or, which they love worse, to hold their peace. For geometry hath in it somewhat like wine, which, when new, is windy; but afterwards though less pleasant, yet more wholesome. Whatsoever therefore is true, young geometricians think demonstrable; but elder not. Wherefore I purposely pass over the questions of infinite and eternal; contenting myself with that doctrine concerning the beginning and magnitude of the world, which I have been persuaded to by the holy Scriptures and fame of the miracles which confirm them; and by the custom of my country, and reverence due to the laws. And so I pass on to such things as it is not unlawful to dispute of.

No place in the world empty.

2. Concerning the world it is further questioned, whether the parts thereof be contiguous to one another, in such manner as not to admit of the least empty space between; and the disputation both for and against it is carried on with probability enough. For the taking away of vacuum, I will instance in only one experiment, a common one, but I think unanswerable.

Let A B (in fig. 1) represent a vessel, such as gardeners use to water their gardens withal; whose bottom B is full of little holes; and whose mouth A may be stopped with one's finger, when there shall be need. If now this vessel be filled with water, the hole at the top A being stopped, the

water will not flow out at any of the holes in the
·bottom B. But if the finger be removed to let in
the air above, it will run out at them all; and as
soon as the finger is applied to it again, the water
will suddenly and totally be stayed again from
running out. The cause whereof seems to be no
other but this, that the water cannot by its natural
endeavour to descend drive down the air below it,
because there is no place for it to go into, unless
either by thrusting away the next contiguous air,
it proceed by continual endeavour to the hole A,
where it may enter and succeed into the place of
the water that floweth out, or else, by resisting the
endeavour of the water downwards, penetrate the
same and pass up through it. By the first of these
ways, while the hole at A remains stopped, there
is no possible passage; nor by the second, unless
the holes be so great that the water, flowing out
at them, can by its own weight force the air at the
same time to ascend into the vessel by the same
holes : as we see it does in a vessel whose mouth
is wide enough, when we turn suddenly the bottom
upwards to pour out the water ; for then the air
being forced by the weight of the water, enters, as
is evident by the sobbing and resistance of the
water, at the sides or circumference of the orifice.
And this I take for a sign that all space is full;
for without this, the natural motion of the water,
which is a heavy body, downwards, would not be
hindered.

3. On the contrary, for the establishing of va-
cuum, many and specious arguments and experi-
ments have been brought. Nevertheless there
seems to be something wanting in all of them to

PART IV.
26.

The arguments
of Lucretius
for vacuum
invalid.

conclude it firmly. These arguments for vacuum are partly made by the followers of the doctrine of Epicurus; who taught that the world consists of very small spaces not filled by any body, and of very small bodies that have within them no empty space, which by reason of their hardness he calls *atoms;* and that these small bodies and spaces are every where intermingled. Their arguments are thus delivered by Lucretius.

And first he says, that unless it were so, there could be no motion. For the office and property of bodies is to withstand and hinder motion. If, therefore, the universe were filled with body, motion would everywhere be hindered, so as to have no beginning anywhere; and consequently there would be no motion at all. It is true that in whatsoever is full and at rest in all its parts, it is not possible motion should have beginning. But nothing is drawn from hence for the proving of vacuum. For though it should be granted that there is vacuum, yet if the bodies which are inter-mingled with it, should all at once and together be at rest, they would never be moved again. For it has been demonstrated above, in chap. IX, art. 7, that nothing can be moved but by that which is contiguous and already moved. But supposing that all things are at rest together, there can be nothing contiguous and moved, and there-fore no beginning of motion. Now the denying of the beginning of motion, doth not take away present motion, unless beginning be taken away from body also. For motion may be either co-eternal, or concreated with body. Nor doth it seem more necessary that bodies were first at rest,

and afterwards moved, than that they were first moved, and rested, if ever they rested at all, after-wards. Neither doth there appear any cause, why the matter of the world should, for the admission of motion, be intermingled with empty spaces rather than full ; I say full, but withal fluid. Nor, lastly, is there any reason why those hard atoms may not also, by the motion of intermingled fluid matter, be congregated and brought together into compounded bodies of such bigness as we see. Wherefore nothing can by this argument be con-cluded, but that motion was either coeternal, or of the same duration with that which is moved ; neither of which conclusions consisteth with the doctrine of Epicurus, who allows neither to the world nor to motion any beginning at all. The necessity, therefore, of vacuum is not hitherto de-monstrated. And the cause, as far as I understand from them that have discoursed with me of vacuum, is this, that whilst they contemplate the nature of fluid, they conceive it to consist, as it were, of small grains of hard matter, in such manner as meal is fluid, made so by grinding of the corn ; when nevertheless it is possible to conceive fluid to be of its own nature as homogeneous as either an atom, or as vacuum itself.

The second of their arguments is taken from weight, and is contained in these verses of Lu-cretius :

> Corporis officium est quoniam premere omnia deorsum ;
> Contra autem natura manet sine pondere inanis ;
> Ergo, quod magnum est æque, leviusque videtur,
> Nimirum plus esse sibi declarat inanis.—I. 363-66.

That is to say, *seeing the office and property of*

PART IV.
26.

The arguments
of Lucretius
for vacuum
invalid.

body is to press all things downwards; and on the contrary, seeing the nature of vacuum is to have no weight at all; therefore when of two bodies of equal magnitude, one is lighter than the other, it is manifest that the lighter body hath in it more vacuum than the other.

To say nothing of the assumption concerning the endeavour of bodies downwards, which is not rightly assumed, because the world hath nothing to do with downwards, which is a mere fiction of ours ; nor of this, that if all things tended to the same lowest part of the world, either there would be no coalescence at all of bodies, or they would all be gathered together into the same place : this only is sufficient to take away the force of the argument, that air, intermingled with those his atoms, had served as well for his purpose as his intermingled vacuum.

The third argument is drawn from this, that lightning, sound, heat and cold, do penetrate all bodies, except atoms, how solid soever they be. But this reason, except it be first demonstrated that the same things cannot happen without vacuum by perpetual generation of motion, is altogether invalid.　But that all the same things may so happen, shall in due place be demonstrated.

Lastly, the fourth argument is set down by the same Lucretius in these verses :

Duo de concursu corpora lata
Si cita dissiliant, nempe aer omne necesse est,
Inter corpora quod fuerat, possidat inane.
Is porro quamvis circum celerantibus auris
Confluat, haud poterit tamen uno tempore totum
Compleri spatium ; nam primum quemque necesse est
Occupet ille locum, deinde omnia possideantur.—I. 385-91.

That is, *if two flat bodies be suddenly pulled asunder, of necessity the air must come between them to fill up the space they left empty. But with what celerity soever the air flow in, yet it cannot in one instant of time fill the whole space, but first one part of it, then successively all.* Which nevertheless is more repugnant to the opinion of Epicurus, than of those that deny vacuum. For though it be true, that if two bodies were of infinite hardness, and were joined together by their superficies which were most exactly plane, it would be impossible to pull them asunder, in regard it could not be done but by motion in an instant; yet, if as the greatest of all magnitudes cannot be given, nor the swiftest of all motions, so neither the hardest of all bodies; it might be, that by the application of very great force, there might be place made for a successive flowing in of the air, namely, by separating the parts of the joined bodies by succession, beginning at the outermost and ending at the innermost part. He ought, therefore, first to have proved, that there are some bodies extremely hard, not relatively as compared with softer bodies, but absolutely, that is to say, infinitely hard; which is not true. But if we suppose, as Epicurus doth, that atoms are indivisible, and yet have small superficies of their own; then if two bodies should be joined together by many, or but one only small superficies of either of them, then I say this argument of Lucretius would be a firm demonstration, that no two bodies made up of atoms, as he supposes, could ever possibly be pulled asunder by any force whatsoever. But this is repugnant to daily experience.

E E 2

4. And thus much of the arguments of Lucretius. Let us now consider the arguments which are drawn from the experiments of later writers.

I. The first experiment is this : that if a hollow vessel be thrust into water with the bottom upwards, the water will ascend into it ; which they say it could not do, unless the air within were thrust together into a narrower place ; and that this were also impossible, except there were little empty places in the air.   Also, that when the air is compressed to a certain degree, it can receive no further compression, its small particles not suffering themselves to be pent into less room.   This reason, if the air could not pass through the water as it ascends within the vessel, might seem valid.   But it is sufficiently known, that air will penetrate water by the application of a force equal to the gravity of the water.   If therefore the force, by which the vessel is thrust down, be greater or equal to the endeavour by which the water naturally tendeth downwards, the air will go out that way where the resistance is made, namely, towards the edges of the vessel.   For, by how much the deeper is the water which is to be penetrated, so much greater must be the depressing force.   But after the vessel is quite under water, the force by which it is depressed, that is to say, the force by which the water riseth up, is no longer increased. There is therefore such an equilibration between them, as that the natural endeavour of the water downwards is equal to the endeavour by which the same water is to be penetrated to the increased depth.

II. The second experiment is, that if a concave

cylinder of sufficient length, made of glass, that
the experiment may be the better seen, having
one end open and the other close shut, be filled
with quicksilver, and the open end being stopped
with one's finger, be together with the finger
dipped into a dish or other vessel, in which
also there is quicksilver, and the cylinder be set
upright, we shall, the finger being taken away to
make way for the descent of the quicksilver, see it
descend into the vessel under it, till there be only
so much remaining within the cylinder as may fill
about twenty-six inches of the same; and thus it
will always happen whatsoever be the cylinder,
provided that the length be not less than twenty-
six inches. From whence they conclude that the
cavity of the cylinder above the quicksilver remains
empty of all body. But in this experiment I find
no necessity at all of vacuum. For when the
quicksilver which is in the cylinder descends, the
vessel under it must needs be filled to a greater
height, and consequently so much of the conti-
guous air must be thrust away as may make place
for the quicksilver which is descended. Now if it
be asked whither that air goes, what can be an-
swered but this, that it thrusteth away the next
air, and that the next, and so successively, till
there be a return to the place where the propulsion
first began. And there, the last air thus thrust
on will press the quicksilver in the vessel with the
same force with which the first air was thrust away;
and if the force with which the quicksilver descends
be great enough, which is greater or less as it
descends from a place of greater or less height, it
will make the air penetrate the quicksilver in the

vessel, and go up into the cylinder to fill the place which they thought was left empty. But because the quicksilver hath not in every degree of height force enough to cause such penetration, therefore in descending it must of necessity stay somewhere, namely, there, where its endeavour downwards, and the resistance of the same to the penetration of the air, come to an equilibrium. And by this experiment it is manifest, that this equilibrium will be at the height of twenty-six inches, or thereabouts.

III. The third experiment is, that when a vessel hath as much air in it as it can naturally contain, there may nevertheless be forced into it as much water as will fill three quarters of the same vessel. And the experiment is made in this manner. Into the glass bottle, represented (in figure 2) by the sphere F G, whose centre is A, let the pipe B A C be so fitted, that it may precisely fill the mouth of the bottle ; and let the end B be so near the bottom, that there may be only space enough left for the free passage of the water which is thrust in above. Let the upper end of this pipe have a cover at D, with a spout at E, by which the water, when it ascends in the pipe, may run out. Also let H C be a cock, for the opening or shutting of the passage of the water between B and D, as there shall be occasion. Let the cover D E be taken off, and the cock H C being opened, let a syringe full of water be forced in ; and before the syringe be taken away, let the cock be turned to hinder the going out of the air. And in this manner let the injection of water be repeated as often as it shall be requisite, till the water rise within the bottle ; for example, to G F. Lastly, the cover being

fastened on again, and the cock H C opened, the
water will run swiftly out at E, and sink by little
and little from G F to the bottom of the pipe B. Other argu-
ments for esta-
blishing of va-
cuum invalid.

From this phenomenon, they argue for the neces-
sity of vacuum in this manner. The bottle, from
the beginning, was full of air ; which air could
neither go out by penetrating so great a length of
water as was injected by the pipe, nor by any other
way. Of necessity, therefore, all the water as high
as F G, as also all the air that was in the bottle
before the water was forced in, must now be in the
same place, which at first was filled by the air
alone ; which were impossible, if all the space
within the bottle were formerly filled with air pre-
cisely, that is, without any vacuum. Besides,
though some man perhaps may think the air, being
a thin body, may pass through the body of the
water contained in the pipe, yet from that other
phenomenon, namely, that all the water which is
in the space B F G is cast out again by the spout at
E, for which it seems impossible that any other
reason can be given besides the force by which the
air frees itself from compression, it follows, that
either there was in the bottle some space empty,
or that many bodies may be together in the same
place. But this last is absurd ; and therefore the
former is true, namely, that there was vacuum.

This argument is infirm in two places. For first,
that is assumed which is not to be granted ; and
in the second place, an experiment is brought,
which I think is repugnant to vacuum. That
which is assumed is, that the air can have no pas-
sage out through the pipe. Nevertheless, we see
daily that air easily ascends from the bottom to the

Other argu-
ments for esta-
blishing of va-
cuum invalid.

superficies of a river, as is manifest by the bubbles
that rise ; nor doth it need any other cause to give
it this motion, than the natural endeavour down-
wards of the water.   Why, therefore, may not the
endeavour upwards of the same water, acquired by
the injection, which endeavour upwards is greater
than the natural endeavour of the water down-
wards, cause the air in the bottle to penetrate in
like manner the water that presseth it downwards ;
especially, seeing the water, as it riseth in the
bottle, doth so press the air that is above it, as that
it generateth in every part thereof an endeavour
towards the external superficies of the pipe, and
consequently maketh all the parts of the enclosed
air to tend directly towards the passage at B ?   I
say, this is no less manifest, than that the air which
riseth up from the bottom of a river should pene-
trate the water, how deep soever it be. Wherefore
I do not yet see any cause why the force, by which
the water is injected, should not at the same time
eject the air.

    And as for their arguing the necessity of vacuum
from the rejection of the water ; in the first place,
supposing there is vacuum, I demand by what
principle of motion that ejection is made. Certainly,
seeing this motion is from within outwards, it must
needs be caused by some agent within the bottle ;
that is to say, by the air itself.   Now the motion
of that air, being caused by the rising of the water,
begins at the bottom, and tends upwards ; whereas
the motion by which it ejecteth the water ought to
begin above, and tend downwards.   From whence
therefore hath the enclosed air this endeavour to-
wards the bottom ?   To this question I know not

what answer can be given, unless it be said, that
the air descends of its own accord to expel the
water. Which, because it is absurd, and that the
air, after the water is forced in, hath as much room
as its magnitude requires, there will remain no
cause at all why the water should be forced out.
Wherefore the assertion of vacuum is repugnant
to the very experiment which is here brought to
establish it.

Many other phenomena are usually brought for
vacuum, as those of *weather-glasses, æolipyles,
wind-guns,* &c. which would all be very hard to be
salved, unless water be penetrable by air, without
the intermixture of empty space. But now, seeing
air may with no great endeavour pass through not
only water, but any other fluid body though never
so stubborn, as quicksilver, these phenomena prove
nothing. Nevertheless, it might in reason be
expected, that he that would take away vacuum,
should without vacuum show us such causes of
these phenomena, as should be at least of equal, if
not greater probability. This therefore shall be
done in the following discourse, when I come to
speak of these phenomena in their proper places.
But first, the most general hypotheses of natural
philosophy are to be premised.

And seeing that suppositions are put for the true
causes of apparent effects, every supposition, except
such as be absurd, must of necessity consist of
some supposed possible motion ; for rest can never
be the efficient cause of anything ; and motion sup-
poseth bodies moveable ; of which there are three
kinds, *fluid, consistent,* and *mixed of both. Fluid*
are those, whose parts may by very weak endeavour

be separated from one another; and *consistent* those for the separation of whose parts greater force is to be applied. There are therefore degrees of consistency; which degrees, by comparison with more or less consistent, have the names of *hardness* or *softness*. Wherefore a fluid body is always divisible into bodies equally fluid, as quantity into quantities; and soft bodies, of whatsoever degree of softness, into soft bodies of the same degree. And though many men seem to conceive no other difference of *fluidity*, but such as ariseth from the different magnitudes of the parts, in which sense dust, though of diamonds, may be called fluid; yet I understand by *fluidity*, that which is made such by nature equally in every part of the fluid body; not as dust is fluid, for so a house which is falling in pieces may be called fluid; but in such manner as water seems fluid, and to divide itself into parts perpetually fluid. And this being well understood, I come to my suppositions.

Six suppositions for the salving of the phenomena of nature.

5. First, therefore, I suppose that the immense space, which we call the world, is the aggregate of all bodies which are either consistent and visible, as the earth and the stars; or invisible, as the small atoms which are disseminated through the whole space between the earth and the stars; and lastly, that most fluid ether, which so fills all the rest of the universe, as that it leaves in it no empty place at all.

Secondly, I suppose with Copernicus, that the greater bodies of the world, which are both consistent and permanent, have such order amongst themselves, as that the sun hath the first place,

Mercury the second, Venus the third, the Earth with the moon going about it the fourth, Mars the fifth, Jupiter with his attendants the sixth, Saturn the seventh; and after these, the fixed stars have their several distances from the sun.

Thirdly, I suppose that in the sun and the rest of the planets there is and always has been a simple circular motion.

Fourthly, I suppose that in the body of the air there are certain other bodies intermingled, which are not fluid; but withal that they are so small, that they are not perceptible by sense; and that these also have their proper simple motion, and are some of them more, some less hard or consistent.

Fifthly, I suppose with Kepler that as the distance between the sun and the earth is to the distance between the moon and the earth, so the distance between the moon and the earth is to the semidiameter of the earth.

As for the magnitude of the circles, and the times in which they are described by the bodies which are in them, I will suppose them to be such as shall seem most agreeable to the phenomena in question.

6. The causes of the different seasons of the year, and of the several variations of days and nights in all the parts of the superficies of the earth, have been demonstrated, first by Copernicus, and since by Kepler, Galileus, and others, from the supposition of the earth's diurnal revolution about its own axis, together with its annual motion about the sun in the ecliptic according to the order of the signs; and thirdly, by the annual

Possible causes of the motions annual and diurnal; and of the apparent direction, station, and retrogradation of the planets.

PART IV.
26.

Possible causes
of the motions,
annual and
diurnal, &c.
revolution of the same earth about its own centre, contrary to the order of the signs. I suppose with Copernicus, that the diurnal revolution is from the motion of the earth, by which the equinoctial circle is described about it. And as for the other two annual motions, they are the efficient cause of the earth's being carried about in the ecliptic in such manner, as that its axis is always kept parallel to itself. Which parallelism was for this reason introduced, lest by the earth's annual revolution its poles should seem to be necessarily carried about the sun, contrary to experience. I have, in art. 10, chap. xxi, demonstrated, from the supposition of simple circular motion in the sun, that the earth is so carried about the sun, as that its axis is thereby kept always parallel to itself. Wherefore, from these two supposed motions in the sun, the one simple circular motion, the other circular motion about its own centre, it may be demonstrated that the year hath both the same variations of days and nights, as have been demonstrated by Copernicus.

For if the circle $a\,b\,c\,d$ (in fig. 3) be the ecliptic, whose centre is e, and diameter $a\,e\,c$; and the earth be placed in a, and the sun be moved in the little circle $f\,g\,h\,i$, namely, according to the order f, g, h, and i, it hath been demonstrated, that a body placed in a will be moved in the same order through the points of the ecliptic a, b, c, and d, and will always keep its axis parallel to itself.

But if, as I have supposed, the earth also be moved with simple circular motion in a plane that passeth through a, cutting the plane of the ecliptic so as that the common section of both the planes

be in $a\,c$, thus also the axis of the earth will be kept always parallel to itself. For let the centre of the earth be moved about in the circumference of the epicycle, whose diameter is $l\,a\,k$, which is a part of the strait line $l\,a\,c$; therefore $l\,a\,k$, the diameter of the epicycle, passing through the centre of the earth, will be in the plane of the ecliptic. Wherefore seeing that by reason of the earth's simple motion both in the ecliptic and in its epicycle, the strait line $l\,a\,k$ is kept always parallel to itself, every other strait line also taken in the body of the earth, and consequently its axis, will in like manner be kept always parallel to itself; so that in what part soever of the ecliptic the centre of the epicycle be found, and in what part soever of the epicycle the centre of the earth be found at the same time, the axis of the earth will be parallel to the place where the same axis would have been, if the centre of the earth had never gone out of the ecliptic.

Now as I have demonstrated the simple annual motion of the earth from the supposition of simple motion in the sun; so from the supposition of simple motion in the earth may be demonstrated the monthly simple motion of the moon. For if the names be but changed, the demonstration will be the same, and therefore need not be repeated.

7. That which makes this supposition of the sun's simple motion in the epicycle $f\,g\,h\,i$ probable, is first, that the periods of all the planets are not only described about the sun, but so described, as that they are all contained within the zodiac, that is to say, within the latitude of about sixteen degrees; for the cause of this seems to

The supposition of simple motion, why likely.

PART IV.
26.

The supposi-
tion of simple
motion, why
likely.

depend upon some power in the sun, especially in that part of the sun which respects the zodiac. Secondly, that in the whole compass of the heavens there appears no other body from which the cause of this phenomenon can in probability be derived. Besides, I could not imagine that so many and such various motions of the planets should have no dependance at all upon one another. But, by supposing motive power in the sun, we suppose motion also ; for power to move without motion is no power at all. I have therefore supposed that there is in the sun for the governing of the primary planets, and in the earth for the governing of the moon, such motion, as being received by the primary planets and by the moon, makes them necessarily appear to us in such manner as we see them. Whereas, that circular motion, which is commonly attributed to them, about a fixed axis, which is called conversion, being a motion of their parts only, and not of their whole bodies, is insufficient to salve their appearances. For seeing whatsoever is so moved, hath no endeavour at all towards those parts which are without the circle, they have no power to propagate any endeavour to such bodies as are placed without it. And as for them that suppose this may be done by magnetical virtue, or by incorporeal and immaterial species, they suppose no natural cause ; nay, no cause at all. For there is no such thing as an incorporeal movent, and magnetical virtue is a thing altogether unknown ; and whensoever it shall be known, it will be found to be a motion of body. It remains, therefore, that if the primary planets be carried about by the sun, and the moon by the earth, they

have the simple circular motions of the sun and the earth for the causes of their circulations. Otherwise, if they be not carried about by the sun and the earth, but that every planet hath been moved, as it is now moved, ever since it was made, there will be of their motions no cause natural. For either these motions were concreated with their bodies, and their cause is supernatural; or they are coeternal with them, and so they have no cause at all. For whatsoever is eternal was never generated.

I may add besides, to confirm the probability of this simple motion, that as almost all learned men are now of the same opinion with Copernicus concerning the parallelism of the axis of the earth, it seemed to me to be more agreeable to truth, or at least more handsome, that it should be caused by simple circular motion alone, than by two motions, one in the ecliptic, and the other about the earth's own axis the contrary way, neither of them simple, nor either of them such as might be produced by any motion of the sun. I thought best therefore to retain this hypothesis of simple motion, and from it to derive the causes of as many of the phenomena as I could, and to let such alone as I could not deduce from thence.

It will perhaps be objected, that although by this supposition the reason may be given of the parallelism of the axis of the earth, and of many other appearances, nevertheless, seeing it is done by placing the body of the sun in the centre of that orb which the earth describes with its annual motion, the supposition itself is false; because this annual orb is eccentric to the sun. In the first

place, therefore, let us examine what that eccentricity is, and whence it proceeds.

The cause of the eccentricity of the annual motion of the earth.

8. Let the annual circle of the earth *a b c d* (in fig. 3) be divided into four equal parts by the strait lines *a c* and *b d*, cutting one another in the centre *e* ; and let *a* be the beginning of Libra, *b* of Capricorn, *c* of Aries and *d* of Cancer ; and let the whole orb *a b c d* be understood, according to Copernicus, to have every way so great distance from the zodiac of the fixed stars, that it be in comparison with it but as a point. Let the earth be now supposed to be in the beginning of Libra at *a*. The sun, therefore, will appear in the beginning of Aries at *c*. Wherefore, if the earth be moved from *a* to *b*, the apparent motion of the sun will be from *c* to the beginning of Cancer in *d* ; and the earth being moved forwards from *b* to *c*, the sun also will appear to be moved forwards to the beginning of Libra in *a* ; wherefore *c d a* will be the summer arch, and the winter arch will be *a b c*. Now, in the time of the sun's apparent motion in the summer arch, there are numbered 186¾ days ; and, consequently, the earth makes in the same time the same number of diurnal conversions in the arch *a b c* ; and, therefore, the earth in its motion through the arch *c d a* will make only 178½ diurnal conversions. Wherefore the arch *a b c* ought to be greater than the arch *c d a* by 8¼ days, that is to say, by almost so many degrees. Let the arch *a r*, as also *c s*, be each of them an arch of two degrees and $\frac{1}{10}$. Wherefore the arch *r b s* will be greater than the semicircle *a b c* by 4¼ degrees, and greater than the arch *s d r* by 8¼ degrees. The equinoxes, therefore, will be

in the points *r* and *s ;* and therefore also, when
the earth is in *r*, the sun will appear in *s*. Where-
fore the true place of the sun will be in *t*, that is
to say, without the centre of the earth's annual
motion by the quantity of the sine of the arch *a r*,
or the sine of two degrees and 16 minutes. Now
this sine, putting 100,000 for the radius, will be
near 3580 parts thereof. And so much is the ec-
centricity of the earth's annual motion, provided
that that motion be in a perfect circle ; and *s* and
r are the equinoctial parts. And the strait lines
s r and *e a*, produced both ways till they reach the
zodiac of the fixed stars, will fall still upon the same
fixed stars ; because the whole orb *a b e d* is sup-
posed to have no magnitude at all in respect of
the great distance of the fixed stars.

Supposing now the sun to be in *e*, it remains
that I show the cause why the earth is nearer to
the sun, when in its annual motion it is found to
be in *d*, than when it is in *b*. And I take the cause
to be this. When the earth is in the beginning of
Capricorn at *b*, the sun appears in the beginning
of Cancer at *d ;* and then is the midst of summer.
But in the midst of summer, the northern parts of
the earth are towards the sun, which is almost all
dry land, containing all Europe and much the
greatest part of Asia and America. But when the
earth is in the beginning of Cancer at *d*, it is the
midst of winter, and that part of the earth is towards
the sun, which contains those great seas called the
South Sea and the Indian Sea, which are of far
greater extent than all the dry land in that hemi-
sphere. Wherefore by the last article of chapter
XXI, when the earth is in *d*, it will come nearer to

its first movent, that is, to the sun which is in t; that is to say, the earth is nearer to the sun in the midst of winter when it is in d, than in the midst of summer when it is in b; and, therefore, during the winter the sun is in its *Perigæum*, and in its *Apogæum* during the summer. And thus I have shown a possible cause of the eccentricity of the earth; which was to be done.

I am, therefore, of Kepler's opinion in this, that he attributes the eccentricity of the earth to the difference of the parts thereof, and supposes one part to be affected, and another disaffected to the sun. And I dissent from him in this, that he thinks it to be by magnetic virtue, and that this magnetic virtue or attraction and thrusting back of the earth is wrought by immateriate species; which cannot be, because nothing can give motion but a body moved and contiguous. For if those bodies be not moved which are contiguous to a body unmoved, how this body should begin to be moved is not imaginable; as has been demonstrated in art. 7, chap. IX, and often inculcated in other places, to the end that philosophers might at last abstain from the use of such unconceivable connexions of words. I dissent also from him in this, that he says the similitude of bodies is the cause of their mutual attraction. For if it were so, I see no reason why one egg should not be attracted by another. If, therefore, one part of the earth be more affected by the sun than another part, it proceeds from this, that one part hath more water, the other more dry land. And from hence it is, as I showed above, that the earth comes nearer to the sun when it shines upon that part where there is more water,

than when it shines upon that where there is more
dry land.

9. This eccentricity of the earth is the cause why the way of its annual motion is not a perfect circle, but either an elliptical, or almost an elliptical line; as also why the axis of the earth is not kept exactly parallel to itself in all places, but only in the equinoctial points.

Now seeing I have said that the moon is carried about by the earth, in the same manner that the earth is by the sun; and that the earth goeth about the sun in such manner as that it shows sometimes one hemisphere, sometimes the other to the sun; it remains to be enquired, why the moon has always one and the same face turned towards the earth.

Suppose, therefore, the sun to be moved with simple motion in the little circle $f g h i$, (in fig. 4) whose centre is t; and let ♈ ♋ ♎ ♑ be the annual circle of the earth; and a the beginning of Libra. About the point a let the little circle $l k$ be described; and in it let the centre of the earth be understood to be moved with simple motion; and both the sun and the earth to be moved according to the order of the signs. Upon the centre a let the way of the moon $m n o p$ be described; and let $q r$ be the diameter of a circle cutting the globe of the moon into two hemispheres, whereof one is seen by us when the moon is at the full, and the other is turned from us.

The diameter therefore of the moon $q o r$ will be perpendicular to the strait line $t a$. Wherefore the moon is carried, by reason of the motion of the earth, from o towards p. But by reason of the

PART IV.
26.

The cause why
the moon hath
&c.
motion of the sun, if it were in p it would at the
same time be carried from p towards o ; and by
these two contrary movents the strait line $q\,r$ will
be turned about; and, in a quadrant of the circle
$m\,n\,o\,p$, it will be turned so much as makes the
fourth part of its whole conversion. Wherefore
when the moon is in p, $q\,r$ will be parallel to the
strait line $m\,o$. Secondly, when the moon is in m,
the strait line $q\,r$ will, by reason of the motion
of the earth, be in $m\,o$. But by the working of the
sun's motion upon it in the quadrant $p\,m$, the same
$q\,r$ will be turned so much as makes another quarter
of its whole conversion. When, therefore, the moon
is in m, $q\,r$ will be perpendicular to the strait line
$o\,m$. By the same reason, when the moon is in n,
$q\,r$ will be parallel to the strait line $m\,o$; and, the
moon returning to o, the same $q\,r$ will return to
its first place; and the body of the moon will in
one entire period make also one entire conversion
upon her own axis. In the making of which, it is
manifest, that one and the same face of the moon
is always turned towards the earth. And if any
diameter were taken in that little circle, in which
the moon were supposed to be carried about with
simple motion, the same effect would follow ;
for if there were no action from the sun, every
diameter of the moon would be carried about
always parallel to itself. Wherefore I have given
a possible cause why one and the same face of the
moon is always turned towards the earth.

But it is to be noted, that when the moon is
without the ecliptic, we do not always see the same
face precisely. For we see only that part which is
illuminated. But when the moon is without the

ecliptic, that part which is towards us is not exactly
the same with that which is illuminated.

10. To these three simple motions, one of the
sun, another of the moon, and the third of the
earth, in their own little circles f g h i, l k, and
q r, together with the diurnal conversion of the
earth, by which conversion all things that adhere
to its superficies are necessarily carried about with
it, · may be referred the three phenomena concern-
ing the tides of the ocean.   Whereof the first is
the alternate elevation and depression of the water
at the shores, twice in the space of twenty-four hours
and near upon fifty-two minutes; for so it has
constantly continued in all ages. The second, that
at the new and full moons, the elevations of the
water are greater than at other times between.
And the third, that when the sun is in the equi-
noctial, they are yet greater than at any other
time.   For the salving of which phenomena, we
have already the four above-mentioned motions;
to which I assume also this, that the part of the
earth which is called America, being higher than
the water, and extended almost the space of a
whole semicircle from north to south, gives a stop
to the motion of the water.

This being granted, in the same 4th figure, where
l b k c is supposed to be in the plane of the moon's
monthly motion, let the little circle l d k e be de-
scribed about the same centre a in the plane of the
equinoctial. This circle therefore will decline from
the circle l b k c in an angle of almost $28\frac{1}{2}$ degrees;
for the greatest declination of the ecliptic is $23\frac{1}{2}$,
to which adding 5 for the greatest declination of
the moon from the ecliptic, the sum will be $28\frac{1}{4}$

PART IV.
26.

The cause
of the tides
of the ocean.
degrees.   Seeing now the waters, which are
under the circle of the moon's course, are by
reason of the earth's simple motion in the plane of
the same circle moved together with the earth, that
is to say, together with their own bottoms, neither
outgoing nor outgone; if we add the diurnal
motion, by which the other waters which are under
the equinoctial are moved in the same order, and
consider withal that the circles of the moon and
of the equinoctial intersect one another ; it will be
manifest, that both those waters, which are under
the circle of the moon, and under the equinoctial,
will run together under the equinoctial ; and con-
sequently, that their motion will not only be swifter
than the ground that carries them ; but also that
the waters themselves will have greater elevation
whensoever the earth is in the equinoctial.   Where-
fore, whatsoever the cause of the tides may be,
this may be the cause of their augmentation at
that time.

Again, seeing I have supposed the moon to be
carried about by the simple motion of the earth in
the little circle $l\,b\,k\,c$; and demonstrated, at the
4th article of chapter xxi, that whatsoever is
moved by a movent that hath simple motion, will
be moved always with the same velocity ; it follows,
that the centre of the earth will be carried in the
circumference $l\,b\,k\,c$ with the same velocity with
which the moon is carried in the circumference
$m\,n\,o\,p$.   Wherefore the time, in which the moon
is carried about in $m\,n\,o\,p$, is to the time, in which
the earth is carried about in $l\,b\,k\,c$, as one circum-
ference to the other, that is, as $a\,o$ to $a\,k$.   But
$a\,o$ is observed to be to the semidiameter of the

earth as 59 to 1 ; and therefore the earth, if $a\,k$ be put for its semidiameter, will make fifty-nine revo- tions in $l\,b\,k\,c$ in the time that the moon makes one monthly circuit in $m\,n\,o\,p$. But the moon makes her monthly circuit in little more than twenty-nine days. Wherefore the earth shall make its circuit in the circumference $l\,b\,k\,c$ in twelve hours and a little more, namely, about twenty-six minutes more ; that is to say, it shall make two circuits in twenty-four hours and almost fifty-two minutes ; which is observed to be the time between the high-water of one day and the high-water of the day following.   Now the course of the waters being hindered by the southern part of America, their motion will be interrupted there ; and consequently, they will be elevated in those places, and sink down again by their own weight, twice in the space of twenty-four hours and fifty-two minutes.   And thus I have given a possible cause of the diurnal reciprocation of the ocean.

Now from this swelling of the ocean in those parts of the earth, proceed the flowings and ebbings in the Atlantic, Spanish, British, and German seas; which though they have their set times, yet upon several shores they happen at several hours of the day,   And they receive some augmentation from the north, by reason that the shores of China and Tartary, hindering the general course of the waters, make them swell there, and discharge themselves in part through the strait of Anian into the Northern Ocean, and so into the German Sea.

As for the spring tides which happen at the time of the new and full moons, they are caused by that simple motion, which at the beginning I

PART IV.
26.

The cause
of the tides
of the ocean.

supposed to be always in the moon. For as, when I showed the cause of the eccentricity of the earth, I derived the elevation of the waters from the simple motion of the sun; so the same may here be derived from the simple motion of the moon. For though from the generation of clouds, there appear in the sun a more manifest power of elevating the waters than in the moon; yet the power of increasing moisture in vegetables and living creatures appears more manifestly in the moon than in the sun; which may perhaps proceed from this, that the sun raiseth up greater, and the moon lesser drops of water. Nevertheless, it is more likely, and more agreeable to common observation, that rain is raised not only by the sun, but also by the moon; for almost all men expect change of weather at the time of the conjunctions of the sun and moon with one another and with the earth, more than in the time of their quarters.

In the last place, the cause why the spring tides are greater at the time of the equinoxes hath been already sufficiently declared in this article, where I have demonstrated, that the two motions of the earth, namely, its simple motion in the little circle $l\,b\,k\,c$, and its diurnal motion in $l\,d\,k\,e$, cause necessarily a greater elevation of waters when the sun is about the equinoxes, than when he is in other places. I have therefore given possible causes of the phenomenon of the flowing and ebbing of the ocean.

11. As for the explication of the yearly *precession of the equinoctial points*, we must remember that, as I have already shown, the annual motion of the earth is not in the circumference of a circle,

but of an ellipsis, or a line not considerably dif-
ferent from that of an ellipsis.   In the first place,
therefore, this elliptical line is to be described.

Let the ecliptic ♎ ♑ ♈ ♋ (in fig. 5) be divided
into four equal parts by the two strait lines *a b* and
♑ ♋, cutting one another at right angles in the
centre *c*.   And taking the arch *b d* of two degrees
and sixteen minutes, let the strait line *d e* be
drawn parallel to *a b*, and cutting ♑ ♋ in *f*; which
being done, the eccentricity of the earth will be
c f. Seeing therefore the annual motion of the
earth is in the circumference of an ellipsis, of
which ♑ ♋ is the greater axis, *a b* cannot be the
lesser axis ; for *a b* and ♑ ♋ are equal.   Where-
fore the earth passing through *a* and *b*, will either
pass above ♑, as through *g*, or passing through ♑,
will fall between *c* and *a ;* it is no matter which.
Let it pass therefore through *g ;* and let *g l* be
taken equal to the strait line ♑ ♋ ; and dividing
g l equally in *i*, *g i* will be equal to ♑ *f*, and *i l*
equal to *f* ♋ ; and consequently the point *i* will
cut the eccentricity *c f* into two equal parts ; and
taking *i h* equal to *i f*, *h i* will be the whole
eccentricity. If now a strait line, namely, the
line ♎ *i* ♈, be drawn through *i* parallel to the
strait lines *a b* and *e d*, the way of the sun in
summer, namely, the arch ♎ *g* ♈, will be greater
than his way in winter, by $8\frac{1}{4}$ degrees.   Where-
fore the true equinoxes will be in the strait line
♎ *i* ♈ ; and therefore the ellipsis of the earth's
annual motion will not pass through *a, g, b*, and *l ;*
but through ♎ *g*, ♈ and *l*.   Wherefore the annual
motion of the earth is in the ellipsis ♎ *g* ♈ *l ;* and
cannot be, the eccentricity being salved, in any

other line. And this perhaps is the reason, why Kepler, against the opinion of all the astronomers of former time, thought fit to bisect the eccentricity of the earth, or, according to the ancients, of the sun, not by diminishing the quantity of the same eccentricity, (because the true measure of that quantity is the difference by which the summer arch exceeds the winter arch), but by taking for the centre of the ecliptic of the great orb the point c nearer to f, and so placing the whole great orb as much nearer to the ecliptic of the fixed stars towards ♋, as is the distance between c and i. For seeing the whole great orb is but as a point in respect of the immense distance of the fixed stars, the two strait lines ♎ ♈ and $a\,b$, being produced both ways to the beginnings of Aries and Libra, will fall upon the same points of the sphere of the fixed stars. Let therefore the diameter of the earth $m\,n$ be in the plane of the earth's annual motion. If now the earth be moved by the sun's simple motion in the circumference of the ecliptic about the centre i, this diameter will be kept always parallel to itself and to the strait line $g\,l$. But seeing the earth is moved in the circumference of an ellipsis without the ecliptic, the point n, whilst it passeth through ♎ ♑ ♈, will go in a lesser circumference than the point m; and consequently, as soon as ever it begins to be moved, it will lose its parallelism with the strait line ♑ ♋ ; so that $m\,n$ produced will at last cut the strait line $g\,l$ produced. And contrarily, as soon as $m\,n$ is past ♑, the earth making its way in the internal elliptical line ♈ l ♎, the same $m\,n$ produced towards m, will cut $l\,g$ produced. And when the

earth hath almost finished its whole circumfer-
ence, the same $m\,n$ shall again make a right angle
with a line drawn from the centre i, a little short
of the point from which the earth began its motion.
And there the next year shall be one of the equi-
noctial points, namely, near the end of ♍; the
other shall be opposite to it near the end of ♓.
And thus the points in which the days and nights
are made equal do every year fall back; but with
so slow a motion, that, in a whole year, it makes but
51 first minutes. And this relapse being contrary
to the order of the signs, is commonly called the
precession of the equinoxes. Of which I have
from my former suppositions deduced a possible
cause; which was to be done.

According to what I have said concerning the
cause of the eccentricity of the earth; and according
to Kepler, who for the cause thereof supposeth one
part of the earth to be affected to the sun, the other
part to be disaffected; the apogæum and peri-
gæum of the sun should be moved every year in
the same order, and with the same velocity, with
which the equinoctial points are moved; and their
distance from them should always be the quadrant
of a circle; which seems to be otherwise. For
astronomers say, that the equinoxes are now, the
one about 28 degrees gone back from the first star
of Aries, the other as much from the beginning of
Libra; so that the apogæum of the sun or the
aphelium of the earth ought to be about the 28th
degree of Cancer. But it is reckoned to be in the
7th degree. Seeing, therefore, we have not suffi-
cient evidence of the ὅτι (that so it is,) it is in vain
to seek for the διότι (why it is so.) Wherefore, as

long as the motion of the apogæum is not observ-able by reason of the slowness thereof, and as long as it remains doubtful whether their distance from the equinoctial points be more or less than a quadrant precisely; so long it may be lawful for me to think they proceed both of them with equal velocity.

Also, I do not at all meddle with the causes of the eccentricities of Saturn, Jupiter, Mars, and Mercury. Nevertheless, seeing the eccentricity of the earth may, as I have shewn, be caused by the unlike constitution of the several parts of the earth which are alternately turned towards the sun, it is credible also, that like effects may be produced in these other planets from their having their superficies of unlike parts.

And this is all I shall say concerning Sidereal Philosophy. And, though the causes I have here supposed be not the true causes of these phenomena, yet I have demonstrated that they are sufficient to produce them, according to what I at first propounded.

CHAPTER XXVII.

OF LIGHT, HEAT, AND OF COLOURS.

1. Of the immense magnitude of some bodies, and the unspeakable littleness of others.—2. Of the cause of the light of the sun.—3. How light heateth.—4. The generation of fire from the sun.—5. The generation of fire from collision.—6. The cause of light in glow-worms, rotten wood, and the Bolognan stone.— 6. The cause of light in the concussion of sea water.—8. The cause of flame, sparks, and colliquation.—9. The cause why wet hay sometimes burns of its own accord ; also the cause of lightning.—10. The cause of the force of gunpowder ; and what is to be ascribed to the coals, what to the brimstone, and what to the nitre.—11. How heat is caused by attrition.—12. The distinction of light into first, second, &c.—13. The causes of the colours we see in looking through a prisma of glass, namely, of red, yellow, blue, and violet colour. 14. Why the moon and the stars appear redder in the horizon than in the midst of the heaven. — 15. The cause of whiteness.—16. The cause of blackness.

PART IV.
27.

1. BESIDES the stars, of which I have spoken in the last chapter, whatsoever other bodies there be in the world, they may be all comprehended under the name of intersidereal bodies. And these I have already supposed to be either the most fluid æther, or such bodies whose parts have some degree of cohesion. Now, these differ from one another in their several *consistencies, magnitudes, motions,* and *figures.* In consistency, I suppose some bodies to be harder, others softer through all the several degrees of *tenacity.* In magnitude, some to be greater, others less, and many unspeakably little. For we must remember that, by the understanding,

Of the immense magnitude of some bodies, and the unspeakable littleness of others.

quantity is divisible into divisibles perpetually. And, therefore, if a man could do as much with his hands as he can with his understanding, he would be able to take from any given magnitude a part which should be less than any other magnitude given.   But the Omnipotent Creator of the world can actually from a part of any thing take another part, as far as we by our understanding can conceive the same to be divisible.   Wherefore there is no impossible smallness of bodies.   And what hinders but that we may think this likely ? For we know there are some living creatures so small that we can scarce see their whole bodies.   Yet even these have their young ones ; their little veins and other vessels, and their eyes so small as that no microscope can make them visible.   So that we cannot suppose any magnitude so little, but that our very supposition is actually exceeded by nature. Besides, there are now such microscopes commonly made, that the things we see with them appear a hundred thousand times bigger than they would do if we looked upon them with our bare eyes.   Nor is there any doubt but that by augmenting the power of these microscopes (for it may be augmented as long as neither matter nor the hands of workmen are wanting) every one of those hundred thousandth parts might yet appear a hundred thousand times greater than they did before.   Neither is the smallness of some bodies to be more admired than the vast greatness of others.   For it belongs to the same Infinite Power, as well to augment infinitely as infinitely to diminish.   To make the great orb, namely, that whose radius reacheth from the earth to the sun, but as a

point in respect of the distance between the sun and the fixed stars; and, on the contrary, to make a body so little, as to be in the same proportion less than any other visible body, proceeds equally from one and the same Author of Nature. But this of the immense distance of the fixed stars, which for a long time was accounted an incredible thing, is now believed by almost all the learned. Why then should not that other, of the smallness of some bodies, become credible at some time or other? For the Majesty of God appears no less in small things than in great; and as it exceedeth human sense in the immense greatness of the universe, so also it doth in the smallness of the parts thereof. Nor are the first elements of compositions, nor the first beginnings of actions, nor the first moments of times more credible, than that which is now believed of the vast distance of the fixed stars.

Some things are acknowledged by mortal men to be very great, though finite, as seeing them to be such. They acknowledge also that some things, which they do not see, may be of infinite magnitude. But they are not presently nor without great study persuaded, that there is any mean between infinite and the greatest of those things which either they see or imagine. Nevertheless, when after meditation and contemplation many things which we wondered at before are now grown more familiar to us, we then believe them, and transfer our admiration from the creatures to the Creator. But how little soever some bodies may be, yet I will not suppose their quantity to be less than is requisite for the salving of the phenomena. And in like manner I shall suppose their motion, namely,

their velocity and slowness, and the variety of their figures, to be only such as the explication of their natural causes requires. And lastly, I suppose, that the parts of the pure æther, as if it were the first matter, have no motion at all but what they receive from bodies which float in them, and are not themselves fluid.

2. Having laid these grounds, let us come to speak of causes ; and in the first place let us inquire what may be the cause of the light of the sun. Seeing, therefore, the body of the sun doth by its simple circular motion thrust away the ambient ethereal substance sometimes one way sometimes another, so that those parts, which are next the sun, being moved by it, do propagate that motion to the next remote parts, and these to the next, and so on continually ; it must needs be that, notwithstanding any distance, the foremost part of the eye will at last be pressed ; and by the pressure of that part, the motion will be propagated to the innermost part of the organ of sight, namely, to the heart ; and from the reaction of the heart, there will proceed an endeavour back by the same way, ending in the endeavour outwards of the coat of the eye, called the *retina*. But this endeavour outwards, as has been defined in chapter xxv, is the thing which is called light, or the phantasm of a lucid body. For it is by reason of this phantasm that an object is called lucid. Wherefore we have a possible cause of the light of the sun ; which I undertook to find.

3. The generation of the light of the sun is accompanied with the generation of heat. Now every man knows what heat is in himself, by feeling

it when he grows hot; but what it is in other
things, he knows only by ratiocination. For it is one thing to grow hot, and another thing to heat or make hot. And therefore though we perceive that the fire or the sun heateth, yet we do not perceive that it is itself hot. That other living creatures, whilst they make other things hot, are hot themselves, we infer by reasoning from the like sense in ourselves. But this is not a necessary inference. For though it may truly be said of living creatures, that *they heat, therefore they are themselves hot;* yet it cannot from hence be truly inferred that *fire heateth, therefore it is itself hot;* no more than this, *fire causeth pain, therefore it is itself in pain.* Wherefore, that is only and properly called hot, which when we feel we are necessarily hot.

Now when we grow hot, we find that our spirits and blood, and whatsoever is fluid within us, is called out from the internal to the external parts of our bodies, more or less, according to the degree of the heat; and that our skin swelleth. He, therefore, that can give a possible cause of this evocation and swelling, and such as agrees with the rest of the phenomena of heat, may be thought to have given the cause of the heat of the sun.

It hath been shown, in the 5th article of chapter XXI, that the fluid medium, which we call the air, is so moved by the simple circular motion of the sun, as that all its parts, even the least, do perpetually change places with one another; which change of places is that which there I called fermentation. From this fermentation of the air, I have, in the 8th article of the last chapter, demon-

strated that the water may be drawn up into the clouds.

And I shall now show that the fluid parts may, in like manner, by the same fermentation, be drawn out from the internal to the external parts of our bodies. For seeing that wheresoever the fluid medium is contiguous to the body of any living creature, there the parts of that medium are, by perpetual change of place, separated from one another ; the contiguous parts of the living creature must, of necessity, endeavour to enter into the spaces of the separated parts. For otherwise those parts, supposing there is no vacuum, would have no place to go into. And therefore that, which is most fluid and separable in the parts of the living creature which are contiguous to the medium, will go first out ; and into the place thereof will succeed such other parts as can most easily transpire through the pores of the skin. And from hence it is necessary that the rest of the parts, which are not separated, must altogether be moved outwards, for the keeping of all places full. But this motion outwards of all parts together must, of necessity, press those parts of the ambient air which are ready to leave their places ; and therefore all the parts of the body, endeavouring at once that way, make the body swell. Wherefore a possible cause is given of heat from the sun ; which was to be done.

The generation of fire from the sun.

4. We have now seen how light and heat are generated ; heat by the simple motion of the medium, making the parts perpetually change places with one another ; and light by this, that by the same simple motion action is propagated in a

strait line.   But when a body hath its parts so
moved, that it sensibly both heats and shines at
the same time, then it is that we say fire is
generated.

Now by fire I do not understand a body distinct
from matter combustible or glowing, as wood or
iron, but the matter itself, not simply and always,
but then only when it shineth and heateth.   He,
therefore, that renders a cause possible and agree-
able to the rest of the phenomena, namely, whence,
and from what action, both the *shining* and *heating*
proceed, may be thought to have given a possible
cause of the generation of *fire*.

Let, therefore, A B C (in the first figure) be a
sphere, or the portion of a sphere, whose centre is
D ; and let it be transparent and homogeneous, as
crystal, glass, or water, and objected to the sun.
Wherefore, the foremost part A B C will, by the
simple motion of the sun, by which it thrusts
forwards the medium, be wrought upon by the
sunbeams in the strait lines E A, F B, and G C ;
which strait lines may, in respect of the great dis-
tance of the sun, be taken for parallels.   And
seeing the medium within the sphere is thicker
than the medium without it, those beams will be
refracted towards the perpendiculars.   Let the
strait lines E A and G C be produced till they cut
the sphere in H and I ; and drawing the perpen-
diculars A D and C D, the refracted beams E A and
G C will of necessity fall, the one between A H
and A D, the other between C I and C D.   Let
those refracted beams be A K and C L.   And again,
let the lines D K M and D L N be drawn perpen-
dicular to the sphere ; and let A K and C L be

produced till they meet with the strait line B D
produced in O. Seeing, therefore, the medium
within the sphere is thicker than that without it,
the refracted line A K will recede further from the
perpendicular K M than K O will recede from the
same. Wherefore K O will fall between the re-
fracted line and the perpendicular. Let, therefore,
the refracted line be K P, cutting F O in P; and
for the same reason the strait line L P will be the
refracted line of the strait line C L. Wherefore,
seeing the beams are nothing else but the ways in
which the motion is propagated, the motion about
P will be so much more vehement than the motion
about A B C, by how much the base of the portion
A B C is greater than the base of a like portion in
the sphere, whose centre is P, and whose magnitude
is equal to that of the little circle about P, which
comprehendeth all the beams that are propagated
from A B C; and this sphere being much less than
the sphere A B C, the parts of the medium, that is,
of the air about P, will change places with one
another with much greater celerity than those
about A B C. If, therefore, any matter combustible,
that is to say, such as may be easily dissipated, be
placed in P, the parts of that matter, if the pro-
portion be great enough between A C and a like
portion of the little circle about P, will be freed
from their mutual cohesion, and being separated
will acquire simple motion. But vehement simple
motion generates in the beholder a phantasm of
lucid and hot, as I have before demonstrated of
the simple motion of the sun; and therefore the
combustible matter which is placed in P will be
made lucid and hot, that is to say, will be fire.

Wherefore I have rendered a possible cause of fire ;
which was to be done.

5. From the manner by which the sun generateth
fire, it is easy to explain the manner by which fire
may be generated by the collision of two flints.
For by that collision some of those particles of
which the stone is compacted, are violently sepa-
rated and thrown off ; and being withal swiftly
turned round, the eye is moved by them, as it is in
the generation of light by the sun. Wherefore they
shine ; and falling upon matter which is already half
dissipated, such as is tinder, they thoroughly dis-
sipate the parts thereof, and make them turn round.
From whence, as I have newly shown, light and
heat, that is to say fire, is generated.

6. The shining of glow-worms, some kinds of The cause o
light in glow-
worms, rotten
wood, and the
Bolognan stone
rotten wood, and of a kind of stone made at Bo-
logna, may have one common cause, namely, the
exposing of them to the hot sun.   We find by expe-
rience that the Bologna stone shines not, unless it be
so exposed ; and after it has been exposed it shines
but for a little time, namely, as long as it retains
a certain degree of heat.   And the cause may be
that the parts, of which it is made, may together
with heat have simple motion imprinted in them
by the sun.   Which if it be so, it is necessary that
it shine in the dark, as long as there is sufficient heat
in it ; but this ceasing, it will shine no longer.
Also we find by experience that in the glow-worm
there is a certain thick humour, like the crystalline
humour of the eye ; which if it be taken out and
held long enough in one's fingers, and then be
carried into the dark, it will shine by reason of the
warmth it received from the fingers ; but as soon

as it is cold it will cease shining. From whence, therefore, can these creatures have their light, but from lying all day in the sunshine in the hottest time of summer? In the same manner, rotten wood, except it grow rotten in the sunshine, or be afterwards long enough exposed to the sun, will not shine. That this doth not happen in every worm, nor in all kinds of rotten wood, nor in all calcined stones, the cause may be that the parts, of which the bodies are made, are different both in motion and figure from the parts of bodies of other kinds.

The cause of light in the concussion of sea water.

7. Also the sea water shineth when it is either dashed with the strokes of oars, or when a ship in its course breaks strongly through it; but more or less, according as the wind blows from different points. The cause whereof may be this, that the particles of salt, though they never shine in the salt-pits, where they are but slowly drawn up by the sun, being here beaten up into the air in greater quantities and with more force, are withal made to turn round, and consequently to shine, though weakly. I have, therefore, given a possible cause of this phenomenon.

The cause of flame, sparks, & colliquation.

8. If such matter as is compounded of hard little bodies be set on fire, it must needs be, that, as they fly out in greater or less quantities, the flame which is made by them will be greater or less. And if the ethereal or fluid part of that matter fly out together with them, their motion will be the swifter, as it is in wood and other things which flame with a manifest mixture of wind. When, therefore, these hard particles by their flying out move the eye strongly, they shine bright; and a

great quantity of them flying out together, they
make a great shining body.   For flame being
nothing but an aggregate of shining particles, the
greater the aggregate is, the greater and more
manifest will be the flame.   I have, therefore,
shown a possible cause of flame.   And from hence
the cause appears evidently, why glass is so easily
and quickly melted by the small flame of a candle
blown, which will not be melted without blowing
but by a very strong fire.

Now, if from the same matter there be a part
broken off, namely, such a part as consisteth of
many of the small particles, of this is made a spark.
For from the breaking off it hath a violent turning
round, and from hence it shines.   But though
from this matter there fly neither flame nor sparks,
yet some of the smallest parts of it may be carried
out as far as to the superficies, and remain there
as ashes ; the parts whereof are so extremely small,
that it cannot any longer be doubted how far na-
ture may proceed in dividing.

Lastly, though by the application of fire to this
matter there fly little or nothing from it, yet
there will be in the parts an endeavour to simple
motion ; by which the whole body will either be
melted, or, which is a degree of melting, softened.
For all motion has some effect upon all matter
whatsoever, as has been shown at art. 3, chap. xv.
Now if it be softened to such a degree, as that the
stubbornness of the parts be exceeded by their
gravity, then we say it is melted ; otherwise, soft-
ened and made pliant and ductile.

Again, the matter having in it some particles
hard, others ethereal or watery ; if, by the appli-

cation of fire, these latter be called out, the former will thereby come to a more full contact with one another; and, consequently, will not be so easily separated; that is to say, the whole body will be made harder. And this may be the cause why the same fire makes some things soft, others hard.

The cause why wet hay sometimes burns of its own accord; also the cause of lightning.

9. It is known by experience that if hay be laid wet together in a heap, it will after a time begin to smoke, and then burn as it were of itself. The cause whereof seems to be this, that in the air, which is enclosed within the hay, there are those little bodies, which, as I have supposed, are moved freely with simple motion. But this motion being by degrees hindered more and more by the descending moisture, which at the last fills and stops all the passages, the thinner parts of the air ascend by penetrating the water; and those hard little bodies, being so thrust together that they touch and press one another, acquire stronger motion; till at last by the increased strength of this motion the watery parts are first driven outwards, from whence appears vapour; and by the continued increase of this motion, the smallest particles of the dried hay are forced out, and recovering their natural simple motion, they grow hot and shine, that is to say, they are set on fire.

The same also may be the cause of lightning, which happens in the hottest time of the year, when the water is raised up in greatest quantity and carried highest. For after the first clouds are raised, others after others follow them; and being congealed above, they happen, whilst some of them ascend and others descend, to fall one upon another in such manner, as that in some places all their parts

are joined together, in others they leave hollow
spaces between them; and into these spaces, the
ethereal parts being forced out by the compressure
of the clouds, many of the harder little bodies are
so pent together, as they have not the liberty of
such motion as is natural to the air. Wherefore
their endeavour grows more vehement, till at last
they force their way through the clouds, sometimes
in one place, sometimes in another; and, breaking
through with great noise, they move the air vio-
lently, and striking our eyes, generate light, that
is to say, they shine. And this shining is that we
call lightning.

10. The most common phenomenon proceeding The cause of the force
from fire, and yet the most admirable of all others, of gunpowder; and what is to
is the force of gunpowder fired; which being com- be ascribed to
pounded of nitre, brimstone and coals, beaten small, the coals, what to the brim-
hath from the coals its first taking fire; from the stone, and what to the nitre.
brimstone its nourishment and flame, that is to say,
light and motion, and from the nitre the vehe-
mence of both. Now if a piece of nitre, before it
is beaten, be laid upon a burning coal, first it melts,
and, like water, quencheth that part of the coal it
toucheth. Then vapour or air, flying out where the
coal and nitre join, bloweth the coal with great
swiftness and vehemence on all sides. And from
hence it comes to pass, that by two contrary mo-
tions, the one, of the particles which go out of the
burning coal, the other, of those of the ethereal
and watery substance of the nitre, is generated
that vehement motion and inflammation. And,
lastly, when there is no more action from the nitre,
that is to say, when the volatile parts of the nitre
are flown out, there is found about the sides a cer-

PART IV.
27.

The cause of
the force of
gunpowder,&c.
tain white substance, which being thrown again into the fire, will grow red-hot again, but will not be dissipated, at least unless the fire be augmented. If now a possible cause of this be found out, the same will also be a possible cause why a grain of gunpowder set on fire doth expand itself with such vehement motion, and shine. And it may be caused in this manner.

Let the particles, of which nitre consisteth, be supposed to be some of them hard, others watery, and the rest ethereal. Also let the hard particles be supposed to be spherically hollow, like small bubbles, so that many of them growing together may constitute a body, whose little caverns are filled with a substance which is either watery, or ethereal, or both. As soon, therefore, as the hard particles are dissipated, the watery and ethereal particles will necessarily fly out; and as they fly, of necessity blow strongly the burning coals and brimstone which are mingled together; whereupon there will follow a great expansion of light, with vehement flame, and a violent dissipation of the particles of the nitre, the brimstone and the coals. Wherefore I have given a possible cause of the force of fired gunpowder.

It is manifest from hence, that for the rendering of the cause why a bullet of lead or iron, shot from a piece of ordnance, flies with so great velocity, there is no necessity to introduce such rarefaction, as, by the common definition of it, makes the same matter to have sometimes more, sometimes less quantity; which is inconceivable. For every thing is said to be greater or less, as it hath more or less quantity. The violence with which a bullet

is thrust out of a gun, proceeds from the swiftness
of the small particles of the fired powder ; at least
it may proceed from that cause without the suppo-
sition of any empty space.

11. Besides, by the attrition or rubbing of one How heat
body against another, as of wood against wood, we attrition.
find that not only a certain degree of heat, but fire
itself is sometimes generated.   For such motion
is the reciprocation of pressure, sometimes one way,
sometimes the other ; and by this reciprocation
whatsoever is fluid in both the pieces of wood is
forced hither and thither ; and consequently, to an
endeavour of getting out ; and at last by breaking
out makes fire.

12. Now light is distinguished into, first, second, The distinction
third, and so on infinitely.   And we call that first first,second,&c
light, which is in the first lucid body ; as the sun,
fire, &c. : second, that which is in such bodies, as
being not transparent are illuminated by the sun ;
as the moon, a wall, &c.: and third, that which is in
bodies not transparent, but illuminated by second
light, &c.

13. *Colour* is light, but troubled light, namely, The causes of
such as is generated by perturbed motion ; as shall see in looking
be made manifest by the red, yellow, blue and pur- ma of glass,
ple, which are generated by the interposition of a namely, of red
diaphanous prisma, whose opposite bases are violet colour.
triangular, between the light and that which is
enlightened.

For let there be a prisma of glass, or of any other
transparent matter which is of greater density than
air ; and let the triangle A B C be the base of this
prisma.   Also let the strait line D E be the dia-
meter of the sun's body, having oblique position to

the strait line A B; and let the sunbeams pass in
the lines D A and E B C. And lastly, let the strait
lines D A and E C be produced indefinitely to F
and G. Seeing therefore the strait line D A, by
reason of the density of the glass, is refracted to-
wards the perpendicular; let the line refracted at
the point A be the strait line A H. And again,
seeing the medium below A C is thinner than that
above it, the other refraction, which will be made
there, will diverge from the perpendicular. Let
therefore this second refracted line be A I. Also
let the same be done at the point C, by making the
first refracted line to be C K, and the second C L.
Seeing therefore the cause of the refraction in the
point A of the strait line of A B is the excess of the
resistance of the medium in A B above the resist-
ance of the air, there must of necessity be reaction
from the point A towards the point B; and conse-
quently the medium at A within the triangle A B C
will have its motion troubled, that is to say, the
strait motion in A F and A H will be mixed with
the transverse motion between the same A F and
A H, represented by the short transverse lines in
the triangle A F H. Again, seeing at the point A
of the strait line A C there is a second refraction
from A H in A I, the motion of the medium will
again be perturbed by reason of the transverse re-
action from A towards C, represented likewise by
the short transverse lines in the triangle A H I.
And in the same manner there is a double pertur-
bation represented by the transverse lines in the
triangles C G K and C K L. But as for the light
between A I and C G, it will not be perturbed;
because, if there were in all the points of the strait

lines A B and A C the same action which is in the
points A and C, then the plane of the triangle C G K
would be everywhere coincident with the plane of
the triangle A F H ; by which means all would ap-
pear alike between A and C.   Besides, it is to be
observed, that all the reaction at A tends towards
the illuminated parts which are between A and C,
and consequently perturbeth the first light.   And
on. the contrary, that all the reaction at C tends
towards the parts without the triangle or without
the prisma A B C, where there is none but second
light ; and that the triangle A F H shows that per-
turbation of light which is made in the glass itself ;
as the triangle A H I shows that perturbation of
light which is made below the glass. In like manner,
that C G K shows the perturbation of light within
the glass ; and C K L that which is below the glass.
From whence there are four divers motions, or four
different illuminations or colours, whose differences
appear most manifestly to the sense in a prisma,
whose base is an equilateral triangle, when the
sunbeams that pass through it are received upon a
white paper.   For the triangle A F H appears red
to the sense ; the triangle A H I yellow ; the tri-
angle C G K green, and approaching to blue ; and
lastly, the triangle C K L appears purple.   It is
therefore evident that when weak but first light
passeth through a more resisting diaphanous body,
as glass, the beams, which fall upon it transversely,
make redness ; and when the same first light is
stronger, as it is in the thinner medium below the
strait line A C, the transverse beams make yellow-
ness.   Also when second light is strong, as it is in
the triangle C G K, which is nearest to the first

light, the transverse beams make greenness; and when the same second light is weaker, as in the triangle C K L, they make a purple colour.

Why the moon and the stars appear redder in the horizon than in the midst of the heaven.

14. From hence may be deduced a cause, why the moon and stars appear bigger and redder near the horizon than in the mid-heaven. For between the eye and the apparent horizon there is more impure air, such as is mingled with watery and earthy little bodies, than is between the same eye and the more elevated part of heaven. But vision is made by beams which constitute a cone, whose base, if we look upon the moon, is the moon's face, and whose vertex is in the eye; and therefore, many beams from the moon must needs fall upon little bodies that are without the visual cone, and be by them reflected to the eye. But these reflected beams tend all in lines which are transverse to the visual cone, and make at the eye an angle which is greater than the angle of the cone. Wherefore, the moon appears greater in the horizon, than when she is more elevated. And because those reflected beams go transversely, there will be generated, by the last article, redness. A possible cause therefore is shown, why the moon as also the stars appear greater and redder in the horizon, than in the midst of heaven. The same also may be the cause, why the sun appears in the horizon greater and of a colour more degenerating to yellow, than when he is higher elevated. For the reflection from the little bodies between, and the transverse motion of the medium, are still the same. But the light of the sun is much stronger than that of the moon; and therefore, by the last article, his splendour

must needs by this perturbation degenerate into
yellowness.

But for the generation of these four colours, it is not necessary that the figure of the glass be a prisma; for if it were spherical it would do the same. For in a sphere the sunbeams are twice refracted and twice reflected. And this being observed by Des Cartes, and withal that a rainbow never appears but when it rains; as also, that the drops of rain have their figures almost spherical; he hath shown from thence the cause of the colours in the rainbow; which therefore need not be repeated.

15. *Whiteness* is light, but light perturbed by the reflections of many beams of light coming to the eye together within a little space. For if glass or any other diaphanous body be reduced to very small parts by contusion or concussion, every one of those parts, if the beams of a lucid body be from any one point of the same reflected to the eye, will represent to the beholder an idea or image of the whole lucid body, that is to say, a phantasm of white. For the strongest light is the most white; and therefore many such parts will make many such images. Wherefore, if those parts lie thick and close together, those many images will appear confusedly, and will by reason of the confused light represent a white colour. So that from hence may be deduced a possible cause, why glass beaten, that is, reduced to powder, looks white. Also why water and snow are white; they being nothing but a heap of very small diaphanous bodies, namely, of little bubbles, from whose several convex superficies there are by reflection made several confused phan-

The cause of whiteness.

PART IV.
27.

tasms of the whole lucid body, that is to say, white-
ness. For the same reason, salt and nitre are white,
as consisting of small bubbles which contain within
them water and air; as is manifest in nitre, from
this, that being thrown into the fire it violently
blows the same; which salt also doth, but with less
violence. But if a white body be exposed, not to
the light of the day, but to that of the fire or of a
candle, it will not at the first sight be easily judged
whether it be white or yellow; the cause whereof
may be this, that the light of those things, which
burn and flame, is almost of a middle colour between
whiteness and yellowness.

The cause of blackness.

16. As whiteness is light, so *blackness* is the pri-
vation of light, or darkness. And, from hence it is,
first, that all holes, from which no light can be re-
flected to the eye, appear black. Secondly, that
when a body hath little eminent particles erected
straight up from the superficies, so that the beams
of light which fall upon them are reflected not to
the eye but to the body itself, that superficies
appears black; in the same manner as the sea
appears back when ruffled by the wind. Thirdly,
that any combustible matter is by the fire made to
look black before it shines. For the endeavour of
the fire being to dissipate the smallest parts of such
bodies as are thrown into it, it must first raise and
erect those parts before it can work their dissipa-
tion. If, therefore, the fire be put out before the
parts are totally dissipated, the coal will appear
black; for the parts being only erected, the beams
of light falling upon them will not be reflected to
the eye, but to the coal itself. Fourthly, that burn-
ing glasses do more easily burn black things than

white.   For in a white superficies the eminent
parts are convex, like little bubbles; and there-
fore the beams of light, which fall upon them, are
reflected every way from the reflecting body.   But
in a black superficies, where the eminent particles
are more erected, the beams of light falling upon
them are all necessarily reflected towards the body
itself; and, therefore, bodies that are black are
more easily set on fire by the sun beams, than
those that are white.   Fifthly, that all colours
that are made of the mixture of white and black
proceed from the different position of the particles
that rise above the superficies, and their different
forms of asperity.   For, according to these differ-
ences, more or fewer beams of light are reflected
from several bodies to the eye.   But in regard
those differences are innumerable, and the bodies
themselves so small that we cannot perceive them;
the explication and precise determination of the
causes of all colours is a thing of so great difficulty,
that I dare not undertake it.

CHAPTER XXVIII.

OF COLD, WIND, HARD, ICE, RESTITUTION OF BODIES BENT, DIAPHANOUS, LIGHTNING AND THUNDER; AND OF THE HEADS OF RIVERS.

1. Why breath from the same mouth sometimes heats and sometimes cools.—2. Wind, and the inconstancy of winds, whence. 3. Why there is a constant, though not a great wind, from east to west, near the equator.—4. What is the effect of air pent in between the clouds.—5. No change from soft to hard, but by motion.—6. What is the cause of cold near the poles. 7. The cause of ice; and why the cold is more remiss in rainy than in clear weather. Why water doth not freeze in deep wells as it doth near the superficies of the earth. Why ice is not so heavy as water; and why wine is not so easily frozen as water.—8. Another cause of hardness from the fuller contact of atoms; also, how hard things are broken.—9. A third cause of hardness from heat.—10. A fourth cause of hardness from the motion of atoms enclosed in a narrow space.—11. How hard things are softened. — 12. Whence proceed the spontaneous restitution of things bent.—13. Diaphanous and opacous, what they are, and whence.—14. The cause of lightning and thunder.—15. Whence it proceeds that clouds can fall again after they are once elevated and frozen.—16. How it could be that the moon was eclipsed, when she was not diametrically opposite to the sun.—17. By what means many suns may appear at once.—18. Of the heads of rivers.

PART IV.
28.

Why breath from the same mouth sometimes heats and sometimes cools.

1. As, when the motion of the ambient ethereal substance makes the spirits and fluid parts of our bodies tend outwards, we acknowledge heat; so, by the endeavour inwards of the same spirits and humours, we feel cold. So that to cool is to make the exterior parts of the body endeavour inwards,

by a motion contrary to that of calefaction, by which the internal parts are called outwards. He, therefore, that would know the cause of cold, must find by what motion or motions the exterior parts of any body endeavour to retire inwards. To begin with those phenomena which are the most familiar; there is almost no man but knows, that breath blown strongly, and which comes from the mouth with violence, that is to say, the passage being strait, will cool the hand; and that the same breath blown gently, that is to say, through a greater aperture, will warm the same. The cause of which phenomenon may be this, the breath going out hath two motions; the one, of the whole and direct, by which the foremost parts of the hand are driven inwards; the other, simple motion of the small particles of the same breath, which (as I have shown in the 3rd article of the last chapter) causeth heat. According, therefore, as either of these motions is predominant, so there is the sense sometimes of cold, sometimes of heat. Wherefore, when the breath is softly breathed out at a large passage, that simple motion which causeth heat prevaileth, and consequently heat is felt; and when, by compressing the lips, the breath is more strongly blown out, then is the direct motion prevalent, which makes us feel cold. For, the direct motion of the breath or air is wind; and all wind cools or diminisheth former heat.

2. And seeing not only great wind, but almost any ventilation and stirring of the air, doth refrigerate; the reason of many experiments concerning cold cannot well be given without finding, first, what are the causes of wind. Now, wind is

nothing else but the direct motion of the air thrust
forwards; which, nevertheless, when many winds
concur, may be circular or otherwise indirect, as
it is in whirlwinds. Wherefore, in the first place
we are to enquire into the causes of winds. Wind
is air moved in a considerable quantity, and that
either in the manner of waves, which is both for-
wards and also up and down, or else forwards
only.

Supposing, therefore, the air both clear and
calm for any time how little soever, yet, the
greater bodies of the world being so disposed and
ordered as has been said, it will be necessary that
a wind presently arise somewhere. For, seeing
that motion of the parts of the air, which is made
by the simple motion of the sun in his own epicycle,
causeth an exhalation of the particles of water
from the seas and all other moist bodies, and those
particles make clouds; it must needs follow, that,
whilst the particles of water pass upwards, the
particles of air, for the keeping of all spaces full,
be jostled out on every side, and urge the next par-
ticles, and these the next; till having made their
circuit, there comes continually so much air to the
hinder parts of the earth as there went water from
before it. Wherefore, the ascending vapours move
the air towards the sides every way; and all direct
motion of the air being wind, they make a wind.
And if this wind meet often with other vapours
which arise in other places, it is manifest that the
force thereof will be augmented, and the way or
course of it changed. Besides, according as the
earth, by its diurnal motion, turns sometimes the
drier, sometimes the moister part towards the sun,

so sometimes a greater, sometimes a less, quantity of vapours will be raised; that is to say, sometimes there will be a less, sometimes a greater wind. Wherefore, I have rendered a possible cause of such winds as are generated by vapours; and also of their inconstancy.

From hence it follows that these winds cannot be made in any place, which is higher than that to which vapours may ascend. Nor is that incredible which is reported of the highest mountains, as the Peak of Teneriffe and the Andes of Peru, namely, that they are not at all troubled with these inconstant winds. And if it were certain that neither rain nor snow were ever seen in the highest tops of those mountains, it could not be doubted but that they are higher than any place to which vapours use to ascend.

3. Nevertheless, there may be wind there, though not that which is made by the ascent of vapours, yet a less and more constant wind, like the continued blast of a pair of bellows, blowing from the east. And this may have a double cause; the one, the diurnal motion of the earth; the other, its simple motion in its own epicycle. For these mountains being, by reason of their height, more eminent than all the rest of the parts of the earth, do by both these motions drive the air from the west eastwards. To which, though the diurnal motion contribute but little, yet seeing I have supposed that the simple motion of the earth, in its own epicycle, makes two revolutions in the same time in which the diurnal motion makes but one, and that the semidiameter of the epicycle is double to the semidiameter of the diurnal conversion, the

motion of every point of the earth in its own epicycle will have its velocity quadruple to that of the diurnal motion; so that by both these motions together, the tops of those hills will sensibly be moved against the air; and consequently a wind will be felt. For whether the air strike the sentient, or the sentient the air, the perception of motion will be the same. But this wind, seeing it is not caused by the ascent of vapours, must necessarily be very constant.

What is the effect of air pent in between the clouds.

4. When one cloud is already ascended into the air, if another cloud ascend towards it, that part of the air, which is intercepted between them both, must of necessity be pressed out every way. Also when both of them, whilst the one ascends and the other either stays or descends, come to be joined in such manner as that the ethereal substance be shut within them on every side, it will by this compression also go out by penetrating the water. But in the meantime, the hard particles, which are mingled with the air and are agitated, as I have supposed, with simple motion, will not pass through the water of the clouds, but be more straitly compressed within their cavities. And this I have demonstrated at the 4th and 5th articles of chapter XXII. Besides, seeing the globe of the earth floateth in the air which is agitated by the sun's motion, the parts of the air resisted by the earth will spread themselves every way upon the earth's superficies; as I have shown at the 8th article of chapter XXI.

No change from soft to hard, but by motion.

5. We perceive a body to be hard, from this, that, when touching it, we would thrust forwards that part of the same which we touch, we cannot

do it otherwise than by thrusting forwards the
whole body. We may indeed easily and sensibly thrust forwards any particle of the air or water which we touch, whilst yet the rest of its parts remain to sense unmoved. But we cannot do so to any part of a stone. Wherefore I define a *hard* body to be that whereof no part can be sensibly moved, unless the whole be moved. Whatsoever therefore is soft or fluid, the same can never be made hard but by such motion as makes many of the parts together stop the motion of some one part, by resisting the same.

6. Those things premised, I shall show a possible cause why there is greater cold near the poles of the earth, than further from them. The motion of the sun between the tropics, driving the air towards that part of the earth's superficies which is perpendicularly under it, makes it spread itself every way; and the velocity of this expansion of the air grows greater and greater, as the superficies of the earth comes to be more and more straitened, that is to say, as the circles which are parallel to the equator come to be less and less. Wherefore this expansive motion of the air drives before it the parts of the air, which are in its way, continually towards the poles more and more strongly, as its force comes to be more and more united, that is to say, as the circles which are parallel to the equator are less and less; that is, so much the more, by how much they are nearer to the poles of the earth. In those places, therefore, which are nearer to the poles, there is greater cold than in those which are more remote from them. Now this expansion of the air upon the superficies of the

earth, from east to west, doth, by reason of the sun's perpetual accession to the places which are successively under it, make it cold at the time of the sun's rising and setting ; but as the sun comes to be continually more and more perpendicular to those cooled places, so by the heat, which is generated by the supervening simple motion of the sun, that cold is again remitted ; and can never be great, because the action by which it was generated is not permanent. Wherefore I have rendered a possible cause of cold in those places that are near the poles, or where the obliquity of the sun is great.

The cause of ice ; and why the cold is more remiss in rainy than in clear weather. Why water doth not freeze in deep wells, as it doth near the superficies of the earth. Why ice is not so heavy as water ; and why wine is not so easily frozen as water.

7. How water may be congealed by cold, may be explained in this manner. Let A (in figure 1) represent the sun, and B the earth. A will therefore be much greater than B. Let E F be in the plane of the equinoctial ; to which let G H, I K, and L C be parallel. Lastly, let C and D be the poles of the earth. The air, therefore, by its action in those parallels, will rake the superficies of the earth ; and that with motion so much the stronger, by how much the parallel circles towards the poles grow less and less. From whence must arise a wind, which will force together the uppermost parts of the water, and withal raise them a little, weakening their endeavour towards the centre of the earth. And from their endeavour towards the centre of the earth, joined with the endeavour of the said wind, the uppermost parts of the water will be pressed together and coagulated, that is to say, the top of the water will be skinned over and hardened. And so again, the water next the top will be hardened in the same manner, till at length the ice be thick. And this

ice, being now compacted of little hard bodies, must also contain many particles of air received into it.

As rivers and seas, so also in the same manner may the clouds be frozen. For when, by the ascending and descending of several clouds at the same time, the air intercepted between them is by compression forced out, it rakes, and by little and little hardens them. And though those small drops, which usually make clouds, be not yet united into greater bodies, yet the same wind will be made; and by it, as water is congealed into ice, so will vapours in the same manner be congealed into snow. From the same cause it is that ice may be made by art, and that not far from the fire. For it is done by the mingling of snow and salt together, and by burying in it a small vessel full of water. Now while the snow and salt, which have in them a great deal of air, are melting, the air, which is pressed out every way in wind, rakes the sides of the vessel; and as the wind by its motion rakes the vessel, so the vessel by the same motion and action congeals the water within it.

We find by experience, that cold is always more remiss in places where it rains, or where the weather is cloudy, things being alike in all other respects, than where the air is clear. And this agreeth very well with what I have said before. For in clear weather, the course of the wind which, as I said even now, rakes the superficies of the earth, as it is free from all interruption, so also it is very strong. But when small drops of water are either rising or falling, that wind is repelled,

broken, and dissipated by them ; and the less the wind is, the less is the cold.

We find also by experience, that in deep wells the water freezeth not so much as it doth upon the superficies of the earth.   For the wind, by which ice is made, entering into the earth by reason of the laxity of its parts, more or less, loseth some of its force, though not much.   So that if the well be not deep, it will freeze ; whereas if it be so deep, as that the wind which causeth cold cannot reach it, it will not freeze.

We find moreover by experience, that ice is lighter than water. The cause whereof is manifest from that which I have already shown, namely, that air is received in and mingled with the particles of the water whilst it is congealing.

Lastly, wine is not so easily congealed as water, because in wine there are particles, which, being not fluid, are moved very swiftly, and by their motion congelation is retarded.   But if the cold prevail against this motion, then the outermost parts of the wine will be first frozen, and afterwards the inner parts ; whereof this is a sign, that the wine which remains unfrozen in the midst will be very strong.

Another cause of hardness from the fuller contact of atoms. Also how hard things are broken.   8. We have seen one way of making things hard, namely, by congelation.   Another way is thus. Having already supposed that innumerable atoms, some harder than others and that have several simple motions of their own, are intermingled with the ethereal substance ; it follows necessarily from hence, that by reason of the fermentation of the whole air (of which I have spoken in chapter xxi) some of those atoms meeting with others will

cleave together, by applying themselves to one
another in such manner as is agreeable to their
motions and mutual contacts ; and, seeing there is
no vacuum, cannot be pulled asunder, but by
so much force as is sufficient to overcome their
hardness.

Now there are innumerable degrees of hardness.
As for example, there is a degree of it in water,
as is manifest from this, that upon a plane it may
be drawn any way at pleasure by one's finger.
There is a greater degree of it in clammy liquors,
which, when they are poured out, do in falling
downwards dispose themselves into one continued
thread ; which thread, before it be broken, will by
little and little diminish its thickness, till at last it
be so small, as that it seems to break only in a
point ; and in their separation the external parts
break first from one another, and then the more
internal parts successively one after another.   In
wax there is yet a greater degree of hardness. For
when we would pull one part of it from another,
we first make the whole mass slenderer, before we
can pull it asunder.   And how much the harder
anything is which we would break, so much the
more force we must apply to it.   Wherefore, if we
go on to harder things, as ropes, wood, metals,
stones, &c. reason prompteth us to believe that the
same, though not always sensibly, will necessarily
happen ; and that even the hardest things are
broken asunder in the same manner, namely, by
solution of their continuity begun in the outermost
superficies, and proceeding successively to the
innermost parts.   In like manner, when the parts
of bodies are to be separated, not by pulling them

asunder, but by breaking them, the first separation will necessarily be in the convex superficies of the bowed part of the body, and afterwards in the concave superficies. For in all bowing there is in the convex superficies an endeavour in the parts to go one from another, and in the concave superficies to penetrate one another.

This being well understood, a reason may be given how two bodies, which are contiguous in one common superficies, may by force be separated without the introduction of vacuum ; though Lucretius thought otherwise, believing that such separation was a strong establishment of vacuum. For a marble pillar being made to hang by one of its bases, if it be long enough, it will by its own weight be broken asunder ; and yet it will not necessarily follow that there should be vacuum, seeing the solution of its continuity may begin in the circumference, and proceed successively to the midst thereof.

A third cause of hardness, from heat.

9. Another cause of hardness in some things may be in this manner. If a soft body consist of many hard particles, which by the intermixture of many other fluid particles cohere but loosely together, those fluid parts (as hath been shown in the last article of chapter XXI) will be exhaled ; by which means each hard particle will apply itself to the next to it according to a greater superficies, and consequently they will cohere more closely to one another, that is to say, the whole mass will be made harder.

A fourth cause of hardness, from the motion of atoms

10. Again, in some things hardness may be made to a certain degree in this manner. When any fluid substance hath in it certain very small bodies

intermingled, which, being moved with simple mo-
tion of their own, contribute like motion to the
parts of the fluid substance, and this be done in a
small enclosed space, as in the hollow of a little
sphere, or a very slender pipe, if the motion be
vehement and there be a great number of these
small enclosed bodies, two things will happen ; the
one, that the fluid substance will have an endeavour
of dilating itself at once every way ; the other,
that if those small bodies can nowhere get out,
then from their reflection it will follow, that the
motion of the parts of the enclosed fluid substance,
which was vehement before, will now be much
more vehement. Wherefore, if any one particle
of that fluid substance should be touched and
pressed by some external movent, it could not yield
but by the application of very sensible force.
Wherefore the fluid substance, which is enclosed
and so moved, hath some degree of hardness.
Now, greater and less degree of hardness depends
upon the quantity and velocity of those small
bodies, and upon the narrowness of the place both
together.

11. Such things as are made hard by sudden
heat, namely such as are hardened by fire, are
commonly reduced to their former soft form by
maceration. For fire hardens by evaporation, and
therefore if the evaporated moisture be restored
again, the former nature and form is restored
together with it. And such things as are frozen
with cold, if the wind by which they were frozen
change into the opposite quarter, they will be un-
frozen again, unless they have gotten a habit of
new motion or endeavour by long continuance in

that hardness. Nor is it enough to cause thawing, that there be a cessation of the freezing wind ; for the taking away of the cause doth not destroy a produced effect ; but the thawing also must have its proper cause, namely, a contrary wind, or at least a wind opposite in some degree. And this we find to be true by experience. For, if ice be laid in a place so well enclosed that the motion of the air cannot get to it, that ice will remain unchanged, though the place be not sensibly cold.

Whence proceeds the spontaneous restitution of things bent.

12. Of hard bodies, some may manifestly be bowed ; others not, but are broken in the very first moment of their bending. And of such bodies as may manifestly be bended, some being bent, do, as soon as ever they are set at liberty, restore themselves to their former posture ; others remain still bent. Now if the cause of this restitution be asked, I say, it may be in this manner, namely, that the particles of the bended body, whilst it is held bent, do nevertheless retain their motion ; and by this motion they restore it as soon as the force is removed by which it was bent. For when any thing is bent, as a plate of steel, and as soon as the force is removed restores itself again, it is evident that the cause of its restitution cannot be referred to the ambient air ; nor can it be referred to the removal of the force by which it was bent ; for in things that are at rest the taking away of impediments is not a sufficient cause of their future motion ; there being no other cause of motion, but motion. The cause therefore of such restitution is in the parts of the steel itself. Wherefore, whilst it remains bent, there is in the parts, of which it consisteth, some motion though invisible ; that is to

say, some endeavour at least that way by which
the restitution is to be made; and therefore this
endeavour of all the parts together is the first
beginning of restitution; so that the impediment
being removed, that is to say, the force by which
it was held bent, it will be restored again. Now
the motion of the parts, by which this done, is
that which I called simple motion, or motion
returning into itself. When therefore in the bend-
ing of a plate the ends are drawn together, there
is on one side a mutual compression of the parts;
which compression is one endeavour opposite to
another endeavour: and on the other side a divul-
sion of the parts. The endeavour therefore of the
parts on one side tends to the restitution of the
plate from the middle towards the ends; and on
the other side, from the ends towards the middle.
Wherefore the impediment being taken away, this
endeavour, which is the beginning of restitution,
will restore the plate to its former posture. And
thus I have given a possible cause why some bodies,
when they are bent, restore themselves again;
which was to be done.

As for stones, seeing they are made by the
accretion of many very hard particles within the
earth; which particles have no great coherence,
that is to say, touch one another in small latitude,
and consequently admit many particles of air; it
must needs be that, in bending of them, their
internal parts will not easily be compressed, by
reason of their hardness. And because their co-
herence is not firm, as soon as the external hard
particles are disjoined, the ethereal parts will

necessarily break out, and so the body will sud-
denly be broken.

Diaphanous
and opacous,
what they are,
and whence.
13. Those bodies are called *diaphanous*, upon
which, whilst the beams of a lucid body do work,
the action of every one of those beams is propa-
gated in them in such manner, as that they still
retain the same order amongst themselves, or the
inversion of that order; and therefore bodies,
which are perfectly diaphanous, are also perfectly
homogeneous. On the contrary, an *opacous* body
is that, which, by reason of its heterogeneous
nature, doth by innumerable reflections and refrac-
tions in particles of different figures and unequal
hardness, weaken the beams that fall upon it before
they reach the eye. And of diaphanous bodies,
some are made such by nature from the beginning;
as the substance of the air, and of the water, and
perhaps also some parts of stones, unless these
also be water that has been long congealed. Others
are made so by the power of heat, which congre-
gates homogeneous bodies. But such, as are made
diaphanous in this manner, consist of parts which
were formerly diaphanous.

The cause of
lightning and
thunder.
14. In what manner clouds are made by the
motion of the sun, elevating the particles of water
from the sea and other moist places, hath been
explained in chapter xxvi. Also how clouds come
to be frozen, hath been shown above at the 7th
article. Now from this, that air may be enclosed
as it were in caverns, and pent together more and
more by the meeting of ascending and descending
clouds, may be deduced a possible cause of *thunder*
and *lightning*. For seeing the air consists of two

parts, the one ethereal, which has no proper motion of its own, as being a thing divisible into the least parts; the other hard, namely, consisting of many hard atoms, which have every one of them a very swift simple motion of its own: whilst the clouds by their meeting do more and more straiten such cavities as they intercept, the ethereal parts will penetrate and pass through their watery substance; but the hard parts will in the meantime be the more thrust together, and press one another; and consequently, by reason of their vehement motions, they will have an endeavour to rebound from each other. Whensoever, therefore, the compression is great enough, and the concave parts of the clouds are, for the cause I have already given, congealed into ice, the cloud will necessarily be broken; and this breaking of the cloud produceth the first clap of thunder. Afterwards the air, which was pent in, having now broken through, makes a concussion of the air without, and from hence proceeds the roaring and murmur which follows; and both the first clap and the murmur that follows it · make that noise which is called *thunder*. Also, from the same air breaking through the clouds and with concussion falling upon the eye, proceeds that action upon our eye, which causeth in us a perception of that light, which we call *lightning*. Wherefore I have given a possible cause of thunder and lightning.

15. But if the vapours, which are raised into clouds, do run together again into water or be congealed into ice, from whence is it, seeing both ice and water are heavy, that they are sustained in the air? Or rather, what may the cause be, that

being once elevated, they fall down again? For there is no doubt but the same force which could carry up that water, could also sustain it there. Why therefore being once carried up, doth it fall again? I say it proceeds from the same simple motion of the sun, both that vapours are forced to ascend, and that water gathered into clouds is forced to descend. For in chapter xxi, article 11, I have shown how vapours are elevated; and in the same chapter, article 5, I have also shown how by the same motion homogeneous bodies are congregated, and heterogeneous dissipated; that is to say, how such things, as have a like nature to that of the earth, are driven towards the earth; that is to say, what is the cause of the descent of heavy bodies. Now if the action of the sun be hindered in the raising of vapours, and be not at all hindered in the casting of them down, the water will descend. But a cloud cannot hinder the action of the sun in making things of an earthly nature descend to the earth, though it may hinder it in making vapours ascend. For the lower part of a thick cloud is so covered by its upper part, as that it cannot receive that action of the sun by which vapours are carried up; because vapours are raised by the perpetual fermentation of the air, or by the separating of its smallest parts from one another, which is much weaker when a thick cloud is interposed, than when the sky is clear. And therefore, whensoever a cloud is made thick enough, the water, which would not descend before, will then descend, unless it be kept up by the agitation of the wind. Wherefore I have rendered a possible cause, both why the clouds may

be sustained in the air, and also why they may fall
down again to the earth ; which was propounded
to be done.

16. Granting that the clouds may be frozen, it is How it could be that the moon was eclipsed, when she was not diametrically opposite to the sun.
no wonder if the moon have been seen eclipsed at
such time as she hath been almost two degrees
above the horizon, the sun at the same time appear-
ing in the horizon ; for such an eclipse was ob-
served by Mæstlin, at Tubingen, in the year 1590.
For it might happen that a frozen cloud was then
interposed between the sun and the eye of the
observer. And if it were so, the sun, which was
then almost two degrees below the horizon, might
appear to be in it, by reason of the passing of his
beams through the ice. And it is to be noted that
those, that attribute such refractions to the atmos-
phere, cannot attribute to it so great a refraction
as this. Wherefore not the atmosphere, but either
water in a continued body, or else ice, must be the
cause of that refraction.

17. Again, granting that there may be ice in the By what means many suns may appear at once.
clouds, it will be no longer a wonder that many
suns have sometimes appeared at once. For look-
ing-glasses may be so placed, as by reflections to
show the same object in many places. And may
not so many frozen clouds serve for so many look-
ing-glasses? And may they not be fitly disposed for
that purpose? Besides, the number of appearances
may be increased by refractions also ; and there-
fore it would be a greater wonder to me, if such
phenomena as these should never happen.

And were it not for that one phenomenon of the
new star which was seen in Cassiopea, I should
think comets were made in the same manner,

namely, by vapours drawn not only from the earth but from the rest of the planets also, and congealed into one continued body. For I could very well from hence give a reason both of their hair, and of their motions. But seeing that star remained sixteen whole months in the same place amongst the fixed stars, I cannot believe the matter of it was ice. Wherefore I leave to others the disquisition of the cause of comets; concerning which nothing that hath hitherto been published, besides the bare histories of them, is worth considering.

Of the heads of rivers.

18. The heads of rivers may be deduced from rain-water, or from melted snows, very easily; but from other causes, very hardly, or not at all. For both rain-water and melted snows run down the descents of mountains; and if they descend only by the outward superficies, the showers or snows themselves may be accounted the springs or fountains; but if they enter the earth and descend within it, then, wheresoever they break out, there are their springs. And as these springs make small streams, so, many small streams running together make rivers. Now, there was never any spring found, but where the water which flowed to it, was either further, or at least as far from the centre of the earth, as the spring itself. And whereas it has been objected by a great philosopher, that in the top of Mount Cenis, which parts Savoy from Piedmont, there springs a river which runs down by Susa; it is not true. For there are above that river, for two miles length, very high hills on both sides, which are almost perpetually covered with snow; from which innumerable little streams running down do manifestly supply that river with water sufficient for its magnitude.

CHAP. XXIX.

OF SOUND, ODOUR, SAVOUR, AND TOUCH.

1. The definition of sound, and the distinctions of sounds.
2. The cause of the degrees of sounds.—3. The difference between sounds acute and grave.—4. The difference between clear and hoarse sounds, whence.—5. The sound of thunder and of a gun, whence it proceeds.—6. Whence it is that pipes, by blowing into them, have a clear sound.—7. Of reflected sound.—8. From whence it is that sound is uniform and lasting.—9. How sound may be helped and hindered by the wind. 10. Not only air, but other bodies how hard soever they be, convey sound.—11. The causes of grave and acute sounds, and of concent.—12. Phenomena for smelling.—13. The first organ and the generation of smelling.—14. How it is helped by heat and by wind.—15. Why such bodies are least smelt, which have least intermixture of air in them.—16. Why odorous things become more odorous by being bruised.—17. The first organ of tasting; and why some savours cause nauseousness.— 18. The first organ of feeling; and how we come to the knowledge of such objects as are common to the touch and other senses.

1. SOUND *is sense generated by the action of the medium, when its motion reacheth the ear and the rest of the organs of sense.* Now, the motion of the medium is not the sound itself, but the cause of it. For the phantasm which is made in us, that is to say, the reaction of the organ, is properly that which we call *sound.*

PART IV.
29.

The definition of sound, and the distinction of sounds.

The principal distinctions of sounds are these; first, that one sound is stronger, another weaker. Secondly, that one is more grave, another more acute. Thirdly, that one is clear, another hoarse. Fourthly, that one is primary, another derivative.

Fifthly, that one is uniform, another not. Sixthly, that one is more durable, another less durable. Of all which distinctions the members may be sub-distinguished into parts distinguishable almost infinitely. For the variety of sounds seems to be not much less than that of colours.

As vision, so hearing is generated by the motion of the medium, but not in the same manner. For sight is from pressure, that is, from an endeavour; in which there is no perceptible progression of any of the parts of the medium; but one part urging or thrusting on another propagateth that action successively to any distance whatsoever; whereas the motion of the medium, by which sound is made, is a stroke. For when we hear, the drum of the ear, which is the first organ of hearing, is stricken; and the drum being stricken, the *pia mater* is also shaken, and with it the arteries which are inserted into it; by which the action being propagated to the heart itself, by the reaction of the heart a phantasm is made which we call sound; and because the reaction tendeth outwards, we think it is without.

The cause of the degrees of sounds.

2. And seeing the effects produced by motion are greater or less, not only when the velocity is greater or less, but also when the body hath greater or less magnitude though the velocity be the same; a sound may be greater or less both these ways. And because neither the greatest nor the least magnitude or velocity can be given, it may happen that either the motion may be of so small velocity, or the body itself of so small magnitude, as to produce no sound at all; or either of them may be so

great, as to take away the faculty of sense by hurting the organ.

From hence may be deduced possible causes of the strength and weakness of sounds in the follow- ing phenomena.

The first whereof is this, that if a man speak through a trunk which hath one end applied to the mouth of the speaker, and the other to the ear of the hearer, the sound will come stronger than it would do through the open air. And the cause, not only the possible, but the certain and manifest cause is this, that the air which is moved by the first breath and carried forwards in the trunk, is not diffused as it would be in the open air, and is consequently brought to the ear almost with the same velocity with which it was first breathed out. Whereas, in the open air, the first motion diffuseth itself every way into circles, such as are made by the throwing of a stone into a standing water, where the velocity grows less and less as the undulation proceeds further and further from the beginning of its motion.

The second is this, that if the trunk be short, and the end which is applied to the mouth be wider than that which is applied to the ear, thus also the sound will be stronger than if it were made in the open air. And the cause is the same, namely, that by how much the wider end of the trunk is less distant from the beginning of the sound, by so much the less is the diffusion.

The third, that it is easier for one, that is within a chamber, to hear what is spoken without, than for him, that stands without, to hear what is spoken within. For the windows and other inlets of the

moved air are as the wide end of the trunk.   And
for this reason some creatures seem to hear the
better, because nature has bestowed upon them
wide and capacious ears.

The fourth is this, that though he, which standeth
upon the sea-shore, cannot hear the collision of
the two nearest waves, yet nevertheless he hears
the roaring of the whole sea.   And the cause seems
to be this, that though the several collisions move
the organ, yet they are not severally great enough
to cause sense ; whereas nothing hinders but that
all of them together may make sound.

The difference
between sounds
acute and grave
3. That bodies when they are stricken do yield
some a more grave, others a more acute sound, the
cause may consist in the difference of the times in
which the parts stricken and forced out of their
places return to the same places again.   For in
some bodies, the restitution of the moved parts is
quick, in others slow.   And this also may be the
cause, why the parts of the organ, which are moved
by the medium, return to their rest again, some-
times sooner, sometimes later.   Now, by how much
the vibrations or the reciprocal motions of the
parts are more frequent, by so much doth the
whole sound made at the same time by one stroke
consist of more, and consequently of smaller parts.
For what is acute in sound, the same is subtle in
matter ; and both of them, namely acute sound
and subtle matter, consist of very small parts, that
of time, and this of the matter itself.

The third distinction of sounds cannot be con-
ceived clearly enough by the names I have used of
clear and *hoarse,* nor by any other that I know ;
and therefore it is needful to explain them by

examples. When I say hoarse, I understand whis- PART IV. 29.
pering and hissing, and whatsoever is like to these,
by what appellation soever it be expressed. And The difference between sounds
sounds of this kind seem to be made by the force acute and grave
of some strong wind, raking rather than striking
such hard bodies as it falls upon.   On the con-
trary, when I use the word clear, I do not under-
stand such a sound as may be easily and distinctly
heard; for so whispers would be clear; but such
as is made by somewhat that is broken, and such
as is clamour, tinkling, the sound of a trumpet, &c.
and to express it significantly in one word, noise.
And seeing no sound is made but by the concourse
of two bodies at the least, by which concourse it is
necessary that there be as well reaction as action,
that is to say, one motion opposite to another; it
follows that according as the proportion between
those two opposite motions is diversified, so the
sounds which are made will be different from one
another. And whensoever the proportion between
them is so great, as that the motion of one of the
bodies be insensible if compared with the motion
of the other, then the sound will not be of the same
kind; as when the wind falls very obliquely upon
a hard body, or when a hard body is carried swiftly
through the air; for then there is made that sound
which I call a hoarse sound, in Greek συριγμος.
Therefore the breath blown with violence from the
mouth makes a hissing, because in going out it
rakes the superficies of the lips, whose reaction
against the force of the breath is not sensible.
And this is the cause why the winds have that
hoarse sound. Also if two bodies, how hard soever,
be rubbed together with no great pressure, they

make a hoarse sound. And this hoarse sound, when it is made, as I have said, by the air raking the superficies of a hard body, seemeth to be nothing but the dividing of the air into innumerable and very small files. For the asperity of the superficies doth, by the eminences of its innumerable parts, divide or cut in pieces the air that slides upon it.

The difference between clear & hoarse sounds, whence.

4. *Noise,* or that which I call clear sound, is made two ways; one, by two hoarse sounds made by opposite motions; the other, by collision, or by the sudden pulling asunder of two bodies, whereby their small particles are put into commotion, or being already in commotion suddenly restore themselves again; which motion, making impression upon the medium, is propagated to the organ of hearing. And seeing there is in this collision or divulsion an endeavour in the particles of one body, opposite to the endeavour of the particles of the other body, there will also be made in the organ of hearing a like opposition of endeavours, that is to say, of motions; and consequently the sound arising from thence will be made by two opposite motions, that is to say, by two opposite hoarse sounds in one and the same part of the organ. For, as I have already said, a hoarse sound supposeth the sensible motion of but one of the bodies. And this opposition of motions in the organ is the cause why two bodies make a noise, when they are either suddenly stricken against one another, or suddenly broken asunder.

The sound of thunder and of a gun, whence it proceeds.

5. This being granted, and seeing withal that thunder is made by the vehement eruption of the air out of the cavities of congealed clouds, the

cause of the great noise or clap may be the sudden
breaking asunder of the ice. For in this action it
is necessary that there be not only a great concus-
sion of the small particles of the broken parts, but
also that this concussion, by being communicated
to the air, be carried to the organ of hearing, and
make impression upon it. And then, from the
first reaction of the organ proceeds that first and
greatest sound, which is made by the collision of
the parts whilst they restore themselves. And
seeing there is in all concussion a reciprocation
of motion forwards and backwards in the parts
stricken; for opposite motions cannot extinguish
one another in an instant, as I have shown in the
11th article of chapter VIII; it follows necessarily
that the sound will both continue, and grow weaker
and weaker, till at last the action of the recipro-
cating air grow so weak, as to be imperceptible.
Wherefore a possible cause is given both of the
first fierce noise of the thunder, and also of the
murmur that follows it.

The cause of the great sound from a discharged
piece of ordnance is like that of a clap of thunder.
For the gunpowder being fired doth, in its en-
deavour to go out, attempt every way the sides of
the metal in such manner, as that it enlargeth the
circumference all along, and withal shorteneth the
axis; so that whilst the piece of ordnance is in
discharging, it is made both wider and shorter
than it was before; and therefore also presently
after it is discharged its wideness will be dimi-
nished, and its length increased again by the resti-
tution of all the particles of the matter, of which it
consisteth, to their former position. And this is

The sound of
thunder and of
a gun, whence
it proceeds.

done with such motions of the parts, as are not only very vehement, but also opposite to one another ; which motions, being communicated to the air, make impression upon the organ, and by the reaction of the organ create a sound, which lasteth for some time ; as I have already shown in this article.

I note by the way, as not belonging to this place, that the possible cause why a gun recoils when it is shot off, may be this ; that being first swollen by the force of the fire, and afterwards restoring itself, from this restitution there proceeds an endeavour from all the sides towards the cavity ; and consequently this endeavour is in those parts which are next the breech ; which being not hollow, but solid, the effect of the restitution is by it hindered and diverted into the length ; and by this means both the breech and the whole gun is thrust backwards ; and the more forcibly by how much the force is greater, by which the part next the breech is restored to its former posture, that is to say, by how much the thinner is that part. The cause, therefore, why guns recoil, some more some less, is the difference of their thickness towards the breech ; and the greater that thickness is, the less they recoil ; and contrarily.

Whence it is that pipes, by blowing into them, have a clear sound.
6. Also the cause why the sound of a pipe, which is made by blowing into it, is nevertheless clear, is the same with that of the sound which is made by collision. For if the breath, when it is blown into a pipe, do only rake its concave superficies, or fall upon it with a very sharp angle of incidence, the sound will not be clear, but hoarse. But if the

angle be great enough, the percussion, which is made against one of the hollow sides, will be reverberated to the opposite side; and so successive repercussions will be made from side to side, till at last the whole concave superficies of the pipe be put into motion; which motion will be reciprocated, as it is in collision; and this reciprocation being propagated to the organ, from the reaction of the organ will arise a clear sound, such as is made by collision, or by breaking asunder of hard bodies.

In the same manner it is with the sound of a man's voice. For when the breath passeth out without interruption, and doth but lightly touch the cavities through which it is sent, the sound it maketh is a hoarse sound. But if in going out it strike strongly upon the *larynx*, then a clear sound is made, as in a pipe. And the same breath, as it comes in divers manners to the palate, the tongue, the lips, the teeth, and other organs of speech, so the sounds into which it is articulated become different from one another.

7. I call that *primary sound*, which is generated by motion from the sounding body to the organ in a strait line without reflection; and I call that *reflected sound*, which is generated by one or more reflections, being the same with that we call *echo*, and is iterated as often as there are reflections made from the object to the ear. And these reflections are made by hills, walls, and other resisting bodies, so placed as that they make more or fewer reflections of the motion, according as they are themselves more or fewer in number; and they make them more or less frequently, according

Of reflected sound.

as they are more or less distant from one another. Now the cause of both these things is to be sought for in the situation of the reflecting bodies, as is usually done in sight. For the laws of reflection are the same in both, namely, that the angles of incidence and reflection be equal to one another. If, therefore, in a hollow elliptic body, whose inside is well polished, or in two right parabolical solids, which are joined together by one common base, there be placed a sounding body in one of the burning points, and the ear in the other, there will be heard a sound by many degrees greater than in the open air; and both this, and the burning of such combustible things, as being put in the same places are set on fire by the sun-beams, are effects of one and the same cause. But, as when the visible object is placed in one of the burning points, it is not distinctly seen in the other, because every part of the object being seen in every line, which is reflected from the concave superficies to the eye, makes a confusion in the sight; so neither is sound heard articulately and distinctly when it comes to the ear in all those reflected lines. And this may be the reason why in churches which have arched roofs, though they be neither elliptical nor parabolical, yet because their figure is not much different from these, the voice from the pulpit will not be heard so articulately as it would be, if there were no vaulting at all.

From whence it is that sound is uniform and lasting.

8. Concerning the *uniformity* and *duration* of sounds, both which have one common cause, we may observe, that such bodies as being stricken yield an unequal or harsh sound, are very heterogeneous, that is to say, they consist of parts which

are very unlike both in figure and hardness, such
as are wood, stones, and others not a few.   When
these are stricken, there follows a concussion of
their internal particles, and a restitution of them
again.   But they are neither moved alike, nor
have they the same action upon one another;
some of them recoiling from the stroke, whilst
others which have already finished their recoilings
are now returning; by which means they hinder
and stop one another.   And from hence it is that
their motions are not only unequal and harsh, but
also that their reciprocations come to be quickly
extinguished.   Whensoever, therefore, this motion
is propagated to the ear, the sound it makes is
unequal and of small duration.   On the contrary,
if a body that is stricken be not only sufficiently
hard, but have also the particles of which it con-
sisteth like to one another both in hardness and
figure, such as are the particles of glass and metals,
which being first melted do afterwards settle and
harden; the sound it yieldeth will, because the
motions of its parts and their reciprocations are
like and uniform, be uniform and pleasant, and be
more or less lasting, according as the body stricken
hath greater or less magnitude.   The possible
cause, therefore, of sounds uniform and harsh, and
of their longer or shorter duration, may be one
and the same likeness and unlikeness of the inter-
nal parts of the sounding body, in respect both of
their figure and hardness.

Besides, if two plane bodies of the same matter
and of equal thickness, do both yield an uniform
sound, the sound of that body, which hath the
greatest extent of length, will be the longest heard.

PART IV.
29.

From whence it is that sound is uniform and lasting.

For the motion, which in both of them hath its beginning from the point of percussion, is to be propagated in the greater body through a greater space, and consequently that propagation will require more time; and therefore also the parts which are moved, will require more time for their return. Wherefore all the reciprocations cannot be finished but in longer time; and being carried to the ear, will make the sound last the longer. And from hence it is manifest, that of hard bodies which yield an uniform sound, the sound lasteth longer which comes from those that are round and hollow, than from those that are plane, if they be like in all other respects. For in circular lines the action, which begins at any point, hath not from the figure any end of its propagation, because the line in which it is propagated returns again to its beginning; so that the figure hinders not but that the motion may have infinite progression. Whereas in a plane, every line hath its magnitude finite, beyond which the action cannot proceed. If, therefore, the matter be the same, the motion of the parts of that body whose figure is round and hollow, will last longer than of that which is plane.

Also, if a string which is stretched be fastened at both ends to a hollow body, and be stricken, the sound will last longer than if it were not so fastened; because the trembling or reciprocation which it receives from the stroke, is by reason of the connection communicated to the hollow body; and this trembling, if the hollow body be great, will last the longer by reason of that greatness. Where-

fore also, for the reason above mentioned, the PART IV.
sound will last the longer.   29.

9. In hearing it happens, otherwise than in How sound
seeing, that the action of the medium is made may be helped
and hindered
stronger by the wind when it blows the same by the wind.
way, and weaker when it blows the contrary way.
The cause whereof cannot proceed from anything
but the different generation of sound and light.
For in the generation of light, none of the parts
of the medium between the object and the eye are
moved from their own places to other places sen-
sibly distant; but the action is propagated in
spaces imperceptible; so that no contrary wind
can diminish, nor favourable wind encrease the
light, unless it be so strong as to remove the
object further off or bring it nearer to the eye.
For the wind, that is to say the air moved, doth
not by its interposition between the object and the
eye work otherwise than it would do, if it were
still and calm. For, where the pressure is perpetual,
one part of the air is no sooner carried away, but
another, by succeeding it, receives the same impres-
sion, which the part carried away had received before.
But in the generation of sound, the first collision
or breaking asunder beateth away and driveth out
of its place the nearest part of the air, and that to
a considerable distance, and with considerable
velocity; and as the circles grow by their remote-
ness wider and wider, so the air being more and
more dissipated, hath also its motion more and
more weakened. Whensoever therefore the air is
so stricken as to cause sound, if the wind fall upon
it, it will move it all nearer to the ear, if it blow

that way, and further from it, if it blow the contrary way ; so that according as it blows from or towards the object, so the sound which is heard will seem to come from a nearer or remoter place; and the reaction, by reason of the unequal distances, be strengthened or debilitated.

From hence may be understood the reason why the voice of such as are said to speak in their bellies, though it be uttered near hand, is nevertheless heard, by those that suspect nothing, as if it were a great way off. For having no former thought of any determined place from which the voice should proceed, and judging according to the greatness, if it be weak they think it a great way off, if strong near. These ventriloqui, therefore, by forming their voice, not as others by the emission of their breath, but by drawing it inwards, do make the same appear small and weak ; which weakness of the voice deceives those, that neither suspect the artifice nor observe the endeavour which they use in speaking ; and so, instead of thinking it weak, they think it far off.

Not only air, but other bodies, how hard soever they be, convey sound.   10. As for the medium, which conveys sound, it is not air only. For water, or any other body how hard soever, may be that medium. For the motion may be propagated perpetually in any hard continuous body; but by reason of the difficulty, with which the parts of hard bodies are moved, the motion in going out of hard matter makes but a weak impression upon the air. Nevertheless, if one end of a very long and hard beam be stricken, and the ear be applied at the same time to the other end, so that, when the action goeth out of the beam, the

air, which it striketh, may immediately be received
by the ear, and be carried to the tympanum, the
sound will be considerably strong.

In like manner, if in the night, when all other
noise which may hinder sound ceaseth, a man lay
his ear to the ground, he will hear the sound of
the steps of passengers, though at a great distance;
because the motion, which by their treading they
communicate to the earth, is propagated to the ear
by the uppermost parts of the earth which receiveth
it from their feet.

11. I have shown above, that the difference be- The causes of
tween grave and acute sounds consisteth in this, sounds, and of
that by how much the shorter the time is, in which concent.
the reciprocations of the parts of a body stricken
are made, by so much the more acute will be the
sound. Now by how much a body of the same
bigness is either more heavy or less stretched, by
so much the longer will the reciprocations last;
and therefore heavier and less stretched bodies,
if they be like in all other respects, will yield a
graver sound than such as be lighter and more
stretched.

As for the concent of sounds, it is to be con-
sidered that the reciprocation or vibration of the
air, by which sound is made, after it hath reached
the drum of the ear, imprinteth a like vibration
upon the air that is inclosed within it; by which
means the sides of the drum within are stricken
alternately. Now the concent of two sounds
consists in this, that the tympanum receives its
sounding stroke from both the sounding bodies in
equal and equally frequent spaces of time; so that
when two strings make their vibrations in the same.

PHYSICS.

PART IV.
29.

The causes of
grave and acute
sounds, and of
concent.

times, the concent they produce is the most exquisite of all other. For the sides of the tympanum, that is to say of the organ of hearing, will be stricken by both those vibrations together at once, on one side or other. For example, if the two equal strings A B and C D be stricken together, and the latitudes of their vibrations E F and G H be also equal, and the points E, G, F and H be in the concave superficies of the tympanum, so that it receive strokes from both the strings together in E and G, and again together in F and H, the sound, which is made by the vibrations of each string, will be so like, that it may be taken for the same sound, and is called *unison;* which is the greatest concord. Again, the string A B retaining still its former vibration E F, let the string C D be stretched till its vibration have double the swiftness it had before, and let E F be divided equally in I. In what time therefore the string C D makes one part of its vibration from G to H, in the same time the string A B will make one part of its vibration from E to I; and in what time the string C D hath made the other part of its vibration back from H to G, in the same time another part of the vibration of the string A B will be made from I to F. But the points F and G are both in the sides of the organ, and therefore they will strike the organ both together, not at every stroke, but at every other stroke. And this is the nearest concord to unison, and makes that sound which is called an *eighth.*

A————B
C————D
G   E

K

I

L

H   F

Again, the vibration of the string A B remaining still the same it was, let C D be stretched till its vibration be swifter than that of the string A B in the proportion of 3 to 2, and let E F be divided into three equal parts in K and L.   In what time therefore the string C D makes one third part of its vibration, which third part is from G to H, the string A B will make one third part of its vibration, that is to say, two-thirds of E F, namely, E L. And in what time the string C D makes another third part of its vibration, namely H G, the string A B will make another third part of its vibration, namely from L to F, and back again from F to L. Lastly, whilst the string C D makes the last third part of its vibration, that is from G to H, the string A B will make the last third part of its vibration from L to E.   But the points E and H are both in the sides of the organ.   Wherefore, at every third time, the organ will be stricken by the vibration of both the strings together, and make that concord which is called a *fifth*.

12. For the finding out the cause of *smells*, I shall make use of the evidence of these following phenomena.   First, that smelling is hindered by cold, and helped by heat.   Secondly, that when the wind bloweth from the object, the smell is the stronger ; and, contrarily, when it bloweth from the sentient towards the object, the weaker ; both which phenomena are, by experience, manifestly found to be true in dogs, which follow the track of beasts by the scent.   Thirdly, that such bodies, as are less pervious to the fluid medium, yield less smell than such as are more pervious ; as may be seen in stones and metals, which, compared with

plants and living creatures, and their parts, fruits and excretions, have very little or no smell at all. Fourthly, that such bodies, as are of their own nature odorous, become yet more odorous when they are bruised. Fifthly, that when the breath is stopped, at least in men, nothing can be smelt. Sixthly, that the sense of smelling is also taken away by the stopping of the nostrils, though the mouth be left open.

The first organ and the generation of smelling.

13. By the fifth and sixth phenomenon it is manifest, that the first and immediate organ of smelling is the innermost cuticle of the nostrils, and that part of it, which is below the passage common to the nostrils and the palate. For when we draw breath by the nostrils we draw it into the lungs. That breath, therefore, which conveys smells is in the way which passeth to the lungs, that is to say, in that part of the nostrils which is below the passage through which the breath goeth. For, nothing is smelt, neither beyond the passage of the breath within, nor at all without the nostrils.

And seeing that from different smells there must necessarily proceed some mutation in the organ, and all mutation is motion ; it is therefore also necessary that, in smelling, the parts of the organ, that is to say of that internal cuticle and the nerves that are inserted into it, must be diversely moved by different smells. And seeing also, that it hath been demonstrated, that nothing can be moved but by a body that is already moved and contiguous ; and that there is no other body contiguous to the internal membrane of the nostrils but breath, that is to say attracted air, and such little solid invisible bodies, if there be any such, as

are intermingled with the air; it follows neces-
sarily, that the cause of smelling is either the
motion of that pure air or ethereal substance, or
the motion of those small bodies.   But this motion
is an effect proceeding from the object of smell,
and, therefore, either the whole object itself or its
several parts must necessarily be moved.   Now,
we know that odorous bodies make odour, though
their whole bulk be not moved.   Wherefore the
cause of odour is the motion of the invisible parts
of the odorous body.   And these invisible parts do
either go out of the object, or else, retaining their
former situation with the rest of the parts, are
moved together with them, that is to say, they have
simple and invisible motion.   They that say, there
goes something out of the odorous body, call it
an effluvium; which effluvium is either of the
ethereal substance, or of the small bodies that are
intermingled with it.   But, that all variety of
odours should proceed from the effluvia of those
small bodies that are intermingled with the ethe-
real substance, is altogether incredible, for these
considerations ; first, that certain unguents, though
very little in quantity, do nevertheless send forth
very strong odours, not only to a great distance of
place, but also to a great continuance of time, and
are to be smelt in every point both of that place
and time ; so that the parts issued out are sufficient
to fill ten thousand times more space, than the
whole odorous body is able to fill; which is impos-
sible.   Secondly, that whether that issuing out be
with strait or with crooked motion, if the same
quantity should flow from any other odorous body
with the same motion, it would follow that all
odorous bodies would yield the same smell. Thirdly,

that seeing those effluvia have great velocity of motion (as is manifest from this, that noisome odours proceeding from caverns are presently smelt at a great distance) it would follow, that, by reason there is nothing to hinder the passage of those effluvia to the organ, such motion alone were sufficient to cause smelling ; which is not so ; for we cannot smell at all, unless we draw in our breath through our nostrils.   Smelling, therefore, is not caused by the effluvium of atoms ; nor, for the same reason, is it caused by the effluvium of ethereal substance ; for so also we should smell without the drawing in of our breath.   Besides, the ethereal substance being the same in all odorous bodies, they would always affect the organ in the same manner; and, consequently, the odours of all things would be alike.

It remains, therefore, that the cause of smelling must consist in the simple motion of the parts of odorous bodies without any efflux or diminution of their whole substance.   And by this motion there is propagated to the organ, by the intermediate air, the like motion, but not strong enough to excite sense of itself without the attraction of air by respiration.   And this is a possible cause of smelling.

How smelling is helped by heat and by wind.

14. The cause why smelling is hindered by cold and helped by heat may be this ; that heat (as hath been shown in chapter xxi) generateth simple motion ; and therefore also, wheresoever it is already, there it will increase it ; and the cause of smelling being increased, the smell itself will also be increased.   As for the cause why the wind blowing from the object makes the smell the stronger, it is all one with that for which the at-

traction of air in respiration doth the same.   For,
he that draws in the air next to him, draws with it
by succession that air in which is the object.
Now, this motion of the air is wind, and, when
another wind bloweth from the object, will be in-
creased by it.

15. That bodies which contain the least quan- Why such bo-
dies are least
tity of air, as stones and metals, yield less smell smelt, which
than plants and living creatures ; the cause may have the least
intermixture
be, that the motion, which causeth smelling, is a of air in them.
motion of the fluid parts only ; which parts, if
they have any motion from the hard parts in which
they are contained, they communicate the same to
the open air, by which it is propagated to the
organ.   Where, therefore, there are no fluid parts
as in metals, or where the fluid parts receive no
motion from the hard parts, as in stones, which
are made hard by accretion, there can be no smell.
And therefore also the water, whose parts have
little or no motion, yieldeth no smell.   But, if the
same water, by seeds and the heat of the sun, be
together with particles of earth raised into a plant,
and be afterwards pressed out again, it will be
odorous, as wine from the vine.   And as water
passing through plants is by the motion of the
parts of those plants made an odorous liquor ; so
also of air, passing through the same plants whilst
they are growing, are made odorous airs.   And
thus also it is with the juices and spirits, which are
bred in living creatures.

16. That odorous bodies may be made more Why odorous
things become
odorous by contrition proceeds from this, that more odorous,
being broken into many parts, which are all odor- when bruised.
ous, the air, which by respiration is drawn from
the object towards the organ, doth in its passage

touch upon all those parts, and receive their motion. Now, the air toucheth the superficies only ; and a body having less superficies whilst it is whole, than all its parts together have after it is reduced to powder, it follows that the same odorous body yieldeth less smell whilst it is whole, than it will do after it is broken into smaller parts.   And thus much of smells.

The first organ of tasting ; and why some savours cause nauseousness.   17. The taste follows ; whose generation hath this difference from that of the sight, hearing, and smelling, that by these we have sense of remote objects ; whereas, we taste nothing but what is contiguous, and doth immediately touch either the tongue or palate, or both.   From whence it is evident, that the cuticles of the tongue and palate, and the nerves inserted into them are the first organ of taste ; and, because from the concussion of the parts of these, there followeth necessarily a concussion of the *pia mater*, that the action communicated to these is propagated to the brain, and from thence to the farthest organ, namely, the heart, in whose reaction consisteth the nature of sense.

Now, that savours, as well as odours, do not only move the brain but the stomach also, as is manifest by the loathings that are caused by them both ; they, that consider the organ of both these senses, will not wonder at all ; seeing the tongue, the palate and the nostrils, have one and the same continued cuticle, derived from the *dura mater*.

And that effluvia have nothing to do in the sense of tasting, is manifest from this, that there is no taste where the organ and the object are not contiguous.

By what variety of motions the different kinds of tastes, which are innumerable, may be distinguished, I know not. I might with others derive them from the divers figures of those atoms, of which whatsoever may be tasted consisteth; or from the diverse motions which I might, by way of supposition, attribute to those atoms; conjecturing, not without some likelihood of truth, that such things as taste sweet have their particles moved with slow circular motion, and their figures spherical; which makes them smooth and pleasing to the organ; that bitter things have circular motion, but vehement, and their figures full of angles, by which they trouble the organ; and that sour things have strait and reciprocal motion, and their figures long and small, so that they cut and wound the organ. And in like manner I might assign for the causes of other tastes such several motions and figures of atoms, as might in probability seem to be the true causes. But this would be to revolt from philosophy to divination.

18. By the *touch*, we feel what bodies are cold or hot, though they be distant from us. Others, as hard, soft, rough, and smooth, we cannot feel unless they be contiguous. The organ of touch is every one of those membranes, which being continued from the *pia mater* are so diffused throughout the whole body, as that no part of it can be pressed, but the *pia mater* is pressed together with it. Whatsoever therefore presseth it, is felt as hard or soft, that is to say, as more or less hard. And as for the sense of rough, it is nothing else but innumerable perceptions of hard and hard succeeding one another by short intervals both of

The first organ of feeling; and how we came to the knowledge of such objects as are common to the touch and to other senses,

time and place.   For we take notice of rough and
smooth, as also of magnitude and figure, not only
by the touch, but also by memory.   For though
some things are touched in one point, yet rough
and smooth, like quantity and figure, are not per-
ceived but by the flux of a point, that is to say,
we have no sense of them without time ; and we
can have no sense of time without memory.

CHAPTER XXX.

OF GRAVITY.

1. A thick body doth not contain more matter, unless also more
place, than a thin.—2. That the descent of heavy bodies pro-
ceeds not from their own appetite, but from some power of
the earth.—3. The difference of gravities proceedeth from the
difference of the impetus with which the elements, whereof
heavy bodies are made, do fall upon the earth.—4. The cause
of the descent of heavy bodies.—5. In what proportion the
descent of heavy bodies is accelerated.—6. Why those that
dive do not, when they are under water, feel the weight of the
water above them.—7. The weight of a body that floateth, is
equal to the weight of so much water as would fill the space,
which the immersed part of the body takes up within the
water.—8. If a body be lighter than water, then how big
soever that body be, it will float upon any quantity of water,
how little soever.—9. How water may be lifted up and forced
out of a vessel by air.—10. Why a bladder is heavier when
blown full of air, than when it is empty.—11. The cause of
the ejection upwards of heavy bodies from a wind-gun.
12. The cause of the ascent of water in a weather-glass.
13. The cause of motion upwards in living creatures.—14. That
there is in nature a kind of body heavier than air, which never-
theless is not by sense distinguishable from it.—15. Of the
cause of magnetical virtue.

A thick body
doth not con-
tain more mat-
1. In chapter XXI I have defined thick and thin,
as that place required, so, as that by thick was

signified a more resisting body, and by thin, a body
less resisting ; following the custom of those that
have before me discoursed of refraction. Now if
we consider the true and vulgar signification of
those words, we shall find them to be names col-
lective, that is to say, names of multitude ; as *thick*
to be that, which takes up more parts of a space
given, and *thin* that, which contains fewer parts of
the same magnitude in the same space, or in a
space equal to it. Thick therefore is the same
with frequent, as a thick troop ; and thin the same
with unfrequent, as a thin rank, thin of houses ;
not that there is more matter in one place than in
another equal place, but a greater quantity of some
named body. For there is not less matter or body,
indefinitely taken, in a desert, than there is in a
city ; but fewer houses, or fewer men. Nor is
there in a thick rank a greater quantity of body,
but a greater number of soldiers, than in a thin.
Wherefore the multitude and paucity of the parts
contained within the same space do constitute
density and rarity, whether those parts be sepa-
rated by vacuum or by air. But the consideration
of this is not of any great moment in philosophy ;
and therefore I let it alone, and pass on to the
search of the causes of *gravity*.

2. Now we call those bodies *heavy*, which,
unless they be hindered by some force, are carried
towards the centre of the earth, and that by their
own accord, for aught we can by sense perceive to
the contrary. Some philosophers therefore have
been of opinion, that the descent of heavy bodies
proceeded from some internal appetite, by which,
when they were cast upwards, they descended

again, as moved by themselves, to such place as was agreeable to their nature. Others thought they were attracted by the earth. To the former I cannot assent, because I think I have already clearly enough demonstrated that there can be no beginning of motion, but from an external and moved body; and consequently, that whatsoever hath motion or endeavour towards any place, will always move or endeavour towards that same place, unless it be hindered by the reaction of some external body. Heavy bodies, therefore, being once cast upwards, cannot be cast down again but by external motion. Besides, seeing inanimate bodies have no appetite at all, it is ridiculous to think that by their own innate appetite they should, to preserve themselves, not understanding what preserves them, forsake the place they are in, and transfer themselves to another place; whereas man, who hath both appetite and understanding, cannot, for the preservation of his own life, raise himself by leaping above three or four feet from the ground. Lastly, to attribute to created bodies the power to move themselves, what is it else than to say that there be creatures which have no dependance upon the Creator? To the latter, who attribute the descent of heavy bodies to the attraction of the earth, I assent. But by what motion this is done, hath not as yet been explained by any man. I shall therefore in this place say somewhat of the manner and of the way by which the earth by its action attracteth heavy bodies.

The difference of gravities proceedeth from the differ-

3. That by the supposition of simple motion in the sun, homogeneous bodies are congregated and heterogeneous dissipated, has already been demon-

strated in the 5th article of chapter XXI. I have
also supposed, that there are intermingled with the
pure air certain little bodies, or, as others call them, ence of the impetus with
atoms; which by reason of their extreme small- which the elements, whereof
ness are invisible, and differing from one another heavy bodies
in consistence, figure, motion, and magnitude; are made, do fall upon the
from whence it comes to pass that some of them earth.
are congregated to the earth, others to other
planets, and others are carried up and down in the
spaces between. And seeing those, which are car-
ried to the earth, differ from one another in figure,
motion, and magnitude, they will fall upon the
earth, some with greater, others with less impetus.
And seeing also that we compute the several
degrees of gravity no otherwise than by this their
falling upon the earth with greater or less impetus;
it follows, that we conclude those to be the more
heavy that have the greater impetus, and those to be
less heavy that have the less impetus. Our inquiry
therefore must be, by what means it may come to
pass, that of bodies, which descend from above to
the earth, some are carried with greater, others
with less impetus; that is to say, some are more
heavy than others. We must also inquire, by what
means such bodies, as settle upon the earth, may
by the earth itself be forced to ascend.

4. Let the circle made upon the centre C (in The cause of
fig. 2) be a great circle in the superficies of the the descent of heavy bodies.
earth, passing through the points A and B. Also
let any heavy body, as the stone A D, be placed
anywhere in the plane of the equator; and let it
be conceived to be cast up from A D perpendicu-
larly, or to be carried in any other line to E, and
supposed to rest there. Therefore, how much

PART IV.
30.

The cause of
the descent of
heavy bodies.

space soever the stone took up in A D, so much
space it takes up now in E. And because all place
is supposed to be full, the space A D will be filled
by the air which flows into it first from the nearest
places of the earth, and afterwards successively
from more remote places.   Upon the centre C let
a circle be understood to be drawn through E ; and
let the plane space, which is between the superficies
of the earth and that circle, be divided into plane
orbs equal and concentric ; of which let that be the
first, which is contained by the two perimeters that
pass through A and D.   Whilst therefore the air,
which is in the first orb, filleth the place A D, the
orb itself is made so much less, and consequently
its latitude is less than the strait line A D. Where-
fore there will necessarily descend so much air
from the orb next above.   In like manner, for the
same cause, there will also be a descent of air from
the orb next above that ; and so by succession
from the orb in which the stone is at rest in E.
Either therefore the stone itself, or so much air,
will descend.   And seeing air is by the diurnal
revolution of the earth more easily thrust away
than the stone, the air, which is in the orb that
contains the stone, will be forced further upwards
than the stone.   But this, without the admission
of vacuum, cannot be, unless so much air descend
to E from the place next above ; which being done,
the stone will be thrust downwards.   By this
means therefore the stone now receives the begin-
ning of its descent, that is to say, of its gravity.
Furthermore, whatsoever is once moved, will be
moved continually (as hath been shown in the
19th article of chapter VIII) in the same way, and

with the same celerity, except it be retarded or
accelerated by some external movent. Now the
air, which is the only body that is interposed be-
tween the earth A and the stone above it E, will
have the same action in every point of the strait
line E A, which it hath in E. But it depressed the
stone in E; and therefore also it will depress it
equally in every point of the strait line E A. Where-
fore the stone will descend from E to A with acce-
lerated motion. The possible cause therefore of
the descent of heavy bodies under the equator, is
the diurnal motion of the earth. And the same
demonstration will serve, if the stone be placed in
the plane of any other circle parallel to the equator.
But because this motion hath, by reason of its
greater slowness, less force to thrust off the air in
the parallel circles than in the equator, and no
force at all at the poles, it may well be thought
(for it is a certain consequent) that heavy bodies
descend with less and less velocity, as they are
more and more remote from the equator; and that
at the poles themselves, they will either not descend
at all, or not descend by the axis; which whether
it be true or false, experience must determine. But
it is hard to make the experiment, both because
the times of their descents cannot be easily mea-
sured with sufficient exactness, and also because
the places near the poles are inaccessible. Never-
theless, this we know, that by how much the
nearer we come to the poles, by so much the
greater are the flakes of the snow that falls; and
by how much the more swiftly such bodies descend
as are fluid and dissipable, by so much the smaller
are the particles into which they are dissipated.

PART IV.
30.

In what pro-
portion the de-
scent of heavy
bodies is accele-
rated.

5. Supposing, therefore, this to be the cause of the descent of heavy bodies, it will follow that their motion will be accelerated in such manner, as that the spaces, which are transmitted by them in the several times, will have to one another the same proportion which the odd numbers have in succession from unity. For if the strait line E A be divided into any number of equal parts, the heavy body descending will, by reason of the perpetual action of the diurnal motion, receive from the air in every one of those times, in every several point of the strait line E A, a several new and equal impulsion ; and therefore also in every one of those times, it will acquire a several and equal degree of celerity. And from hence it follows, by that which Galileus hath in his *Dialogues of Motion* demonstrated, that heavy bodies descend in the several times with such differences of transmitted spaces, as are equal to the differences of the square numbers that succeed one another from unity ; which square numbers being 1, 4, 9, 16, &c. their differences are 3, 5, 7, &c. ; that is to say, the odd numbers which succeed one another from unity. Against this cause of gravity which I have given, it will perhaps be objected, that if a heavy body be placed in the bottom of some hollow cylinder of iron or adamant, and the bottom be turned upwards, the body will descend, though the air above cannot depress it, much less accelerate its motion. But it is to be considered that there can be no cylinder or cavern, but such as is supported by the earth, and being so supported is, together with the earth, carried about by its diurnal motion. For by this means the bottom of

the cylinder will be as the superficies of the earth; and by thrusting off the next and lowest air, will make the uppermost air depress the heavy body, which is at the top of the cylinder, in such manner as is above explicated.

6. The gravity of water being so great as by experience we find it is, the reason is demanded by many, why those that dive, how deep soever they go under water, do not at all feel the weight of the water which lies upon them. And the cause seems to be this, that all bodies by how much the heavier they are, by so much the greater is the endeavour by which they tend downwards. But the body of a man is heavier than so much water as is equal to it in magnitude, and therefore the endeavour downwards of a man's body is greater than that of water. And seeing all endeavour is motion, the body also of a man will be carried towards the bottom with greater velocity than so much water. Wherefore there is greater reaction from the bottom; and the endeavour upwards is equal to the endeavour downwards, whether the water be pressed by water, or by another body which is heavier than water. And therefore by these two opposite equal endeavours, the endeavour both ways in the water is taken away; and consequently, those that dive are not at all pressed by it.

Why those that dive, do not, when they are under water, feel the weight of the water above them.

Coroll. From hence also it is manifest, that water in water hath no weight at all, because all the parts of water, both the parts above, and the parts that are directly under, tend towards the bottom with equal endeavour and in the same strait lines.

The weight of a body that floateth, is equal to so much water as would fill the space which the immersed part of the body takes up within the water.

7. If a body float upon the water, the weight of that body is equal to the weight of so much water as would fill the place which the immersed part of the body takes up within the water.

Let EF (in fig. 3) be a body floating in the water A B C D; and let the part E be above, and the other part F under the water. I say, the weight of the whole body E F is equal to the weight of so much water as the space F will receive. For seeing the weight of the body E F forceth the water out of the space F, and placeth it upon the superficies A B, where it presseth downwards; it follows, that from the resistance of the bottom there will also be an endeavour upwards. And seeing again, that by this endeavour of the water upwards, the body E F is lifted up, it follows, that if the endeavour of the body downwards be not equal to the endeavour of the water upwards, either the whole body E F will, by reason of that inequality of their endeavours or moments, be raised out of the water, or else it will descend to the bottom. But it is supposed to stand so, as neither to ascend nor descend. Wherefore there is an equilibrium between the two endeavours; that is to say, the weight of the body E F is equal to the weight of so much water as the space F will receive; which was to be proved.

If a body be lighter than water, then how big soever that body be, it will float upon any quantity of water, how little soever.

8. From hence it follows, that any body, of how great magnitude soever, provided it consist of matter less heavy than water, may nevertheless float upon any quantity of water, how little soever.

Let A B C D (in fig. 4) be a vessel; and in it let E F G H be a body consisting of matter which is less heavy than water; and let the space A G C F

be filled with water. I say, the body E F G H will not sink to the bottom D C. For seeing the matter of the body E F G H is less heavy than water, if the whole space without A B C D were full of water, yet some part of the body E F G H, as E F I K, would be above the water; and the weight of so much water as would fill the space I G H K would be equal to the weight of the whole body E F G H; and consequently G H would not touch the bottom D C. As for the sides of the vessel, it is no matter whether they be hard or fluid; for they serve only to terminate the water; which may be done as well by water as by any other matter how hard soever; and the water without the vessel is terminated somewhere, so as that it can spread no farther. The part therefore E F I K will be extant above the water A G C F which is contained in the vessel. Wherefore the body E F G H will also float upon the water A G C F, how little soever that water be; which was to be demonstrated.

9. In the 4th article of chapter XXVI, there is brought for the proving of vacuum the experiment of water enclosed in a vessel; which water, the orifice above being opened, is ejected upwards by the impulsion of the air. It is therefore demanded, seeing water is heavier than air, how that can be done. Let the second figure of the same, chapter XXVI be considered, where the water is with great force injected by a syringe into the space F G B. In that injection, the air (but pure air) goeth with the same force out of the vessel through the injected water. But as for those small bodies, which formerly I supposed to be intermingled with air and to be moved with simple motion, they

How water may be lifted up and forced out of a vessel by air.

cannot, together with the pure air, penetrate the water; but remaining behind are necessarily thrust together into a narrower place, namely into the space which is above the water F G. The motions therefore of those small bodies will be less and less free, by how much the quantity of the injected water is greater and greater; so that by their motions falling upon one another, the same small bodies will mutually compress each other, and have a perpetual endeavour of regaining their liberty, and of depressing the water that hinders them. Wherefore, as soon as the orifice above is opened, the water which is next it will have an endeavour to ascend, and will therefore necessarily go out. But it cannot go out, unless at the same time there enter in as much air; and therefore both the water will go out, and the air enter in, till those small bodies which were left within the vessel have recovered their former liberty of motion; that is to say, till the vessel be again filled with air, and no water be left of sufficient height to stop the passage at B. Wherefore I have shown a possible cause of this phenomenon, namely, the same with that of thunder. For as in the generation of thunder, the small bodies enclosed within the clouds, by being too closely pent together, do by their motion break the clouds, and restore themselves to their natural liberty; so here also the small bodies enclosed within the space which is above the strait line F G, do by their own motion expel the water as soon as the passage is opened above. And if the passage be kept stopped, and these small bodies be more vehemently compressed

by the perpetual forcing in of more water, they will at last break the vessel itself with great noise.

10. If air be blown into a hollow cylinder, or into a bladder, it will increase the weight of either of them a little, as many have found by experience, who with great accurateness have tried the same. And it is no wonder, seeing, as I have supposed, there are intermingled with the common air a great number of small hard bodies, which are heavier than the pure air. For, the ethereal substance, being on all sides equally agitated by the motion of the sun, hath an equal endeavour towards all the parts of the universe; and, therefore, it hath no gravity at all.

11. We find also by experience, that, by the force of air enclosed in a hollow cannon, a bullet of lead may with considerable violence be shot out of a gun of late invention, called the wind-gun. In the end of this cannon there are two holes, with their valves on the inside, to shut them close; one of them serving for the admission of air, and the other for the letting of it out. Also, to that end which serves for the receiving in of air, there is joined another cannon of the same metal and bigness, in which there is fitted a rammer which is perforated, and hath also a valve opening towards the former cannon. By the help of this valve the rammer is easily drawn back, and letteth in air from without; and being often drawn back and returned again with violent strokes, it forceth some part of that air into the former cannon, so long, till at last the resistance of the enclosed air is greater than the force of the stroke. And by this

means men think there is now a greater quantity of air in the cannon than there was formerly, though it were full before. Also, the air thus forced in, how much soever it be, is hindered from getting out again by the aforesaid valves, which the very endeavour of the air to get out doth necessarily shut. Lastly, that valve being opened which was made for the letting out of the air, it presently breaketh out with violence, and driveth the bullet before it with great force and velocity.

As for the cause of this, I could easily attribute it, as most men do, to condensation, and think that the air, which had at the first but its ordinary degree of rarity, was afterwards, by the forcing in of more air, condensed, and last of all, rarified again by being let out and restored to its natural liberty. But I cannot imagine how the same place can be always full, and, nevertheless, contain some times a greater, sometimes a less quantity of matter; that is to say, that it can be fuller than full. Nor can I conceive how fulness can of itself be an efficient cause of motion. For both these are impossible. Wherefore we must seek out some other possible cause of this phenomenon. Whilst, therefore, the valve which serves for the letting in of air, is opened by the first stroke of the rammer, the air within doth with equal force resist the entering of the air from without; so that the endeavours between the internal and external air are opposite, that is, there are two opposite motions whilst the one goeth in and the other cometh out; but no augmentation at all of air within the cannon. For there is driven out by the stroke as much pure air, which passeth between the rammer and the sides

of the cannon, as there is forced in of air impure by the same stroke. And thus, by many forcible strokes, the quantity of small hard bodies will be increased within the cannon, and their motions also will grow stronger and stronger, as long as the matter of the cannon is able to endure their force ; by which, if it be not broken, it will at least be urged every way by their endeavour to free themselves ; and as soon as the valve, which serves to let them out, is opened, they will fly out with violent motion, and carry with them the bullet which is in their way. Wherefore, I have given a possible cause of this phenomenon.

12. Water, contrary to the custom of heavy bodies, ascendeth in the weather-glass ; but it doth it when the air is cold : for when it is warm it descendeth again. And this organ is called a thermometer or thermoscope, because the degrees of heat and cold are measured and marked by it. It is made in this manner :—Let A B C D (in fig. 5) be a vessel full of water, and E F G a hollow cylinder of glass, closed at E and open at G. Let it be heated, and set upright within the water to F ; and let the open end reach to G. This being done, as the air by little and little grows colder, the water will ascend slowly within the cylinder from F towards E ; till at last the external and internal air coming to be both of the same temper, it will neither ascend higher nor descend lower, till the temper of the air be changed. Suppose it, therefore, to be settled anywhere, as at H. If now the heat of the air be augmented, the water will descend below H ; and if the heat be diminished, it will ascend above

The cause of the ascent of water in a weather-glass

it. Which, though it be certainly known to be true by experience, the cause, nevertheless, hath not as yet been discovered.

In the sixth and seventh articles of chapter XXVII, where I consider the cause of cold, I have shown, that fluid bodies are made colder by the pressure of the air, that is to say, by a constant wind that presseth them. For the same cause it is, that the superficies of the water is pressed at F; and having no place, to which it may retire from this pressure, besides the cavity of the cylinder between H and E, it is therefore necessarily forced thither by the cold, and consequently it ascendeth more or less, according as the cold is more or less increased. And again, as the heat is more intense or the cold more remiss, the same water will be depressed more or less by its own gravity, that is to say, by the cause of gravity above explicated.

Cause of motion upwards in living creatures

13. Also living creatures, though they be heavy, can by leaping, swimming and flying, raise themselves to a certain degree of height. But they cannot do this except they be supported by some resisting body, as the earth, the water and the air. For these motions have their beginning from the contraction, by the help of the muscles, of the body animate. For to this contraction there succeedeth a distension of their whole bodies; by which distension, the earth, the water, or the air, which supporteth them, is pressed; and from hence, by the reaction of those pressed bodies, living creatures acquire an endeavour upwards, but such as by reason of the gravity of their bodies is presently

lost again. By this endeavour, therefore, it is, that living creatures raise themselves up a little way by leaping, but to no great purpose : but by swimming and flying they raise themselves to a greater height; because, before the effect of their endeavour is quite extinguished by the gravity of their bodies, they can renew the same endeavour again.

That by the power of the soul, without any antecedent contraction of the muscles or the help of something to support him, any man can be able to raise his body upwards, is a childish conceit. For if it were true, a man might raise himself to what height he pleased.

14. The diaphanous medium, which surrounds the eye on all sides, is invisible ; nor is air to be seen in air, nor water in water, nor anything but that which is more opacous. But in the confines of two diaphanous bodies, one of them may be distinguished from the other. It is not therefore a thing so very ridiculous for ordinary people to think all that space empty, in which we say is air; it being the work of reason to make us conceive that the air is anything. For by which of our senses is it, that we take notice of the air, seeing we neither see, nor hear, nor taste, nor smell, nor feel it to be anything ? When we feel heat, we do not impute it to the air, but to the fire : nor do we say the air is cold, but we ourselves are cold ; and when we feel the wind, we rather think something is coming, than that any thing is already come. Also, we do not at all feel the weight of water in water, much less of air in air. That we come to know that to be a body, which we call air, it is by

That there is in nature a kind of body heavier than air, which nevertheless is not by sense distinguishable from it.

reasoning ; but it is from one reason only, namely, because it is impossible for remote bodies to work upon our organs of sense but by the help of bodies intermediate, without which we could have no sense of them, till they come to be contiguous. Wherefore, from the senses alone, without reasoning from effects, we cannot have sufficient evidence of the nature of bodies.

For there is underground, in some mines of coals, a certain matter of a middle nature between water and air, which nevertheless cannot by sense be distinguished from air ; for it is as diaphanous as the purest air ; and, as far as sense can judge, equally penetrable. But if we look upon the effect, it is like that of water. For when that matter breaks out of the earth into one of those pits, it fills the same either totally or to some degree ; and if a man or fire be then let down in it, it extinguishes them in almost as little time as water would do. But for the better understanding of this phenomenon, I shall describe the 6th figure. In which let A B represent the pit of the mine; and let part thereof, namely C B, be supposed to be filled with that matter. If now a lighted candle be let down into it below C, it will as suddenly be extinguished as if it were thrust into water. Also, if a grate filled with coals thoroughly kindled and burning never so brightly, be let down, as soon as ever it is below C, the fire will begin to grow pale, and shortly after, losing its light, be extinguished, no otherwise than if it were quenched in water. But if the grate be drawn up again presently, whilst the coals are still very hot, the fire will, by little and little, be kindled

again, and shine as before. There is, indeed, be-
tween this matter and water this considerable dif-
ference, that it neither wetteth, nor sticketh to such
things as are put down into it, as water doth;
which, by the moisture it leaveth, hindereth the
kindling again of the matter once extinguished.
In like manner, if a man be let down below C, he
will presently fall into a great difficulty of breath-
ing, and immediately after into a swoon, and die
unless he be suddenly drawn up again. They,
therefore, that go down into these pits, have this
custom, that as soon as ever they feel themselves
sick, they shake the rope by which they were let
down, to signify they are not well, and to the end
that they may speedily be pulled up again. For
if a man be drawn out too late, void of sense and
motion, they dig up a turf, and put his face and
mouth into the fresh earth ; by which means,
unless he be quite dead, he comes to himself again,
by little and little, and recovers life by breathing
out, as it were, of that suffocating matter, which he
had sucked in whilst he was in the pit ; almost in
the same manner as they that are drowned come
to themselves again by vomiting up the water. But
this doth not happen in all mines, but in some only ;
and in those not always, but often. In such pits
as are subject to it, they use this remedy. They
dig another pit, as D E, close by it, of equal depth,
and joining them both together with one common
channel, E B, they make a fire in the bottom E,
which carries out at D the air contained in the pit
D E ; and this draws with it the air contained in
the channel E B ; which, in like manner, is fol-

PART IV.
30.

lowed by the noxious matter contained in C B; and, by this means, the pit is for that time made healthful. Out of this history, which I write only to such as have had experience of the truth of it, without any design to support my philosophy with stories of doubtful credit, may be collected the following possible cause of this phenomenon : namely, that there is a certain matter fluid and most transparent, and not much lighter than water, which, breaking out of the earth, fills the pit to C ; and that in this matter, as in water, both fire and living creatures are extinguished.

Of the cause of magnetical virtue.

15. About the nature of heavy bodies, the greatest difficulty ariseth from the contemplation of those things which make other heavy bodies ascend to them ; such as jet, amber, and the loadstone. But that which troubles men most is the loadstone, which is also called *Lapis Herculeus ;* a stone, though otherwise despicable, yet of so great power that it taketh up iron from the earth, and holds it suspended in the air, as Hercules did Antæus. Nevertheless, we wonder at it somewhat the less, because we see jet draw up straws, which are heavy bodies, though not so heavy as iron. But as for jet, it must first be excited by rubbing, that is to say, by motion to and fro ; whereas the loadstone hath sufficient excitation from its own nature, that is to say, from some internal principle of motion peculiar to itself. Now, whatsoever is moved, is moved by some contiguous and moved body, as hath been formerly demonstrated. And from hence it follows evidently, that the first endeavour, which iron hath towards the loadstone, is caused by the

motion of that air which is contiguous to the iron : also, that this motion is generated by the motion of the next air, and so on successively, till by this succession we find that the motion of all the intermediate air taketh its beginning from some motion which is in the loadstone itself; which motion, because the loadstone seems to be at rest, is invisible. It is therefore certain, that the attractive power of the loadstone is nothing else but some motion of the smallest particles thereof. Supposing, therefore, that those small bodies, of which the loadstone is in the bowels of the earth composed, have by nature such motion or endeavour as was above attributed to jet, namely, a reciprocal motion in a line too short to be seen, both those stones will have one and the same cause of attraction. Now in what manner and in what order of working this cause produceth the effect of attraction, is the thing to be enquired. And first we know, that when the string of a lute or viol is stricken, the vibration, that is, the reciprocal motion of that string in the same strait line, causeth like vibration in another string which hath like tension. We know also, that the dregs or small sands, which sink to the bottom of a vessel, will be raised up from the bottom by any strong and reciprocal agitation of the water, stirred with the hand or with a staff. Why, therefore, should not reciprocal motion of the parts of the loadstone contribute as much towards the moving of iron? For, if in the loadstone there be supposed such reciprocal motion, or motion of the parts forwards and backwards, it will follow that the like motion will be

propagated by the air to the iron, and consequently
that there will be in all the parts of the iron the
same reciprocations or motions forwards and back-
wards.   And from hence also it will follow, that
the intermediate air between the stone and the
iron will, by little and little, be thrust away; and
the air being thrust away, the bodies of the load-
stone and the iron will necessarily come together.
The possible cause therefore why the loadstone
and jet draw to them, the one iron, the other
straws, may be this, that those attracting bodies
have reciprocal motion either in a strait line, or in
an elliptical line, when there is nothing in the na-
ture of the attracted bodies which is repugnant to
such a motion.

But why the loadstone, if with the help of cork
it float at liberty upon the top of the water, should
from any position whatsoever so place itself in the
plane of the meridian, as that the same points,
which at one time of its being at rest respect the
poles of the earth, should at all other times respect
the same poles, the cause may be this; that the reci-
procal motion, which I supposed to be in the parts
of the stone, is made in a line parallel to the axis
of the earth, and has been in those parts ever since
the stone was generated.   Seeing therefore, the
stone, whilst it remains in the mine, and is carried
about together with the earth by its diurnal mo-
tion, doth by length of time get a habit of being
moved in a line which is perpendicular to the line
of its reciprocal motion, it will afterwards, though
its axis be removed from the parallel situation it
had with the axis of the earth, retain its endeavour

of returning to that situation again; and all en-
deavour being the beginning of motion, and nothing
intervening that may hinder the same, the load-
stone will therefore return to its former situation.
For, any piece of iron that has for a long time rested
in the plane of the meridian, whensoever it is forced
from that situation and afterwards left to its own
liberty again, will of itself return to lie in the
meridian again; which return is caused by the
endeavour it acquired from the diurnal motion of
the earth in the parallel circles which are perpen-
dicular to the meridians.

If iron be rubbed by the loadstone drawn from
one pole to the other, two things will happen; one,
that the iron will acquire the same direction with
the loadstone, that is to say, that it will lie in the
meridian, and have its axis and poles in the same
position with those of the stone; the other, that
the like poles of the stone and of the iron will
avoid one another, and the unlike poles approach
one another. And the cause of the former may be
this, that iron being touched by motion which is
not reciprocal, but drawn the same way from pole
to pole, there will be imprinted in the iron also an
endeavour from the same pole to the same pole.
For seeing the loadstone differs from iron no other-
wise than as ore from metal, there will be no
repugnance at all in the iron to receive the same
motion which is in the stone. From whence it
follows, that seeing they are both affected alike by
the diurnal motion of the earth, they will both
equally return to their situation in the meridian,
whensoever they are put from the same. Also, of

the latter this may be the cause, that as the load-stone in touching the iron doth by its action imprint in the iron an endeavour towards one of the poles, suppose towards the North Pole; so reciprocally, the iron by its action upon the loadstone doth imprint in it an endeavour towards the other pole, namely towards the South Pole. It happens therefore in these reciprocations or motions forwards and backwards of the particles of the stone and of the iron betwixt the north and the south, that whilst in one of them the motion is from north to south, and the return from south to north, in the other the motion will be from south to north, and the return from north to south; which motions being opposite to one another, and communicated to the air, the north pole of the iron, whilst the attraction is working, will be depressed towards the south pole of the loadstone; or contrarily, the north pole of the loadstone will be depressed towards the south pole of the iron; and the axis both of the loadstone and of the iron will be situate in the same strait line. The truth whereof is taught us by experience.

As for the propagation of this magnetical virtue, not only through the air, but through any other bodies how hard soever, it is not to be wondered at, seeing no motion can be so weak, but that it may be propagated infinitely through a space filled with body of any hardness whatsoever. For in a full medium, there can be no motion which doth not make the next part yield, and that the next, and so successively without end; so that there is no effect whatsoever, but to the production thereof

something is necessarily contributed by the several motions of all the several things that are in the world.

And thus much concerning the nature of body in general; with which I conclude this my first section of the Elements of Philosophy. In the first, second, and third parts, where the principles of ratiocination consist in our own understanding, that is to say, in the legitimate use of such words as we ourselves constitute, all the theorems, if I be not deceived, are rightly demonstrated. The fourth part depends upon hypotheses; which unless we know them to be true, it is impossible for us to demonstrate that those causes, which I have there explicated, are the true causes of the things whose productions I have derived from them.

Nevertheless, seeing I have assumed no hypotheses, which is not both possible and easy to be comprehended; and seeing also that I have reasoned aright from those assumptions, I have withal sufficiently demonstrated that they may be the true causes; which is the end of physical contemplation. If any other man from other hypotheses shall demonstrate the same or greater things, there will be greater praise and thanks due to him than I demand for myself, provided his hypotheses be such as are conceivable. For as for those that say anything may be moved or produced by *itself*, by *species*, by *its own power*, by *substantial forms*, by *incorporeal substances*, by *instinct*, by *antiperistasis*, by *antipathy, sympathy, occult quality*, and other empty words of schoolmen, their saying so is to no purpose.

PART IV.
30.
Conclusion.

And now I proceed to the phenomena of man's body; where I shall speak of the *optics*, and of the *dispositions*, *affections*, and *manners* of men, if it shall please God to give me life, and show their causes.

END OF VOL. I.

Tab.I.

Fig. 2.

Fig. 1.

Fig.4.

Fig.3.

Fig. 6.

Fig. 5.