Expected Utility Theory

Expected utility theory is a branch of preference theory that analyses the utility (understood as happiness, pleasure, or preference satisfaction) of complex choices, each of which consists of a variety of possible outcomes of varying probability. A choice situation that may have several possible outcomes of different probability is called a "lottery," and the choice is then between a number of competing lotteries. (If there is only one possible outcome and its probability is certain, that that is a trivial lottery of only one outcome.)

Let a *lottery* L consist of a finite set of *outcomes* $\{o_1,\ldots,o_n\}$, and let each have a *utility* $u(o_i)$, and a *probability* $p(o_i)$ such that the sum of the probabilities is 1.

The *expected utility* of L is then defined as

$$eu(L) = \Sigma_{i\leq n}[p(o_i)\times u(o_i)]$$

or in other words,

$$[p(o_1)\times u(o_1)]+ \ldots+[p(o_n)\times u(o_n)]$$

Pascal's Wager

Let us consider two lotteries L_B in which we choose to believe in God, and $L_{\sim B}$ in which you choose not to believe in God. Let there be two possible outcomes e in which God exists, and $\sim e$ in which he does not. We now attach probabilities and utilities to each of these.

Let us grant the skeptic's opinion that the probability of e, of God's existing, is very low, say .001. That is, $p(e)$=.001. Hence, since the probability of e and $\sim e$ add up to 1, the probability of $\sim e$, of God's not existing, is correspondingly high, .999. That is, $p(\sim e)$=.999.

Pascal says that the utility of God's existing if you believe in him is infinite because you will enjoy the infinite joy of heaven. Let us represent an infinite value by the letter ω (omega). That is, $u(e)= \omega$, On the other hand, the disutility of God existing if you do not believe in him is a negative infinity, $-\omega$, because you would suffer the infinite loss of being deprived of heaven. Hence, $u(\sim e)= -\omega$. Moreover, the utility of God's not existing if you do not believing would be a positive quantity, for example, the pleasure derived from the life of a hedonistic libertine, say 50 "utiles." The disutility of God's not existing if you nevertheless believed in him would be that of, let us say, a grim Christian life style. Let us be pessimistic and set it at a negative value, say -25 "utiles".

Now, let us note several features of the arithmetic of the infinite. Any finite positive value, no matter how small, if taken an infinite number of times, is itself a positive infinity, i.e. for any positive value x, $\omega\times x=\omega$. Hence, even if $p(e)$ is very small, if it is positive, $\omega\times p(e)=\omega$.

Conversely, a negative quantity taken an infinite number of times is a negative infinity, i.e. $-\omega \times x = -\omega$. Hence, even if $p(\sim e)$ is small, $-\omega \times p(e) = -\omega$. Also, subtracting a finite negative value from a positive infinity does not diminish it, nor does adding a finite positive quantity to a negative infinitiy increase it.

With this background we can now calculate the expected utility of the two lotteries L_B and $L_{\sim B}$, believing and not believing in God respectively:

$$
\begin{aligned}
eu(L_B) &= [p(e) \times u(e)] + [p(\sim e) \times u(\sim e)] \\
&= [.001 \times \omega] + [.999 \times -25] \\
&= \omega + [.999 \times -25] \\
&= \omega
\end{aligned}
$$

$$
\begin{aligned}
eu(L_{\sim B}) &= [p(e) \times u(e)] + [p(\sim e) \times u(\sim e)] \\
&= [.001 \times -\omega] + [.999 \times 50] \\
&= -\omega + [.999 \times 50] \\
&= -\omega
\end{aligned}
$$

Clearly, the argument goes, L_B, the choice of believing in God, is hands down the better bet.