Complexity as Defined in Mathematics and Computer Science

One of the issues (since Darwin) in the "design" debates is the claim that evolution tends to the more "complex."  But most of these discussions tend not to define "complex."    
  
In serious literature of the sciences one of the important attempts to define complexity is found in mathematics and computer science. Here complexity is understood roughly a measure of the number of steps it takes to complete an algorithm or decision procedure.  Moreover, in modern biology when the concept of complexity is invoked it is often this mathematical idea that is being employed or alluded to. That is, in the biological literature discussions of biological complexity sometimes seems to be making the assumption that computation complexity is what manifested in biological systems when they are said to be complex. [[1]](#footnote-1)

Unfortunately , it seems to be a huge and largely false assumption in such discussions that biological systems are algorithmic (i.e. modeled by calculable (aka recursive functions, Turing machines, algorithms) and that therefore the computational concept of complexity applies.  Most biological systems fail to exhibit the appropriate properties for the application of the mathematical concept. In particular, they are largely

1. The system is non-deterministic in the mathematical sense of that is properly modeled by a non-functional relation as defined in set theory, i.e. by relations that map a single input to more than one output. Frequently, as is often the case in the natural sciences, the relevant biological mechanism maps a single input to has multiple outputs each weighted by a probability.
2. Even if a mechanism is functional (deterministic), it is rarely modeled by what is called a calculable function in mathematics. That is, it is rarely appropriate to model it by a function that is solvable by an algorithm.

If a biological system or mechanism is are non-deterministic or non-calculable, then the computational notion of complexity is not well-defined for it.  
  
 The issue is a bit more complicated because there is a non-deterministic concept of complexity in mathematics, but it is technical and relevant to issues in mathematics and computer science we no counterpart in biology. Much computational theory that deals with a notion of complexity is a measure of the number of steps in an calculation that solves not just any algorithm  (calculable function) but specifically those in a *decision procedure.* A decision procedure is a calculable function that is the “characteristic function” of some set *C* in the sense that it answer the question of whether a given input *x* is in some set *C* by returning *f*(*x*)=1 if *x*∈*C* *f*(*x*)=1 if *x*∉*C* .  Sometimes a decision problem is determinate, i.e. there is a calculable characteristic function *f* with output 0 or1 for any input.  Then the number of steps for the longest calculation of *f* for input *x* (t*he complexity of* *f* *for* *x*) is well-defined.  Sometimes there is a calculable function that will return 1 if *x* is in *C* but may not return an output at all (is undefined) if *x* is not in *C*.  Then *the decision problem* is indeterminate.  It is still well-defined to count (an hence measure) the number of steps for longest calculation that returns 1, and for it is theoretically useful and well-define to define concepts of complexity  for even this non-deterministic case of "partial" decision functions.    
  
The crux is that – this is where biology comes in – decisions problems (i.e.  questions of whether something is in a set) are not important in biology.  Biologists do not investigate whether biological sets are decidable.) Rather, what is important in biology are functional and quasi-functional relations (i.e. systems, mechanisms, "laws" relating causes to effects) with inputs and outputs that may be deterministic, more or less probabilistic, or random.  These are not "decision problems," and concepts of complexity defined by reference to decision functions are irrelevant. If a biological system were aptly modeled by a calculable function, then the numbers of steps and some notion of computational complexity would be well-defined for it.  But because we live in a non-deterministic world, I bet few real biological systems fit calculable functions. So, biologists are stuck with non-functional relational systems, one for which the notion of deterministic calculation isn't well-defined, and hence the neither is the idea of counting the number of steps in calculations.     
  
Thus, though there are notions of complexity relevant to non-deterministic functions in computation theory, these are defined relative to the special case of non-deterministic decision functions for class membership, which these are issues irrelevant to biology.  There is no well-defined notion of complexity for the simple case of a non-calculable function, or (*a fortiori*) for a relation that is not a function, which is the sort of relation that is most relevant to modeling biological systems.

1. Sometimes in discussions of “design” participants seems by *more complex* to mean *less probable*. In such contexts the notion relevant to unpacking “complex” is really entropy. [↑](#footnote-ref-1)