

TIME, CREATION AND THE CONTINUUM

theories in antiquity and the early middle ages

Richard Sorabji



Duckworth

having existed previously, namely, when the heavens themselves (his words, not mine) began to exist.¹⁷

It is an interesting twist in Philoponus' attack that he declares a successive infinity harder to accommodate than a simultaneous infinity, not easier as Aristotle supposed, precisely because it immediately raises the problems of traversal, addition and multiplication.¹⁸ We may be reminded of Augustine's claim that God can count all the innumerably many things without any succession (*alternatio*) in his thought:¹⁹ an infinite count is evidently thought more possible, if it is not successive.

Simplicius replies to Philoponus' attack by drawing attention to an idea which crops up in Aristotle *Phys.* 3.6 (206a33-b3) and 3.8 (208a20-1).

But in extended objects, this [taking of one thing after another] occurs in such a way that what is taken stays behind; whereas in the case of time and of men, what is taken ceases to exist, though in such a way that the series does not fail.

Time and movement are infinite, and so is thinking, in such a way that what is taken does not stay.

Simplicius takes Aristotle's point to be that, because past days and past generations of men do not stay, but have perished, you do not get an infinity of them existing. Moreover, he adds, you will not find an infinity being increased, because there is no infinity there in the first place.²⁰

At first sight, Simplicius' point seems arresting: what made it possible for an infinity of divisions to be a merely *potential* infinity was that the divisions not yet made did not *exist*. Has not Simplicius shown that years are like divisions, in that no more than a finite number exists? Unfortunately, we have seen that the situation is more complex, because Aristotle is willing to think of points and potential divisions as entities of a sort, capable of forming collections. And he would have to allow the same for past years – all the more so because the sense in which past years no longer exist is only the rather weak sense of no longer being present. They are still entities enough to form a collection, and Aristotle ought therefore to avoid their forming an actually infinite one, just as he does in *Phys.* 3 and *GC* 1.2 for potential divisions and points. Otherwise, he will be back with the Anaxagorean problem, which he hoped to avoid, concerning an infinity whose parts are infinite, and with the problem of a line composed of nothing but points. Admittedly, we have seen him allowing an actual infinity of potential divisions in *Phys.* 8.8, but first, that was argued to be an inconsistency, and secondly, potential divisions have less claim on existence than past years, so hardly set a precedent for them.

Things will be no better if Simplicius' point is transposed and phrased in terms of actuality instead of existence, so as to say that past years are no longer *actual*. We should then have had at least three senses of actuality:

¹⁷ 6th of 6 books against Aristotle, as recorded by Simpl. in *Phys.* 1179, 15-21.

¹⁸ *Aet.* 9-11 as above, and in *Phys.* 429,20-430,10.

¹⁹ *City* XII.17: *innumera omnia sine cogitationis alternatione numerantem.*

²⁰ in *Phys.* 506, 3-18; cf. 1180, 29-31.

divisions are actual when *marked out*, years when *present* and infinities when they are *more than a finitude*. But the answer would be that the status of the years does not settle the status of their infinity. Even through the past years in a beginningless universe are not actual, their infinity must be. And for some purposes Aristotle needs to avoid any actual infinity.

My conclusion so far is that Philoponus' arguments are successful as an objection to Aristotle and the pagans. But the question remains whether we can answer his arguments by freeing ourselves in some way from Aristotelian ideas. To this question I shall now turn, and I shall start with Philoponus' arguments about *increasing* infinity.

Philoponus against increasing infinity

In order to answer the adding and multiplying objections, we must see what is right and what wrong about them. We can do so without entering at all into the complication of transfinite numbers. There are perfectly good analogues of adding and multiplying in relation to infinity. The only restriction is that in a certain sense these processes will not have the usual consequence of making the collection larger. In order to see why not, we can imagine an infinite series of past years terminating at the present and a corresponding infinite series of past days. Suppose we imagine the column of past years stretching away from our left eye infinitely far into the distance, and parallel to it, stretching away from our right eye, the column of past days, also receding infinitely far. The two columns should be aligned at the near end, starting at the present, and the members of the two columns should be matched against each other one to one. I can now explain the sense in which the column of past days is not larger than the column of past years: it will not *stick out beyond the far end* of the other column, since neither column has a far end.

Provided we understand this, it will not too much matter whether we talk of adding or multiplying. The context may make it very natural to say that in a year's time an extra year will have been *added* to the collection. But no objection can be raised on this basis to the hypothesis that there has been an infinity of years. For we are not committed to the only thing that is objectionable, that is, to the idea that the collection of years will soon be, or that the collection of days already is, larger in the sense just proscribed.

It was in the fourteenth century that some of these points began to be appreciated, as John Murdoch has shown.²¹ An attempt then began to find a sense in which one (denumerable) infinite set might be called *greater* than another, and a sense in which it might not. It might be called *greater* in the sense of containing all the members of the other and some members *besides* (*preter*, elsewhere *praeter*). But it would not be right to talk of one infinite set containing members beyond (*ultra*) the other: 'besides' is legitimate, 'beyond'

²¹ 'Mathesis in philosophiam scholasticam introducta: the rise and development of the application of mathematics in fourteenth-century Philosophy and Theology', *Arts Libéraux et Philosophie au Moyen Âge, Actes du Quatrième Congrès de Philosophie Médiévale*, Paris 1969, 222-3; 'The "equality" of infinities in the Middle Ages', *Actes du XIe Congrès International d'Histoire des Sciences*, Warsaw-Cracow 1968, vol. 3, pp. 171-4. Philoponus, however, is not here given the credit for applying the infinity question to the hypothesis of a beginningless universe.

Philoponus turns Aristotle against himself

The two restrictions on infinity, that it is never actual or traversed, create a difficulty for Aristotle. For he argues that there is no beginning for time, motion, the universe, or the generations of man. But if there has already been an infinity of days, does this not provide an example of an infinity which exists *actually*, in a stronger sense than Aristotle wants? And will not the infinite set of past days have been *traversed*? This was the point on which Philoponus was to fasten nearly nine hundred years later, in his defence of the Christian belief in a beginning. In *de Aeternitate Mundi* (Rabe, pp. 9-11) and elsewhere,¹³ he makes the following complaints. If the universe has no beginning, then an infinity of years or generations will already be both actual and traversed. Next, that infinity will shortly have been added to, which is absurd. Moreover, besides addition, we could get multiplication of infinity. This last complaint is most graphically illustrated in Philoponus' attack on Aristotle, as recorded by Simplicius (*in Phys.* 1179, 15-21). If Saturn has performed an infinity of revolutions already, then Jupiter, the moon and the fixed stars will have revolved many times that infinite number of times. Finally, Philoponus makes use of Aristotle's proof from *Phys.* 3.5, 204b7-10, that there cannot be an infinite number, because number is countable.¹⁴ I have already indicated that these arguments were used again and again by Islamic and Jewish philosophers from the beginning of the ninth century onwards, and repeated in thirteenth-century Paris.¹⁵ Let me begin by quoting two of the leading passages from Philoponus.

So since past time will be actually infinite, if the *kosmos* is uncreated, the individuals which have come into being in that infinite time must also be actually infinite in number. Hence, if the *kosmos* is uncreated, the result will be that there exists and has occurred an actually infinite number. But it is in no way possible for the infinite to exist in actuality, neither by existing all at once, nor by coming into being part at a time, as we shall show more completely, God willing, in what follows. For after refuting all the puzzles designed to show

¹³ The same arguments are used by Philoponus in his commentaries on Aristotle's *Physics* (428, 14-430, 10; 467, 5-468, 4) and *Meteorologica* (16, 36ff.), portions of which are translated by S. Sambursky, 'Note on John Philoponus' rejection of the infinite' in S.M. Stern, Albert Hourani, Vivian Brown (eds), *Islamic Philosophy and the Classical Tradition*, Essays Presented to Richard Walzer, Oxford 1972, 351-3, and by Todd, *op. cit.* The arguments concerning addition and multiplication are used again in *Aet.* 619, and in Philoponus' attack on Aristotle, *ap. Simpl. in Phys.* 1179, 12-26.

¹⁴ Philoponus *in Phys.* 428

¹⁵ See, as above, Davidson, *op. cit.*; H.A. Wolfson, *The Philosophy of the Kalam*, 410-34; 452-5. Davidson cites thirteen Islamic and Jewish sources for discussions of the argument that infinity cannot be increased, thirteen for discussions of the argument that infinity cannot be traversed and of its variants, and four for use of the argument that number is finite, because countable. To these may be added the unpublished treatise of Avicenna, summarised in English by S. Pines, at the end of 'An Arabic summary of a lost work of John Philoponus', *IOS* 2, 1972, 320-52. The most accessible of the sources in English translation are: (1) Averroes *Tahāfut al-Tahāfut* (ed. Bouyges), which replies to, and incorporates, most of Ghazālī *Tahāfut al-Falāsifa*, translated by S. van den Bergh, London 1954. See pp. 16-19 (Bouyges' pagination) on increasing infinity and pp. 19-21 on traversing infinity. (2) Maimonides *Guide For The Perplexed* 1.74 (7th argument), which adds that Farabi offered replies to these arguments.

that the *kosmos* is everlasting, we then establish for our part that the *kosmos* cannot be everlasting. And I shall add to the exposition Aristotle himself establishing this particular point. I say that the infinite cannot in any way exist in actuality, and I think this is clear from the following. Since the infinite cannot exist all together and at once, for the very same reason it cannot emerge into actuality by existing part at a time. For if it were at all possible for the infinite to exist part at a time, and so to emerge in actuality, what reason would there be to prevent it from existing in actuality all at once? For saying that it is brought to birth in actuality part at a time, and counted, so to speak, unit by unit, one after another, would appear much more impossible than saying that it exists all together and at once. For if it exists all at once perhaps it will not have to be traversed unit by unit and, so to speak, enumerated. But if it comes into being part at a time, one unit always existing after another, so that eventually an actual infinity of units will have come into being, then even if it does not exist all together at once (since some units will have ceased when others exist), none the less it will have come to be traversed. And that is impossible: traversing the infinite and, so to speak, counting it off unit by unit, even if the one who does the counting is everlasting. For by nature the infinite cannot be traversed, or it would not be infinite. Hence if the infinite cannot be traversed, but the succession of the race has proceeded one individual at a time, and come down through an infinity of individuals to those who exist now, then the infinite has come to be traversed, which is impossible. So the number of earlier individuals is not infinite. If it were, the succession of the race would not have come down as far as each of us, since it is impossible to traverse the infinite.

Moreover, suppose the *kosmos* had no beginning, then the number of individuals down, say, to Socrates will have been infinite. But there will have been added to it the individuals who came into existence between Socrates and the present, so that there will be something greater than infinity, which is impossible.

Again, the number of men who have come into existence will be infinite, but the number of horses which have come into existence will also be infinite. You will double the infinity; if you add the number of dogs, you will triple it, and the number will be multiplied as each of the other species is added. This is one of the most impossible things. For it is not possible to be larger than infinity, not to say many times larger. Thus if these strange consequences must occur, and more besides, as we shall show elsewhere, if the *kosmos* is uncreated, then it cannot be uncreated or lack a beginning.¹⁶

Suppose (Philoponus says) the spheres do not revolve at equal speeds, but one takes thirty years, another twelve, and another in turn less, so that the sphere of the moon takes a month, and that of the fixed stars a day and a night. Suppose too that the movement of the heavens has no beginning, then the sphere of Saturn must have revolved an infinity of circuits, that of Jupiter nearly three times as many, while the circuits of the sun will be thirty times those of Saturn, those of the moon three hundred and sixty times as many, and those of the fixed stars more than ten thousand times. Is this not beyond all absurdity, if the infinite cannot be traversed even once, to entertain ten thousand times infinity, or rather infinity times infinity. Hence it is necessary (he says) that the revolution of the heavens should have had a beginning of its existence, without

¹⁶ *Aet.* (Rabe) pp. 9, 14-11, 17.