Aristotelian Infinity
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ARISTOTELIAN INFINITY

IN AN earlier paper, I have argued that Aristotle accepted and used, in some form or other, the principle which has been called by A. O. Lovejoy "the principle of plenitude."[1] That is to say, he accepted the principle that every genuine possibility is sometimes actualized or, possibly, another form of the same principle according to which no genuine possibility can remain unactualized through an infinity of time.[2]

Aristotle's theory of infinity might seem to constitute a counter-example to this interpretation. In many expositions of this theory, it is said that according to Aristotle infinity had merely potential existence but was never actualized. (The very terms "actual infinity" and "potential infinity" which are still used hail from Aristotle's terminology.) As one author puts it, for Aristotle infinity is "never realized, though conceivable." If this is all there is to the subject, we would in fact have here a clear-cut counterinstance to the principle of plenitude in Aristotle.

This interpretation is encouraged by certain remarks Aristotle himself makes. For instance, in *De Interpretatione* 13, 23a23-26 he writes: "Some things are actualities without capability . . ., others with capability . . . and others are never actualities but

[1] See my paper, "Necessity, Universality, and Time in Aristotle," *Ajatus*, XX (1957), 65-90. In this paper, I did not use the term "principle of plenitude." For the principle, see A. O. Lovejoy, *The Great Chain of Being* (Cambridge, Mass., 1936).

[2] What is here supposed to count as a "genuine" possibility is not obvious from Aristotle's formulations. It lies close at hand to suggest that for Aristotle each *general* possibility (each possible *kind of* individuals or of events) was actualized sooner or later. There are indications that Aristotle went further than this, however. The second formulation given in the text seems to catch his intentions fairly accurately without violating such well-known Aristotelian examples as the coat that could be cut but is worn out before it is (see *De Int.* 9, 19a12-15). Just because the individual coat in question perishes, we do not have a possibility here which would remain unactualized through an infinity of time.

only capabilities."[3] Aristotle's remarks in *Metaphysica* IX, 6, 1048b9-17 are sometimes construed in the same spirit (mistakenly, I shall argue later).

In view of the wealth of evidence that there is for ascribing the principle of plenitude to Aristotle, his views on infinity need a closer scrutiny.[4] I shall argue that Aristotle did not give up this principle in his theory of infinity, but rather assumed it in certain important parts of the theory. I shall also argue that by noticing this we can understand better certain issues that come up in the course of Aristotle's discussion of infinity in his *Physica* III, 4-8.

As so often in Aristotle, we cannot take his preliminary discussion of reasons for and against the existence of infinity at its face value.[5] The arguments presented in this discussion, like the corresponding preparatory arguments in other Aristotelian discussions, primarily serve to set the stage for Aristotle's own solution of the difficulty. Usually, such preliminary arguments give rise to apparently contradictory conclusions. These contradictions are normally resolved by means of a conceptual distinction. In the case at hand, it is a distinction between the different senses in which the infinite may be asserted or denied to exist (206a12-14).

Aristotle first indicates that an infinite potentiality must be said to exist (206a14-18). He goes on to suggest, however, that in order to understand the sense in which the infinite exists potentially we have to heed the different senses of existence (206a21-23). In other words, it is not true (*pace* Evans)[6] that "potentiality has

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[3] In quoting *De Interpretatione*, I shall use J. L. Ackrill's new translation (*Aristotle's "Categoriae" and "De Interpretatione"* [Oxford, 1963]). In quoting other works of Aristotle, I shall usually follow the familiar Oxford translation (ed. by W. D. Ross), with changes that are not always explicitly indicated.

[4] In fact, a form of the principle of plenitude is assented to by Aristotle at the beginning of his discussion of infinity: "In the case of eternal things what may be must be" (203b30).

[5] The structure of Aristotle's discussion of the infinite may be compared, e.g., with the structure of his discussion of the problem of future contingents in *De Int.* The latter discussion is analyzed in my paper, "The Once and Future Sea Fight," *Philosophical Review*, LXXII (1964), 461-492, esp. 468-472. In reading Aristotle, it is vital to keep constantly in mind his characteristic method of approaching a problem.

[6] Cf. Melbourne G. Evans, *The Physical Philosophy of Aristotle* (Albuquerque, 1964), p. 47.

here a special sense," different from the sense in which finite things may be potential. Rather, the infinite *is* (potentially *and* actually) in a sense different from the one in which a finite thing *is*. In the latter sense of being, the infinite does not exist even potentially: "The infinite does not have, even potentially, the independent (*καθ' αὑτό*) being which the finite has" (206b15-16; trans. by Wicksteed and Cornford).

In what sense, then, does the infinite exist? It exists, Aristotle says, in the sense in which a day "is" or the Olympic Games "are." These are not actualized in their entirety at any given moment of time in the way an individual is. Rather, their parts come to existence successively one by one. As Aristotle says, "one thing after another is always coming into existence" (206a22-23). In other words, infinity is not a term which applies to individual things, such as men or houses, in any sense, either actually or potentially. Rather, it is an attribute of certain sequences of individual things or individual events—"definite if you like at each stage, yet always different" (206a32-33). This is the gist of the Aristotelian theory of infinity.

Saying that the infinite exists potentially might perhaps be used to express that it exists in this derivative sense. Occasionally Aristotle allows himself the luxury of this locution. It is a very misleading way of speaking, however, not merely because it does not fully express the mode of existence of the infinite according to Aristotle, but even more so because it muddles an important distinction. As Aristotle is well aware, the distinction between actuality and potentiality applies also to the kind of existence which is enjoyed, *inter alia*, by the infinite, by a day, and by the Olympic Games: "For of these things too the distinction between potential and actual existence holds. We say that there are Olympic Games, both in the sense that they may occur and that they are actually occurring" (206a23-25). When this distinction is made clear, the principle of plenitude is seen to apply. Although there perhaps is a (rather loose and inappropriate) sense in which the infinite may be said to exist only potentially, in the exact and proper sense in which, according to Aristotle, it exists potentially, it also exists actually. "The infinite is actual in the sense in which a day or the games are said to be actual"

(206b13-14); and this, we have seen, is just the proper sense in which the Aristotelian infinite exists.[7] For instance, the infinity of time does not mean for Aristotle merely that later and later moments of time are possible; it implies that there will actually be later and later moments of time.

In a way, the Aristotelian theory of infinity has thus been found to entail exactly the opposite to what it is usually said to assert. Usually it is said that for Aristotle infinity exists potentially but never actually. In the *precise* sense, however, in which the infinite was found to exist potentially for Aristotle, it also exists actually. Far from discrediting my attribution of the principle of plenitude to Aristotle, an analysis of Aristotle's theory of infinity serves to confirm it.

The fact that Aristotle abides by the principle of plenitude in developing his theory of infinity is not without consequences for the theory. One of these is that he cannot accept any infinite (except in a relative sense as the inverse of infinite divisibility), not even in the "potential" sense of the infinite in which an infinite division or an infinity of numbers is possible. For the potential infinity of extension would mean that arbitrarily large extensions are possible. But if they were possible, they would have to be actual at some time or other. There cannot, however, be any actually existing extended magnitude greater than the universe itself (says Aristotle at 207b19-21), hence there are no arbitrarily large (actual) extensions; and hence there is not even a potential infinity with respect to extension. As Aristotle puts it, "A potential extension can be only as large as the greatest possible actual extension" (207b17-18).[8] Aristotle's universe is thus

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[7] Hence Aristotle in fact assumed the existence of actually infinite sets of objects (in the modern sense of actual infinity), though not the existence of infinite sets whose members all exist simultaneously.

[8] This feature of Aristotle's theory of infinity is pointed out by Harold Cherniss in *Aristotle's Criticism of Presocratic Philosophy* (Baltimore, 1935), p. 34: "That is, infinity by addition, in the sense that any given magnitude may be surpassed, does not exist even potentially [according to Aristotle]. And the reason he himself gives is that it is impossible for an infinite body to exist actually."

What is being added to Cherniss' account here is an explanation *why* Aristotle inferred the nonexistence of arbitrarily large *potential* magnitudes finite in an especially strong sense: no extension beyond it is even possible.[9]

Aristotle's argument would make no sense if he were not actually making use of the principle of plenitude. By possibility he could not mean here mere conceivability, for he admits at 203b23-25 that we can think of extensions extending beyond the boundaries of the physical universe.

What we have found about Aristotle's theory of spatial magnitude shows that the problem of reconciling his theory of infinity with mathematical practice is a much more serious one than commentators have usually realized. Aristotle thought that he could get away with saying merely this:

> Our account does not rob the mathematicians of their study, by disproving the actual existence of the infinite in the direction of increase, in the sense of the untraversable. In point of fact they do not need the infinite and do not use it. They postulate only that the finite straight line may be produced as far as they wish. It is possible to have divided in the same ratio as the largest quantity another magnitude of any size you like. Hence, for the purposes of proof, it will make no difference to them to have such an infinite instead, while its existence will be in the sphere of real magnitudes [*Phys.* III, 7, 207b27-34].[10]

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from the nonexistence of arbitrarily large *actual* extensive magnitudes. This inference was clearly mediated by the principle of plenitude.

A closely related explanation is offered (without explicitly mentioning the general principle on which Aristotle is relying) by Friedrich Solmsen in *Aristotle's System of the Physical World* (Ithaca, N. Y., 1960), p. 168, esp. n. 35.

[9] Aristotle's way of thinking is rather amusingly illustrated by the words *ἔξω τοῦ ἄστεος* at 208a18 which were taken by Alexander, Themistius, and Philoponus (*apud* Ross, *Aristotle's Physics* [Oxford, 1936], p. 562) to mean "outside the city." They suggest that Aristotle was worried about too large a magnitude's "sticking out" of the boundaries of the physical universe, in the same way too large a man would have to be "outside the city." If this is right, there do not seem to be good reasons for omitting *τοῦ ἄστεος* and *ἤ* from 208a18.

[10] Evans (*op. cit.*, p. 49) and Sir Thomas L. Heath, in his *Manual of Greek Mathematics* (Oxford, 1931), p. 199, omit, when quoting this passage, the sentence "It is possible . . . any size you like." They are thus presupposing that the infinite extension which according to Aristotle's last sentence suffices "for the purpose of proof" is the possibility of producing lines "as far as one wishes." This is not, however, what the passage says; the last sentence of the quotation clearly refers to the penultimate one, saying that what suffices

Aristotle is not saying here merely that a mathematician does not need an infinite magnitude all of whose parts are simultaneously actualized. If this were all that he were saying, he would have a plausible argument. The quoted passage shows him doing much more, however; he is also arguing that a geometer does not even need arbitrarily large potential extensions. He is suggesting in effect that all that the geometer needs is the kind of infinite extension that exists merely as the inverse of infinite divisibility, and that a geometer therefore does not even need arbitrarily large potential extensions. All that he needs according to Aristotle is that there be *arbitrarily small* potential magnitudes.

What Aristotle's statement therefore amounts to is to say that for each proof of a theorem, dealing with a given figure, there is a sufficiently small similar figure for which the proof can be carried out. In short, each geometrical theorem holds in a sufficiently small neighborhood. From this it does not follow, however, that the theorem really holds. There are in fact geometrical assumptions requiring arbitrarily large extensions. The best-known case in point is of course Euclid's fifth postulate, the famous "axiom of parallels": "If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, *if produced indefinitely*, meet on that side on which are the angles less than the two right angles" (trans. by Heath). If there is a maximum to the extent to which lines can be produced, this postulate fails. What we can justify on Aristotle's principles is merely the statement that, given the situation described by Euclid (line *AB* falling on the

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for the purposes of proof is the infinite extension that exists merely as the inverse of infinite divisibility. This kind of infinity Aristotle discusses at 206b3-12, a passage which is echoed by the penultimate sentence of our quotation.

The point is correctly made by Cherniss (*op. cit.*, p. 35, n. 129) and to some extent also by Heath in his *History of Greek Mathematics* (Oxford, 1921), I, 344. Heath there suggests that Aristotle's statement is incompatible with the mathematical practice of his time. This is criticized by Sir David Ross in his edition of *Aristotle's Physics* (note 9 above), p. 52. Ross is right in the case of the particular assumption he is discussing (the so-called axiom of Archimedes), but there are other mathematical assumptions that are in fact vitiated by Aristotle's theory.

straight lines *AC* and *BD*, angle *CAB* + angle *ABD* being less than two right angles) there is a point *A'* on *AB* sufficiently near *A* such that a parallel to *BD* through *A'* meets *AC* on the side Euclid specifies. This does not, however, guarantee that the resulting geometry is Euclidean (in the present-day sense of the word); it only guarantees that it is *locally* Euclidean, as a non-Euclidean geometry can of course be.

It might be thought that this nevertheless makes no difference to the truths which we can prove about those geometrical configurations that in fact exist, that is to say, about those configurations that are "in the sphere of real magnitudes" (207b33-34), in other words, that are wholly contained within the finite Aristotelian universe. This claim is not justified, however, for very often we can prove something about a given figure only by means of auxiliary constructions. (Aristotle was aware of this need and in fact keenly appreciated the role of these constructions.) These auxiliary constructions may require the existence of longer lines than any of the ones involved in the given figure. Hence we are back at the same difficulty.

Was Aristotle perhaps misled by his own terminology? In *Metaphysica* IX, 9, he refers to a certain auxiliary construction as a "division" (διαίρεσις), and similar locutions occur elsewhere, too.[11] This suggests that he might have thought that our auxiliary constructions are mere "divisions" in the sense that they never transgress the limits of the given figure. This assumption is gratuitous, however.

Thus we have to conclude that Aristotle's peculiar doctrine of of the existence of a maximal spatial extension made it impossible for him to justify fully the practice of the geometers of his time. In particular, the use of Euclid's fifth postulate could not be reconciled with his doctrine. If understood according to the letter of Aristotle's statements, his physical universe is non-Euclidean: the axiom of parallels is not satisfied in it.[12]

Naturally, Aristotle could not have been himself aware of this

[11] The same word is used by Aristotle at 203b17 of those "divisions" of mathematicians which sometimes induce belief in the existence of the infinite. Here, too, mathematical *constructions* are probably meant.

[12] Cf. Solmsen, *op. cit.*, p. 173, n. 57.

conclusion. In fact, it may be doubted whether he would have argued in the way he did if he had known that Euclid's fifth postulate is even prima facie an indispensable part of the usual system of geometrical postulates and axioms. In other words, Aristotle's theory of infinity makes us doubt whether the indispensability of this postulate was realized in Aristotle's time (or at any rate by Aristotle) as clearly as it was realized by Euclid. It may be indicative that in *Physica* II, 9, 200a16-18, Aristotle traces one of the theorems that turns on the axiom of parallels to the straight line's being "such as it is" without specifying its nature in any more detail and without mentioning the axiom of parallels. In the form in which it was stated by Euclid, the axiom of parallels would scarcely have been taken by Aristotle to express a part of the essence of the straight line, as the subsequent criticism of the postulate brings out. It has a form entirely different from the Aristotelian definitions that according to him expressed the essence of this or that thing.[13]

Far from containing "a sort of prophetic idea" of a non-Euclidean geometry, as Heath suggests,[14] this passage might on the contrary indicate that the role of the axiom of parallels was not particularly clear to Aristotle. Elsewhere Heath argues himself, with reference to Aristotle, that the fifth postulate was not known before Euclid.[15]

Aristotle's compunctions about geometrical constructions were apparently shared by at least one well-known mathematician of antiquity. Heron *mechanicus* tried to dispense with the production of particular straight lines as much as possible, motivated by the idea that there might not always be enough space available to carry out such a production.[16] (It does not matter for my purposes whether Heron was himself worried about this or whether he was trying to reassure others.) In fact, Heron gave proofs of certain propositions in Euclid alternative to Euclid's own proofs. They

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[13] For one thing, typically Aristotelian definitions were equivalences, whereas the converse of the fifth postulate was demonstrable in Euclid's system.

[14] T. L. Heath, *Mathematics in Aristotle* (Oxford, 1949), pp. 100-101.

[15] T. L. Heath, *The Thirteen Books of Euclid's Elements*, 2nd. ed., (Cambridge, 1925), 1, 202.

[16] See Heath, *op. cit.*, pp. 22-23.

were designed to dispense with the applications of Euclid's second postulate which justifies the production of straight lines. This line of thought is potentially very interesting, for if it had been pushed far enough, it would have led into difficulties not only in connection with Euclid's second postulate but also in connection with the fifth.

It is not impossible, however, that mathematicians' attention was unfortunately directed away from the fifth postulate by its explicitly hypothetical form: "*if* [the two straight lines are] produced indefinitely . . . ." This may have led Heron and others to think that the second postulate is the only one in Euclid that leads into trouble in connection with the finitude of the universe.

There are statements elsewhere in the Aristotelian corpus which seem to contradict the doctrine of the largest possible geometrical extension that we have found in *Physica* III, 7. The most important one is *De Caelo* I, 5, 271b28-272a7, which was in fact referred to by Proclus in his attempt to prove the fifth postulate.[17] There Aristotle says that the space between two divergent straight lines is infinite. This passage is, however, inconclusive. The principle Aristotle there seems to appeal to, if it can be accepted, suffices for Proclus' purposes. Aristotle's apparent argument for it is fallacious, however. Moreover, Aristotle is in any case conducting there a *reductio ad absurdum* argument against the alleged infinity of the world, and hence may have appealed to the principle in question merely because he thought that his opponents were committed to it. In any case, his argument is carefully couched in explicitly hypothetical terms: "*If* the revolving body be infinite, the straight lines radiating from the centre must be infinite."

Apostle suggests that, according to Aristotle, a mathematician need not worry about problems occasioned by the finitude of the universe because "it belongs to the physicist to investigate the shape and magnitude of the universe."[18] The remarks of Aristotle to which he refers (*Phys.* II, 2, 193b22-35) do not warrant this complacency, however. Aristotle's doctrine is that "the geometer deals with physical lines, but not *qua* physical" (194a9-11). In other words, a geometer deals with physical lines by abstracting

[17] Cf. Heath, *op. cit.*, p. 207.

[18] Hippocrates George Apostle, *Aristotle's Philosophy of Mathematics* (Chicago, 1952), p. 79.

from certain of their attributes. (This is also suggested by *Parva Naturalia* [*De Mem.*] 449b31-450a6.) Now the real problem here is that some of the lines which a geometer needs do not seem to be forthcoming at all, and of course *this* existential problem is not alleviated by the possibility of abstracting from certain attributes of lines. If the requisite lines do not exist, there is nothing to abstract from. In fact, Aristotle's words at 204a34 ff. show that he included the mathematical senses of infinity within the scope of his discussion, at least when he was arguing λογικῶς.

To return to the problems connected with the principle of plenitude, the doctrine that the infinite is in a sense actualized is apparently denied by Aristotle at 206a18-21: "But possibility can be understood in more than one way. A statue exists possibly in that it will in fact exist. But the infinite will not exist actually." This does not yet give us Aristotle's settled view of the matter, however, for he hastens to emphasize the peculiar sense of existence which is involved here, not the sense of potentiality that is being used. Hence the quotation does not disprove my interpretation.

In the last analysis, Aristotle's references to the "merely potential" existence of the infinite tell us less of his notion of infinity than of his idea of genuine full-fledged existence. This was the separate, independent (καθ' αὑτό) existence of an individual substance. All the other modes of existence were viewed by him with some amount of suspicion, and were sometimes liable to be assimilated to "merely potential" existence. The mode of being that belongs to the infinite is a case in point. Here the crucial consideration was clearly that there is no moment of time at which one can truthfully say: the infinite is *now* actualized, in the way we can say of an existing individual that it is *now* actually existent. Hence the burden of such Aristotelian remarks as the one just quoted is perhaps not so much that the infinite is not actualized but that it does not exist *as an individual*—that no infinite body exists or can exist. As Aristotle formulates his point:

> We must not regard the infinite as a "this" (τόδε τι), like a man or a house, but must suppose it to exist in the sense in which a day or the games are said to exist—things whose being has not come to them like that of a particular substance (οὐσία τις), but consists in a process of coming to be and passing away [206a29-33].

When Aristotle says that the infinite "will not exist actually," what he has primarily in mind is therefore merely the fact that there will not be any moment of time at which it can be said to be actualized. This does not go to show, however, that the infinite is not actualized in some other sense.

It would nevertheless be too rash to disregard contrary evidence altogether. There are indications that Aristotle is himself hesitating between different views. It may be significant that some of the clearest statements to the effect that for Aristotle the infinite was in a sense actualized come from passages which appear somewhat parenthetical. This is clearly the case with the passage which was just quoted from 206a29-33; it is in fact considered by Ross as "an alternative version which ... was at an early date incorporated in the text" (p. 556). The same may be the case with the words "*ἄλλως μὲν ... πεπερασμένον*" at 206b12-15 which do not contribute anything to what Aristotle is discussing there. It is also interesting to note that the passage we quoted earlier from 206a23-25 is missing from one of the manuscripts (*sc.* E). It almost looks as if we had caught Aristotle here in the process of changing his mind, or perhaps rather changing his emphasis. This assumption would also serve to explain Aristotle's reliance on the descriptions of the infinite as "potential but not actual" which we have found to be unrepresentative of Aristotle's definitive statements on the subject.

The line of thought which these statements to some extent replace seems to turn on assimilating the mode of existence which the infinite enjoys to that of the material which, for example, may become a statue. This line of thought is seen from 206b14-16, 207a21-32, and 207b34-208a4. It is not obvious that it has to contradict the emphasis on the principle of plenitude which we found elsewhere; the two ideas seem to coexist happily in 206b14-16. The contrast between matter and form, however, is elsewhere (for example *Met.* IX, 6, 1048b1-8) assimilated by Aristotle to that between potentiality and actuality. Hence this seems to lead at least to a different emphasis in the case of infinity.

Another problem with wider implications comes up in the course of Aristotle's discussion of infinity. I quoted a statement to the effect that for Aristotle infinity is "conceivable though never realized." This was found to be a misleading formulation in that the infinite is in a sense realizable for Aristotle. It may now be asked whether the formulation is perhaps equally misleading in so far as the conceivability of the infinite is concerned. I shall argue that it is.

In general, there appears to have been little difference for Aristotle between actual physical realizability and realizability in thought. The difference between these two, should one make a distinction here, would be in effect a distinction between two senses of possibility, a distinction which bears some resemblance to our distinction between logical possibility and physical possibility. In most cases Aristotle completely fails to appreciate distinctions of this sort, even in cases where he would find it convenient to use it. How foreign the general trend of his thought is to such a distinction is perhaps seen by considering the equivalent distinction between two senses of necessity (the impossibility of conceiving of the contradictory to something versus the impossibility of actually realizing the contradictory). The main burden of *Analytica Posteriora* is to make definitions, or truths essentially like definitions, the ultimate starting points of each science.[19] The way of coming to know the basic principles of a science is described by Aristotle as a way of coming to have the basic concepts of that science.

This idea has a neat counterpart in Aristotle's psychology. There we learn that thinking is an actuality and that the thinking mind is formally identical with the object of which it is thinking. "Knowledge when actively operative is identical with its object" (*De Anima* III, 6, 431a1-2). In other words, in thinking of x the mind assumes the form of x and even in a sense *becomes* this form.[20] For this reason, the conceivability of a form entails that this form is in a sense actualizable. In being thought of, this form is actualized in the mind of the thinker: being conceivable is a form of being realizable.[21]

[19] See, e.g., *An. Post.* I, 8, 75b31; II, 3, 90b23; II, 17, 99a22.

[20] See, e.g., *De Anima* III, 4-5.

[21] Apostle (*op. cit.*, p. 79) claims that, according to Aristotle, "we may have thoughts of impossibilities." In support of this view he refers to *Met.* VI, 3, 1027b25-27. But in this passage Aristotle is not discussing possibility and impossibility at all, merely truth and falsity, which are said to be "not in things . . . but in thought."

Of course, what is realizable in the mind need not for that reason be realizable outside the mind, according to Aristotle. What makes the difference in such cases is apparently the material factor; what for Aristotle was realizable in one medium was not necessarily realizable in another. This is shown, for instance, by Aristotle's remarks on the Socratic paradox in *Ethica Eudemia* I, 5, 1216b6 ff. A man may know what virtue is—that is, the form of virtue may be present in his mind—but he may nevertheless fail to become virtuous. This is explained by Aristotle in terms of the material factor, which therefore is to be blamed for the failure of realization in this case. A similar point is made in *Physica* II, 2, 194a21 ff.

When we discuss realizability without qualifications, however, we are discussing actualization in any material whatsoever, and for this purpose actualization in one's mind seems to serve perfectly well in Aristotle's view. In being able to bring about a certain result x the main thing was to have in one's mind the form of x. This was taken for granted by Aristotle; what he argues for in *Ethica Eudemia* is the further point that one must *also* have knowledge of the material "out of which" x is to be formed.

It is not quite clear, however, exactly how Aristotle thought of the actualization of the various forms in one's mind. In what kind of material are these forms realized? Is a bodily change involved? Is Aristotle dealing with the images which he says must accompany all thinking, or with thinking proper? What exactly is the distinction between these two? We cannot discuss these difficult and involved questions here. It may be pointed out, in any case, that the realization in one's mind takes place in a material different from the ones in which forms are normally embodied outside us. Hence knowing an individual x, which involves having its form in one's mind, does not necessarily give us a capacity of realizing the same (numerically the same) individual in one's mind or in any other medium, but only the capacity of reproducing the same form in some material or other.

What is also clear is that Aristotle repeatedly insists that actualization in one's mind is in principle as good a sort of actualization as any other. Aristotle wants to apply his principle that "everything comes out of that which actually is" (*De Anima*

III, 7, 431a3-4) to artificial products like houses or to such results of skillful activity as the health which a doctor has brought about. In order to do so, he has to say that the process of building or of healing has as its starting point another actual instance of the form of house or of the form of health—namely, the form which exists in the mind of the builder or of the healer. In *Metaphysica* XII, 4, 1070b33-34 he writes: "For the medical art is in some sense health, and the building art is the form of the house, and man begets man." The analogy presented here shows that the form of a house that exists in the builder's mind is for Aristotle as good an instantiation of the form in question as the father of a son is an instance of the form of man. Essentially the same point is made more fully in *Metaphysica* VII, 7, 1032b1-15 (cf. also *Met.* VII, 9, 1034a22-24). The obvious connection between these passages and Aristotle's discussion of the temporal priority of the actual in *Metaphysica* IX, 8, 1049b18-29 shows that the thought (or image) which one has in one's mind when one knows *x* is for Aristotle as fully actual an instance of the form of *x* as an external object exemplifying this form.

This parity of actualization in thought with actualization in external reality is what leads me to say that for Aristotle conceivability implied actualizability. According to Aristotle, to conceive of a form in one's mind was *ipso facto* to actualize it.

This idea is also applied by Aristotle to mathematical entities. They exist only in thinking, but since thinking is an actuality, they are not any less real for this reason.[22]

A case in point is the existence of the auxiliary constructions or "divisions" that are often needed in a geometrical proof. These divisions are obviously of the same kind as the divisions that are contemplated by Aristotle when he discusses infinite divisibility and are hence of immediate relevance to his theory

[22] Cf. esp. *Met.* IX, 9, 1051a21-33, to be quoted (in part) below, and *Met.* XIII, 3. There is something of a contrast between these two passages, however. At *Met.* XIII, 3, 1078a30-31 it is implied that mathematical objects exist ὑλικῶς—i.e., by way of matter—whereas in the *Met.* IX, 9 passage it is stressed that "a geometer's thinking is an actuality." This appears to be a matter of emphasis, however. Cf. also Apostle, *op. cit.*, pp. 11-17, and Anders Wedberg, *Plato's Philosophy of Mathematics* (Stockholm, 1956), pp. 88-89.

of infinity. Of the "divisions" or constructions needed in geometrical proofs Aristotle writes in *Metaphysica* IX, 9, 1051a21-31:

It is by an activity also that geometrical constructions [or theorems, *διαγράμματα*] are discovered, for we discover them by dividing. If the figures had been already divided, the constructions [theorems] would have been obvious; but as it is they are present only potentially. ... Obviously, therefore the potentially existing constructions are discovered by being brought to actuality: the reason is that a geometer's thinking is an actuality.[23]

Now this idea of conceivability as realizability in one's mind seems to fare very badly in Aristotle's discussion of infinity. Initially, Aristotle appeals to it in the way he might be expected to use it on the basis of what we have found:

Most of all, a reason which is peculiarly appropriate and presents a difficulty that is felt by everybody—not only number but also mathematical magnitudes and what is outside the heaven are supposed to be infinite because they never give out in our thought [203b22-25].

Aristotle's words reflect the importance he attached to this argument. Nevertheless, it seems to be rejected in *Physica* III, 8, 208a14-19:

To rely on mere thinking is absurd, for then the excess and defect is not in the thing but in thinking (*ἐπὶ τῆς νοήσεως*). One might think that one of us was bigger than he is and magnify him *ad infinitum*. But it does not follow that he is bigger than the size we are, just because someone thinks he is.

In fact, these words seem to indicate that Aristotle made a clear distinction between conceivability and actual realizability. If this were really the case, this passage would have important consequences for our interpretation of Aristotle's thought in general.

[23] In interpreting *diagrammata* as theorems, I am following Heath (*Mathematics in Aristotle*, pp. 216-217) who refers to *Cat.* 12, 14a39 and *Met.* V, 10, 1014a36 for further evidence. The context itself shows rather clearly that this is what Aristotle has in mind here. Cf. also Eckhard Niebel, "*Untersuchungen über die Bedeutung der geometrischen Konstruktionen in der Antike*," *Kant-Studien*, Ergänzungshefte, LXXVI (1959), esp. 92-95, where further references to the literature on this subject are given.

We can see, however, that Aristotle's purpose here is severely restricted, and that it cannot therefore support any general conclusions concerning the relation of conceivability and realizability in Aristotle. The fact that Aristotle formulates his point in terms of "excess or defect" shows that he has in mind only quantities, and indeed only spatial magnitudes. He hastens to point out (at 208a20-21) that his remarks do not apply to movement or time. There is no trace anywhere of an application of this idea to the other concepts which Aristotle had been considering and which he had mentioned at 203b22-25—for example, to number or to divisibility. On the contrary, at 207b10-13 he seems to rely on the conceivability of a higher and higher number of divisions to establish infinite divisibility:

But it is always possible to think of a larger number (ἐπὶ δὲ τὸ πλεῖον ἀεὶ ἔστι νοῆσαι); for the number of bisections of a magnitude is infinite. Hence (ὥστε) the infinite exists potentially, although never actually, in that the number [of bisections] always surpasses any assigned number.

It is also significant that in reassuring us of the infinity of time and movement at 208a20 Aristotle adds thinking to the list. His point seems to be that in the case of time and movement, infinity in thought and infinity in fact go together. In fact, time and movement are at once contrasted by Aristotle to magnitude, which "is not infinite either in the way of division *or of magnification in thought*" (208a21-22).

What Aristotle has in mind in 208a14-19 is his idea that although in thinking of x one's mind assumes the form of x (or, alternatively, makes use of an image having the form of x), it need not assume this form in the same size as the original. The replicas of outside forms that one has in one's mind are merely scale models of these forms, as it were. That this is what Aristotle had in mind is shown by *Parva Naturalia* (*De Mem.*) 2, 452b13 ff.:

How, then, when the mind thinks of bigger things, will its thinking of them differ from its thinking of smaller things? For all internal things are smaller, and as it were proportional to those outside. Perhaps, just as we may suppose that there is something in man proportional to the forms, we may assume that there is something similarly proportionate to their distances.

Hence Aristotle's whole point turns out to be this. Because of limitations of size ("all internal things are smaller"), a human mind can think of large things only as being large in relation to something else. Because of this, it does not follow that an imaginable size is realizable, because what is realized in one's mind is merely large in relation to something else, but not absolutely. It does follow that there is no limit to *relative* size; and this is in fact a conclusion in which Aristotle acquiesces at 206b3-9.

Thus Aristotle does not give up the general principle that conceivability (or imaginability) implies realizability, but only that this principle applies to the realizability of (absolute) sizes. It is only in the case of spatial "excess and defect" that Aristotle can say that they lie "not in the thing but in conceiving." The form that is being thought of ordinarily lies *both* in the thing and in the conceiving or imagining mind.

There is even more direct evidence that for Aristotle infinity was not "conceivable though never realized." Properly speaking, for Aristotle infinity was inconceivable. In *Metaphysica* II, 2, 994b20-27, Aristotle denies in so many words that we can apprehend an infinity. "The notion of infinity is not infinite," Aristotle says, thus emphasizing that in the sense in which the infinite is not actualized in external reality it is not realized in thinking, either, for that would involve the realization of an infinite form in one's finite mind. In the sense in which the infinite was for Aristotle unactualizable, in that sense it was also inconceivable.[24]

In general it may be said that Aristotle has a distinction which prima facie looks very much like a distinction between conceiv-

[24] By the same token, in the sense in which the infinite was in Aristotle's view conceivable, it was also actualizable. At 208a20-21 *noesis* is accordingly said to be infinite in the same sense as time and movement, viz., "in the sense that each part that is taken passes in succession out of existence." It is well known that time and movement are according to Aristotle's doctrine in a perfectly good sense *actually* infinite, viz., in the sense that there actually has been and will be an infinite number of moments of time and movements of bodies, although Aristotle does not usually express himself in this way.

ability and actual realizability (or perhaps our modern distinction between logical and physical possibility) and which serves some of the same purposes but which from a theoretical point of view is entirely different. This distinction is used, *inter alia*, in *De Motu Animalium* 4, 699b17-22, and *De Anima* II, 10, 422a26-29. The corresponding (in effect, equivalent) distinction between two different kinds of necessity is even more familiar. It is often referred to as a distinction between absolute and hypothetical necessity. It is explained, *inter alia*, in *Physica* II, 9, *De Partibus Animalium* I, 1, 639b25 and 642a8, as well as in *An. Pr.* I, 10, 30b31-34, 38-40.

The distinction between two senses of possibility can be characterized as a distinction between what is possible absolutely speaking (that is, in so far as we merely consider its own nature) and what is possible on certain conditions—for example, possible to us in our present circumstances. To use Aristotle's own example, if there are men on the moon, they will be visible in the ordinary unqualified sense of the word, though *we* cannot see them. It is important to realize that Aristotle is not here postulating two different irreducible senses of possibility but rather two senses one of which is in effect definable in terms of the other. This interpretation is confirmed by Aristotle's terminology; he refers to the distinction by means of such locutions as ποσαχῶς λέγεται (204a2-3) and λέγεται πλεοναχῶς (699b17). As I have shown elsewhere, Aristotle uses these expressions not of outright ambiguities, but rather of interrelated but different uses of one and the same word.[25]

The fact that Aristotle deals in this way with cases which we might characterize in terms of a difference between logical and physical possibility suggests that he either had no recourse to the latter distinction or else did not want to use it. In the unqualified sense of the word, conceivability implied for him realizability somewhat in the same way as it did later for Descartes.

There are in any case indications that Aristotle's distinction between intrinsic possibility and possibility under certain circumstances is different from the distinction he makes in his

[25] Jaakko Hintikka, "Aristotle and the Ambiguity of Ambiguity," *Inquiry*, II (1959), 137-151.

discussion of infinity between possibility in thought and possibility in actual physical reality. In the course of this very discussion, Aristotle also uses the former distinction, applied to a special case:

We must begin by distinguishing the various ways in which the term "infinite" is used. (1) What is incapable of being gone through, because it is not its nature to be gone through. . . . (4) What naturally admits of being gone through, but is not actually gone through or does not actually reach an end [*Phys.* III, 4, 204a2-6].

A comparison with Aristotle's remarks elsewhere suggests that this distinction between the different senses in which it is impossible to go through something is an instance of the distinction he makes elsewhere between different uses of possibility. Nevertheless, it is in no way related by Aristotle to the distinction between realizability in thought and realizability in actual reality which he makes later in his discussion of infinity. This strongly suggests that the two distinctions were not connected by Aristotle with each other.

Our discussion of the sense in which the infinite was conceivable for Aristotle shows that his theory of infinity does not constitute a counterexample to this relation between conceivability and realizability in Aristotle.

Our conclusions also help us to understand what Aristotle is really up to in his brief pronouncement on infinity in *Metaphysica* IX, 6, 1048b14-17, and are confirmed by what we find there. If I am right, what Aristotle says in this passage may be expressed as follows:

The infinite does not exist potentially in the sense that it will ever exist actually and separately; it exists only in thinking. The potential existence of this activity ensures that the process of division never comes to an end, but not that the infinite exists separately.

This version follows Ross's translation fairly closely. Nevertheless it requires a few explanatory comments.

(1) The first and foremost thing to be noted in this passage is Aristotle's main conclusion. Understanding this conclusion is completely independent of the difficulties which we may have in understanding the passage in other respects. The conclusion is

that the infinite does not exist *separately* (Aristotle's word is χωριστόν). Now what would such a separate existence mean for Aristotle? It is contrasted by him not with potential existence but to the kind of nonseparate existence which, for example, qualities enjoy in relation to the substances whose qualities they are.[26] The Platonists had supposed that the forms exist separately, Aristotle tells us, and goes on to argue that they were wrong and that the forms exist only in those things whose forms they are and on whose existence their being is dependent.[27] In the same way, Aristotle is here pointing out the peculiar way in which the infinite exists. According to him, it depends for its existence on the finite beings which one after another come into being.

In short, he makes here the same point we found him making in *Physica* III, 7. He is pointing out that the infinite *exists* in an unusual sense of existence, not that it is potential in a new sense of potentiality. As he says himself in introducing the subject of infinity, "but also the infinite and the void and all similar things are said to exist potentially *and actually* in a sense different from that which applies to many other things" (1048b9-11).

(2) The second clause of the first sentence is sometimes taken to mean that according to Aristotle the infinite exists *separately* only in thinking (or knowledge, γνώσει). This is surely wrong. As we have seen, Aristotle's doctrine is that the infinite does not exist as an individual (that is, separately) in any sense at all.

Ross translates the clause by "it exists *potentially* only for knowledge." This may quite well be right, although it does not quite square with Aristotle's avowed purpose of showing that the infinite exists potentially *and actually* in an unusual sense. The difference does not matter, however, since for Aristotle each potentiality eventually actualizes. For then we might equally well render Aristotle's thought by saying "it exists (potentially and therefore also actually) only in thinking." What Aristotle is bringing out here is not any special way in which the infinite

[26] It is also contrasted with the mode of existence of mathematical objects which do not exist apart from sensible particulars and which can be separated from them only in thinking; see *Met.* VI, 1, 1025b27; XI, 1, 1059b13; XIII, 3, 1078a21-31; XIII, 7, 1080b17; XIII, 9, 1086a33. Cf. (2) *infra*.

[27] Cf. e.g., *Met.* VII, 14-16.

exists, but rather the way in which all mathematical objects exist according to him.

(3) The second sentence of our quotation is very difficult to understand and to translate. We shall not discuss here the philological details but refer the reader once and for all to Ross's comments in his edition of *Metaphysica* (II, 252-253). The main problem is whether the subject of the sentence is the phrase which may be translated "the potential existence of this actuality" (or "activity") or the phrase which may be translated "the fact that the division never comes to an end." Accordingly, we shall have a choice of two translations which run somewhat as follows: "for the potential existence of this activity ensures that the process of division never comes to an end" and "for the fact that the process of dividing never comes to an end ensures that this activity always exists potentially." Ross points out that the philological evidence favors the former interpretation, but he finds in favor of the latter on topical grounds. These grounds are inconclusive, however, for they amount to taking Aristotle's statement at 203b22-25 as being accepted by him and viewed "as a given fact." We have already seen that Aristotle returns to the same subject in 208a14-19 and qualifies his earlier statement in certain respects. It is true that we have also seen that these qualifications are not nearly as sweeping as commentators have often taken them to be; but perhaps they should nevertheless warn us not to rely too much on 203b22-25.

The question is really this. In *Physica* III, 4, 203b18-20 Aristotle mentions as a putative proof of the actual (better: separate) existence of the infinite the idea that an endless coming-to-be (*μὴ ὑπολείπειν γένεσιν*) can only take place if there (actually) exists an infinite supply from which the things that are coming to be are coming from. In *Physica* III, 8, 208a8-11 Aristotle points out that this explanation is not needed (*Οὔτε γὰρ ἵνα ἡ γένεσις μὴ ἐπιλείπῃ, ἀναγκαῖον ἐνεργείᾳ ἄπειρον εἶναι σῶμα αἰσθητόν*). At 208a8-11, however, he does not give any alternative explanation, maybe because such an explanation is implicit in the rest of his discussion of infinity in *Physica*.

Now the statement at 1048b15-17 can be understood as offering just such an alternative explanation, formulated in terms of infinite division. The endless coming-to-be of further and further divisions is "ensured" (Aristotle's verb is ἀποδίδωμι) by the potential existence of the activity of dividing. Hence it is compatible with everything Aristotle says to follow the philological evidence and to parse Aristotle's sentence in a way different from the one Ross endorses.

It is seen, however, that something is still missing here. (This insufficiency of our interpretation so far may have been instrumental in leading Ross to the other reading.) How can the merely potential existence of the activity of dividing ensure that the actual process of dividing never comes to an end? The answer is of course that no genuine potentiality is for Aristotle a *mere* possibility: if it continues to exist as a potentiality, it will ultimately be actualized. Hence the principle of plenitude supplies the link which our interpretation might prima facie seem to fail to provide.

Far from being incompatible with the principle of plenitude, the passage we have been discussing again turns out to presuppose it.[28]

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[28] In writing the present version of this paper I have profited from the friendly criticism to which Richard Sorabji and Peter Geach have subjected an earlier version of it, although I have probably failed to meet most of their criticism. I greatly regret that I could not take into account Professor Friedrich Solmsen's interesting and pertinent comments, which (through a fault of my own) reached me too late for the purpose.