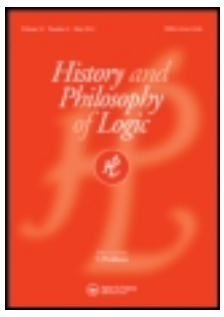

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A Tense Logic for Boethius

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An interpretation in modal and tense logic is proposed for Boethius's reconciliation of God's foreknowledge with human freedom from *The consolation of philosophy*, Book V. The interpretation incorporates a suggestion by Paul Spade that God's special status in time be explained as a restriction of God's knowledge to eternal sentences. The argument proves valid, and the seeming restriction on omnipotence is mitigated by the very strong expressive power of eternal sentences.

1. Boethius on divine foreknowledge and human freedom

In Book V of *The consolation of philosophy*, written in the 6th century, Boethius argues that human free will is compatible with divine foreknowledge in a famous argument that employs notions of modality and tense.[1] In this paper I present a formal version of that argument using the concepts of modal and tense logic, and show that under this reading the argument is valid. The argument turns on an analysis of the eternality of God and his knowledge that casts some new light on how 'the God of the philosophers', as Kenny calls him, might be defined.

The interpretation of the text on which the formalized argument is based has several components, some standard, some novel. The first is the standard reading that attributes to Boethius the idea that humans can only have future knowledge of what is now determined. The idea is found in Aristotle's discussion of the sea battle in *De interpretatione* 9 (18^a27 *ff.*), and is discussed by Boethius as a view of Aristotle's in his commentary on that work.[2] In his own discussion of related ideas in *Consolation*, V, Boethius seems to adopt this Aristotelian position himself. He does not explain what is involved in grounding knowledge of the future in the present. He certainly does not spell out the conditions for the epistemic justification of future knowledge claims nor the metaphysical mechanism underlying their reliability. In the later Middle Ages these would be explained in terms of the natural potencies present in those objects that exist now.[3] At a minimum, however, Boethius does seem to be committed to the view that if we know something about the future now, then that fact is in some sense necessary. We make use of a formalized version of this premise in the argument below.

1 There are two readily available editions of the *Consolation* in English: Boethius, *The consolation of philosophy* (int. by V. E. Watts; Baltimore: Penguin Books, 1978); and Boethius, *The consolation of philosophy* (trans. with int. and notes by Richard Green; Indianapolis: Bobbs-Merrill, 1962). The Latin text with English translation is to be found in *Boethius, the theological tractates* (trans. H. F. Stewart and E. K. Rand) and *The consolation of philosophy* (trans. S. J Tester) published together as one volume (Cambridge, Mass: Harvard University Press, 1973).

2 See the discussion of Boethius's commentaries in John Marenbon, *Early medieval philosophy (480–1150)* (London: Routledge, 1983).

3 For a discussion of one later view see William Ockham, *Predestination, God's foreknowledge, and future contingents* (trans. Marilyn McCord Adams and Norman Kretzmann; second ed. Indianapolis: Hackett, 1983). Note also the commentary by Adams.

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A second piece of the interpretation is Boethius's view that God has a special relation to time and that therefore he is not bound like humans only to knowledge of what is now necessary. That Boethius holds some such view is standardly recognized. Gilson quotes Boethius and explains the idea this way:

> God is eternal and eternity is the complete possession, perfect and simultaneous, of life without end (*aeternitas est interminabilis vitae tota simul et perfecta possessio*): God lives, then, in a perpetual present . . . There is, then, before-ness and after-ness in events, but not in the totally-present knowledge that God has of them.[4]

Marenbon renders the idea as follows:

> Different types of being know things in a manner according to their capacities: divine knowledge should not be thought of in the terms applicable to human knowledge. The eternity in which God has his being is not a matter of living for ever, but of being outside the very process of time. He therefore knows all things, which appear to us as past, present and future, at once.[5]

In a well-known criticism of a similar view held by Aquinas, Kenny has argued that any such view that makes God simultaneous with different moments of time is absurd.[6] But exactly what is involved in God's special temporal status is by no means clear, and Boethius does not provide anything like an adequate explanation of the idea. The appropriate analysis of the idea suitable to the argument, if there is one, must be regarded as an open question.

In the version of the argument advanced below, we adopt a suggestion due to Paul Spade that has the advantage of conceptual coherence—none of Kenny's points apply to it—and of letting the argument go through. Spade speculates that the argument may be made to work if we understand God's eternality to consist in the fact that his knowledge is limited to what Quine calls eternal sentences, sentences which if true at any time are true at all times.[7] In the account given below, we implement Spade's suggestion by means of a temporal 'eternality' operator E which attaches to a sentence p in such a way that Ep is true at t if, and only if, p is true at all times in actual history. Boethius's view that God is apart from time is then captured in terms of an axiom to the effect that God only knows sentences that are eternally true. No appeal is made to the problematic idea of God's simultaneity, and the resulting theory is perfectly consistent.

The theory also has implications for the analysis of omniscience. Since in the theory God can know only eternal truths, omniscience cannot be understood to be knowledge of every truth, but only knowledge of every eternal truth. This restriction of God's knowledge is less problematic than it might seem because the expressive power of the various eternal sentences is rendered capable of describing every fact at every point in time by the inclusion in the language of a special modal feature for

4 Etienne Gilson, *History of Christian philosophy in the middle ages* (London: Sheed & Ward, 1955), 103.

5 Marenbon (footnote 2), 41.

6 See Anthony Kenny, 'Divine foreknowledge and human freedom', in his (ed.), *Aquinas: a collection of critical essays* (London: Macmillan, 1970).

7 See Paul Vincent Spade, 'Boethius: foreknowledge and free will', ch. 22 of his *A survey of mediaeval philosphy*, vol. I, 1985, typescript, Department of Philosophy, Indiana University, Bloomington. I am indebted to the author for permission to consult his text.

capturing one variety of eternal sentence: time specifiers, rather like dates, that render basic statements of fact eternal. While 'Reagan is the U.S. President' may differ in truth-value from time to time, 'Reagan is the U.S. President in 1988' is true always, if it ever is.

Finally, the formalization makes use of some additional ideas from modal and tense semantics that when combined render the assumptions previously mentioned true and also insure the validity of the argument. The first of these is an interpretation of 'necessity' as an alethic modality ranging over just possible future times.[8] In his discussion in *Consolation*, V, Boethius does seem to limit the necessities in question to items in the future. It is only these, after all, that are relevant to the issue at hand. Thus in the semantics $\Box p$ is said to be true at t if p is true at all possible times later than p.[9] Secondly, a distinction is drawn between those times that fall in actual history from those that do not, and the eternality operator and its dual the inevitability operator are interpreted as ranging over just actual history. By this means a distinction is drawn between what is eternal and what is necessary. It will turn out to be perfectly possible that God knows eternal truths that are not necessary, while man is limited to future knowledge of necessities.

2. Grammar

Let us postulate three basic parts of speech:

(1) a set $\{a,b,c,...,g,...\}$ of *subject terms* or *proper names*;
(2) a set $\{G,H,...\}$ of *predicate terms* or *common nouns*;
(3) a set $\{\tau,\sigma,...\}$ of *time constants*.

Intuitively, proper names stand for individual objects, common nouns stand for sets, and time constants stand for dates or instants in history. We single out the proper name g to refer to God.

From these basic parts of speech we first construct a set of *atomic* or *simple sentences*. These are formed by concatenating a subject term behind a predicate term, and are understood to be in the present tense.

The set of *sentences* is defined so as to include all atomic sentences plus all *molecular* or *complex* sentences made up in the following ways:

(1) attaching a time constant to the front of a sentence (thus if p is a sentence and τ a time constant, then τp is a sentence and is read 'It is true at τ that p');

8 The combination of temporal specifiers τ with the necessity operator $\llcorner$ allows for expressions like $\llcorner \tau p$ which reads 'It is necessary that p is true at τ', and this may suggest some similarity between the analysis given here with that by Martin Davies in 'Boethius and others on divine foreknowledge', *Pacific philosophical quarterly*, **64** (1983), 313–329. Davies makes use of sentences of similar form. Within our theory, however, the nearest thing to Davies's $\llcorner \tau p$ is probably our $\llcorner p$, inasmuch as both embody a fixed prior history. Other features are different. Davies seems to regard the operator $\llcorner \tau$ as a unit, unlike our treatment. Certainly, our use of time specifiers to generate eternal sentences has no counterpart in Davies's theory. A more important difference of a rather basic sort is that Davies's approach to the problem of providing a modal account of Boethius is non-semantic. His method turns on postulating somewhat *ad hoc* modal principles which are supposed to be taken as plausible as they stand without the benefit of being fixed within a broad semantical theory in which they are defined. The approach is difficult because of the profusion of modal ideas. Given the many senses of the terms that are possible, it is very hard to have clear intuitions about unexplained principles. Our approach is rather to tie down the principles at issue by giving a clear semantics.

9 Boethius does not always talk this way. Sometimes he uses necessity in the more usual sense in which it ranges over all possible times, past as well as future. See the discussion in section 37 of Karl Dürr, *The propositional logic of Boethius* (Amsterdam: North-Holland, 1951).

(2) attaching to any sentence the past tense operator P (thus if p is a sentence, Pp is a sentence and is read 'It was the case that p at some time in the past of actual history');

(3) attaching to any sentence the future tense operator F (thus if p is a sentence, Fp is a sentence and is read 'It will be the case that p at some time in the future of actual history');

(4) attaching to any sentence the necessity operator $\Box$ (thus if p is a sentence, $\Box p$ is a sentence and is read 'It must be the case that p' or 'p is true at all possible future times');

(5) attaching to any sentence the eternality operator E (thus if p is a sentence, Ep is a sentence and is read 'It is true at each point in actual history that p');

(6) attaching to any sentence the inevitability operator I (thus if p is a sentence, Ip is a sentence and is read 'It was, is now, or will be the case that p' or 'p is true at some point in actual history', and I is the dual of E);

(7) attaching to any sentence the knowledge operator K and a name s (thus if p is a sentence and s is a proper name, Ksp is a sentence and is read 's knows that p').

3. Semantics

We understand possible worlds to be times gathered into a set $\mathbf{T} = \{t, t', t'', \ldots\}$. We further impose some structure on $\mathbf{T}$. We assume in particular that points, or what we shall call *instances* or *times* in $\mathbf{T}$, fall into a tree structure organized by a partial ordering $\leqslant$. We understand $\leqslant$ to represent temporal order, and accordingly understand $t \leqslant t'$ to mean that t is earlier in time than t'. The fact that t may have more than one immediate temporal successor in the tree structure represents the intuitive idea that history may unfold in various ways in the instant following t. Even though each point in time may have more than one possible immediate temporal successor, it must have only one actual successor. More precisely, if t were to become actual then it would have only one actual immediate successor in time. We represent this structural feature of time by identifying for each t in $\mathbf{T}$ a unique branch $A(t)$ made up of points in $\mathbf{T}$ among which t is included. Intuitively $A(t)$ is the path that history would take through time if t were to become actual.

We assume that at each time t all the subject terms (proper names) and the predicate terms (common nouns) each have a referent. We require that if n is a name, then its referent at t is an object that exists at time t, and we use the notation $R_t(n)$ to refer to the object referred to by n at t. Likewise we require that the referent at t of a predicate G to be a set of objects that exist at t, and we refer to this set by $R_t(G)$. We also require that each temporal constant τ refers at t to a time that falls someplace in history as determined by time t. Define $R(\tau)$ *relative to* $A(t)$ to be an element of $A(t)$, and $R(t', \tau)$ *relative to* $A(t)$ to be an element of $\mathbf{T}$ such that if $t' \in A(t)$, $R(t', \tau)$ is $R(\tau)$ relative to $A(t)$.

We define an *assignment R of truth-values T or F to the sentences of the grammar* as follows. If p is a sentence, we read $R_t(p)$ as 'the truth-value assigned to p at t'. We now define $R_t(p)$ for each type of sentence as follows.

(0) For an atomic sentence Gn: R assigns the truth-value T to Gn at t if, and only if, the referent of n at t is a member of the referent of G at t:

$R_t(Gn) = T$ iff $R_t(n) \in R_t(G)$.

(1) For a sentence τp composed of a sentence p and the temporal constant τ: R assigns T to τp *at* t if, and only if, R assigns T to p at that time $R(t,\tau)$ referred to by τ at t: for $R(t,\tau)$ relative to some $A(t')$,

$$R_t(\tau p) = T \text{ iff } R_{R(t,\tau)}(p) = T.$$

(For example, to an atomic sentence Gn, we may thus attach the constant τ to get the sentence τGn which is evaluated as follows:

$$R_t(\tau Gn) = T \text{ iff } R(t,\tau) = t' \;\&\; R_{t'}(Gn) = T$$
$$\text{iff } R_{R(t,\tau)}(Gn) = T.)$$

(2) For the sentence Pp made up of the past tense operator P and the sentence p: R assigns T to Pp at t if, and only if, R assigns T to p at some time t' earlier than t:

$$R_t(Pp) = T \text{ iff } (\exists t')(t' < t \;\&\; R_{t'}(p) = T).$$

(Note that if $t' < t$, then t' automatically falls into the actual history $A(t)$ of t.)

(3) For the sentence Fp made of the future tense operator F and the sentence p: R assigns T to Fp at t if, and only if, R assigns T to p at some time t' later than t in the actual history $A(t)$ of t:

$$R_t(Fp) = T \text{ iff } (\exists t')(t' > t \;\&\; t' \in A(t) \;\&\; R_{t'}(p) = T).$$

(4) For the sentence $\Box p$ made up of the necessity operator $\Box$ and the sentence p: R assigns T to $\Box p$ at t if, and only if, R assigns T to p at all times t' later than t:

$$R_t(\Box p) = T \text{ iff } (\forall t')(t' > t \rightarrow R_{t'}(p) = T).$$

(5) For the sentence Ep made of the eternality operator E and the sentence p: R assigns T to Ep at t if, and only if, R assigns T to p at all times t' in the actual history $A(t)$ determined by t:

$$R_t(Ep) = T \text{ iff } (\forall t')(t' \in A(t) \rightarrow R_{t'}(p) = T).$$

(6) We now explain the semantics of the inevitability operator. If our language had the sentence connective 'or' in it, it would be possible to define Ip as 'Pp or p or Fp'. But since it lacks 'or', we must give a semantic interpretation directly. For the sentence Ip made up of the inevitability operator I and the sentence p: R assigns T to Ip at t if, and only if, R assigns T to p at some time t' in the actual history $A(t)$ determined by t:

$$R_t(Ip) = T \text{ iff } (\exists t')(t' \in A(t) \;\&\; R_{t'}(p) = T).$$

For any p containing τ, we say $R_t(p) = T$(*simpliciter*) iff relative to A(t), $R_t(p) = T$. We now define the logical notions of logical truth and implication:

p is a *logical truth* (briefly $\models p$) if, and only if, for any structure **T** of time and any t in **T**, $R_t(p) = T$:

$\models p$ iff $(\forall \mathbf{T})(\forall t)(t \in \mathbf{T} \rightarrow R_t(p) = T)$.

p *logically implies* q (briefly $p \models q$) if, and only if, for any structure **T** of time and any t in **T**, if $R_t(p) = T$ then $R_t(q) = T$:

$p \models q$ iff $(\forall \mathbf{T})(\forall t)[(t \in \mathbf{T} \ \& \ R_t(p) = T) \rightarrow R_t(q) = T]$.

Thus far we have neglected to explain the truth conditions of knowledge statements Ksp. We do so now as follows. Consider the sentence Ksp composed of the knowledge operator K with subject s. Intuitively, we should say that R assigns T to Ksp at t if, and only if, the person $R_t(s)$ referred to by s at t believes p, the person is well justified in believing that p, and p has the truth-value T at t. In addition, we should also build into the analysis of knowledge a special detail about the justification of future knowledge that was accepted by Boethius: if we know something about the future it must be because we know something about the present and that those facts about the present in some sense necessitate the fact about the future that we know. Rather than trying to capture these ideas in the form of truth conditions, we will instead lay down two axioms constraining time and the assignment of truth-values. These axioms or 'meaning postulates' may be looked upon as spelling out in part the meaning of the word 'knowledge':

MP1 K$sp \models p$,
MP2 KsF$p \models \Box p$.

Since we are to incorporate into the theory the idea that God's knowledge is limited to eternal sentences, we also add:

MP3 K$gp \models Ep$.

Examples:

$R_t(FGn) = T$ iff Gn is T at some later time in the history $A(t)$ of t:
iff $(\exists t')(t' > t \ \& \ t' \in A(t) \ \& \ R_{t'}(Gn) = T$.

$R_t(\Box PGn) = T$ iff at any time later than t, PGn, is T
iff at any time t' later than t, there is time t'' earlier than t' at which Gn is T
iff $(\forall t')(t' > t \rightarrow R_{t'}(PGn) = T)$
iff $(\forall t')(t' > t \rightarrow (\exists t'')(t'' < t' \ \& \ R_{t''}(Gn) = T))$.

$R_t(F\tau Gn) = T$ iff at some time later than t, τGn is T
iff at some actual time t' later than t, Gn is true at time $R(t', \tau)$.
iff $(\exists t')(t' > t \ \& \ t' \in A(t) \ \& \ R_{t'}(\tau Gn) = T)$
iff $(\exists t')(t' > t \ \& \ t' \in A(t) \ \& \ R_{R(t',\tau)}(Gn) = T)$.

$R_t(\Box P\tau Gn) = T$ iff $(\forall t')(t' > t \rightarrow R_{t'}(P\tau Gn) = T)$
iff $(\forall t')(t' > t \rightarrow (\exists t'')(t'' < t' \ \& \ R_{t''}(\tau Gn) = T))$
iff $(\forall t')(t' > t \rightarrow (\exists t'')(t'' < t' \ \& \ R_{R(t'',\tau)}(Gn) = T))$.

4. Consequences for necessity and God's foreknowledge

Theorem: *There are two senses of conditional necessity, one of which is independent of the other.* It is possbile that q must follow from p yet it not be the case that q's necessity follow from p. That is, it does not follow from the fact that p logically implies q that p logically implies $\Box q$. *Briefly*, for some p and q, both $\models q$ and not($\models \Box q$).

Proof: $Gn \models Gn$

not($Gn \models \Box Gn$). Q.E.D.

Theorem: *The past is necessary.* Pp logically implies $\Box Pp$. *Briefly*, $Pp \models \Box Pp$.

Proof: $R_t(Pp) = T$

$(\exists t')(t' < t \;\&\; R_{t'}(p) = T$

$t' < t \;\&\; R_{t'}(p) = T$

Let $t < t''$. Then $t' < t'' \;\&\; R_{t'}(p) = T$

$(\exists t')(t' < t'' \;\&\; R_{t'}(p) = T)$

$R_{t''}(Pp) = T$

$(\forall t'')(t < t'' \rightarrow R_{t''}(Pp) = T)$

$R_t(\Box Pp) = T$. Q.E.D.

Corollary: $KsPp$ logically implies $\Box Pp$. *Briefly*: $KsPp \models \neg Pp$.

Proof: $KsPp \models Pp$

$KsPp \models \Box Pp$. Q.E.D.

Theorem: *Knowledge of the past does not need to be of necessary truths.* $KsPp$ does not logically imply $\Box p$. *Briefly*, not($KsPp \models P\Box p$).

Proof: $R_t(KsPGn) = T$

$R_t(PGn) = T$

Let $t < t'' \;\&\; R_{t''}(Gn) = F$

Let $t' < t$. Then $t' < t''$.

$(\exists t'' > t')(R_{t''}(Gn) = F)$

$\sim(\forall t'' > t')(R_{t''}(Gn) = T)$

$R_{t'}(\Box Gn) = F$

$(\forall t' < t)(R_{t'}(\Box Gn) = F)$

$\sim(\exists t' < t)(R_{t'}(\Box Gn) = T)$

$R_t(P\Box Gn) = F$. Q.E.D.

Theorem: *Temporally specified sentences need not be necessary.* τp does not imply $\Box \tau p$. *Briefly*, no ($\tau p = \Box \tau p$).

Proof: Let $t < t'$, $t' \notin A(t)$, $t'' \in A(t)$, $R(\tau) = t''$ relative to $A(t)$, $R(t', \tau) = t'$, $R_t(Gn) = F$, and $R_{t''}(Gn) = T$.

$R(t, \tau) = t''$

$R_t(\tau Gn) = T$

$R_{t'}(\tau Gn) = F$

$\exists t' < t$, $R'_t(Gn) = F$

$R_t(\Box \tau Gn) = F$. Q.E.D.

Theorem: *Eternal truths need not be necessary.* Ep does not logically imply $\Box p$. *Briefly*, not($Ep \models \Box p$).

Proof: Let $R_{t'}(Gn) = T$ at all t' in $A(t)$
Let $t'' \notin A(t)$, $t < t''$ and $R_{t''}(Gn) = F$
$R_t(EGn) = T$ and $R_t(\Box Gn) = F$. Q.E.D.

Corollary: *God may have knowledge of a future time that is not necessary*. For some p, Kgp does not logically imply that $\Box p$. *Briefly*, not($Kgp \models \Box p$).

Theorem: *All true temporally specified sentences are eternally true*. τp logically implies $E\tau p$. *Briefly*, $\tau p \models E\tau p$.

Proof: $R_t(\tau p) = T$ relative to $A(t)$
$R_{R(t,\tau)}(p) = T$
Let $t' \in A(t)$. Then $R(t,\tau) = R(t',\tau)$
$R_{R(t',\tau)}(p) = T$
$t' \in A(t) \rightarrow R_{R(t',\tau)}(p) = T$
$(\forall t')(t' \in A(t) \rightarrow R_{R(t',\tau)}(p) = T)$
$R_t(E\tau p) = T$. Q.E.D.

Corollary: $Ks\tau p$ logically implies τp, which in turn logically implies $E\tau p$. *Briefly*, $Ks\tau p \models \tau p \models E\tau p$.

Theorem: $Ks\tau p$ does not logically imply $\tau\Box p$. *Briefly*, not($Ks\tau p \models \tau\Box p$).

Proof: Let $R_t(Ks\tau Gn) = T$, and $R(t,\tau) < t'$ & $R(n) \notin R(G)$
$R_{t'}(Gn) = F$
$R_{R(t',\tau)}(\Box Gn) = F$
$R(\tau Gn) = F$. Q.E.D.

5. Omniscience

Once it is required that God know only eternal truths, it is not possible to ascribe to God omniscience in what is perhaps the most obvious sense:

$R_t(p) = T$ iff $R_t(Kgp) = T$.

Within the system, this property fails:

Theorem: *God need not know every truth*. p does not logically imply Kgp. *Briefly*, not($p \models Kgp$).

Proof: Let $t > t'$, $R_t(Gn) = T$ and $R_{t'}(Gn) = F$
$R_t(EGn) = F$
$R_t(KgGn) = F$. Q.E.D.

Within the semantics, however, it is certainly possible for God to know every truth in some guise or other. As in all recursive semantic theories, truth conditions of every complex sentence can be formulated in semantic rules that mention only the truth-values of atomic expressions at various times. Moreover, though God may not know the atomic sentences themselves, he can know their eternalized, time-specified form. Though in general Kgp may not be true at t, $Kg\tau p$ may be true where $R(t,\tau) = t$. We could, therefore, add an additional meaning postulate saying as much:

MP4 $R_t(p) = T$ iff ($R(t,\tau) = t$ only if $R_t(Kg\tau p) = T$).

The role of the postulate is then the following. Within the context of the general

semantic theory that explains the meaning of the ideas of modality and tense, we may then understand MP4 as capturing the relevant notion of omniscience, much as MP3 unpacks the special temporal status of God.

The question now arises whether MP4 is a significant constraint on God's knowledge, and whether a god conforming to it could fairly be called omniscient. The answer is not straightforward and turns on the relation of p to τp where $R(t,\tau) = t$. Let Let $R(t,\tau) = t$. Clearly, in general p is not logically equivalent to τp because p may vary in truth-value from time to time whereas τp will not. On the other hand, it is not an accident that p and τp are either both true or both false at t. The question of whether a God conforming to MP4 could be called omniscient, then, turns on the sense in which p and τp relate and on the legitimacy of substituting the one for the other on the strength of this relation.

The relationship between p and τp can be explained, but to do so in the most perspicuous way, it is best to imagine a language with greater expressive power than the one we have been working with thus far. Without going into the mathematical details, let us suppose that our language has been supplemented so as to include indexicals and the two-dimensional intensional semantics they require. In such an account times would serve two purposes. First they would continue to serve as possible worlds described by the language. They thus continue to serve as the reference points at which expressions have extensions. They are the points relative to which constants refer and sentences take on truth-values. The intension of an expression is then identified with the function from times (in this role) to the extensions. The second role times play is as contexts of utterance. Depending on the context, certain indexical expressions change their intensions, and this is represented by assigning intensions relative to times. For example, the indexical pronoun 'I' refers to the speaker of a sentence. In one context it would pick out one individual and its intension would follow that individual through possible times. In a different context it would pick out a different individual and have a different intension. As a result of the variability of the intension of the indexical pronoun, any atomic sentence using the pronouns, like 'I am wise', would likewise have an intension that varies with context. At one time when I refers to Socrates it would mean the same as 'Socrates is wise' and at other times when it referred to other individuals it would correspondingly have different intensions.

We may use a device of Robert Stalnaker to illustrate this situation and also to explain the relation of p to τp.[10] Let us stipulate what we may call the *semantic matrix* for p in which the columns represent the intensions of p relative to contexts of utterance:

t_3	T	T	F	
t_2	F	F	F	the intensions of p
t_1	T	F	T	relative to contexts
	t_1	t_2	t_3	
	contexts			

10 These remarks on two-dimensional indexical semantics break no new ground beyond relating the ideas to eternal sentences and to Boethius. For a full statement of Stalnaker's theory see his 'Context and possible worlds', typescript; an abbreviated version is given in his 'Indexical belief', *Synthese*, **49** (1981), 129–151. For the actuality operator and its semantics see David K. Lewis, 'Anselm and actuality', *Noûs*, **4** (1970), 175–188. *N.B.* Both $R(t,\tau)$ relative $A(t')$ and$R_t(p)$ relative to $A(t')$ are essentially two-dimensional and could equally well be symbolized $R_{t,t'}(\tau)$ and $R_{t,t'}(p)$.

Here p has three different intensions in three different contexts. Assuming this matrix for p, let us construct similiar matrices for τp for the three cases in which $R(t,\tau)$ is t_1, t_2, and t_3 respectively:

t_3	T	F	T
t_2	T	F	T
t_1	T	F	T
	t_1	t_2	t_3

τp, $R(t,\tau)=t_1$

t_3	F	F	F
t_2	F	F	F
t_1	F	F	F
	t_1	t_2	t_3

τp, $R(t,\tau)=t_2$

t_3	T	T	F
t_2	T	T	F
t_1	T	T	F
	t_1	t_2	t_3

τp, $R(t,\tau)=t_3$

Now, let us construct a matrix in which in each context t we place the intension from the matrix above in which $R(t,\tau)=t$, and let us assign the semantic analysis represented by this matrix to an operator A attaching to p:

t_3	T	F	F	
t_2	T	F	F	the intensions of Ap
t_1	T	F	F	relative to contexts
	t_1	t_2	t_3	

Intuitively, what Ap says in context t is that τp is true where $R(t,\tau)=t$. An ordinary language reading of Ap is 'Actually p' or 'Now p'.

For our purposes, this operator is relevant in two ways. First of all, notice that Ap is eternal in every context. Moreover, it is not implausible to ascribe to the God of the philosophers the ability to know eternal sentences that contain indexicals, and have their meanings fixed by context. After all, all terms have their meanings fixed by context if context is interpreted broadly enough. Certainly, MP4 rules out no such knowledge.

Secondly, in the manner of Stalnaker we may use the A operator to explain the relation of p to Ap. To do so, Stalnaker points out the importance of the diagonal from lower left to upper right in the matrix for p. It records the truth-values for p at times t in which the context of utterance is also t. Such situations in which the world being described is also the context of utterance may be singled out as having a special status. They are normal or standard in a way other combinations of context and world are not. Indeed, it is possible to define an operator / attaching to p such that no matter what the context t, the intension of /p at t is the diagonal of the semantic matrix for p. What we discover is that p and its eternalized version Ap are equivalent in the sense that /p and /Ap have the same intension in every context:

t_3	F	F	F	
t_2	F	F	F	the intensions of /p
t_1	T	T	T	and /Ap
	t_1	t_2	t_3	

Thus, in a very strong sense the 'standardized' versions of p and Ap are semantically equivalent.

The situation with regard to God is, then, as follows. Though God may not know that p at the various times t, he may know for each t, the sentence τp where $R(t,\tau)=t$, and he may summarize this knowledge by knowing Ap. Moreover, there is a strong sense in which p and Ap say the same thing, namely in their standardized forms they always mean the same.