DISCUSSION AND EXPOSITION

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SOME MISCONCEPTIONS IN THE CRITIQUE OF SEMANTIC PRESUPPOSITION

An attempt to refute recent critiques of semantic pressupposition. It is argued that the formal, semantic notion is correctly understood more narrowly than is current in linguistics, that its semantic study is fully compatible with pragmatic investigation and a large degree of contextual cancellation. Linguistic definitions of pressupposition are criticised, especially those in terms of entialment (a contraposition supporting logical implication) and necessitation, because they misapply the technical ideas. The success of the formal notion is appraised by giving a short summary of the development of many-valued pressuppostion theories (including supervaluations) and pointing out the virtues of the later theories.

I. The Problem

Recently the concept of semantic presupposition has been subjected to two lengthy and sophisticated critiques: Deidre Wilson, *Presupposition and Non-truth-conditional Semantics*; and Steven Boër and William Lycan, *The Myth of Semantic Presupposition*. Nobody has responded and some of their mistakes are now being repeated and having influence. Presupposition is a concept whose home is in logic. Frege had an incipient version, and it was first explicitly defined, if in a somewhat muddled way, by Strawson. It was then taken up by many-valued logic and formal semantics. Somewhat later it found its way into linguistics. It is my impression that many linguists use the term in a way intended to be consistent with formal work. They assume that they are studying the same phenomenon and that in principle their informal observations might well be backed up by formal theory. What I want to argue for in this paper is that the linguistic critique of semantic presupposition falls far short of disproving the power or success of its formal versions. In identifying the object of criticism with theories of formal presupposition, I am making a simplification not completly fair to Wilson, or Boër and Lycan. Their audiences

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were intended to be linguists, and to a large extent the opinions they attack are opinions of other linguists. But since the roots of presupposition theory lie in the logical tradition and since it is there that the most developed theories of presupposition exist, any serious critique of semantic presupposition must be understood as an attack on the formal idea. It is the developed theory, not its informal and sometimes sketchy versions, that must be appraised.

My thesis will be that though the critics have much of interest to say of a positive nature on pragmatic explanations of presupposition, they fail rather badly in their negative attack on the semantic idea. Among the particular points I make are the following. Both sets of critics elevate an obscure inference relation which they call entailment to a pivotal position in logic best filled by more conventional ideas. Both misdefine presuppostion. Boër and Lycan so misrepresent the definition that many of the arguments they use against presupposition are in fact traditional arguments for it. They also misuse the idea of necessitation. Wilson mistakenly argues that presupposition and logical entailment cannot overlap, and on the basis of this formal error argues for many false conclusions. Both sets of critics rather badly misunderstand the relation between formal semantics and pragmatics, and mistakenly assume that something given an empirical pragmatic explanation cannot profitably also be studied by abstract formal methods. Both also fail to appreciate the degree to which cancellation of presupposition is consistent with its abstract study.

The discussion begins with a fairly elementary presentation of the standard many-valued theories of presupposition. If these basic matters were more widely disseminated in linguistics many fewer mistakes about presupposition would be made. The presentation here is designed as an introduction, and except in the case of a few examples, concepts from formal logic are kept to a minimum.

II. Abstract Theory

1. Structures

In the ultimate sense what formal semantics tries to study is natural language. But at best it touches on real speech only indirectly. Its direct object of study consists of formal constructs called abstract structures, and the thought process that goes into defining these structures is called abstraction. Understanding structures and abstraction is crucial to appreciating the goals and methods of formal semantics, and I shall argue that much of the linguistic critique of presupposition theory misses its mark because of a failure to grasp these ideas.

Structures vary considerably but their building blocks are all the same: sets, relations and functions on these sets, and distinguished elements from these sets. Why a melange of such elements is called a structure may be explained by analogy with a blueprint. The sets and distinguished objects correspond to the list of materials, and the relations and functions to the building diagram. Just as a blueprint describes a structure so does its abstract cousin. Indeed, 'structure' in its most abstract sense is well captured in the formal notion. Traditionally, structures are ordered in n-tuples.

Definition:

> An *abstract structure* is any $\langle C_1, \ldots C_j; R_1, \ldots, R_k; f_1, \ldots, f_l; O_1, \ldots, O_m\rangle$ such that $0 \leqq j, k, l, m$, and each C_i is a chlass, each R_i is a relation on elements form these classes, each f_i is a function on elements of these classes, and each O_i is an object from these classes.

Ordinarily, structures are not studied singly but in groups. Indeed, an *abstract theory* may be defined as consisting of a definition of a particular set or family of structures and a list of logical consequences following from this definition. For concreteness theorists occasionally single out a particular representative of the family to talk about, but they are best understood to have in mind the whole class. In algebra, for example, the set of lattices is studied. These are any structures $\langle c, O_1, O_2\rangle$ such that c is closed O_1 and O_2, and

(1) $O_1(x,y) = O_1(y,x), O_2(x,y) = O_2(y,x)$

(2) $O_1(x, O_1(y,z)) = O_1(O_1(x,y),z)$
$O_2(x, O_2(y,z)) = O_2(O_2(x,y),z)$

(3) $O_1(x,x) = O_2(x,x) = x$

(4) $O_1(x, O_2(x,y)) = O_2(x, O_1(x,y)) = x$

Mathematicians study all sorts of structures (e.g. groups, rings, and Boolean algebras) and logicians define structures of their own to study language.

The investigation of a sort of structure depends first of all on the derivation of consequences from the definition. Such deductions constitute theories. Thus, the definition of lattice implies a connection between lattices and partial orderings:

(5) If $\langle c, O_1, O_2\rangle$ is a lattice then $\langle c, \leqq\rangle$ is a partial ordering where $x \leqq y$ iff $O_1(x,y) = y$ iff $O_2(x,y) = x$.

Such principles or rules hold of any structure in the set and can like other "laws" be used in "explanation". The subset relation for example can be shown to be a partial ordering because the structure made up of sets, intersection and union constitutes a lattice. A particular phenomenon is subsumed under a general law in a fashion not unlike empirical science. Again as in empirical science, the order of investigation tends to reverse the order of the eventual explanation. One has a particular structure and property in mind, and then seeks a class of structures embracing the particular case and exhibiting the relevant property. Thus, lattice theory was historically motivated in part by abstraction from operations on sets. If there were no more to abstraction than the seeking of wider sets of structures to explain the properties of an interesting case, then it would be fairly easy to understand. But unfortunately abstract theory is also used to explain empirical phenomena and then matters become very murky indeed.

2. Empirical Structures

Notice that the definition of abstract structure is broad enough to permit structures containing sets, relations etc. of empirical objects. In fact some of the best examples of successfully applied mathematics are explanations in terms of abstract structures. What happens roughly is that some empirically definable structure is explained to have a property because it meets the conditions for membership in a set of abstract structures that can be logically shown to have that property. Land surveying, for example, works according to geometric principles because the empirical space of points, lines, and planes qualifies for membership in the set of geometrical structures. In economics, to give another example, it is well known that price reflects supply and demand in structures called perfect markets. It is then argued that this or that empirically identifiable market meets the conditions for perfect competition. Even in these empirical explanations, the defining conditions on the set of structures are usually stated in mathematical language and the derivation of the set's properties are also formal. The novelty lies in the use of empirical terms to characterize the particular structure under investigation. Its identification and the evidence that it satisfies the membership conditions for the set of structures are both empirical matters, and, like most of the workings of empirical science, are best left vague. Certainly, the nature of empirical justification is a deep and open question. Consider, for example, what is involved in showing that terrestrial geography meets certain geometrical axioms, or that the British economy of 1870 was perfectly competitive. What is important here, however, is less the nature of verification than the use of abstraction from empirical data. As these cases show, generalization to sets of structures may be inspired equally by an empirical structure as a formal one. Unfortunately, exactly what is being abstracted from and what is being explained is often left quite inexact in such empirical abstraction.

3. Conditions of Abstraction

Empirical structures must be formed from the confusion of observable reality which often needs judicious suppression of detail to yield elegant structures. More precisely, empirical structures are often defined relative to certain standard or idealized conditions. Empirical planes form euclidean geometries, for example, only under conditions that they are small and flat. Under different conditions like those in navigation or astronomy, non-euclidian geometry may be more appropriate. Likewise market economics applies only under the conditions of perfect competition, an ideal state if there ever was one. In formal semantics the observable manifold is natural language, and only by rather severe limitation to special conditions does language yield neat structures. Grammar, for example, must ignore half-formed and misspoken expressions, and semantics must presume some ideal of standard use. Strictly speaking, the precise statement of the conditions of application should be part of the science. In economics the conditions for perfect competition have been studied in depth. But in practice formal semantics (and much of linguistics) ignores the statement of conditions and concentrates exclusively on the formal tasks of defining sets of structures and deducing their properties. The intended applications of theories are therefore sometimes not very clear. One of my major points will be that linguistic critics of presupposition exploit one such vagueness. Many of the sorts of presupposition they attack are not, nor could they be, within the empirical scope of formal accounts.

Abstraction from empirical data will also provide a framework for rejecting another assumption of the critics, the presumed rivalry between semantics and pragmatics. Much of what I have been describing as the empirical manifold underlying formal semantics is what others would call pragmatics. Strictly speaking, natural language as it appears in pragmatics is not undisciplined data but a sort of science itself, making use of its own concepts and rules. These derive largely from speech-act theory and recently from Grice's ideas on conversational implicature. But abstraction from pragmatic phenomena is in principle possible, and such is exactly the role Morris originally conceived for semantics.[1] Given that formal semantics is an abstraction from pragmatics, there is no rivalry between the two. I shall argue in particular that it is fallacious to reason, as the critics do again and again, that if something is explicable pragmatically, then there is no need for a semantic explanation.

III. Abstract Theory of Language

1. Syntax

The most basic part of language that formal semantics studies by means of structures is syntax. Syntax here differs from its form in linguistics only in its packaging. As in any abstract theory, a *set* of syntaxes is defined. Each syntax is a structure and, though they differ from theory to theory, each contains sets of expressions, variously called descriptive, lexical, or categorematic terms, which are intended to have no fixed meaning. In addition to these, each syntax contains a series of formation rules and a closely connected set of logical or syncategorematic terms. Each descriptive term may vary in its interpretation quite widely within a set of possible interpretations open to that set. Names, for example, may vary over objects, and sentences over truth-values. Logical terms on the other hand are intended to have a fixed sense and are paired one-one with the formation rules. Each formation rule consists of taking some input descriptive expressions and yielding a longer descriptive expression by combining its inputs with the logical term characteristic of the rule. The formation rule for conjunction, for example, forms a descriptive expression 'p & q' from two descriptive inputs 'p' and 'q', and one logical term '&'. The distinction between logical and descriptive terms is really quite hard to draw precisely and I shall return to it below. It shall underlie my claim that critics tend to misconstrue the intended application of presupposition theory: formal theories attempt to explain presuppositions rooted in logical terms, and not presuppositions dependent on the meanings of descriptive terms.

Syntax exhibits two features of all formal theories that are important to this discussion. First, it offers an excellent example of explanatory power, one of the criteria of evaluation used in formal theories generally. Normally a theory must produce consequences that match the data, and in a formal

[1] See Morris [32].

theory this means that the properties provable of the set of structures in general must match the observed properties of the empirical structure being explained. The property under investigation in syntax is well-formedness and the data being explained are our intuitions about well-formedness. A theory is adequate only if judgments of well-formedness derivable in the theory match these intuitions.

A second feature of abstract theory well illustrated by syntax is the difficulty of stating its conditions of application to natural language. Not every utterance is well-formed, and thus not every utterance is intended to be explained. Nor is every well-formed expression ever uttered. Thus, actual cases of utterance coincide very imperfectly with the predicted cases of well-formedness. The situation is well known, but hard to explain. Chomsky's ideas of competence and performance are designed to try. The situation is one in which it is very hard to formulate under what conditions a bit of natural language is supposed to be in the explanatory range of the theory. We shall meet similar difficult cases of presupposition.

2. Language

In addition to syntax, formal semantics also studies the concept of language by means of abstract structures. The notion of language used is that of an interpreted syntax, and a language is identified with a structure consisting of its syntax and possible interpretations. The sense of interpretation I shall use here is broad enough to embrace all presuppositional languages and is rather abstract.

Definition:

> An *uninterpreted formal language* L is any pair ⟨Syn, Val⟩ such that Syn is a syntax and Val (called the set of *admissible valuations* for L) is a set of functions mapping the sentences of Syn into truth-values

Val is intended to represent the set of all logically possible interpretations or worlds consistent with the intended reading of the logical terms of Syn. Thus, Val is a formal version of the philosopher's set of logically possible worlds. Each valuation records the truth-values of sentences in the "world" it represents. Often Val is defined by first defining a set of formal structures called *models* designed to capture more directly the idea of possible world, and then valuations are defined to represent the models.

Such a language is called 'uninterpreted' because it attaches no particular interpretation to its non-logical terms. The interpretation of descriptive terms requires a restriction on the set of logical possibilities in a manner to be discussed below.

Strictly speaking a theory always defines a *set* of languages, one of which is supposed to approximate at least a part of natural language. In practice, however, when only one class of languages is under discussion, one typical member is all that is mentioned, or the set is spoken of as a single language. When there is no possibility of confusion, I shall adopt this practice. Before considering any particular presuppositional language, I shall outline briefly what the goals are of an abstract theory of language and how it should be appraised.

3. Implication

Like any theory a formal language is evaluated by its explanatory power, but the first criterion of adequacy I would like to point out concerns not which consequences are produced, but how they are produced, and I call it *conceptual adequacy*. In addition to producing the right consequences, the production mechanism must be plausible. A theory is more than a consequence producing black box. It is a series of definitions and principles, and these internal workings of the theory must be convincing. This particular requirement falls heavily on a theory of language because ordinarily it provides a series of definitions that must take stands on concepts with a long history of controversy. In the process of defining Val such difficult concepts as 'predication', 'possible world', and 'truth' are defined, and by means of Val others are in turn defined like 'logical truth' and 'implication'. Such definitions must be adequate, and though some details are negotiable, the argument must take place in part on a conceptual plane. The student of empirical language must assume the role of a philosopher and argue with the tools and methods of analytic philosophy. It is not unusual for empirical theories to build upon philosophical foundations, and it is one of the tasks of the philosophy of science to point out when this happens. In formal semantics the intersection of philosophy and science occurs most clearly in its theoretical definitions. In presupposition theory one of the main debates is essentially philosophical and concerns the proper definition of truth and falsity, and their relation to negation.

Formal languages must in addition meet a second criterion of adequacy, explanatory power. I have already indicated how syntax explains linguistic intuitions about well-formedness. Among our linguistic intuitions there are also ones concerning the validity of inference, and the many familiar examples from logic with a strong "ring" of validity should be viewed as data to be explained. Formal language attempts an explanation by deducing particular cases of implication from one definition. In all theories I shall discuss this definition takes a common form. It depends only on the specification for a language L of a set D_L of *designated* truth-values, values which are thought to be preserved in reasoning from premise to conclusion. In three-valued theories, truth is always designated, falsity is never designated, and the third value is or is not depending on whether the theory understands logical inference to preserve truth or non-falsity.

Definition:

> A *logically implies* B in L, briefly A ||— B, iff for any $\vee$ in Val, if $\vee$ (A) is in D_L, then $\vee$ (B) is in D_L.

Definition:

> A is a *logical truth* in L iff for all $\vee$, $\vee$ (A) is in D_L.

A formal language may then be assessed according to how well predictions for implication that follow from these definitions conform to intuition.

It is quickly seen that not all cases of felt implication count as logical in any theory, nor are they intended to. Logical implication is only one of many varieties, and in presupposition theory there are about a dozen different implication relations that have been distinguished. I shall give examples of non-logical implications shortly, but first it is important for later discussion to see the natural division that there is supposed to be between logical implications and the rest. The distinction lies at the heart of the traditional conceptual framework of semantics, but like other conceptual issues it is difficult to explain outside a particular abstract theory. Recall the distinction sketched earlier between logical or syncategorematic terms, on the one hand, and descriptive, lexical, or categorematic terms, on the other. It is unfortunately rather difficult to give an explicit account of this distinction in a manner general enough to embrace all formal languages. Like Quine one could do so by giving a list of the logical terms, like the classical 'not', 'and' 'or', 'if-then', 'all' and 'some'. But any list is theory relative. Generally logical terms are distinguished as particles with a constant rule-like interpretation and are distinguished from the various types of descriptive terms with variable meanings. Natural language descriptive terms typically include nouns, verbs, adjectives, and various sorts of complex phrases including sentences. Descriptive terms of formal language include constants, predicates, and formulas. One property that may seem to characterize logical terms, if only in a vague way, is their rule-like interpretation. Somewhere in the conceptual framework leading to the definition of valuation, every language assigns a characteristic semantical rule to each logical term. These rules are always rather simple and stated purely in mathematical or set theoretic vocabulary. They explain how the logical term contributes to the meaning of longer expressions given the interpretations of its descriptive parts. This rule remains unchanged even though the interpretations of the descriptive parts may vary. A class of descriptive terms is generally limited in its interpretation to a class of entities appropriate to that term. Sentences, for example, refer to truth-values, names to objects, and predicates to properties. But any possible combination of terms with entities of the relevant sort constitutes an acceptable interpretation. Descriptive interpretations provide the "parameters" that need to be filled in the rule. It is in this sense that logical terms have a fixed meaning but descriptive terms do not. In the familiar recursive definitions of valuation, the semantical rules are just the various clauses of the definition. For example, in 'A & B' the logical term '&' is always interpreted by the truth-table for conjunctions whereas the descriptive sentences 'A' and 'B' may take on any possible truth-value. In classical quantification theory, the formula 'A' in '(x) A' may vary in the objects which satisfy it, but the quantifier is always interpreted by a rule something like: '(x) A' is true iff 'A' is satisfied by everything in the domain. The notion of logical or formal implication may now be loosely characterized. It is implication that depends on the form of sentence – the arrangement of logical terms and descriptive parts of speech – but is independent of any particular interpretation of the descriptive terms.

4. Analyticity

Not all inference, however, appears to be formal. Intuition condones some inferences that appear to depend on particular interpretations of descriptive terms. In any world in which (6) is true, so is (7):

(6) John is a bachelor.

(7) John is unmarried.

A married bachelor is a conceptual impossibility. Further, both 'bachelor' and 'married' seem to be descriptive terms. There are no obvious rules which given the interpretation of 'John' and the concepts of mathematics or set theory, would yield the truth-value of (6) or (7). Unfortunately, most terms seem to be descriptive in this sense. Relatively few terms contribute by formal regularity to the meaning of longer expressions. How then is a concept of implication broad enough to capture that from (6) to (7) to be defined within the framework of a formal language?

There is a method for defining non-logical inference and it is particularly interesting because of what it shows about abstraction. Its rationale goes as follows. The existence of non-logical inference shows that in some sense the notion of possible world in an uninterpreted language is too generous. There is some logically possible world in which the extension of 'bachelor' is not a subset of that of 'unmarried' and in its valuation (6) would be true but (7) false. Such an interpretation is just one of the possible combinations of predicates with entities. But clearly no such interpretation is compatible with the usual meanings of the terms. Thus, the set Val should be restricted so as not to violate any conceptual inference. Though we may not know in detail what the conceptual inferences of a language are nor the precise subset of Val that is conceptually possible, we may use the fact that assigning meanings to descriptive terms does restrict Val to characterize the notion of a language with both logical and non-logical terms having particular senses.

Definition:

> An *interpreted language* L* relative to an uninterpreted language L = $\langle$Syn, Val$\rangle$ is any $\langle$Syn, Val*$\rangle$ such that Val* is a subset of Val.

Each L* has its own implication relation.

Definition:

> A *analytically implies* B in L* relative to L, briefly $A \Vdash^{*} B$, iff for all $\vee$ in Val*, if $\vee$ (A) is in D_L, then $\vee$ (B) is in D_L.

Definition:

> A is an *analytic truth* in L* relative to L iff for all $\vee$ in Val*, $\vee$ (A) is in D_L.

Though we may not know which L* comes closest to a natural language like English, we can be sure that some approximation of English is there. This knowledge will not satisfy those seeking semantical analysis of descriptive terms, but it is sufficient for studying the phenomenon of analytic implication in general. The student of language as such is not so much interested in what is peculiar about English or German as he is in the general properties common to all languages, and the abstract definition of analytic implication allows logicians to investigate one such property. Thus, the analytic implications of English are explained in the peculiar way characteristic of formal abstraction. In doing so, the theory does not predict the particular analytic implications of English. It rather predicts that there will be a language with the analytic implications of English. It differs from the abstraction to uninterpreted languages in that it predicts not that this or that argument will be universally valid in every language, but only that each language has its own relation of universal validity. We know independently that English is among these. The situation is typical of abstraction. A particular structure is identified, in this case English. Then a set of structures is defined that assumes the particular structure has the property under investigation, in this case analyticity. Note that it is not the job of the formal theory as such to identify the particular structure. As explained above, such particular identification is the task of empirical science. The formal method rather presupposes a study of empirical meaning. In practice, of course, abstraction may proceed before many of the details of the empirical account are completed. But it is not the job, nor is it the intention, of logicians to elucidate the meanings of descriptive terms. Having said as much, however, I would like to point out how results from such empirical study can be added to the formal theory to provide a characterization of a particular natural language like English.

The particular set of conceptual possibilities Val* of an interpreted language may be singled out by the sentences they render analytically true. It is a technical truth that on all normal definitions of valuation, there is a one to one correspondence between subsets of Val and sets of sentences. Corresponding to Val* is the set of sentences A such that A is true in every member of Val*. Such sentences are said to *axiomatize* Val* and may be called *meaning postulates*.[2] For example, let Val* be the set of all elements in Val that assign true to (8).

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2 The idea that some non-logical truths may axiomatize a set of consequences and in turn correspond to a restricted set of possible worlds is captured in formal logic by the concepts of axiomatizable theory and the set of models which satisfy it. Carnap first introduces meaning postulates to formal semantics in [5] where they are conceived as axioms added to the logical axioms so as to extend the set of logical truths to a wider set of analytic truths. In different ways, van Fraassen [42], [43] and Montague [31] study how to relativize the notion of interpreted language to a subset of logically possible worlds. The

(8) All bachelors are unmarried.

Then (8) is a meaning postulate for L* and in L* the inference from (6) to (7) is analytically valid.

Meaning postulates, though seldom formulated in formal semantics, are much like dictionary entries and are very closely linked to the linguists' conception of semantic theory. They may in fact be looked upon as formal versions of rules assigning "semantic markers" to descriptive terms. Their relative unimportance in formal work shows a major difference in emphasis between linguistics and logic.

5. Paraphrase and Entailment

Indeed, Deidre Wilson and other linguistic critics tend to emphasize the search for empirical meaning and synonymous paraphrase so much that they manage only a superficial theory of implication. No difference is more striking in the approaches to semantics of linguistics and logicians. In a desire to explain intuition about inference, logicians require a rigorous definition of implication. Without a clear definition no instance of intuition can be clearly tested against the theory. Wilson clearly intends that the analysis of implication should depend on the theory of paraphrase. The idea is roughly that A implies B if B is part of the paraphrase of A. But there is no serious attempt to actually define the dependence of implication on paraphrase. On a sympathetic reading, however, this vagueness does not seem to present a problem in principle. As meaning postulates show, a theory of synonymous paraphrase is pefectly compatible with the rigorous definition of logical and analytic implication. The best way to fill out Wilson's semantics is propably by presupposing some version of the abstract theory of language common to formal semantics.

Curiously Wilson and others provide in addition to the theory of paraphrase another and rather problematic analysis of implication.[3] According

discussion here essentially follows Montague for simplicity. Van Fraassen's theory, however, employs the concepts of logical space and is a considerably richer abstraction from natural language meaning relations.

3 The *zeitgeist* is evidently ready for this concept of entailment, because having never been mentioned before in many-valued semantics, it appears almost simultaneously in Wilson [47], p. 4; Kempson [17], p. 48 and [18], p. 142; and Boër and Lycan [3], pp. 10, 11. The historical root of the idea seems to be found in Strawson [36], pp. 20–21, 175. Strawson argues that since B may be false while A is neither true nor false, A cannot entail B. He therefore calls the suggested implication from A to B presupposition rather

to this second account, there is a particular form that must be fit by any implication relation that is to be a serious attempt to explain intuitions about inference. The envisioned relation goes under the technical name of *entailment* and has the following rather informal definition:

Definition:

A *entails* B iff whenever A is true, so is B, and whenever B is false, so is A.

To be testable against intuition such a definition must be much more precise, and though the linguists do not themselves attempt any refinement, the necessary resources are clearly available in abstract semantics:

Definition:

A *entails* B in L = ⟨Syn, Val⟩ iff for any ∨ in Val,

(1) if ∨ (A) = T, then ∨ (B) = T,

(2) if ∨ (B) = F, then ∨ (A) = F.

I shall call entailment *logical* if L is an uninterpreted language, and *analytic* if L is an interpreted language. Though Wilson and others who use entailment are not terribly clear, what they seem to maintain is:

(i) Any relation sufficient to capture all intuitive cases of formal implication will be a variety of logical entailment.

(ii) Any relation sufficient to capture all intuitive cases of implications depending on the meanings of descriptive as well as logical terms will be a variety of analytic entailment.

I shall argue here that neither of these claims is particularly convincing and that entailment is really much too restrictive to embrace all serious candidates for logical or analytic implication.

What is plausible about entailment is that in the special case of classical logic, implication and it coincide:

(9) If L is a classical language, then A entails B in L if A ||— B.

than entailment. But Strawson is not being very rigorous. He certainly should not be understood to mean that in any acceptable semantics the *analysans* of logical implication will support conversion. See van Frasssen [40], p. 154 for a discussion of the clarity of Strawson's usage in the relevant passage.

Thus, entailment is a possible candidate for generalization to all serious accounts of implication. In addition by clause (2) of its definition, entailment assures that the principle of conversion familiar from classical logic is preserved. If $\sim$ is any normal negation, then

(10) A entails B iff $\sim$B entails $\sim$A.

But merely including classical implication as a special case and preserving conversion is not enough to justify thinking that all adequate accounts of implication will also be entailments. The most important criterion of adequacy for a definition of implication is whether it accounts for all intuitive cases of valid argument. In practice this criterion amounts to the preservation of all classically valid arguments and the admission of additional validities only on sound intuitive grounds. According to this more basic criterion the more successful 3-valued theories fare quite well. But all 3-valued theories reject conversion, and none of their implication relations $||\!\!—$ are entailments. Conversion turns out to be a relatively minor feature of classical logic whose plausibility diminishes with new notions of negation and implication. It has always been quite self-consciously one of the first classical properties dropped in 3-valued semantics. It is therefore poor metatheory to require without argument that any acceptable language will preserve conversion. The proper general approach is to define implication in a broad enough way to embrace 3-valued as well as classical theories, and then to argue the advantage of each according to the fundamental criteria of adequacy. Such an all-inclusive concept of inference is already provided in the definition of $||\!\!—$; this is the common metatheoretic concept of inference used in formal semantics, and the imposition of entailment by its linguistic critics is both question-begging and parochial. Rejecting entailment as a core explanatory idea amounts, however, to rejecting one of the major novel features of Wilson's theory. The detailed demonstration of exactly how question-begging the approach is must await the review below of the major 3-valued semantics. First, however, it is necessary to discuss a further variety of implication that is a subspecies of its logical and analytic cousins, presupposition.

6. Presupposition

Let me begin with examples. Though many cases of presupposition have been mooted by linguists, it is striking that formal work has concentrated on four types: existential and sortal presupposition, and to a lesser extent non-paradoxical and factive presupposition. To my mind the burgeoning of presuppositional varieties among linguists and their very tight restriction among logicians requires an explanation. I shall argue that only a restricted type of presupposition is amenable to formal treatment and that much of the critique of presupposition is frustrated when applied to its formal kinds.

By non-paradoxical presupposition I mean the sort attributed to self-referring expressions in presuppositional explanations of the semantic paradoxes. The topic is technical and rather far from the interests of linguists, so I shall not discuss it further here. It should not be forgotten, however, that both historically and theoretically it is one of the major underpinnings of presupposition theory. No appraisal of presupposition can be complete without a review of its successes in the area. Examples of the other sorts of presupposition are familiar.

Existential Presupposition

(11) John is bald
(12) John is not bald
imply
(13) John exists

Sortal Presupposition

(14) John is red
(15) John is not red
imply
(16) John is colored

Factive Presupposition

(17) John discovered that the earth is round
(18) John did not discover that the earth is round
imply
(19) The earth is round

An adequate formal theory will provide a concept or concepts of implication that will account for such data. All the formal accounts of presupposition define the term in a broadly similar way.

Definition:

A presupposes B iff both A and not-A imply B.

Theories however differ radically on how to unpack this *definiens*, especially in their analyses of negation and implication. To show the range of possibilities, I shall now briefly sketch the major varieties of formal presupposition theories.

IV. Review of Standard Many-valued Theories of Presupposition

1. Classical Theory and *Exponibilia*

One of the earliest and still prevalent theories of presupposition analyzes the concept within classical logic. In presupposition theory *classical logic* is used to refer to any of a number of formal languages that contain the usual propositional connectives in the syntax and in which every acceptable valuation assigns truth-values to longer expressions in accordance with the classical truth-tables.

	~	∧	T	F	∨	T	F	→	T	F
T	F		T	F		T	T		T	F
F	T		F	F		T	F		T	T

The classical truth-tables

Any effort to define presupposition for a classical language faces an immediate and well-known problem. If negation is as above, and implication is logical implication (or even analytic implication) for classical valuations, then presupposition is trivialized. Only logical (or analytic) truths can be presupposed since it is only these that are implied by both a sentence and its negation. The usual classical way around this problem is to posit a second sense of negation as the one intended in the definition of presupposition. The new negation must be such that it is not always the case that A or its negation is true, but that they nevertheless cannot be both true together. A and its new negation are to be contraries rather than contradictories. Further they must both imply some non-trivial sentences in common.

These goals are reached by reading sentences which carry presupposition as logically complex and replacing them in formal languages by their complex rendering. It is the application of the more general medieval concept of *exponibles* and is similar to linguistic theories that translate short surface forms into logically complex "logical forms"[4]. According to this method

[4] For a discussion of *exponiblia* and Russell's theory of description as an example see Herzberger [11].

many natural language sentences which look simple are actually a conjunction of different assertions. For example, the deceptively simple (20) gives way to its logically equivalent form (21).

(20) John is a green giant.

(21) John is green and John is a giant.

The method is applied to sentences carrying presupposition by listing the presupposition as one of the conjuncts in its translation. Further, presuppositional negation is analyzed as attaching not to the presupposed conjunct but only to the others. Thus natural language assertions A and not-A are rendered by formal language conjunctions $B \wedge C$ and $\sim B \wedge C$. Here C is the presupposition and B is what is being asserted or denied, but not presupposed. Further, the relevant concept of implication is logical implication because in classical logic $B \wedge C \Vdash C$ and $\sim B \wedge C \Vdash C$. A typical treatment of sortal presupposition would be to translate (22) by (23), and (24) by (25). Then (22) presupposes (26) because both its translation and that of its negation (24) logically imply (26).

(22) John is a bachelor.

(23) John is single $\wedge$ John is a man.

(24) John is not a bachelor.

(25) $\sim$ (John is single) $\wedge$ John is a man.

(26) John is a man.

The most famous example of this sort of theory is Russell's account of definite descriptions. In Russell's terminology negations in natural language which are put on the outside of their formal rendering are said to be in *primary occurrence* whereas those translated to the inside are said to be in *secondary occurrence*. (Some writers refer to these as *external* and *internal* negations, but these terms are best reserved for varieties of 3-valued negations that they were originally intended to describe. In casual discussion the restriction would be unnecessary because of vague similarities in classical and 3-valued approaches. But in precise discussions mixing terminology causes confusion). Natural language negation is then ambiguous between negations in primary and secondary occurrence. In classical theories, thus, it is negation in the latter sense that should be understood in the definition of presupposition, and the relevant sense of implication is logical implication.

Non-classical accounts have been developed because of two grave defects in the classical theory. The first of these is syntactic. All things being equal, surface form should correspond as closely as possible to logical form. It is after all the surface form that is being explained, and it is this that if possible should be predicted. Exponible sentences however frequently require such extensive rewriting of surface form that their syntax becomes implausible. To use a case based on Thomason, it is unconvincing to construe (27) which looks like a simple identity statement as the very complex (28).[5] A theory which rendered (27) as a simple identity statement would be better.

(27) The king of France is the king of France.

(28) There is some x such that x is the king of France, and there is some y such that y is the king of France, and x is the same as y.

Non-classical theories accordingly do not employ exponibilia.

A second problem with the classical account concerns the conceptual framework of its semantics. There seem to be attractive conceptual reasons for abandoning a bivalent semantics and introducing a category of sentences that are neither true nor false. In 3-valued theories, presupposition can be given a new and appealing characterization free from the theory of negation:

(29) A presupposes B iff whenever A is true or false, B is true.

Let us turn then to the 3-valued accounts and begin by outlining the case for a third value.

2. The Third Value

Among philosophers and logicians who learn classical logic in school there is some reluctance to accept a third value. It is therefore important to make very clear exactly what formal theorists think they are accomplishing by its introduction.

I would like first of all to mention and dismiss a rather fatuous argument for bivalence that I frequently encounter. It goes as follows. Every sentence is either true or not true. By 'false' is meant 'not true'. Therefore, there are only two truth-values. But this argument misses the point. Granted every sentence is either true or not true. It is however far from clear what

5 Thomason [37].

we mean by 'false'. Tri-valent semantics is in fact built upon conceptual considerations that reject the classical identification of falsity with non-truth. Its claims may be broken down into two:

(I) There is a distinction to be drawn between two broad classes of non-true sentences that is both intuitively recognizable and theoretically productive.

(II) There are equally weighty conceptual reasons for restricting application of 'false' to one of these subcategories of non-truths as there are for bivalence.

The argument for (I) is first of all grounded in data. There is an intuitively recognizable difference that separates at least core cases of presupposition failure, as in existential cases like (30) or sortal cases like (31), from other non-truths.

(30) The present king of France is bald. (Russell)

(31) Einstein's most important discovery supports combustion. (Thomason)

Even the critics of presupposition admit this intuitive data for existential and sortal cases. The first challenge taken up by trivalent semantics is to provide an explanation of this intuitive difference among non-truths. The theories typically are rather complex in their explanatory range. They not only show the difference in truth-conditions between ordinarily non-true sentences and presupposition failures, they also attempt to define presupposition relations that can explain certain kinds of intuition about implication. Particular theories will stand or fall in part on their success at explaining the intuitions, but the important point here is motivational. The intuitive difference in non-truths needs explanation.

Note that satisfaction of (I) alone is adequate justification for tri-valent semantics. In its most abstract form truth-value assignments are just ways of classifying sentences, and need not be committed to any particular traditional understanding of these classes. In particular, it need not be committed to a non-classical position on the nature of falsity, and those who find reasons for (II) unconvincing would in (I) still find justification for non-classical semantics.

To justify (II) let me start by posing the question: how is it decided what 'falsity' means? The standard of ordinary usage useful in easier cases of analysis does not help here. The sorts of cases that might decide the issue just do not arise in ordinary conversation, and we therefore do not have any occasion to practice our usage of 'false' or to develop relevant intuitions. We seldom, for example, ever encounter sentences like (30) or (31). There is no standard ordinary way of dealing with presupposition failures. Their proper classification is rather a theoretical question posed by students of language, and the ordinary man has no worked out views on the subject.

If ordinary usage will not provide the standard, perhaps theoretical usage will. Is there any established theoretical usage of 'false' in which it always means 'non-true'? No. In most linguistic and philosophical theories of language the definitions of 'true' and 'false' are too vague to resolve the issue, and among theorists who take explicit sides, there are long traditions both for and against bivalence. Serious students of the third value include Aristotle, Buridan, Hegel, Frege, Peirce, and literally hundreds of investigators in the twentieth century. Thus tri-valence is not in violation of theoretical usage and is even less a simple conceptual confusion. Certainly restricting the range of falsity to a category of non-truth has strong conceptual plausibility, and the worth of tri-valence is not to be established by looking deeper into the proper definition of falsity. It will be decided by other standards of adequacy that prove more decisive. According to one of these, the explanation of implication, classical logic has been more successful than the simplest 3-valued theories. Let me turn then to the elementary tri-valent accounts of presupposition to illustrate some of these weaknesses and the greater strength of their refinements.

3. The Strong and Weak Matrices

The simplest attempts to abstract a 3-valued semantics from classical logic all interpret the propositional connectives by truth-functions on three values.

Definition:

A *matrix language* L is any uninterpreted language $\langle$Syn, Val$\rangle$ such that

(1) Syn contains some propositional connectives as logical terms (e.g. $\sim$, $\wedge$, $\vee$, $\rightarrow$), and

(2) the value under any valuation in Val for a sentence formed by a connective is calculated from the values of its immediate sentential parts by application of a truth-function characteristic of that connective.

As in other uninterpreted languages valuations are often defined relative to a prior notion of model. Clearly any classical language qualifies as a matrix language. The number of 3-valued matrix languages that have been proposed as serious accounts of presupposition has actually been very limited. They use one of two sets of truth-functions: the internal (Bochvar) or weak (Kleene) matrix, or the strong (Kleene) matrix.[6]

	~	∧	T F N	∨	T F N	→	T F N
T	F		T F N		T T N		T F N
F	T		F F N		T F N		T T N
N	N		N N N		N N N		N N N

The internal or weak matrix

	~	∧	T F N	∨	T F N	→	T F N
T	F		T F N		T T T		T F N
F	T		F F F		T F N		T T T
N	N		N F N		T N N		T N N

The strong matrix

Consideration of projections has been limited to these two types because of an adequacy criterion that any many-valued semantics must meet. Its truth-tables must give convincing interpretation of the connectives. This requirement may be viewed as a desire to capture the ordinary usage of natural language words like 'not', 'and' and 'or'. It is also in part philosophical because the truth-tables make use of such conceptually sensitive notions as truth and falsity. It is also partly logical and clearly connected to the explanation of intuitions on implication because the logical properties of the connectives are closely tied to their truth-tables. But regardless of how this requirement is classified it is hard to meet. Fairly plausible motivation can be given for the weak and strong connectives, and extensive accounts of this motivation are to be found in the literature. Since it is not the truth-tables themselves that are at issue in the linguistic critique of presupposition, I shall not justify them here. Suffice it to say that the weak connectives view meaninglessness, represented by the third-value, as a creeping disease or infection. When it smites part of an expression, it smites the whole. The strong connectives adopt the

6 For general accounts of matrix semantics see Rescher [33], and van Fraassen [40], [41]. On the weak or internal matrix see Kleene [19] and Bochvar [4]. On the strong matrix see Kleene [20] and Łukasiewicz [24]. Łukasiewicz' matrix is like Kleene's except that T and T in the conditional yield T rather than N.

same view but with a proviso. If a classical truth-value of a part was sufficient to determine the truth-value of the whole in 2-valued logic, it should continue to determine it in 3-valued logic. A disjunction with a true disjunct will be true, a conjunction with a false conjunct false, etc.

For concreteness I shall now provide an example of a simple presuppositional language using these matrices. It captures a minimal existential presupposition.

Example:

Let a *minimal weak (strong) matrix language* be any ⟨Syn, Val⟩ meeting these conditions. Syn must be a language with atomic subject-predicate sentences and longer expressions built up in the usual way from logical terms $\sim$, $\wedge$, $\vee$, and $\rightarrow$. It also contains a further logical term *exists*, the existence predicate, which combines with a subject to yield an atomic sentence. A *model* for Syn is any pair ⟨D, R⟩ such that

(1) D is a set (the domain),

(2) R is a function on *some* subjects and *all* predicates such that
(a) for any subject S, $R(S)$ if defined is in D,
(b) for any predicate P, $R(P)$ is a subset of D,
(c) R (*exists*) = D.

The set Val for Syn consists of all valuations $\vee$ such that for some model ⟨D, R⟩ of Syn, $\vee$ maps sentences into $\{T, F, N\}$ and

(1) for any atomic sentence *S is P*, $\vee$ (*S is P*) is T if R(S) is in R(P), $\vee$ (*S is P*) is F if $R(S)$ is in D but not in $R(P)$, $\vee$(*S is P*) is N otherwise,

(2) for any molecular sentences $\sim A$, $A \wedge B$, $A \vee B$, or $A \rightarrow B$, the value of $\vee(\sim A)$, $\vee(A \wedge B)$, $\vee(A \vee B)$, and $\vee(A \rightarrow B)$ are calculated from $\vee(A)$ and $\vee(B)$ by the weak (strong) truth-tables.

Syntactically this language gives an unambiguous and plausible account of a small part of natural language, and conceptually the introduction of the third value and the truth-tables can be motivated as sketched above. Questions of appraisal for such languages tend to center rather on their explanation of implication. Let presupposition be defined by $\sim$ and $||\!\!-$, where $D_L = \{T\}$. It follows straight from the definitions that *S is P* presupposes *S exists* since both of the following are theorems:

(32) *S is P* $||\!\!-$ *S exists*

(33) $\sim (S\ is\ P) \mid\mid\!\!— S\ exists$

In these languages presupposition is a subrelation of formal implication. But though capturing some presuppositional inferences, languages based on the weak or strong connectives do less well than classical logic in accounting for ordinary logical inference. These discrepancies are only apparent when the semantics including logical implication are fully and rigorously defined. They are well known and I will only mention a few here:

> In a weak language with just T designated, there are no logical truths whatever, and any inference with a new atomic sentence in the conclusion is invalid. Likewise the strong matrix with T designated invalidates some perfectly intuitive classical inferences, e.g. A does not imply $(A \wedge B) \vee (A \wedge \sim B)$.

In the formal tradition the logical quirks of a 3-valued matrix language have been their most noticeable feature and have provided the focus of evaluative debate. It is striking that the linguistic critics of presupposition theory do not even mention the logical hurdles facing any tri-valent semantics. Even more striking is the fact that they do not require of their own positive semantic theories a precise definition of logical implication. This sloppiness leads to a number of errors in both their positive and negative claims as I shall point out below. But what is important from the viewpoint of understanding the logical tradition, is that this disregard for the analysis of implication demonstrates a failure to appreciate what is probably the central issue in formal presupposition theory. It is precisely to improve the analysis of implication that newer and subtler 3-valued semantics have been developed. By ignoring the central role of logical implication the linguists reveal at once that they do not understand what formal theories are trying to explain and that their own positive theories do not even try to rigorously predict logical intuitions. I shall return later to the inadequacies of these linguists' account of logic, but first we must review the main attempts in 3-valued semantics to capture a logical implication more like that of classical logic.

4. The Horizontal and the External Matrix

One class of theories is based on the introduction to the syntax of a truth operator called the *horizontal* after Frege's operator of a similar nature.

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7 See Rescher [33], van Fraassen [41], and Martin [26], [29].

	h
T	T
F	F
N	F

The horizontal

Bochvar first combined this operator with the weak connectives, and others later combined it with the strong connectives, to define a new set of truth-functions available for interpretation. Let $\neg x = \sim h(x)$, $x \mathbin{\hat{\wedge}} y = h(x) \wedge h(y)$, $x \mathbin{\hat{\vee}} y = h(x) \vee h(y)$, and $x \Rightarrow y = h(x) \rightarrow h(y)$. The resulting truth-functions (weak and strong connectives yield the same result) is called the external matrix.[8]

	$\neg$	$\hat{\wedge}$	T	F	N	$\hat{\vee}$	T	F	N	$\Rightarrow$	T	F	N
T	F		T	F	F		T	T	T		T	F	F
F	T		F	F	F		T	F	F		T	T	T
N	T		F	F	F		T	F	F		T	T	T

The external matrix

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8 For Frege's horizontal operator see his [6]. The classical paper on the internal and external connectives is Bochvar [4]. The theory is tied to presupposition in Smiley [34]. Recent extensions of presupposition theory employing the truth-operator are van Fraassen [46], Herzberger [13], Åqvist [1], Humberstone and Bell [14], Woodruff [48], Bergmann [2], and Martin [27].
Humberstone and Bell building on the work of Åqvist define presupposition in the usual way by internal negation, but replace the usual logical implication with material implication. The result is called *material presupposition*. Woodruff's treatment is similar.
Wilson outlines what she thinks are problems for the addition of a truth-operator into a presuppositional language [47], pp. 16, 53, 54, 58, 92. She puzzles about how *S is P* could presuppose *S exists* when *It is true that S is P* does not presuppose *S exists* and both *S is P* and *It is true that S is P* are in some sense equivalent. These questions are essentially logical being matters of formal implication. They are addressed in detail in the logical literature. It is striking that Wilson's discussion makes no reference to this work.
The internal and external connectives are only cursorially discussed if at all, in the linguistic literature. Usually what little is said is well informed as in Karttunen [14]. Occasionally there are mistakes. Kempson [18], for example, without argument admits as the only two possible many-valued projections for presuppositional languages the internal and external matrices. She does not even mention the strong matrix or supervaluations which are projections more favored in logical work on presupposition. Nor does she seem aware that the point of external connectives is that when present in a language with other connectives like the internal ones, they provide a sense in which classical tautologies are preserved. No serious presuppositional semantics has ever argued that the only sense of natural language conjunctions is that of the external connectives. The reason is that no fully external sentence can presuppose anything: they are all bivalent.

Any propositional argument valid in classical logic remains valid when interpreted by the external matrix (T designated), and the converse also holds. Thus, if a propositional syntax were extended to include logical signs $\urcorner$,$\!$ $\curlywedge$, $\curlyvee$, $\Rightarrow$ for the new senses of negation, conjunction, disjunction, and conditionalization, then there would be vocabulary in the syntax for expressing all the classically valid inferences. But even this system has its quirks:[9]

> Natural language connectives are ambiguous, the theory provides no guidelines on proper translation, and thus its prediction of the logical data is incomplete. The theory also suffers from an embarrassment of riches in the form of strange new sentences formed in the formal language by mixing the two sorts of connectives.

Such hybrids seems to lack ordinary language counterparts. Problems such as these are common to matrix theories and have led to the development by van Fraassen of a new sort of 3-valued semantics. By abandoning truth-functionality but retaining the motivation of matrix theories, his theory of supervaluation is able to retain all classically valid arguments.

V. Superlanguages

1. Basic Concepts

The idea underlying supervaluations is based on a novel representation of possible worlds by valuations.[10] In classical semantics every possible world is represented by its unique 2-valued valuation, and in the matrix theories every world is represented by its unique 3-valued valuation. Superlanguages keep the 3-valued idea that possible worlds are in some respects indeterminate but they manage to represent this indeterminateness by bivalence. They do so by breaking the tie of one world, one valuation, representing an indeterminate world instead by a set of 2-valued valuations. The semantics typically proceeds in four stages.

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9 See especially Herzberger [13] and Martin [27].

10 Supervaluations are due to van Fraassen. They are generated from models in [45], and characterized in terms of necessitation in [44] and more fully in [40]. In [8] and [9] Herzberger presents the most definitive metatheoretic statement of superlanguages to date. Among other things, he introduces the idea of a base used here and shows that it yields an equivalent notion of superlanguage to those defined by necessitation.

Stage 1

A class of indeterminate models is defined representing indeterminate worlds.

Stage 2

To each indeterminate model is paired a set of determinate models, the classical models that resolve its indeterminacies in various possible ways.

Stage 3

Each determinate model is given a unique classical valuation, and thus each indeterminate model is paired with a set of classical valuations, those representing its classical resolutions.

Stage 4

Each set of classical valuations yields a unique 3-valued supervaluation recording only the unanimous judgments of the classical valuations. Thus each indeterminate model is finally represented by a 3-valued valuation, the one determined by its set of classical valuations.

A supervaluation is three-valued in the sense that it assigns T to a sentence just in case all the classical valuations generating it do, assigns F if all its classical valuations do, and is undefined otherwise.

As with matrix languages much has been written motivating this projection. Superlanguages may I think be motivated best as an elaboration of the ideas behind the strong matrix. Similarity to the strong matrix is illustrated by the non-functional truth-tables that all supervaluations conform to.[11]

	$\sim$	$\wedge$	T F N	$\vee$	T F N	$\rightarrow$	T F N
T	F		T F N		T T T		T F N
F	T		F F F		T F N		T T T
N	N		N N N, F		T N N, T		T N N, T

Projection of Supervaluations

Both superlanguages and the strong connectives work by first resolving indeterminate values to T or F. Then the classical truth-table is applied to the

11 For a precise characterization of superlanguages in terms of matrices see Martin [26].

determinate values. If all ways of resolution produce the same answer for the whole, the whole sentence receives this determinate value, otherwise the whole is indeterminate. The two projections differ only in the cases when both parts are indeterminate. By abandoning truth-functionality, supervaluations allow some such cases to be determinate as in $A \vee \sim A$, $A \wedge \sim A$, and $A \rightarrow A$ even when A is indeterminate. The result succeeds in retaining all classically valid inferences. Conceptually the most interesting part of the theory is that it posits two senses of possibility and two levels of thinking. In one sense possible worlds are indeterminate. But logical thinking appears to require resolution of these worlds into classically determinate ones. We seem to reason about 3-valued worlds in terms of 2-valued idealizations.

Superlanguages differ greatly in the way they generate sets of bivalent valuations representing indeterminate worlds. Concepts of model vary and are sometimes even dispensed with. It is however possible to give a precise and elegant statement of the theory by abstracting from the particular way the sets of determinate valuations are defined. The statement will also abstract from the particular nature of determinate logic, though in all applications to presuppositions this logic is perfectly classical.

Definition:

> Let a *base relative to a (classical) uninterpreted language* $\langle Syn, Val \rangle$ be any family $\mathscr{B}$ of subsets of Val.

In this definition, $\langle \mathrm{Syn}, \mathrm{Val} \rangle$ is to be understood as the classical language whose logical implication relation (T designated) is to be preserved. Elements of $\mathscr{B}$ are the sets of classical valuations representing indeterminate worlds.

Definition:

> *A superlanguage relative to base* $\mathscr{B}$ *for uninterpreted language* $\langle \mathrm{Syn}, \mathrm{Val} \rangle$ is any $\langle \mathrm{Syn}, \mathrm{Sval} \rangle$ such that Sval is the set of all intersections of elements of $\mathscr{B}$.

Thus, S is in Sval iff, for some $B \in \mathscr{B}$, $S = \cap B$ and for any sentence A, either S agrees with every valuation in B or the value $S(A)$ is undefined.

Notice that when a superlanguage is defined there are two concepts of logical implication. Let $\langle \mathrm{Syn}, \mathrm{Sval} \rangle$ be a superlanguage relative to $\mathscr{B}$ and classical language $\langle \mathrm{Syn}, \mathrm{Val} \rangle$. Then there is the classical entailment relation of $\langle \mathrm{Syn}, \mathrm{Val} \rangle$, which I shall indicate by $\Vert\!\!-$, and the relation of the superlanguage $\langle \mathrm{Syn}, \mathrm{Sval} \rangle$ which I shall call $\Vert\!|\!\!-$. In all applications of superlanguage only T is designated, and it follows straight from this minimal theory that superlanguages preserve classical logic:

(34) If A ||— B, then A |||— B.

The great flexibility of superlanguages lies in the fact that while preserving classical logic, they can also capture presuppositional implications. Let me give as an example a minimal theory of existential presupposition. The classical language presupposed here must be what is called a classically *free* logic. That is, it allows assertions of existence to be contingent non-logical truths. Without this assumption classical logic would trivially capture all existential presuppositions because presuppositions would be trivially true.

Example:

Let a *minimal superlanguage* be any ⟨Syn, Sval⟩ relative to $\mathscr{B}$ an ⟨Syn, Val⟩ meeting these conditions. Let Syn be the subject-predicate syntax in the example above.

Stage 1.

Let a *partial model* for Syn be any pair ⟨D, R⟩ such that

(1) D is a set (the domain),

(2) R is a function on *some* subjects and *all* predicates such that

(a) for any subject S, if R(S) is defined it is in D,
(b) for any predicate P, R(P) is a set (possibly overlapping D),
(c) R (*exists*) = D.

Stage 2.

Let a *determinate extension* of a partial model ⟨D, R⟩ for Syn be any ⟨D, R′⟩ such that R′ is defined for all subjects and predicates, and R is a subset of R′. (Note that R′(S) is allowed to be outside D).

Stage 3.

A *classical valuation* over a determinate extension ⟨D, R′⟩ is any function v from sentences into {T, F} such that

(1) for any atomic sentence S *is* P, v (S *is* P) = T iff R′(S) is in R′ (P).

(2) for any molecular sentence $\sim A$, $A \wedge B$, $A \vee B$ and $A \rightarrow B$, the values $\vee(\sim A)$, $\vee(A \wedge B)$, $\vee(A \vee B)$ and $\vee(A \rightarrow B)$ are calculated from $\vee(A)$ and $\vee(B)$ by the classical truth-tables. Let Val be the set of all classical valuations over any determinate extension of any partial model of Syn, and let the base $\mathscr{B}$ relative to the determinate language $\langle$Syn, Val$\rangle$ be the set of all classical valuation sets B such that for some partial model M for Syn B is the set of classical valuations over determinate extensions of M.

Stage 4.

Sval is the set of all intersections of all elements of $\mathscr{B}$.

By (34), $|||\!\!-$ of $\langle$Syn, Sval$\rangle$ preserves all the implications of $||\!\!-$ in $\langle$Syn, Sval$\rangle$. In addition, *S is P* presupposes *S exists* in the following sense:

(35) *S is P* $|||\!\!-$ *S exists*

(36) $\sim$*(S is P)* $|||\!\!-$ *S exists.*

Superlanguages thus capture presuppositional implications while preserving classical logic. The statement of the theory is elegant and offers on a conceptual level an explanation of the special status of classical reasoning in a three-valued reality. All in all, the theory is very attractive and has been applied rather successfully to several problem areas other than presupposition.[12]

The formal properties of this language can now be used to set aside two misconceptions promulgated by linguistic critics of presupposition.

2. Falsity, Presupposition, and Implication

Deirdre Wilson advances a short and informal argument designed to show that there are no serious cases in which logical implication and presuppo-

12 Supervaluations is just one of several new and powerful presuppositional semantics. Here I can briefly describe two others. Note that any serious rejection of presupposition would require their consideration in detail.
First is an interesting 4-valued, 2 dimensional semantics of Herzberger [10], that defines truth as correspondence plus presupposition satisfaction. Logical inference is defined in terms of correspondence alone. The theory is conceptually plausible, preserves classical logic, and has superlanguages as a special case. See also Herzberger [12], and Martin [25], [26], [28].
Second is a 2-valued non-truth-functional semantics with truth-operator of Bergmann [2], which in its own way gives a reasonable account of logical inference.

sition can overlap.[13] More precisely, her claim is:

(37) If A logically implies B and A presupposes B, then B is never false.

Here presupposition is understood in its usual sence as in (29), but logical implication is understood in Wilson's peculiar sense of logical entailment. On these readings the claim is true. But entailment is not going to be the correct analysis of logical implication in a many-valued theory. Neither the 3-valued matrix semantics for presupposition given above nor superlanguages have implication relations that are also entailments. For all of them, conversion fails for $\Vdash$ and the internal negation. Rather, in the usual formal analysis, logical implications are captured by $\Vdash$ and (37) is false. By its definition presupposition is a subrelation of $\Vdash$. In the case of superlanguages the situation is more complex. There are then two relations of formal inference: classical implication $\Vdash$ and the more generous relation $|||\!-$ capturing formal presuppositions as well. But even if logical implication is interpreted in the narrower sense as the classical $\Vdash$, it is possible to have non-trivial presuppositions that are also logical implications. In the particular superlanguage above, for example, we have:

(38) S *is* $P \Vdash S$ *is* $P \vee S$ *exists*

(39) S *is* P presupposes S *is* $P \vee S$ *exists.*

(38) is true by classical logic, and (39) follows by (35), (36), classical logic and the definition of presupposition in terms of $|||\!-$ and $\sim$. Falsity of (37) would not be very interesting except that Wilson uses (37) as a general metatheoretic club to beat down alleged cases of presupposition. Her strategy is to argue from the fact that an inference is formal, to the conclusion that the inference is not presuppositional. As this case shows, transcendental arguments from vague formal principles are perilous. Superlanguages would be rejected *a priori* without appraisal according to more basic criteria of adequacy.

3. Necessitation

A similar confusion underlies the working definition of presupposition given by Boër and Lycan. They claim that the proper way to analyze presupposition is via a concept called necessitation, and they imply that this is the

13 Wilson [2], pp. 24, 28–31, 38, 40, 50, 59, 123.

same concept of necessitation that is used in the theory of superlanguages: A presupposes B, they say, iff A necessitates B and not-A necessitates B.[14]

The first thing wrong about this definition is the role it assigns to necessitation. Necessitation has a rather limited technical job in supervaluation theory that in no way generalizes to a global definition of presupposition general enough to embrace the many alternative theories. But not only will the concept not generalize, necessitation doesn't even have the role in supervaluations Boër and Lycan assign it: A can presuppose B without necessitating it. The role of necessitation is briefly sketched. Its main function is to allow for the determination of a base for a superlanguage without the need to specify models. Let a *necessitation* relation for Syn be any relation from sentences to sentences.

Definition:

By the *preliminary base relative to a (classical) uninterpreted language* $\langle Syn, Val\rangle$ *and a necessitation relation N on Syn* is meant the family $\mathscr{P}$ of subsets of sentences such that every element of $\mathscr{P}$ is

(1) closed under $||\!\!-$ of $\langle$Syn, Val$\rangle$,

(2) closed under N,

(3) satisfied by some element in Val.

Definition:

The set *generated from a preliminary base* $\mathscr{P}$ *relative to* $\langle Syn, Val\rangle$ *and N* is the set $\mathscr{B}$ of all subsets B of Val such that B is the set of valuations satisfying some element of $\mathscr{P}$.

Clearly, if $\mathscr{B}$ is generated from a preliminary base $\mathscr{P}$, $\mathscr{B}$ is itself a base and together with $\langle$Syn, Val$\rangle$ determines a superlanguage. Such is the role of necessitation. Let me now show that presupposition does not require necessitation by consideration again of the particular superlanguage defined above. Let $|||\!\!-$ and $||\!\!-$ be as there defined and let N be the relative complement of $||\!\!-$ in $|||\!\!-$, i.e. N = $|||\!\!-$ — $||\!\!-$. Then the superlanguage generated from N

14 Boër and Lycan [3], pp. 4, 6–7, 10. On the equivalence of necessitational superlanguages with those presented here see Herzberger [9]. For an example of defining a necessitation relation from a semantics of partial models see van Fraassen [44], p. 89.

would be exactly like the one defined: it would have the same syntax and supervaluations. Then by (38) and the definition of N:

(40) not *(S is P* N *S is P* $\vee$ *S exists)*.

But the presupposition (39) holds. Thus, necessitation should be banished from any definition of presupposition, and should not even be mentioned outside superlanguages. Its use by Boër and Lycan just confuses matters by introducing incorrectly yet another inference relation to a subject where there are already too many.

4. External and Internal Negation

A second major flaw in Boër and Lycan's definitions of presupposition is the sense of negation they use. For some quixotic reason, they decide that presupposition is supposed to be defined in terms of external negation $\neg$, whereas in every precise treatment of presupposition I know of, negation is internal.[15] Admittedly authors unfamiliar with the distinction or not engaged in formal work are often vague, but as the outline of 3-valued semantics given earlier shows, 3-valued formal theories all define presupposition via internal negation. Even Russell's theory of existential presupposition is formulated by negation in secondary occurrence which is the classical version of 3-valued internal negation. But it is not only historically inaccurate to define presupposition by external negation, it is conceptually absurd. In 3-valued semantics presupposition is supposed to conform to the following rules. Let P presuppose Q. Then,

(41) P implies Q,

(42) Not-P implies Q,

(43) If Q is not true, then P is neither true nor false.

Now suppose further that Q is false. Then by (43), P is neither true nor false. Then by the truth-table for external negation, $\neg$ P is true. If we construe negation as external in (42), then Q is true contradicting our assumption. Thus,

[15] The classical statement of the theory is Smiley [34] which the logical tradition follows in using internal negation. For cases of alleged presupposition rejected by Boër and Lycan because they define presupposition by external negation see [3], pp. 22–24, 26–27, 36, 49, 59.

endeavoring to capture (41)–(43) by construing negation as external would be a mistake in elementary logic, a mistake not to be found in the formal tradition. Boër and Lycan thus attack a straw man. Their argument is both destructive and constructive. They argue first that alleged cases of presupposition are not genuine because intuitively the externally negated sentence does not imply the presupposed sentence. This attack attempts to undermine most cases of presupposition they discuss: non-restrictive relative clauses, cleft constructions, factives, implicative verbs, and existential presuppositions. In the positive discussion they point out that at least in the cases of factives and existential presuppositions, there is an implication but only from the internal negation. They also offer a kind of general argument that negation in alleged cases of presupposition must in a sense be internal. It goes as follows. The fact that P presupposes Q treats P as exponible into a conjunction $R \wedge Q$. When we deny P we therefore deny $R \wedge Q$. But by the denial of $R \wedge Q$ we don't mean its classical negation $\sim(R \wedge Q)$ because $\sim(R \wedge Q)$ is equivalent to $\sim R \vee \sim Q$. But we seldom if ever assert $\sim R \vee \sim Q$ because doing so violates Grice's Maxim of Strength. Epistemic situations in which we have evidence for $\sim R \vee \sim Q$ but neither $\sim R$ nor $\sim Q$ are very rare. According to the maxim, we always assert as strong a claim as we have evidence for. Hence by denying $R \wedge Q$ we can't mean the relatively rare $\sim(R \wedge Q)$ but more likely $\sim R \wedge Q$. Now, $\sim R \wedge Q$ is P negated in secondary occurrence. Since negations in secondary occurrence are treated in 3-valued semantics by internal negations, Boër and Lycan in effect argue for the existence of presuppositions, once the concept is correctly defined.

Unfortunately these particular mistakes by Wilson, and Boer and Lycan permeate their critiques, and their negative arguments are rather undermined once they are corrected. As a result, it is inappropriate to consider their arguments line by line. The whole subject must be discussed as I have done from basic assumptions up.

VI. Analytic Presupposition

1. Scope of Abstract Theory

I will now complete the exposition of 3-valued theory by distinguishing between the kind of presupposition discussed above and a new variety that is fundamentally different. It is the former, not the latter that is the main subject matter of formal theories, yet it is mainly examples of the latter that are attacked by the linguistic critics. The type of presupposition we have been discussing thus far and the type explained by the formal theories is what I shall call *logical* presupposition. It is presupposition defined by negation and logical implication. Besides the occurrence of a logical concept in its *definiens*, there is a convenient if informal test that shows whether a presupposition is logical. Generally logical presupposition is distinguished by the fact that it obeys a formal rule. What a sentence presupposes can be stated in a rulelike way by referring to just the grammatical form of the presupposing sentence. Another way to put it is that such presupposition is independent of any particular lexical term. Substituting one lexical term for another in the formal rule yields a genuine case of presupposition. Varieties of existential presupposition provide the best examples. Here are two formal rules that describe legitimate inferences and which are independent of any particular descriptive term:

(44) From a singular subject-predicate sentence *S is P* or its negation *S is not P*, infer *S exists.*

(45) From a universal subject-predicate sentence *All the S are P* or its negation *Not all the S are P*, infer *There are some S*.

Other types of presupposition found in the literature that seem to qualify as logical are those of cleft, pseudo-cleft, and counterfactual constructions. If it can be shown that these varieties are genuine cases of presupposition, then they do seem open to description by formal rules. Whether they can in fact be defined by means of logical implication will depend on development of a suitable formal language and this task has yet to be done.

These cases however are markedly different from presuppositions that depend on the meaning of a particular descriptive term and would be defined in terms of analytic rather than logical implication. Such presuppositions I shall call *analytic*. The best example of this type is sortal presupposition. The fact that *S is red* presupposes *S is colored* depends on the meaning of *red*. The presupposition fails if another predicate, e.g. *is a prime* is substituted for *red*. There is no way of predicting by inspection of the grammar alone what the sortal presupposition will be. Probably the majority of instances of presupposition proposed by linguists are of this sort. Consider the alleged presuppositions, of *stop, manage*, and *condescend*. These implications

(46) John stopped beating his wife,

(47) John beat his wife,

(48) John managed to breathe,

(49) Breathing is not entirely easy,

(50) John condescended to run,

(51) Running is not clearly a suitable thing to do.

are not predictable from syntax alone.

Some examples are not theory-independent and will vary considerably depending on question of syntax. What counts as a logical term and how descriptive terms are classified effects judgements about whether presuppositions are preserved by substitution of descriptive terms of the same class. A good example of a disputed case is factive presupposition, (17) – (19). It was originally hoped that factives formed a syntactically distinct group as well as having their characteristic semantic presuppositions. But their syntactic distinctness has been seriously questioned. If they are grouped together with what Karttunen calls counter-factives and ordinary verbs of propositional attitude, their presupposition must be analytic. A counter-factive is a verb like *pretend* that allegedly presupposes the falsity of its complement as

Counter-factive Presupposition

(52) John pretended that he was rich

(53) John did not pretend that he was rich } imply

(54) John is not rich

Propositional attitude verbs like *believe* presuppose neither the truth nor falsity of its complement. If, on the other hand, factives and counter-factives form distinct syntactic classes, then their presuppositions (if genuine) would be logical.

Regardless of disputed cases, however, there is a major lesson to be learned about the nature of presupposition theory from this distinction. By their nature formal theories are designed to elucidate logical implication rather than analytic, and accordingly formal theories do not even attempt to predict non-logical presuppositions. They are not phenomena formal theories try to explain. Hence any critique of the formal notion that depends on examples of non-logical presupposition grievously misses its mark and betrays a basic misconception of the goal of formal work.

2. Sortal Presupposition

Strictly speaking formal presupposition theory has not totally ignored analytic presupposition. Like formal definitions of analytic implication, it is possible to define concepts of analytic presupposition for an interpreted language. But again as in the case of analytic implication, this definition is for a class of interpreted languages, and no attempt is made in the formal theory to figure out which of the class approximates natural language. Hence there are no predictions about particular cases of analytic presupposition. One variety of analytic presupposition that has been formally defined in this abstract way is sortal presupposition. I shall give an example using supervaluations that first develops an uninterpreted language and then a class of interpreted languages each with its own analytic implication and sortal presupposition. Which of the class approximates English is not decided.

Example[16]

Let an *uninterpreted sortal superlanguage* be any $\langle$Syn, Sval$\rangle$ meeting these conditions. Syn is the subject-predicate syntax used above but without the existence predicate.

Stage 1.

Let a *partial model* for Syn be any $\langle$D, R$\rangle$ such that

(1) D is a set (the domain),

(2) R is a function on all subjects and predicates such that
(a) for any subject S, R(S) is in D,
(b) for any predicate P, R(P) is a pair $\langle R_1(P), R_2(P)\rangle$ of subsets of D. (Here $R_1(P)$ is the objects P is true of and is called its *extension*, and $R_2(P)$ is the objects P is either true or false of and is called its *sort*).

Stage 2.

A *determinate extension* of a partial model $\langle$D, R$\rangle$ is any pair $\langle$D, R$'\rangle$ such that R$'$ is defined for all subjects and predicates, and

(1) for all subjects S, $R'(S) = R(S)$,

16 The example is a simplified version of Thomason [39].

(2) for all predicates P, $R'(P)$ is a subset of D (its *classical extension* such that
(a) $R_1(P) \subseteq R'(P)$ and
(b) $R'(P) \cap (R_2(P) - R_1(P)) = \emptyset$

(Hence, $R'(P)$ contains $R_1(P)$ but does not contain any objects P is false of).

Stages 3 and 4.

The notions of *classical valuations over determinate extensions*, the set *Val*, the base $\mathscr{B}$, and the set Sval of supervaluations are as defined in a minimal superlanguage previously.

This uninterpreted superlanguage however lacks sortal implications just as any uninterpreted language lacks analytic implications. The reason is that the set of partial models is too generous. It captures a notion of logical possibility that does not depend on the meanings of descriptive terms. As defined above, for any predicate, any extension, and any sort, there is some possible partial model such that the predicate is paired with that extension and sort. But conceptually some such models should be ruled out for any given predicate. Indeed it is easy to check that the supervaluations in the uninterpreted language do no significant work. The logical implication relation $|||\!\!-$ of $\langle$Syn, Sval$\rangle$ turns out to be identical to $||\!\!-$ of $\langle$Syn, Val$\rangle$. It is only by restricting the possibilities in an interpreted language that analytic relations are captured. To do so we must first define in general what an interpreted superlanguage would be.

Definition:

An *interpreted superlanguage* relative to an uninterpreted superlanguage SL = $\langle$Syn, Sval$\rangle$, its base B, and its classical language L = $\langle$Syn, Val$\rangle$ is any SL* = $\langle$Syn, Sval*$\rangle$ such that (1) $\mathscr{B}^*$ is the restriction of $\mathscr{B}$ to some subset of Val (i.e. $\exists V \subseteq \text{Val}, \forall X \in \mathscr{B}^*, \exists Y \in \mathscr{B}, X \cup V = Y$).

(2) SL* is a superlanguage defined relative to base B* and classical language L.

In an unambiguous context let SL* be the interpreted superlanguage relative to superlanguage SL, and classical language L, and let the implication relations of these languages be respectively $|||\!\!\overset{*}{-}$, $|||\!\!-$, $||\!\!-$. Consider the application of this general notion to sortal languages. Let SL* be the superlanguage interpreting the uninterpreted SL relative to a classical L of the previous example. Then, analytic implication conserves all logical implications and all classical implications:

(55) $A \mid\mid\!\!- B$ iff $A \mid\mid\mid\!\!- B$

(56) If $A \mid\mid\!\!- B$ or $\mid\mid\mid\!\!- B$, then $A \mid\mid\mid\!\!\overset{*}{-} B$.

But as intended some analytic implications will not be logical and the converse of (56) fails. As in the earlier abstract account of analyticity, this theory predicts that for every uninterpreted superlanguage there is a set of interpretations with its own relation of analytic implication, and one of these interpretations would approximate part of English. Relative to each interpreted language a relation of analytic presupposition is definable capturing sortal implications.

Definition:

A *sortally presupposes* B in SL* iff $A \mid\mid\mid\!\!\overset{*}{-} B$ and $\sim A \mid\mid\mid\!\!\overset{*}{-} B$.

As in earlier cases of analyticity the theory explains such presuppositions not by predicting which sortal presuppositions will hold in natural language, but by predicting that each natural language has its own sortal presuppositions. The theory abstracts for the particular sortal presuppositions of, say, English to define the set of interpreted languages. It is the task of empirical semantics to identify the particular presuppositions of English from which the formal theorist abstracts, and the details of sortal presupposition in English are presupposed rather than entailed by the formal theory.

As in the case of analyticity, however, the results of empirical semantics may be imposed on an interpreted language so as to make it take the form of a given natural language. The method is again to restict the possibilities, in this case the partial models, so that they conform to the meaning relations of the natural language. Let an *acceptable* partial model for a syntax containing predicates *red* and *colored* be one in which:

(57) R_1 (*red*) $\subseteq R_2$ (*colored*).

The same could be accomplished by meaning postulates. If the syntax contained the resources, an acceptable model would be any M in which every classical valuation over every determinate extension of M rendered (58) true.

(58) *Everything which is red or not red is colored.*

In any SL* with only acceptable partial models, the predicated presupposition relation conforms to empirical data:

(59) $S\ is\ red\ |||\!\!\overset{*}{-}\ S\ is\ colored$

$\sim(S\ is\ red)\ |||\!\!\overset{*}{-}\ S\ is\ colored$

But the formulation of meaning postulates is not the task of formal work. The logical theories of sortal presupposition are all of a more abstract nature. They do not endeavor to predict particular instances of sortal implication for a given natural language. Rather they assume this phenomenon and show how it is possible. It is shown to be a consequence in any language of the defining properties of an interpreted language.

VII. Pragmatics and Semantics

1. Semantics as Abstraction

I would like now to turn to the relation between formal semantics and pragmatics. The critics of semantic presupposition assume without argument or explanation, that if a phenomenon can be explained pragmatically, there is no further need for a semantic explanation and that formal explanations with their complicated mathematics can be dispensed with. I shall argue here that this view is misconceived and that the two disciplines should be viewed as mutually supportive rather than as rivals. Morris defines semantics as an abstraction from pragmatics and most people agree on this usage. But in formal work abstraction has a precise meaning.[17] The features of a particular structure

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[17] See Morris [32]. For a precise account of structures and an explicit claim that formal semantics consists of abstraction from language games see van Fraassen [40]. For a somewhat cruder account of how formal analyticity relations abstract from pragmatic data see van Fraassen [42].

The view that I defend here that formal languages are abstractions from natural speech is rejected by many who hold the older view, going back to Russell and Quine, that formal languages are best understood as corrections to or canononical versions of natural language. It is certainly true that languages can be reformed and even created on the model of naturla languages. But the linguistic forms and reasoning patterns of formal languages cannot be wholely invented because then logical theory falls prey to conventionalism. The tie that keeps logic from being arbitrary is the extent to which it is true to the facts of the reasoning we actually do, of the intuitions about validity that we

S with an interesting property P are generalized so as to define a class of structures. That S has P is explained by showing that all structures meeting the defining conditions of the class must have P. The particular is subsumed under the general. In practice, however, the method is not very exact. Often the particular structure abstracted from is known only in empirical terms, and rather imprecisely. But even in such messy instances of abstraction it is clear that empirical specifications of the particular case is compatible with the abstract theory. Indeed, the abstract theory is motivated by the fact that the empirical characterization exists. For example, there would be no point abstracting to first-order logic unless it subsumed some fragment of natural language, however imperfectly understood. Note that the abstract theory is not meant to replace empirical science. The details of the empirical specification must still be filled in. Nor will empirical theory suffice alone. For example, interest in language as such, in "possible" languages, or in the "essence" of language, all require abstraction. If the abstraction is formal, so much the better. The result is a precise mathematical theory with clear assertions and proof procedures, a paradigm of ideal science. From these reflections alone it is easy to see that formal semantics is not rendered unneccessary or superfluous by empirical studies of natural language. Both sorts of research are mutually supportive.

Pragmatics is rather an abused term. Let me distinguish here three of its rather precise applications. In each sense pragmatics is a theory of language, but the three species differ according to the kinds of theoretical vocabulary they employ. In the first sense, pragmatics is a theory characterized by its use of the notion of context. Informally, contexts of use are supposed to embrace people and elements of the world as well as language, and hence a theory employing contexts could be reasonably called pragmatic. But this usage has been extended so that a theory may be called pragmatic if it appeals to any notion of context whatever. In particular, some abstract theories in formal semantics are called pragmatic because they appeal to sets of indices, called

actually have. Given that logic must be tied to the facts of actually reasoning, there does not seem to me to be any room for reforming natural reasoning. To reform in this case would be to deny the very evidence that is being explained. Of course, every scientific theory may deny or surpress some facts in the interest of maximizing other theoretical virtues like elegance or parsimony. But it is much more than compromises of this sort that are envisioned in the reformist view of formal logic, and outside the framework of conventionalism I find it very hard to imagine what these additional reforms would amount to.

contexts, even though these sets are defined mathematically and make no explicit appeal to people or the world. Various theories of demonstratives and contextual meaning, notably those of Montague, are pragmatic in this sense. Numerous logicians have suggested that such theories be called by another name, e.g. *contextual*, and that pragmatics be reserved for its other informal uses. Clearly pragmatics in this sense is in no way incompatible with formal semantics; it is rather one of its species.[18]

A second criterion of the pragmatic is the use of concepts from speech-act theory. Austin, Searle, Grice and others have used notions like action, belief, intention, purpose, expectation, assertion, denial, etc. to study language. Such studies are actually rather hard to classify as purely semantic or purely pragmatic. To some extent such ideas are amenable to abstract formulation. For example, propositional attitude operators and mood indicators have been treated in formal languages.[19] But there is no doubt that some aspects of speech-act theory are genuinely empirical and beyond the definitional extension of mathematics and set theory. Beliefs and assertions, whatever their generalizable features, belong to people in the real world. When singled out in non-mathematical vocabulary, whether for the purposes of subsequent abstraction or for direct investigation, they should be viewed as concepts of empirical science. Certainly much of formal semantics has been intended as generalizations from speech-act descriptions of natural language. In this sense, then, pragmatics in genuinely distinct from semantics, but the two are not rivals. The one attempts the empirical study of actual cases, and the other abstracts to investigate the formal properties of more general structures.

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18 For contextual theories in formal semantics see Montague [31], Kaplan [16], and Martin [28]. In [38] Thomason suggests that theories that use points of reference that are combinations of possible worlds and contexts (as in Montague's work) be called *indexical* rather than *pragmatic*. He does, however, continue to apply *pragmatic* to the species of implication that varies from context to context. He envisions not a formal abstract theory of such implication, but one in speech act theory. Thus, to some extent Thomason has provided the ideas for later research. Note, however, that nothing Thomason says precludes the possibility or desirability of an abstract, formal study.

19 In Lewis [23] moods have a role in the formal semantics, and in Stenius [35] mood indicators actually appear in the syntax. Propositional attitudes are studied in intensional logic.

I have ignored work by linguists on the presupposition of nondeclarative sentences, for two reasons. First, the logic of commands, questions, etc. is not very fully understood, and in most theories then grammar tends to undermine this tie. Whatever the tie to declaratives, their logic (and hence their pressuppositions) are not clear enough to me to say anything very intellegent about it.

In yet a third sense pragmatics is used to refer to a species of speech-act linguistics, studies that use concepts from Grice's theory of implicature. An assumption of both Wilson, and Boër and Lycan is that implications that can be explained as conversational implicatures do not need any further semantic explanation. But semantics cannot be rendered redundant so easily. If a particular relation has been empirically identified there is no reason in principle why it cannot be studied abstractly. I do not want to argue here about what form an abstract theory of implicature might take. Abstractions from speech-act theory may not take the form of implication or presupposition relations, nor even bear on the abstract theory of truth. In such cases it may be better to speak of a formal pragmatics.[20] Even so, linguistic critics would be wrong in suggesting that speech-act explanations could take the place of abstract theory. Their roles are different. I take it as established, then, that pragmatics in any of its senses should not be regarded as a possible replacement for abstract study. Contextual theories can be and to some extent already have been incorporated into formal theories, and speech-act theory may be regarded as the empirical base required for abstraction.

2. Contexts and Cancellation

More, however, needs to be said about context. The concept underlies one of the major critical arguments against semantic presupposition. Presuppositions as species of implication must hold universally. Hence, if a context can be found in which the alleged presupposition is cancellable, then there is in fact no genuine instance of presupposition. A strategy of the critics is then to make up examples in which the presupposition fails.[21] The argument depends on the equivalence of (i)–(iii) and their obvious inconsistency with (iv).

(i) A presupposes B.

(ii) A and its negation each imply B.

(iii) There is no situation in which A or its negation is true, but B is false.

(iv) There is a situation in which A is true or its negation is, but B is false.

But such reasoning is in danger of equivocation on the term situation.

20 Gazdar's work in pragmatics is called formal in this sense. See [7].

21 See Wilson [49] pp. 26, 30, and 99, and Boër and Lycan [3] pp. 40, 46, 50, 53, 67, and 78.

In one sense of situation the meanings of words may be viewed as determined by the situation in which they are used and in fact the particular meanings in use may differ from situation to situation. Situations in this sense are called *contexts of use* and are sometimes used as the indices of the various interpreted languages of a common uninterpreted language. Such is one of their uses in formal contextual pragmatics. Meanings of descriptive terms are all fixed relative to a context. There is however another sense of situation in which it is situations that are described once the descriptive meanings have been fixed. Situations in this sense are the *possible worlds* conceptually compatible with the descriptive sense in effect. The second of these notions of situation is already familiar from the abstract definition of interpreted language; it is represented there by the set Val* of conceptually possible valuations. It is easy to introduce contexts of use as well and to exhibit the different functions of the two ideas. Let a set of *contexts* for an uninterpreted language L be any non-empty set.

Definition:

> Let a *contextual assignment of meaning* for an uninterpreted language L and a set C of contexts of L be any mapping I from C to interpreted languages L* of L.

Situations in the sense of *contexts* are indices of descriptive languages, whereas situations in the sense of *possible worlds* are represented by the set Val* of L* determined by its context of utterance.

Now, the sense of situation in which (iii) is equivalent to (i), and (iv) is contradictory to it, is situation in the sense of possible world. Merely showing that in some sense of situation, there is a situation in which presupposition is cancellable leaves open the possibility that by situation one means context. It is in fact trivially easy to establish cancellability of a sort by shifting contexts to one of rather deviant usage. If by *colored* I mean factorable by 2, then *x is red* does not presuppose *x is colored*. But this meaning shift is still consistent with analytic presupposition in ordinary contexts.

Many alleged cases of cancellation can in fact be explained as results of shift of meaning rather than genuine counter-examples to universal implication. The most important cases require the hypothesizing of an ambiguity in natural language negation. When each of the following sentences are true, their negated parts may be unterstood so as not to imply their ordinary presupposition; (60) and (61) are based on remarks of Wilson, and (62) and (63) on remarks of Boër and Lycan:

(60) Bill Bloggs isn't here because there is no such person.

(61) It won't be America who is going to win this year since nobody is going to win.

(62) It isn't John who caught the thief because nobody caught her.

(63) John didn't *manage* to solve the problem – it was so easy a trained monkey could solve it blindfolded.

Presuppositionalists typically suggest that negation in these contexts be understood as external rather than in its more usual internal form. $\neg$ A could be true and B could be false even though A presupposes B, because A might be neither true nor false. In fact most alleged cases of cancelled presupposition depend on the negated sentence: (iv) is established by showing (v):

(v) There is a situation in which the negation of A is true but B is false.

In all such cases if negation is read as external and situation as context of usage, no refutation of standard presupposition is established. A complete defense of the external reading would be required in a full study of presupposition, but curiously the critics of presupposition are willing to grant that negation may be read as external in these cases. Boër and Lycan work under the misconception that negation is external when (ii) is understood as equivalent to (i). Instead of arguing that $\neg$ A is sometimes true when B is false, they show the $\neg$ A is compatible with B's falsity, a fact the presuppositionalists themselves defend. Wilson just straightforwardly agrees that negation in natural language may well be ambiguous between external and internal uses, and that uses in (60)–(63) are external. She rejects presupposition for other reasons. Let me summarize the results of the discussion so far. Cancellability alone does not undermine claims of presupposition. Meaning shift may be responsible. Further, many of the cases of cancellation considered by critics may be explained as shifts in the meaning of negation to an external reading. The critics moreover endorse the external reading.

Negation is not the only presuppositional term that may be ambiguous. Some factive verbs in particular seem to have presuppositional and non-presuppositional senses. For example, *know* is factive in many ordinary uses but is definitely not factive in technical epistemology. Likewise *report* seems to vary in its factive import, depending on whether the reporter is viewed as fallible or as an eye witness. Methodologically, of course, the multiplications of senses beyond necessity is undesirable, and presupposition theory would become irrefutable if every alleged cancellation was explained by hypothesizing a new sense. But a few such ambiguities, especially that of negation, seem perfectly reasonable.

There is to be balanced against systematic ambiguity one advantage lost when presupposition is ignored as a distinct phenomenon. True, presuppositions are sometimes cancellable. But in most cases there is a strong prima facie suggestion of the presupposed sentence. If a sentence is read alone without any complicating details there is a strong suggestion that its ordinary presuppositions are true. Even the critics of presupposition generally agree that in ordinary and simple cases inference to alleged presuppositions is acceptable. What I want to point out here is that these ordinary cases constitute an important linguistic datum that needs explanation. In throwing out the presuppositional inference because it is sometimes cancellable the critics throw out too much. The question still remains why in ordinary cases the suggested implication is so powerful. Now, the normal cases may be explained in various ways. Much of the constructive theory of Wilson and Boër and Lycan can be viewed as attempts to advance the empirical understanding of both the ordinary and deviant cases. But there still remains a place for an abstract investigation of the ordinary cases. Abstract theories of presupposition may be viewed, at their weakest, as generalization from these ordinary cases. As explained in the introduction to abstract theory, generalization from empirical structures always requires the specification of the empirical structure relative to certain standard conditions. Semantics, for example, may abstract from natural language under conditions of ordinary usage. In practice abstraction tends to take place before either natural language or the conditions of abstraction are well understood empirically. The empirical bases for abstraction to presupposition theory is likewise in a muddled state. We may conclude the discussion of cancellation by observing that cancellation alone does not undermine presupposition, and that there are alternative explanations in terms of ambiguity. Furthermore, even if there are other cases of cancellability not explainable by meaning shift, these cases are non-standard. The existence of ordinary simple cases regardless of these exceptions would be sufficient empirical ground for abstract investigation.

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