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FACTS AND THE SEMANTICS OF GERUNDS*

1. INTRODUCTION

In this paper I propose to explore the idea that the presuppositional properties of gerunds may be explained analogously to the existential use of singular terms. On the basis of some special substitutional properties, it is argued in Section 2 that constructs called facts are needed for the interpretation of some gerunds and that these may be taken as the 'existents' assumed by presuppositional gerunds. In Section 3, I discuss modifications to the notion of fact necessary for its extention to complex gerunds. A formal semantics is then advanced in Section 4 that makes use of a developed version of Bas van Fraassen's notion of fact stated in [8], and results are presented showing previously discussed intuitive requirements are satisfied. Finally, in Section 5 a program is outlined for incorporating this semantics into a fully non-bivalent account of gerundive presupposition; it is shown compatable to both a 3-valued system modeled on the weak connectives of S.C. Kleene and Hans Herzberger's 4-valued product logic.

2. SEMI-TRANSPARANCY AND SIMPLE GERUNDS

In van Fraassen's development, a fact is taken to be a non-linguistic explanatory entity that enters into the analysis of truth according to whether it obtains:

(1) A sentence is true iff the fact it expresses obtains.

The motivating question is whether presuppositional gerunds can be profitably understood to stand for such a notion of facts analogously to the way singular terms presuppose individuals. On this reading, if the gerunds in the following examples correspond to facts that do not exist or, to use the more customary term, obtain, then the contained sentence fails of presupposition as does a sentence with a non-refering singular term:[1]

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(2) *The mining of Haiphong* $\left\{\begin{array}{l}\text{is undermining the national morale.}\\ \text{surprised nobody.}\end{array}\right.$

(3) Boredom encourages *my drinking.*

More important are the non-intensional cases that do not seem to have an adequate interpretation in terms of truth-values or intensions, though they are presuppositional:

(4) I $\left\{\begin{array}{l}\text{photographed}\\ \text{stumbled onto}\\ \text{helped delay}\end{array}\right\}$ *the crowning of the last Inca king.*

(5) *The launching of the ship* is coming off quickly, silently, smoothly.

(6) *The slaughtering of the Huguenots* was bloody.

(7) *The bombing of Hiroshima* was destructive.

(8) *Dropping the stone in the water* frightened the bird.

(9) *Running through the mud* dirtied my trousers.

(10) *Eating a spoonful of arsenic* killed the man.

(11) *The mining of Haiphong* $\left\{\begin{array}{l}\text{will take two months.}\\ \text{delayed the arrival of food.}\end{array}\right.$

The expressions occurring as parts of these gerunds are clearly substitutable *salve veritate*. It does not matter how either the Huguenots or their slaughtering is referred to, the doing of it to them was bloody. Likewise, if the stone was the Hope Diamond, then it was the dropping of the Hope Diamond that frightened the bird. Further, while the parts of the gerunds are extentional, the gerunds as a unit are not. Gerunds transformed from declarative sentences of the same truth-value cannot be substituted *salve veritate*. Suppose you dropped the stone in the water and that in doing so you frightened the bird; suppose also you were wearing red pyjamas. It is not true that your wearing red pyjamas frightened the bird. This substitutional situation is rather unique. The parts of the gerund seem to refer to their usual extentions, but the gerund as a whole cannot be interpreted in either of the usual ways, neither by a truth-value nor a proposition. It cannot stand for a truth-value because then, contrary to the evidence, gerunds associated with the same truth-value would be substitutable. It cannot stand for a proposition because intensionality would then seep down to its parts and they would not be substitutable, again contrary to intuition. Rather, gerunds that are the substantative renderings of non-compound declarative sentences, *simple gerunds* as I shall call them, seem to require a novel interpretation, the necessary and sufficient conditions for which may be infered from the substitutional circumstances. Because the substitutivity of parts is permissible, if the corresponding parts of two simple gerunds have identical interterpretations, the gerunds themselves will as well. Put another way, the references of the parts of a simple gerund functionally determine that of the whole. Conversely, if two simple gerunds have the same interpretation in the appropriate sense, if they 'record the same fact', then their corresponding parts must also stand for the same. If the spying of the man on the beach is the same as Ortcut's, then the man on the beach must be Ortcut. Thus, the interpretations of simple gerunds seem in turn to functionally determine those of their parts.

This bi-functionality between values of the parts and wholes of simple gerunds suggests a straightforward definition of a formal construct, called a fact, to serve as the interpretation of simple gerunds. A fact may be identified with a sequence of values of simple gerundive parts, whatever these may be. The values uniquely determine the sequence and vice versa. Further, these same parts presumably go together to make up the simple declarative sentence corresponding to the gerund, and the truth-conditions for this sentence are statable in terms of the values of the parts within the classical theory of truth. The semantic relations that hold between the values of these parts in the statement of the truth-conditions may be used to define when the fact constructed from them obtains. Let a natural language gerund be syntactically represented as the result of applying a syntactical operation s to basis expressions $e_1, \ldots, e_n$, and let I be the interpretation function for these expressions in a given classical semantics. There will then be a property (set) P such that the declarative sentence corresponding to $s(e_1, \ldots, e_n)$ is true iff $\langle I(e_1), \ldots, I(e_n) \rangle \in P$. In the proposed factual semantics, $I(s(e_1, \ldots, e_n))$ is identified with $\langle I(e_1), \ldots, I(e_n) \rangle$, which is said to *obtain* iff it is in P. For example, in a naive first-order representation of natural language, $I(\textit{Theatetus' flying}) = \langle I(\textit{Theatetus}), I(\textit{fly}) \rangle$, which obtains iff $I(\textit{Theatetus})$ falls within or exemplifies $I(\textit{fly})$. (For such a first-order theory of facts see [6] and [3] as well as [8].) Further, a version of van Fraassen's formula (1) may be satisfied by associating the truth-value T with all and only the gerunds corresponding to obtaining facts.

3. AN ALGEBRA OF FACTS

In this section I shall attempt to extend a factual interpretation to what shall be called *complex* gerunds, compounds built up from simple gerunds by the usual propositional connectives 'not', 'and', 'or', etc. I am assuming, for example, that 'my running and jumping' is, in all important semantic respect, the same as 'my running and my jumping'. The substitutional properties of complex gerunds prove to be more complicated than those of simple gerunds with the result that the corresponding notion of fact becomes more interesting. There are, in fact, reasons to doubt whether a functional relationship exists between the parts and whole of a complex gerund in either direction: parts may not determine the whole and vice versa.

Certainly, if the only guideline of a semantic theory was intuitions about substitution, there would be little reason to doubt that the values of the parts of a complex gerund determine that of the whole. Cicero's ranting and raving is the same as Tully's because Cicero is the same as Tully. Such a functional determination is incorporated into the formal theory presented below. There are, however, global reasons that suggest an alternative development. These would apply if this theory of gerunds were developed within a 3-valued supervaluational account of presupposition. Facts might then be defined so that they determined a truth-value T if they obtained, a value F if they did not obtain, and no truth-value if they neither obtained nor did not. Let P and Q be simple gerunds corresponding to the same fact, one which neither obtains nor does not. Then, $Q \vee \neg P$ cannot stand for the same fact as $P \vee \neg P$ because the former will stand for a fact that neither obtains nor does not, while the latter will always point to an obtaining fact. In Section 5 I discuss how to graft a factual account of gerunds onto a non-bivalent semantics, but there I prefer to retain the simplicity of functional dependence of part on whole and to seek other ways to capture the advantages of supervaluations.

A functional dependence in the other direction, however, is intuitively implausible. There are, of course, more facts than truth-values, but intuitively there should not be so many facts that at most one sentence stands for any given fact. There ought not to be enough facts for a unique decomposition of factual structure corresponding to the unique decomposition of syntactical structure. The evidence rests in the need to identify the facts expressed by certain simple logical equivalents. Cicero's ranting and raving, for example, is the same as his raving and ranting. As far as English gerunds admit the grammatical complexity necessary for forming logical equivalents, the logical operations of commutation, association, double negations, and DeMorgan manipulations do not seem to alter the fact expressed. Thus, the structure of facts seems to mirror some but not all of the structure of grammar. It is difficult to know just how much structural parallel there is. In the following theory which admits gerunds of any complexity, I extrapolate to the somewhat arbitrary but plausible principle that all compounds having the same normal form in classical logic should express the same fact. Together with the requirement that the factual interpretation of the parts of a simple gerunds should determine that of the whole, this idea will serve as the basis for constructing a new, broader notion of fact.

The only serious attempt to define a formal notion of facts for complex expressions is van Fraassen's, but this is intended for other uses, the semantics of entailment, and does not meet either of guiding principles for complex gerunds.[2] But it will serve as the basis of adequate account. By the elimination of certain kinds of redundancy within his notion of fact, it is possible to define Boolean operations on facts and satisfy both requirements.

In the theory that follows simple gerunds will be represented by first-order atomic sentences $P^n c_1 \dots c_n$. An atomic fact will accordingly be an $n + 1$ tuple consisting of an n-place relation followed by n individuals. The complement $-$**a** of an atomic fact **a** will be just like **a** except that it contains the set theoretic complement of the relation in **a**. Hence $-$ **a** is also an atomic fact. A negated atomic sentence $\neg A$ will correspond to the complement of the fact corresponding to A. For complex sentences there will be constructed arrays of $n \times m$ atomic facts. For atomic facts **a**, **b**, **c**, and **d**, an example of a 2×2 atomic fact would be

$$\begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix}.$$

Such molecular facts shall be interpreted as follows. The fact as a whole obtains iff all the atomic facts in at least one row obtain. Each row as a unit is intended to represent a conjunction of atomic sentences or their negations, and the array of rows is to represent a disjunction of such conjunctions. To form the union of two complex facts one reasons as follows: for a disjunction to hold it suffices that one row holds from the fact of either disjunct. Therefore, one fact will be put on top of the other:

$$\begin{bmatrix} a & b \\ \\ \\ c & d \end{bmatrix} \cup \begin{bmatrix} e & f \\ \\ \\ g & h \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \\ g & h \end{bmatrix} .$$

Similarly, in forming the intersection of two complex facts one reasons that one row from each fact must hold. Hence the rows of the intersection are formed by taking all combinations of a row from the first fact with a row from the second.

$$\begin{bmatrix} a & b \\ \\ \\ c & d \end{bmatrix} \cap \begin{bmatrix} e & f \\ \\ \\ g & h \end{bmatrix} = \begin{bmatrix} a & b & e & f \\ a & b & g & h \\ c & d & e & f \\ c & d & g & h \end{bmatrix} .$$

Negation is defined as would be expected: negation of a disjunction of conjunctions is just the conjunction of the disjunction each term of which has been negated:

$$-\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -a \\ -b \end{bmatrix} \cap \begin{bmatrix} -c \\ -d \end{bmatrix} .$$

To facilitate combining two facts with a different number of columns, missing spaces shall be filled by repeating the last defined element of a row a sufficient number of times. For example,

$$\begin{bmatrix} a & b \\ \\ c & d \end{bmatrix} \cup [e \;\; f \;\; g] = \begin{bmatrix} a & b & b \\ c & d & d \\ e & f & g \end{bmatrix} .$$

Though there will be stuttering in this fashion, there is a more important redundancy that will not be tolerated. Clearly, if a sentence A holds when [a b] obtains, it will hold when [a b c] obtains, and to mention [a b c] in addition to [a b] in the fact corresponding to A would be redundant. Apart from matters of style, banishing these redundancies is my only major departure from the construction of van Fraassen. To insure that operations yield a unique result, it will also be required that the elements of a fact be ordered in a straightforward manner.

4. A SEMANTICS FOR GERUNDS

Simple gerunds will be represented as first-order atomic sentences $P^n c_1 \ldots c_n$. As explained above, all that is essential in this representation is that there is some classical analysis of the parts of a simple declarative sentence such that the truth of the whole depends on the extensions of the parts. Complex gerunds will be represented by expression formed from the usual connectives: the least set containing the atomic sentences and $A \wedge B$, $A \vee B$, and $\neg A$ whenever it contains A and B. By an *atomic fact* for a non-empty domain of discourse **U** we shall mean any $\langle R, x_1, \ldots, x_n \rangle$ such that $R \subseteq \mathbf{U}^n$ and $x_1, \ldots, x_n \in \mathbf{U}$. It is assumed that there is a preferred-indexing function ι mapping atomic facts (of **U**), into natural numbers. By $-\langle R, x_1, \ldots, x_n \rangle$ is meant $\langle \bar{R}, x_1, \ldots, x_n \rangle$ where $\bar{R}$ is the set theoretic complement of R (in **U**). Let $f(n, m)$ for **U** be a function (of n columns and m rows) from the set $n \times m$ of pairs of natural numbers less than or equal to n and m respectively onto a set of atomic facts of **U**. The value of that function for column i and row j will be denoted by f^i_j. If $f(n, m)$ is a fact such that f^i_j is defined but no f^k_j, $i < k \leqslant n$ is defined, then a *completed version* of $f(n, m)$ will be constructed, meaning thereby that a fact such as $f(n, m)$ except that f^k_j, for all $i < k \leqslant n$, is defined and equals f^i_j. From this point on in the discussion, all facts referred to shall be assumed to be completed.

Let $f(n, m)$ be *redundant* iff for some x and y, $\{f^1_x, \ldots, f^n_x\} \subset \{f^1_y, \ldots, f^n_y\}$. An order on finite sequences of natural numbers of the same length will now be defined as a preliminary to the definition of an order on facts themselves:

$\langle s_1, \ldots, s_n \rangle < \langle t_1, \ldots, t_n \rangle$ iff

(1) $s_1 < t_1$, or

(2) if for all $j < i$, $s_j = t_j$, then $s_{i+1} < t_{i+1}$.

Then, $f(n, m)$ is in *lexical order* iff

(1) for all $j \leq m$ and $i, k \leq n$, $i < k$ iff $\iota(f^i_j) < \iota(f^k_j)$, and

(2) for all $j, k \leq m$,

$j < k$ iff $\langle \iota(f^1_j), \ldots, \iota(f^n_j) \rangle < \langle \iota(f^1_k), \ldots, \iota(f^n_k) \rangle$.

By a *factual structure for* $\mathbf{U} \neq \Lambda$ will be meant any $\langle \mathfrak{F}, \cap, \cup, - \rangle$ such that

(1) $\mathfrak{F}$ is a set of facts $f(n, m)$ of $\mathbf{U}$

(2) $\cap$ is a closed binary operation on $\mathfrak{F}$ such that
$f(i, j) \cap g(k, p) = h(n, m)$ iff
(a) $m = i + k$
(b) $\forall x \leq m, \exists y \leq j, \exists z \leq p,$
$\{f_j^1, ..., f_y^i\} \cup \{g_z^1, ..., g_z^k\} = \{h_x^1, ..., h_x^n\},$
(c) $\forall x \leq j, \forall y \leq p, \exists z \leq m,$
$\{f_x^1, ..., f_x^i\} \cup \{g_y^1, ..., g_y^k\} = \{h_z^1, ..., h_z^n\},$
(d) $h(m, n)$ is not redundant and is in lexical order;

(3) $\cup$ is a closed binary operation on $\mathfrak{F}$ such that
$f(i, j) \cup g(k, p) = h(n, m)$ iff
(a) $\forall x \leq m \; \exists y \leq j, \{h_x^1, ..., h_x^n\} = \{f_y^1, ..., f_y^i\}$, or
$\exists y \leq p, \{h_x^1, ..., h_x^n\} = \{g_z^1, ..., g_z^k\},$
(b) $\forall x \leq j, \; \exists y \leq m, \{f_x^1, ..., f_x^i\} = \{h_y^1, ..., h_y^n\},$
(c) $\forall x \leq p, \; \exists y \leq m, \{g_x^1, ..., g_x^k\} = \{h_y^1, ..., h_y^n\},$
(d) $h(n, m)$ is not redundant and is lexically ordered.

(4) $-$ is a closed unnary operation on $\mathfrak{F}$ such that

$$-f(n, m) = \begin{matrix} -f_1^1 \\ \cdot \\ \cdot \\ \cdot \\ -f_m^1 \end{matrix} \; \cap \ldots \cap \; \begin{matrix} -f_1^n \\ \cdot \\ \cdot \\ \cdot \\ -f_m^n \end{matrix}$$

The reader may easily check that in the case of small finite numbers, which does not differ essentially from the general case of arbitrary finite numbers, the operations obey the laws of double negation, DeMorgan, commutation, association, and distribution. A *model* may be defined as any $M = \langle \mathbf{U}, R \rangle$ where $\mathbf{U} \neq \Lambda$ and R is a function such that for any constant c, $R(c) \in \mathbf{U}$, and for any predicate P^n of degree n, $R(P^n) \subseteq \mathbf{U}^n$. *A factual structure for M* is $\langle \mathfrak{F}, \cap, \cup, - \rangle$ such that $\langle \mathfrak{F}, \cap, \cup, - \rangle$ is a factual structure and for any atomic sentence $P^n c_1 \ldots c_n$, $\langle R(P^n), R(c_1), ..., R(c_n) \rangle \in \mathfrak{F}$. *The factual assignment for a model M* is the function ϕ_M from sentences to $\mathfrak{F}$, where $\langle \mathfrak{F}, \cap, \cup, - \rangle$ is the factual structure for M such that

(1) if $A = P^n c_1 \ldots c_n$, then $\phi_M(A) = \langle R(P^n), R(c_1), ..., R(c_n) \rangle$,

(2) if $A = B \wedge C$, then $\phi_M(A) = \phi_M(B) \cap \phi(c)$,

(3) if $A = B \vee C$, then $\phi_M(A) = \phi_M(B) \cup \phi(c)$,

(4) if $A = \neg B$, then $\phi_M(A) = -\phi_M(B)$.

An atomic fact $\langle R, x_1, ..., x_n \rangle$ *obtains* iff $\langle x_1, ..., x_n \rangle \in R$, and a fact $f(n, m)$ *obtains* iff $\exists j \leq m, \forall i \leq n, f_j^i$ obtains. If $\langle \mathfrak{F}, \cap, \cup, - \rangle$ is the factual structure for M, let h_M be the characteristic function capturing whether the elements of $\mathfrak{F}$ obtain, i.e., $h_M(f) = T$ iff f obtains, and $h_M(f) = F$ otherwise. Then the *valuation v_M induced by M* is defined as follows: $v_M(A) = h_M(\phi_M(A))$, where ϕ_M is the factual assignment for M. Finally, let $C = \langle \{T, F\}, \wedge, \vee, \neg \rangle$ be the classical structure on truth-values, i.e., $\wedge$, $\vee$, and $\neg$ are the bivalent classical truth-functions for conjunction, disjunction, and negation.

One obvious consequence showing these definitions meet the criteria set earlier is that each factual assignment is a homomorphism from the syntactical structure composed of sentences and the syntactic operations yielding conjunctions, disjunctions, and negations to a factual structure. Also each h_M is a homomorphism from facts to truth-values.

A consequence of some general philosophical interest is that this semantics requires the existence of negative facts. By a negative fact is meant a fact that renders some atomic sentence false but no other atomic sentence true. For example, the fact that Socrates is dead would be a negative fact if there was an atomic sentence 'Socrates is alive' but no atomic sentence 'Socrates is dead'. This concept may be formally analyzed as follows: A fact f is *negative* for a given model M iff (i) there is some atomic sentence A such that $\phi_M(A) = f$ and $v_M(A) = F$ and (ii) there is no atomic sentence B such that $\phi_M(B) = f$ and $v_M(B) = T$. Russell defends this notion of fact in [7], and van Fraassen in [8] argues it is optional. To the extent that the existence of negative facts is a semantic question and not metaphysical, it is interesting that this semantics, which is essentially in the spirit of van Fraassen's, would settle the matter decisively.

T1. There are negative facts in any M with a false atomic sentence.

Proof. Suppose for some atomic sentence A, $v_M(A) = F$. Then, for any B, if $\phi_M(B) = \phi_M(A)$, then $v_M(B) = h_M(\phi_M(B)) = h_M(\phi_M(A)) = v_M(A) = F$. Thus, there is no true sentence, atomic or otherwise, expressing $\phi_M(A)$, and

$\phi_M(A)$ is a negative fact. Van Fraassen's suggestion (p. 482) that there could be an atomic sentence true when a given atomic sentence is false does not alter this result because the two sentences could not express the same fact.

Two results are now presented which show that the major goals of the theory have been achieved. Let $nf(A)$ be the disjunctive normal form of A, and by a classical valuation (a member of V_C) is meant any 2-valued valuation generated by the structure $C = \langle\{T, F\}, \wedge, \vee, \neg\rangle$ where $\wedge, \vee, \neg$ are the classical truth functions. The following fact depends on the elimination of redundancies.

T2. If $nf(A) = nf(B)$, then $\forall M, \phi_M(A) = \phi_M(B)$.

Proof. Suppose the normal form of A is the same as that of B. Then, for every classical valuation v, $v(A) = v(B)$. But suppose for some M, $\phi_M(A) \neq \phi_M(B)$. Then there are some atomic facts of some row of one of them, say $\phi_M(A)$, not present in the other. Where $\phi_M(A) = f(k, l)$ and $\phi_M(B) = g(m, n)$, assume that for some row $d, f_d^j = g_d^j$ unless $j \in \alpha \neq \Lambda$. Construct a model $\langle \mathbf{U}, R\rangle$ in such a way that the only facts that obtain are $f_d^j, j \notin \alpha \neq \Lambda$. It is argued that the classical valuation v such that $v(P^n c_1, ..., c_n) = T$ iff $\langle R(c_1), ..., R(c_n)\rangle \in R(P^n)$ satisfies B but not A. Clearly it satisfies B. Further, it must falsify A because not only does the row $f_d^1 ... f_d^n$ contain non-obtaining atomic facts, every other row does also. For suppose some other row e contains nothing but atomic facts which obtain. But these must be a subset of the $f_d^j, j \notin \alpha$. But then row e is redundant which is impossible. Hence v falsifies A. But this contradicts our assumption. Hence $\phi_M(A) = \phi_M(B)$.
End of Proof.

Another fact that shows the semantics achieves its end is that 2-valued valuations defined from facts are exactly the classical valuations. Let V_C be the set of all functions from sentences to $\{T, F\}$ conforming to C, and let V_M be the set containing v_M for any model M.

T3. $V_C = V_M$.

Proof. That $V_M \subseteq V_C$ follows from the fact, which is easily checked, that the characteristic function of the obtaining property obeys the structure C. That $V_C \subseteq V_M$ follows from the fact that for any $v \in V_C$, we can define a model M such that $v = v_M$. Let $M = \langle \mathbf{U}, R\rangle$ be such that $\mathbf{U}$ is the set of constants and define R on constants such that it is the identity function $R(c) = c$ and let $R(P^n)$ be the $\{\langle c_1, ..., c_n\rangle | v(P^n c_1 ... c_n) = T\}$. A

straightforward induction shows $v = v_M$. Atomic Case: $v(P^n c_1 \ldots c_n) = T$ iff $\langle c_1, \ldots, c_n \rangle \in R(P^n)$ iff $\langle R(P^n), R(c_1), \ldots, R(c_n) \rangle$ obtains iff $\phi_M(P^n c_1 \ldots c_n)$ obtains iff $v_M(P^n c_1 \ldots c_n) = T$. Assume the result for all formulas of length less than A. Let $A = B \wedge C$. $v(A) = T$ iff $v(B) = v(c) = T$ iff $\phi_M(A)$, $\phi_M(B)$ obtain iff $\phi_M(A) \cap \phi_M(A \wedge B)$ obtains iff $v_M(A \wedge B) = T$. Likewise for negation.
End of Proof.

5. A GENERALIZATION OF EXISTENTIAL PRESUPPOSITION

In this final section I would like to sketch informally a program for incorporating a factual interpretation of gerunds into an account of presupposition modeled upon non-bivalent treatments of singular terms. A minimal syntax capable of expressing even the coarsest features of examples (2)–(11) would have some means of constructing simple declarative sentences. These may be thought of, somewhat simplistically, as first-order atomic sentences made up of predicates and constants. There should also be means for constructing molecular sentences out of other sentences by using the usual propositional connectives ‘not’, ‘and’, ‘or’, etc. Gerunds may then be thought of as a special category of expression transformed from sentences. The syntax must also have a variety of predicates that apply to gerunds. Such a syntax is still very crude as a representation of natural language, but it is, I think, accurate enough to illustrate the relevance of factual semantics to fuller treatments of language. Constants and their predicates will accordingly be understood to have their usual first-order interpretations, individuals and classes. Gerunds will, of course, have facts as extentions. Though there is some reason to distinguish sentences from gerunds syntaxtically, there seems to be little reason to do so semantically. Indeed, ordinary usage suggests the contrary. For example, in paradigm uses of declarative sentences, as in scientific reporting or legal testimony, one is naturally said to be ‘describing the facts’. It seems to be understood that there are many facts, corresponging roughly to the various sentences in the description. In a court of law, for instance, one asks for ‘the facts, all the facts’ and for ‘ the whole truth’. It is a satisfying vindication of ordinary usage to have a serious reason for introducing into semantic theory a non-linguistic pluarity that may serve as these ‘facts’. Sentences then will have facts as extentions. Predicates of gerunds then must stand for something that together with a fact yields in a rule like way another fact; set theoretic candidates are not hard to find, but I shall not pause to do so here. There do remain, however, two interesting problems. The first is that some gerundive predicates are clearly intensional, and this fact must be given some sort of semantic explanation. The second is to explain how to use the notion of fact to define a non-bivalent theory of factual presupposition.

Many if not most gerunds occur within intensional, fully opaque contexts, e.g. those cases in (2)–(3). Predicates like 'cause', 'be astonished at', 'is interesting', 'doubts' – almost any predicate suggesting a judgement by less than ominiscient human beings – can apply to a fact under one description but not another: co-extentional gerundive parts are not always substitutable. Within the category of gerundive expressions that is both intensional and presuppositional are the so-called factives, e.g. Plato regretted his flunking of Dionysius. But as with singular terms, presuppositionality and intensionality vary independently, and there seems no reason to tie the explanation of one into that of the other.

	Presuppositional	Non-presuppositional
Intensional	I revere the present king of France. The beheading of Charles I is inexplicable. Plato regretted his flunking of Dionysius.	I believe in Santa Clause. The sinking of Atlantis was first described in the *Timaeus*. Turning on the ignition causes the car to start.
Non-intensional	I tripped the present king of France. I interrupted the crowning of the present king of France.	Santa Clause is hairy. The on-coming of death is quick. The sinking of Atlantis was sudden.

There is no reason why the usual intensional accounts of singular terms and propositional attitudes cannot be adapted to intensional gerunds. One such theory would be founded on a principle of univocity: gerunds stand for facts in and out of intensional contexts. Let the intensions of both gerunds and sentences be understood to be functions from possible worlds w (e.g. models or their indices as in the semantics of Section 4) to facts. The intension g of an intensional gerundive predicate, e.g. 'is alarming', would be defined so that when applied to the intension h of a gerund, it would yield the intension $g(h)$ of a sentence. It would do so in such a way that $h(w)$ would not functionally determine $g(h)(w)$, and hence the predicate would be intensional. An alternative analysis, which has been suggested by Enrique Delacruz [1] for factives, is based on the idea of Frege's that expressions in intensional contexts stand for their usual intension. But I should like to reject such an approach in favor of one respecting the ideal of univocity. Delacruz ties the explanation of intensionality, needless I think, to that of presuppositionality of factives. (He is not concerned with gerunds in general but only with the complements of factive verbs, and makes no use of facts in addition to truth-values and intensions.) A further drawback of his theory is its analysis of presupposition itself, which is essentially Russellian. He reads factives which appear to assert a relation between the subject and its complement as disguised quantificational statements as in Russell's theory of description. Their logical form is $cR(\iota x\colon \phi x)$ where c is a proper name, R is a 2-place predicate, and $(\iota x\colon \phi x)$ is a definite description standing for the complement. But $cR(\iota x\colon \phi x)$ is itself just an abbreviation for $(\exists x)(\phi x \mathbin{\&} (\forall y)(\phi y \to x = y) \mathbin{\&} cRx)$. The entity that satisfies this formula is the usual intension of the complement. Typical of a Russelian theory, presupposition is explained by the fact that the whole and its internal negation logically entail cRx. In part as an alternative to Delacruz's theory, I would like now to take up the second problem of this section: how to extend non-bivalent presuppositional analyses to gerunds interpreted by facts.

I will illustrate how the theory of facts may be extended to capture two non-classical theories of presupposition, a three-valued theory modeled on S. C. Kleene's weak connectives [4] and [5] (p. 334) and Hans Herzberger's four-valued product logic in [2]. In each theory a many-valued concept of 'obtaining' will be defined that partitions facts for sentences and gerunds into three and four categories respectively. Each category will determine a truth value; that which determines T can be read as 'obtaining' and that which determines F as 'not obtaining'. These two categories are thus not exhaustive. In either theory the fact that some gerundive predicates are presuppositional may be built into the intension of those predicates. Let the intensions of gerundive predicates, gerunds, and sentences be as before. What makes a predicate presuppositional is that its intension g applies to that of the intension h of a gerund to yield the intension $g(h)$ of a sentence, and it does so in such a way that for any possible world w, if $h(w)$ does not obtain, then $g(h)(w)$ neither obtains nor does not. But before such a program can be entertained as plausible, three substantive issues must be addressed: (1) the relevant partitioning of facts must be defined, (2) the factual structure must be shown to be homomorphic to the many-valued matrix of its theory, and (3) T2 and the many-valued version of T3 must continue to hold. Appropriate definitions will insure (1) and (2) and slight alterations of previous proofs will guarantee (3).

For the theory based on Kleene's weak connectives the following definitions are used. A model is unchanged except that $R(P^n)$ is a function from $\mathbf{U}^n$ into $\{T, F, N\}$. An atomic fact is any $\langle Q_1^n, Q_2^n, x_1, ..., x_n \rangle$ such that $Q_1^n, Q_2^n \subseteq \mathbf{U}^n$, $Q_1^n \cap Q_2^n = \Lambda$, and $x_i \in \mathbf{U}$. Let $-\langle Q_1^n, Q_2^n, x_1, ..., x_n \rangle$ be $\langle Q_2^n, Q_1^n, x_1, ..., x_n \rangle$, and $h(\langle Q_1^n, Q_2^n, x_1, ..., x_n \rangle)$ equal T if $\langle x_1, ..., x_n \rangle \in Q_1^n$, equal F iff $\langle x_1, ..., x_n \rangle \in Q_2^n$, and equal N if neither. A fact $f(n, m)$ is *bivalent* iff all its atomic elements are either T or F under h, and $h(f(n, m))$ equals T if $h(n, m)$ is bivalent and all elements of some row are T, equals F if bivalent and not T, and equal N if not bivalent. A factual assignment is unchanged except that $\phi_M(P^n e_1 ... c_n) = [\langle Q_1^n, Q_2^n, R(c_1), ..., R(c_n) \rangle]$ where $Q_1^n = \{\langle x_1, ..., x_n \rangle | R(P^n)(\langle x_1 ... x_n \rangle) = T\}$ and $Q_2^n \{\langle x_1, ..., x_n \rangle |$ $R(P^n)(\langle x_1, ..., x_n \rangle) = F\}$. Similar adjustments can be made to capture a three-valued theory based on Kleene's strong connectives ([5], pp. 334–5).

A much more interesting many-valued account of presupposition is Herzberger's. It is possible to give convincing interpretations to his four truth-values and at the same time to define a classical semantic entailment relation. With all the artificiality of Russell's theory of presupposition, its main virtue is its retention of classical logic. Herberger's theory permits the retention of classical logic, the main virtue of Delacruz's account, in a theory of presupposition for gerunds that is both non-bivalent and parallel to that of singular terms. Let a model be as before except $R(P^n)$ is a function from $\mathbf{U}^n$ to $\{T = \langle 1, 1 \rangle, F = \langle 0, 1 \rangle, t = \langle 1, 0 \rangle, f = \langle 0, 0 \rangle\}$. An atomic fact is any $\langle Q_1^n, Q_2^n, x_1, ..., x_2 \rangle$ such that $Q_1^n, Q_2^n \subseteq \mathbf{U}^n$, and $-\langle Q_1^n, Q_2^n,$ $x_1, ..., x_2 \rangle$ is $\langle \overline{Q_1^n}, \overline{Q_2^n}, x_1, ..., x_n \rangle$ where $\overline{Q_1^n}$ and $\overline{Q_2^n}$ are the set theoretic complements of Q_1^n and Q_2^n. For a straightforward interpretation we would require that $Q_2 = \{\langle x_1, ..., x_n \rangle | R(P^n)(\langle x_1, ..., x_n \rangle) \in \{T, F\}\}$ be the extension of the existence predicate and that it be the same for all predicates; $Q_1 = \{\langle x_1, ..., x_2 \rangle | R(P^n)(\langle x_1, ..., x_n \rangle) \in \{T, t\}\}$ is intuitively the extension of P^n. The h-values for $f(n, m)$ are computed analogously to Herzberger's tables.

The first result holds for the two extentions suggested because all sentences with the same normal forms have the same value in every many-valued valuation computed according to the relevant matrix. The second result holds because analogous models required for the proof are definable in each case. For the weak connectives, for example, we let $\{\langle x_1, ..., x_n\rangle \mid R(P^n)(\langle x_1, ..., x_n\rangle) = T\} = \{\langle c_1, ..., c_n\rangle \mid v(P^n c_1 \dots c_n) = T\}$ and $\{\langle x_1, ..., x_n\rangle \mid R(P^n)(\langle x_1, ..., x_n\rangle) = F\} = \{\langle c_1, ..., c_n\rangle \mid v(P^n c_1 \dots c_n) = F\}$. Similarly for Herzberger's theory, $R(P^n)(\langle x_1, ..., x_n\rangle) = \langle y, z\rangle$ iff $v(P^n c_1 \dots c_n) = \langle y, z\rangle \;\&\; R(c_i) = x_i$.

Lastly, I would like to point out that when normal forms are defined in such a way that two sentences are logically equivalent only if they have the same normal form, which is possible in classical logic, then T3 directly entails the converse of T2. The strengthened form of T2, then, holds both for classical factual semantics and for its extension in the manner of Herzberger.

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NOTES

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1 There are, to be sure, non-presuppositional uses of gerunds, and though I shall have nothing to say about their semantics, they do not seem to vitiate the parallel between gerunds and singular terms. On the contrary, they appear to follow the non-presuppositional uses of singular terms. For example, there are negative existentials as in 'Moving the Earth with a lever is impossible', generalizations as in 'Turning on the ignition starts the car', literary uses as in 'the sinking of Atlantis', and intensional uses as in 'John was afraid of meeting Ortcut' parallel to 'John was afraid of Rumpelstiltskin'

2 The factual structure in question will be understood to be that determined by the syntactical structure consisting of sentences and the formation operations and by the function T defined by van Fraassen assigning to each sentence the fact which makes it true. For failure of expressions of the same normal form to have the same interpretation observe $T(A \vee B. \wedge. C \vee B) = \{\{\neg A, \neg C\}, \{A, B\}, \{B, C\}, \{B\}\} \neq \{\{A, C\}, \{B\}\} = T(A \wedge C. \vee B)$. For failure of functionality observe $T(A \wedge. B \vee C) = \{\{A, B\}, \{A, C\}\} = T(A \wedge B. \vee. A \wedge C)$, but $T(\neg(A \wedge. B \vee C)) = F(A \wedge. B \vee C) = \{\{\neg A\}, \{\neg B, \neg C\}\} \neq \{\{\neg A\}, \{\neg A, \neg C\}, \{\neg A, \neg B\}, \{\neg B, \neg C\}\} = F(A \wedge B. \vee. A \wedge C) = T(\neg(A \wedge B. \vee. A \wedge C))$. If $\langle T(A), F(A)\rangle$ rather than $T(A)$ is taken as the interpretation of A, functionality is achieved but at the expense of awkwardness.

REFERENCES

[1] Enrique B. Delacruz, 'Factives and Propositional Level Constructions in Montague Grammar', in *Papers in Montague Grammar*, ed. R. Rodman. (Linguistics Dept., U.C.L.A., (1972).

[2] Hans G. Herzberger, 'Dimensions of Truth', *J. of Philosophical Logic* **2** (1973), 535–556.

[3] Jaegwon Kim, 'Causation, Nomic Substitution, and the Concept of Event', *J.P.* **70** (1973), 217–236.

[4] S.C. Kleene, 'On a Notation for Ordinal Numbers', *Journal of Symbolic Logic* **3** (1938), 150–155.

[5] S.C. Kleene, *Introduction to Metamathematics* (Amsterdam: North Holland, 1959.)

[6] R.M. Martin, 'Facts: What They Are and What They Are Not', *A.P.Q.* **4** (1967), 269–280.

[7] Bertrand Russell, 'The Philosophy of Logical Atomism', in *Logic and Knowledge*, ed. R. Marsh. (London: Allen & Unwin, 1956.)

[8] Bas C. van Fraassen, 'Facts and Tautological Entailment', *J.P.* **66** (1969), 477–487