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Distributive Terms, Truth, and the *Port Royal Logic*

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The paper shows that in the *Art of Thinking (The Port Royal Logic)* Arnauld and Nicole introduce a new way to state the truth-conditions for categorical propositions. The definition uses two new ideas: the notion of *distributive* or, as they call it, *universal term*, which they abstract from distributive supposition in medieval logic, and their own version of what is now called a *conservative quantifier* in general quantification theory. Contrary to the interpretation of Jean-Claude Parienté and others, the truth-conditions do not require the introduction of a new concept of 'indefinite' term restriction because the notion of conservative quantifier is formulated in terms of the standard notion of term intersection. The discussion shows the following. Distributive supposition could not be used in an analysis of truth because it is explained in terms of entailment, and entailment in terms of truth. By abstracting from semantic identities that underlie distribution, the new concept of distributive term is definitionally prior to truth and can, therefore, be used in a non-circular way to state truth-conditions. Using only standard restriction, the *Logic*'s truth-conditions for the categorical propositions are stated solely in terms of (1) *universal (distributive) term*, (2) *conservative quantifier*, and (3) *affirmative* and *negative proposition*. It is explained why the Cartesian notion of extension as a set of ideas is in this context equivalent to medieval and modern notions of extension.

Introduction

In the *Art of Thinking*, Arnauld and Nicole advance a definition of truth for categorical propositions that for the first time states truth-conditions using the concept of distributive term. This paper explains the historical and technical background underlying this definition. The distributive properties of categorical propositions had been part of logical lore since Aristotle. Aristotle knew, for example, that a true universal affirmative has a distributive subject and non-distributive predicate, and it was a standard doctrine in medieval logic that in a universal affirmative the subject has distributive but its predicate a 'merely confused' supposition. Although such rules of thumb provide necessary and sufficient conditions for a proposition's truth, these equivalences were not understood as statements of the truth-conditions. Either distribution was understood syntactically as a non-semantic concept, or if it was understood semantically, it was itself explained in terms of entailment. When explained in terms of entailment, however, distribution could not be understood as conceptually independent of truth because entailment itself was defined in terms of truth.[1] Any explanation of truth by distribution would be circular. What is novel in the *Port Royal Logic* is that it abstracts a definition of distribution that is independent of entailment and truth. It employs what are essentially abstract versions of the traditional 'definitions' of distribution that are stated not in terms of truth, but in terms of identities among referents which are semantically prior to the definition of truth. As a result, it is able to use distribution in truth-conditions in a non-circular way.

The story is in part historical and in part technical. The history has two parts. In Part I, a proto-syntactic sense of distribution is distinguished from various semantic senses, and it

[1] It shall be the practice in this paper to italicize the names of concepts only if they are being defined as part of a formal definition. Expressions that are being mentioned will also be italicized.

is explained how in the *Art of Thinking* and earlier logic it is used in 'tests' of the validity of syllogistic moods. In Part II, the semantic sense is distinguished. Its roots are traced to the medieval notion of the *term having distributive supposition*. It will be shown, however, that the medieval notion is not conceptually independent of the concept of truth. To the extent that *distributive supposition* was ever defined, it was explained in terms of entailments – in terms of consequences called 'ascent' and 'descent' – which were in turn defined by truth. Parts III and IV lay out the technical part of the story. It is shown that by abstraction from its medieval formulation in terms of entailment and truth, the *definiens* of *distributive term* can be given a logically equivalent formulation in terms of the direct referential properties of terms that do not depend on a prior definition of truth. It is then possible to directly state the truth-conditions for the categorical proposition in terms of distribution in the manner of the *Port Royal Logic*.

The results are interesting for three reasons. First of all, they bear on Jean-Claude Parienté's well-known claim that the *Logic* introduces to the grammar and semantics of categorical propositions a new operation of so-called indefinite term restriction. This new variety is not the restriction familiar from medieval logic in which a term like *man* is restricted by a term like *fat* to form the composite *fat man* that stand for the intersection of what its components stand for. Rather, under the new restriction a term like *man* is restricted by the quantifier *some* to form *some man* which is supposed to stand for some undetermined subset of what *man* stands for.[2] On Parienté's interpretation, for example, *some S is P* is true if and only if the extension of *some S*, which is an indeterminate subset of what *S* stands for, is a subset of that of *P*.[3] This interpretation, however, is based in large part on the claim that the *Logic*'s truth-conditions cannot be explained in terms of ordinary restriction. His argument for the new restriction is based on the contention that the single operation is insufficient for characterizing the types of inferences to singular propositions (so-called ascents and descents). These inferences are typical of the various terms of categorical propositions and were classified in medieval logic according to the type of 'supposition' they possess. As will be shown below, however, Parienté's reading fails to recognize the importance of the *Logic*'s distinction between 'universal' or 'distributive' term, which occupies a key role in the *Logic*'s truth-conditions for categorical propositions. It will be shown that the distinction is a direct abstraction from that between distributive and non-distributive supposition. Because the new distinction abstracts away from any conceptual dependence on the concept of truth, it becomes available for use in the analysis of the concept of truth itself. It will become clear in the analysis that only a single notion of restriction occurs, one that is essentially a version of the traditional operation identified in medieval logic.[4]

The paper also makes a supporting metatheoretic claim. To employ the notion of the distributive term in the formulation of truth-conditions and to negotiate issues of the quantifier scope, Arnauld and Nicole found the need to formulate, for the first time, the distinction between conservative and non-conservative quantifiers as understood in modern generalized quantification theory.

The overall significance of the discussion, however, is that it explains how the *Logic* introduces a new formulation of the truth-conditions for the categorical propositions. These

[2] For the traditional operation of restriction see *Buridan*, *Summulae* 5.1.8. (*2001*, pp. 286, 648, 835) and *Fonseca*, *Institytionum dialecticarum*, Liber VIII, Caput 40 (*1964*, pp. 740–741). For restriction in the *Logic* see *Logique et l'Art de Penser* (hereafter *LAP*) I,6; *Kremer and Moreau 2003* (hereafter *KM*) V (pp. 145,40); *Arnauld and Nicole 1996* (hereafter *B*) (p. 40); *LAP* I,7,*KM* V (p. 151), *B* (p. 45); *LAP* II,17, *KM* V (p. 248), *B* (p. 130); and *KM* V (p. 250), *B* (p. 131).

[3] Strictly speaking, on this analysis the truth-conditions are formulated in terms of identity: *some S is P* is true iff the indefinite extension of *some S* is identical to the extension of the (definite) restriction of *P* by that of *some S*.

[4] For Parienté's views on supposition see *Parienté 1985* (pp. 273–274). For the argument for indefinite restriction see page 242 and Chapters 8 and 9. On the double restriction reading see also *Auroux 1993* (pp. 148–149, 87) and *Dominicy 1984* (pp. 167–168).

are stated using the notion of the distributive term. Modern logicians, who look askance at 'Aristotelian' logic, rightly find puzzling the concept of distributive term, which they encounter in logic textbooks as part of the 'rules' for the valid moods. In these texts, the concept is poorly explained. One can learn to identify a distributive term syntactically, but one looks in vain for a clear semantic account. If, on the other hand, the tradition over the last 300 years had preserved what the *Port Royal Logic* said about distributive terms, both in the rules for the valid moods and in its truth theory, not only would the rules have made semantic sense, but a link would have been maintained to the rich metatheory of the Middle Ages.

Part I. Distribution as a syntactic concept: the syllogistic rules

In modern logic a clear distinction is drawn between syntax, on the one hand, understood as including both grammar and proof theory, and semantics, on the other. On Morris's definition, syntax studies the relation of signs to signs, and semantics the relation of signs to both signs and the world.[5] In practice, however, the distinction is marked by concepts and methods foreign to the seventeenth century. Syntactic sets are defined by finite lists of syntactic entities or by inductive definitions that close a previously defined syntactic set under syntactic rules, for example, formation rules in grammar and inference rules in proof theory. In grammar and frequently in proof theory syntactic sets are decidable. Semantics, on the other hand, assumes set theory including at times the axiom of choice, and not infrequently ontological categories outside set theory. It is normal for sets in semantics to be defined by comprehension (abstraction) rather than induction, and for them to be undecidable.

In the pre-nineteenth century logic, the distinction between syntax and semantics is muddled at best, and the muddle affects 'distribution'. An ambiguity between a syntactic and semantic sense dates to the first technical use of the term by Aristotle. In the *De interpretation,* he distinguishes a term that 'stands for many', which he calls *universal* – in later logic, the terms *distributive* and *universal term* came to be used interchangeably – from a term that 'stands for a particular'. Today we would judge the distinction semantic because it concerns the relation of a sign to its referent:

> I call universal [*catholou*] that which is by its nature predicated of a number of things, and particular that which is not; man, for instance, is a universal, Callias a particular. (17a40, Ackrill, trans.)

Here a universal term seems to be what later logicians call a general term or common noun, and the *Logic* retains this usage as one sense of a general or universal term.[6] In the same paragraph, however, Aristotle goes on to explain a universal proposition – one that states 'universally of a universal' [*catholou epi tou catholou*] – by what could be regarded as a syntactic definition:

> what I mean by 'stating universally of a universal' are 'every man is white' and 'no man is white'. (17b5)

Here a universal term is marked syntactically by its syntax within a categorical proposition – it is a subject term modified by *every* or *no*.

Indeed, in the Middle Ages, and for centuries afterwards, logic students memorized what a distributive term is by the syntax of its containing proposition: in a universal proposition the subject is distributed, but in particular it is not; in a negative proposition the predicate is distributed, but in an affirmative, it is not.

[5] *Morris 1939*.
[6] *LAP* I,6; *KM* V (p. 144); and *B* (p. 39).

In the *Logic* Arnauld and Nicole retain this sense of distributive or, as they prefer to call it, universal term. They say that the subject of an affirmative is universal;[7] the predicate of an affirmative proposition is not universal;[8] the predicate of a negative proposition is universal;[9] and the subject of a particular negative is not universal.[10] They explain what they mean by a universal term in both syntactic and semantic terms. Syntactically, they draw the difference between, on the one hand, a universal proposition and the universal subject term and, on the other, a particular proposition and the particular subject term in terms of the particular quantifier that modifies the subject term. They point out that in the universal case subjects are 'joined to universal signs expressed or understood, like *all* (*omnis, tout*)', and that in the particular case they are joined to 'the word *some* (*aliquis*)'.[11] From a modern perspective, it is clear that the distinction between distributive and non-distributive term (between universal and particular term) can easily be drawn syntactically because the defining distinctions, namely between affirmative and negative, and universal and particular propositions, have obvious syntactic definitions in terms of word order, and the occurrence of quantifier terms and the copula.

It is also clear that in this syntactic sense, the universal term is not a common noun because in grammar, unlike a distributive term, a common noun can occupy any term position – it can be the subject or predicate position of both universal and particular propositions.

The *Logic*, moreover, also draws the distinctions between universal and particular proposition, and between a universal and particular term semantically:[12]

> For when the subject of a proposition is a common term taken in its entire extension, the proposition is universal. . . . When the common term is taken only through an indeterminate part of its extension, because it is restricted by the indeterminate word 'some', the proposition is called particular.

And again,

> The universality or particularity of a proposition depends on whether the subject is taken universally or particularly.
> Since the attribute of an affirmative proposition never has a larger extension than the subject, it is always regarded as taken particularly, because it is only accidental if it is sometimes taken generally.
> The attribute of a negative proposition is always taken generally.

[7] Axiom 1. For the discussion of this and later axioms see *LAP* II,17, *KM* V (pp. 248–252), and *B* (pp. 130–133).

[8] Axiom 4.

[9] Axiom 5, remark, and Axiom 6.

[10] Axiom 6, remark.

[11] *LAP* II,3; *KM* V (p. 191); and *B* (p. 83).

[12] Car lorsque le sujet d'une proposition est un terme commun qui est pris dans toute son étendue, la proposition s'appelle universelle …Et lorsque le terme commun n'est pris que selon une partie indéterminée de son étendue, à cause de qu'il est resserré par le mot indéterminé *quelque,* la proposition s'appelle particulière…(*LAP* II,3; *KM* V, p. 191; *B* pp. 83–84).

2. Le sujet d'une proposition, pris universellement ou particulièrement, est ce qui la rend universelle ou particulière.

3. L'attribut d'une proposition affirmative n'ayant jamais plus d'étendue que le sujet, est toujours considéré comme pris particulièrement : parce que ce n'est que par accident s'il est quelquefois pris généralement.

4. L'attribut d'une proposition négative est toujours pris généralement

(*LAP* II,3, *KM* pp. 258–259; B, p. 139).

Mai quoique cette proposition singulière soit différente de l'universelle en ce que son sujet n'est pas commun, elle s'y doit néanmoins plutôt rapporter qu'à la particulière; parce que son sujet, par cela même qu'il est singulier, est nécessairement pris dan toute son étendue, ce qui fait l'essence d'une proposition universelle, & qui la distingue de la particulière. Car il importe peu pour l'universalité d'une proposition, que l'étendue de son sujet soit grande ou petite, pourvu que quelle qu'elle soit on la prenne toute entière. Et c'est pourquoi les propositions singulières tiennent lieu d'universelles dans l'argumentation. Ainsi l'on peut réduire toutes les propositions à quatre sortes, que l'on a marquées par ces quatre voyelles A.E.I.O. pour soulager la mémoire (*LAP* II,3; *KM* V, p. 199; *B* p. 84)

In these passages, a universal term is one that is 'taken universally' and a particular term is one that is 'taken particularly'. Exactly what these and similar semantic explanations mean is a large topic, one which will occupy Parts III and IV. In general, however, it is not at all surprising that the same concept might have both a syntactic and semantic analysis. This is exactly the case in modern metatheory with concepts like entailment and consistency. We shall see that a similar correspondence holds between the *Logic*'s semantic sense of distributive term and its syntactic sense. Indeed, proof theoretic rules formulated in the syntactic sense are valid under the semantic sense.

It is the semantic sense that is key to the Cartesian theory of truth, the main topic of this paper. In what remains of this section, however, we fill out the syntactic sense both because of its prominence in syllogistic practice and because of its correspondence to the latter semantic concept.

The primary use of distributive terms in the syntactic sense is in the statement of syntactic rules characterizing the valid moods. First, more primitive syntactic terms are defined syntactically: *term; quantifier; the copula*; *universal* and *particular proposition*; *affirmative* and *negative proposition*; *syllogism*; *major* and *minor premise*; *conclusion*; *major*, *middle*, and *minor term*. A syntactic sense of *distributive term* can then be defined as any term satisfying the criteria listed in logic student's traditional formula above.

The *Logic* is well known for its six rules characterizing the valid syllogistic moods, the first two of which employ the notion of the distributive (universal) term:[13]

> Rule 1: *The middle term cannot be taken particularly twice, but must be taken universally once.*
> Rule 2. *The terms of the conclusion cannot be taken more universally in the conclusion than in the premises.*
> Rule 3. *No conclusion can be drawn from two negative propositions.*
> Rule 4. *A negative conclusion cannot be proved from two affirmative propositions.*
> Rule 5. *The conclusion always follows the weaker part. That is, if one of the two propositions is negative, the conclusion must be negative; if one of them is particular, it must be particular.*
> Rule 6. *Nothing follows from two particular propositions.*

If the concepts employed in the rules are understood syntactically, then the rule set as a whole characterizes the valid syllogisms in the sense that a mood is valid if, and only if, it does not violate any rule in the set. Moreover, it is straightforward to test whether a syllogism violates a rule. It follows that understood syntactically, the rule set provides a syntactic decision procedure for the valid moods.

It should be remarked that the rule set is not new to the *Port Royal Logic*. Rules 3–6 and many similar rules had been cited in logical treatises since ancient times. Rules 1 and 2, the so-called process rules, grouped together with the other four as a distinct set of six rules for the purpose of characterizing the valid moods, are found in earlier sixteenth-century logic

[13] *LAP* III,3 and *B* (pp. 138–142). Le moyen ne peut être pris deux fois particulièrement, mais il doit être pris au moins une fois universellement.

Les termes de la conclusion ne peuvent point être pris plus universellement dans la conclusion que dans les prémisses.

On ne peut rien conclure de deux propositions négatives.

On ne peut prouver une conclusion négative par deux propositions affirmatives.

La conclusion suit toujours la plus faible partie; c'est-à-dire, que s'il y a une des deux propositions négatives, elle doit être négative, & s'il y en a une particulière, elle doit être particulière.

De deux propositions particulières il ne s'enfuit rien (*KM* V, pp. 259–263).

texts.[14] Rules of this sort are a part of a long tradition of formulating tests for identifying the valid moods, a tradition that includes the various well-known mnemonics featured in logic texts from the Middle Ages onward.[15]

The authors of the *Logic* do seem to appreciate the significance of the fact that the rules characterize the valid moods because they argue for a kind of soundness. As they present the rules one by one in Book III, they argue that a syllogism that violates a rule is invalid. Thus, if a syllogism is valid, it does not violate a rule. By reviewing the 256 moods, it is also easy to check that the converse holds as well – that any mood other than the 24 valid moods violates at least one rule. This characterization, moreover, is not entirely trivial. The rule set characterizes not only the 24 valid moods but the broader set consisting of all arguments formulated in terms of categorical propositions with a finite number of premises. This generalization is due to the fact that, for any finite set of categorical propositions X and categorical proposition A, the argument from X to A is valid if, and only if, there is a finite series of valid syllogisms such that the conclusion of the final syllogism is A and any premise of any syllogism in the series is either in X or is a conclusion of a previous syllogism in the series.[16]

Too much, however, can be made of these syntactic points. In pre-modern logic, including the *Port Royal Logic*, there simply did not exist a clear notion of the difference between syntax and semantics, nor of infinite set, nor of the modern notion of the decision procedure as a calculable characteristic function of a set. Though earlier logicians rightly regarded the rules as a simple test for the valid syllogisms, it would be an anachronism to think that the authors of the *Logic* understood their rules to define an effective syntactic decision procedure for the infinite set of valid categorical arguments or even for just the 24 valid moods. Nevertheless, as an elegant presentation of syllogistic logic, the rules clearly contributed to the *Logic*'s historical influence and are in part responsible for its reputation as a step in the direction of formal logic. Kneale and Kneale, for example, appraise the rules in this way:[17]

> Their quasi-mathematical treatment of these subjects may indeed be the first of its kind, as it is certainly the course from which later writers of logical manuals derive the details of their formal theory, e.g. the determination of the valid moods of the syllogism and their proofs of the special rules of the various figures. . . .the general conception of logic which they expounded in this book was widely accepted and continued to dominate the treatment of logic by most philosophers of the next 200 years.

Taking the *Logic*'s proto-syntactic formulation one step further, Leibniz in fact reformulated the six rules symbolically and proved that they characterize the valid moods in a way highly suggestive of modern formal methods.[18] We may conclude, then, that there is a clear

[14] For example, for Rules 3–7 see *William of Sherwood*, *Introduction to Logic* III, ix (*1966*, p. 67); *Peter of Spain, Tractatus*, IV, 4 (*1990*, p. 40); *De Rijk 1962–1967* (p. 45); *Buridan*, *Summulae* 5.1.8 .(*2001*, pp. 312–313); and *Fonseca*, *Institytionum dialecticarum*, Liber VI, Caput 18 (*1964*, p. 382). For Rules 1 and 2 see *Toletus*, *In Lib I posteriorum analyticorum,* Cap XIX (*1580*, p. 202) and *Fonseca*, *Institytionum dialecticarum*, Liber VI, Caput 20 (*1964*, p. 386). All six rules are presented as a group in *Eustace of St. Paul*'s popular summary of scholastic philosophy, which was read and praised by Descartes for its clarity, *Summa philosophiae quadripartita, Logia* III.2.I (*1609*, p. 117). Letter to Mercenne, *Descartes 1897–1909* (pp. 3, 251).

[15] See, for example, *William of Sherwood*, *Introduction to Logic* III, 9 (*1966*, p. 66); *Peter of Spain*, *Tractatus* IV, 13 (*1990*, p. 46), and *de Rijk 1962–1967* (p. 52); and *Buridan*, *Summulae*, 5.2.2 (*2001*, p. 320).

[16] The relevant semantics for the syllogistic is set out in Part IV. For a proof of this generalization see *Martin1997*, reprinted in *Martin 2004*.

[17] *Kneale and Kneale 1962* (p. 320).

[18] Leibniz lists seven rules, dividing the *Logic*'s fifth rule into two. See *Lenzen 1990* (pp. 29–59). It should be remarked that though Leibniz (and Lenzen) presents the rule set as an 'axiomatization', neither his account nor the *Logic*'s is a true axiomatization of the valid moods. An axiom system characterizes a set of theorems as an inductive set, that is, as a set defined as the closure of a

sense in which distributive term in the *Port Royal Logic* may be understood in a syntactic sense and that its use in the syllogistic rules as a test for the valid moods may be understood as an early version of a decision procedure. As we shall see in the following sections, this syntactic sense is coextensive with the semantic concept of distributive term, which we shall see is key to the *Logic*'s analysis of truth. Part II outlines the historical origins of the semantic concept in medieval logic.

Part II. Distribution in medieval logic

The semantic sense of distribution has its home in medieval supposition theory. Supposition became a standard part of the 'logic of terms' in the twelfth century. Although logicians differed on some details, those parts that influenced the *Port Royal Logic* were widely taught.

Supposition is the tradition's concept of contextually dependent reference. The variety of supposition relevant to this discussion is a subspecies of common supposition distinguished by a term's 'quantity'. Its quantity is marked by the modifiers *every* and *some*, which today we call quantifiers. A term modified by *every* was said to have distributive common supposition and one modified by *some* non-distributive.

Quantification is, of course, tricky to explain. The explanation employed by medieval logicians appealed to what we would call today logical entailment. The details of the explanation varied, but by the time of Arnauld and Nicole, the account was relatively standardized.[19] A proposition containing a term modified by *every* was thought to entail or, in the jargon of the day, 'descend to' all singular instances of that proposition for that term. Equivalently in their view, it descends to the conjunction of these instances. Conversely, this conjunction was said to entail or 'ascend to' the proposition itself. With some important qualifications to be explained shortly, a proposition containing a term modified by *some* was held to entail or descend to at least one singular instance of that proposition for that term or, equivalently in their view, to the disjunction of those instances. Conversely, this disjunction was held to entail or ascend to the proposition itself. Since a valid entailment was universally acknowledged to be defined in terms of truth (because a valid consequence is one that preserves truth), distribution in this sense, like entailment, is definitionally dependent on truth.

The distinction was also held to apply to the predicates of categorical propositions because they too support valid descents and ascents to conjunctions and disjunctions of instances. The details of the theory, however, quickly become technical because the inferences themselves are complex. Because it is from the logical form of the relevant entailments that the Cartesians abstracted their notion of distributive term, it is necessary to explain the relevant details.

For the moment, we shall restrict attention to the four primary categorical forms, representing a subject term by S and a predicate by P. We shall call two terms that occur in the same proposition *collateral*. The entailments in question presuppose that the actual

set of basic elements (axioms) under a set of construction rules (rules of inference). The *Logic*'s rules, on the other hand, provide a decision procedure, not an axiomatization. Curiously, the students' mnemonic poems previous mentioned are theoretically more powerful than the *Logic*'s six rules because they provide not only a decision procedure but also an axiomatization. They encode how to 'reduce' the valid moods to Barbara and Celarent (via four 'inference' rules), and a 'reduction' is easily converted into a proof in the modern sense. See the references in Part IV for details.

[19] Here we shall discuss the version of ascent and descent put forward by John Buridan, not because his medieval text was known by Arnauld and Nicole, but because it is particularly clear about the logical relations at issue and because it contains all the relevant points that were to become common in the standard account from which Arnauld and Nicole abstracted their idea of distributive term. On the standard account see *Corazzon*. For an example of a contemporary account, see *Fonseca 1964* [1575] Liber VIII (Chapters 20–22, pp. 678–688), a text which was part of the Ratio Studiorum of 1599 for philosophy professors at Jesuit colleges of the period.

referents (supposita) of any term in a true proposition are named individually. That is, it is assumed that for any subject term S, there is an associated series of singular terms $s_1, \ldots, s_n$, usually assumed to be finite in number, that stand for all the actual entities that S supposits for. Likewise it is assumed that for any predicate P there is an associated series of singular terms $p_1, \ldots, p_m$ that name all the objects that P actually supposits for. Let i range over $\{1, \ldots, n\}$, and j over $\{1, \ldots, m\}$. An instance of a proposition for a term is then defined in terms of these associated names. In the definitions below, an instance of a proposition for a term is a singular proposition in which an associated singular term is the subject and the term in question is the predicate.

Definitions

> Relative to a subject term S and predicate term P,
> a *positive instance of S for P* is any proposition s_i *is* P;
> a *negative instance of S for P* is any proposition s_i is not P;
> a *positive instance of P for S* is any proposition S *is* p_j;
> a *negative instance of P for S* is any proposition S *is not* p_j.

A *conjunctive instance of a term* is the conjunction of that term's instances for its collateral term, and a *disjunctive instantiation* is the disjunction of these instances.

A term may then be said to have distributive supposition if it entails its conjunctive instance. John Buridan's commentary may serve as a paradigm:[20]

> [Text] Distributive supposition is that in accordance with which from a common term any of its supposita can be inferred separately, or all of them at once conjunctively, in terms of a conjunctive proposition. For example, from 'Every man runs' it follows that therefore 'Socrates runs', ... therefore 'Socrates runs and Plato runs,...' and so on for the rest.
> ...
> [Commentary] distributive supposition differs from the other suppositions, for in its case a common term implies any of its singulars separately, whereas the other suppositions do not. Therefore, if the proposition is true, it has to be true for any suppositum, which is not required in the other cases of supposition.

If a term is used in distributive supposition, the proposition entails – 'descends to' – all the term's immediate instances. Equivalently, it entails their conjunction.

Definition. Relative to a categorical proposition, a term is used in *distributive* supposition iff the proposition entails all of the term's instances for its collateral term or, equivalently, iff it entails their conjunction.

By this criterion, the four terms that count as distributive are the subject of the universal affirmative, the subject and predicate of the universal negative, and the predicate of the

[20] Distributiva est secundum quam ex termino communi potest inferri quodlibet suorum suppositorum seorsum, vel etiam omnia simul copulative, secundum propositionem copulativam, ut 'omnis homo currit', sequitur 'ergo Socrates currit', 'ergo Plato currit', vel etiam. . . .
Et manifestum est quod suppositio distributiva differt ab aliis suppositionibus quia terminus communis secundum eam infert quodlibet suorum singularium seorsum; aliae autem hoc non faciunt. Ideo si propositio sit vera, oportet quod sit vera pro quolibet supposito, quod non requiritur in aliis.
(*Summulae* 4.3.6, p. 264).

particular negative. For comparison later, it is helpful to display the relevant conjunctive entailments. Let $\vDash$ represent syllogistic entailment:[21]

Theorems

$$\begin{array}{l}
\textit{every } S \textit{ is } P \dashv\vDash s_1 \textit{ is } P \wedge \cdots \wedge s_n \textit{ is } P \\
\textit{no } S \textit{ is } P \ \dashv\vDash s_1 \textit{ is not } P \wedge \cdots \wedge s_n \textit{ is not } P \\
\textit{no } S \textit{ is } P \ \dashv\vDash \textit{no } S \textit{ is } p_1 \wedge \cdots \wedge \textit{no } S \textit{ is } p_m \\
\textit{some } S \textit{ is not } P \vDash \textit{some } S \textit{ is not } p_1 \wedge \cdots \wedge \textit{some } S \textit{ is not } p_m
\end{array}$$

Note that the converse of the last entailment fails:
$\textit{some } S \textit{ is not } p_1 \wedge \cdots \wedge \textit{some } S \textit{ is not } p_m \nvDash \textit{some } S \textit{ is not } P$

If it were not for the failure of this last entailment, a distributive term could have been explained as one in which its proposition is analytically equivalent to the conjunction of instances derived by instantiating that term.

The case of non-distributive supposition is similar. It can almost but not quite be explained by saying that its proposition is equivalent to the disjunction of instances derived by instantiating that term. To capture this idea at least in part, non-distributive supposition is traditionally divided into two subtypes: determinate and ‘merely confused’. A term has determinate supposition if the proposition entails at least one of the term’s instances. The criterion is also formulated in terms of disjunction. A term has determinate supposition if the proposition entails the disjunction of all the term’s instances. The entailments also hold in the converse direction. Buridan formulates the distinction as follows:[22]

> [Commentary] I should say that in determinate supposition the proposition need not be true for one suppositum only; indeed, sometimes it is true for any suppositum. But it is necessary and sufficient that it should be true for one. So we have to note immediately that there are two conditions for the determinate supposition of some common term. The first is that from any suppositum of that term it is possible to infer the common term, the other parts of the proposition remaining unchanged. For example, since, in ‘A man runs’, the term ‘man’ supposits determinately, it follows that ‘Socrates runs; therefore, a man runs’, ‘Plato runs; therefore, a man runs’, and so on for any singular contained under the term ‘man’. The second condition is that from a common term suppositing in this manner all singulars can be inferred

[21] Part IV lays out the syllogistic model theory in which $\vDash$ is defined. For the purposes of this paper a singular term may be understood as a special case of a categorical term generally: a singular term is one that happens to supposit for a unique actual object. Thus, in the semantics of Part IV a singular term is a term that stands for a unit set. A universal affirmative with a singular term as subject is then understood as a special case of a universal affirmative, one in which the quantifier *every* is not explicitly expressed – such is the way it is understood in the *Logic* (II,2). It will follow from the semantics of Part IV that a universal affirmative with a singular term as subject and common noun as predicate is true iff the unique object in the set that is the referent of the subject is an element of the set that is the referent of the predicate, and that a proposition with either a singular term or common noun as subject and singular term as predicate is true iff the sets referred to by both terms contain one and the same individual. For perspicuity when the two terms are both singular, we shall use $=$ to represent the copula. The connectives $\wedge$ and $\vee$ here should be understood as conforming to the standard truth tables, as was the common medieval practice.

[22] Commentary: Dico tamen quod in suppositione determinata non oportet veritatem esse pro uno solo supposito, immo aliquando est vera pro quolibet, sed hoc requiritur et sufficit quod sit vera pro aliquo uno. Unde notandum est statim quod duae sunt condiciones suppositionis determinatae alicujus termini communis. Prima est quod ex quolibet supposito illius termini possit inferri terminus communis remanentibus aliis in propositione positis. Verbi gratia, quia in ista ‘homo currit’ iste terminus ‘homo’ supponit determinate, ideo sequitur ‘Socrates currit; ergo homo currit’, ‘Plato currit; ergo homo currit’, et sic de quolibet alio singulari contento sub ‘homine’. Secunda condicio est quod ex termino communi sic supponente possint inferri omnia singularia disjunctive, secundum propositionem disjunctivam; verbi gratia, sequitur ‘homo currit; ergo Socrates currit vel Plato currit vel Johannes currit. . .’ et sic de aliis (*Summulae* 4.3.5, p. 263).

> disjunctively, by a disjunctive proposition. For example, 'A man runs; therefore, Socrates runs, or Plato runs or John runs...' and so on for the rest.

Buridan's remarks suggest a possible analysis:

Definition. A term is used in *determinate* supposition iff the proposition entails and is entailed by at least one of the term's instances for its collateral term or, equivalently, iff it entails and is entailed by the disjunction of those instances.

By this criterion, the three terms that count as distributive are the subject and predicate of the particular affirmative, and the subject of a negative particular. Again it is helpful to display the relevant entailments.

Theorems.

$$\textit{some } S \textit{ is } P \dashv\vDash s_1 \textit{ is } P \vee \cdots \vee s_n \textit{ is } P$$
$$\textit{some } S \textit{ is } P \dashv\vDash \textit{some } S \textit{ is } p_1 \vee \cdots \vee \textit{some } S \textit{ is } p_m$$
$$\textit{some } S \textit{ is not } P \dashv\vDash s_1 \textit{ is not } P \vee \cdots \vee s_n \textit{ is not } P$$

The second variety of non-distributive supposition is called confused non-distributive supposition or, briefly, merely confused (*confusus tantum*) supposition. As Buridan explains, its logical relation to its instances is more complicated than either distributive or determinate supposition:[23]

> [Text] But the merely confused supposition is that in accordance with which none of the singulars follows separately while retaining the other parts of the proposition, and neither do the singulars follow disjunctively, in terms of a disjunctive proposition, although perhaps they do follow by a proposition with a disjunct term.
> [Commentary] ... in the case of confused supposition, the singulars cannot be inferred from the common term by means of a disjunctive proposition, whereas this can correctly be done with determinate supposition. For example, in the proposition 'Every man is an animal' the term 'animal' has merely confused supposition, and the inference 'Every man is an animal; therefore every man is this animal or every man is that animal...' (and so on for the rest) is not valid, for the antecedent is true and all the consequents are false.

Because there are only eight term positions in the four categorical propositions and seven have already been determined to have either distributive or determinate supposition, there is only one term that is neither. It is the predicate of the universal affirmative. Only this could be merely confused, and the category could simply be defined by negation:

Definition. A term has *merely confused* supposition iff it has neither distributive nor determinate supposition.

Buridan, however, holds that merely confused supposition can also be explained by characteristic descent and ascent entailments. The relevant entailment is to an instance that has what Buridan calls a 'disjunctive predicate'. Consider the universal affirmative *every man is an animal*. Let the names of the various individual animals be, as medievals would say, *this animal, that animal*, ... It is clear that *every man is an animal* does not entail

[23] Sed confusa tantum est secundum quam non sequitur aliquod singularium seorsum retentis aliis in propositione positis, nec sequuntur singularia disjunctive, secundum propositionem disjunctivam, licet forte sequantur secundum propositionem de disjuncto extremo....
Suppositio autem confusa tantum differt a suppositione determinata quia secundum suppositionem confusam non inferuntur ex termino communi singularia secundum propositionem disjunctivam, quod bene fit secundum suppositionem determinatam. Verbi gratia, in ista propositione 'omnis homo est animal' ... et sic de aliis, quia prima est vera et omnes aliae sunt falsae (*Summulae* 4.3.6, p. 264).

every man is this animal and every man is that animal,etc. Thus, *animal* does not have distributive supposition. Nor does it have determinate supposition because the proposition does not entail even the disjunction of its instances: *every man is an animal* does not entail *every man is this animal*, *or every man is that animal*, *etc*. What does follow is that Buridan's 'disjunctive predicate' is true of the actual individuals that the subject supposits for.

To construct this predicate, observe that the proposition *every man is an animal* may be reformulated in a logically equivalent way as *every man is such that he is some animal.* The anaphoric syntax of the reformulation is similar to that of the bound variable syntax of the rendering in first-order logic of a universal affirmative:[24]

$$\forall x(Sx \rightarrow Px).$$

Here, though S, which translates *man*, is rightly rendered as syntactically simple, the predicate P represents the complex expression *some animal*. It is this complex that is translated by Buridan's disjunctive predicate. The relevant semantic intuition is that the open sentence *x is some animal* is satisfied iff *animal* is true of at least one individual that it actually supposits for. That is, *x is some animal* is equivalent to the disjunction x *is* $a_1 \vee \cdots \vee x$ *is* a_n, where $a_1, \ldots, a_n$ name all the actual animals. Since x ranges over individuals, the relevant sense of the copula is identity. More generally, if $p_1, \ldots, p_m$ name the individuals in the extension of P, then the universal affirmative $\forall x(Sx \rightarrow Px)$ is equivalent to the first-order formula:

$$\forall x(Sx \rightarrow (x = p_1 \vee \ldots x = p_m)).$$

Using naïve set theory, it is even possible to recast this in a syntax more like that of Buridan,

$$\forall x(Sx \rightarrow x \in \{y|y = p_1 \vee \ldots y = p_m\}).$$

Here the set name $\{y|y = p_1 \vee \ldots y = p_m\}$ represents *some P* and $\in$ represents the copula.

In practice, medieval logicians consider that the logical grammar available to them was much broader than the four categorical propositions. We have seen this already in the use of singular terms and connectives. It is this latitude that enables Buridan to wave his hands at the notion of a complex predicate without pausing to define it carefully. In principle, however, by appeal to singular terms and disjunction, it is not difficult to define 'disjunctive predicate' precisely. For this discussion, however, we need not do so. It will be sufficient for our putative analysis to identify confused supposition negatively as that which is neither distributive nor determinate.

The suppositional properties of the eight propositional term occurrences as normally defined may be summarized in a table:

	Subject	Predicate
A	Common Distributive	Merely Confused
E	Common Distributive	Common Distributive
I	Determinate	Determinate
O	Determinate	Common Distributive

24 Strictly, the syllogistic's existential presupposition should also be expressed here and below, by an additional conjunct $\exists x Sx$. This additional condition is made explicit in the more careful formulation of Part IV.

In later logic, including the *Port Royal Logic*, it was common to collapse the two non-distributive types:

	Subject	Predicate
A	Distributive	Non-distributive
E	Distributive	Distributive
I	Non-distributive	Non-distributive
O	Non-distributive	Distributive

Terms in the *Logic* continue to conform to this pattern although Arnauld and Nicole use *universal* for *distributive* and *particular* for *non-distributive*.

Part III. Corrections

Any attempt to provide what we would today consider a semantic analysis of the distinction between the concepts of distributive and non-distributive term that would validate the six syllogistic rules – an attempt to provide a semantic analysis coextensive to the syntactic concept – would only be partly successful. The distinction would succeed extensionally. All and only the four term occurrences that count as distributive in the syntactic sense count as distributive semantically. The others are non-distributive.

The proposed definitions do less well, however, at achieving an analytic goal. The distinction cannot be drawn in terms of the equivalence of the categorical propositions to characteristic conjunctions or disjunctions of term instances. In the case of distributive supposition, the analysis is vitiated by the predicate of the particular negative. The converse entailment ('the ascent') fails from the disjunction of instances. A more egregious failure occurs in a case of non-distributive supposition. This is the case of the predicate of the universal affirmative. The categorical proposition neither entails nor is entailed by the disjunction of the predicate instances. It is to accommodate terms with these inferential peculiarities that the *ad hoc* class of merely confused supposition is distinguished.

These failures, however, do not mean that the attempt to capture a proposition's meaning by equivalent conjunctions and disjunctions of instances is ill conceived, but only that it has been imperfectly implemented. The non-conforming cases too are open to this kind of analysis if the quantifier scope is properly observed. If the scope is observed, a categorical proposition is fully equivalent to a complex formed by conjunction and disjunction from its term instances, and there is no need to distinguish the *ad hoc* subclass of the merely confused supposition.

A unified account turns on the recognition that to obtain a combination of instances fully equivalent to a categorical proposition, the proposition must be instantiated both for the subject and the predicate. It must express both the instances of the subject relative to the predicate and those of the predicate relative to the subject. Moreover, since the subject determines what the predicate is true of – since, in modern terminology, the subject has wider scope – there is an order in which the terms should be instantiated. The need to supplement the account of ascent and descent by imposing an order on the inferences to and from terms was recognized in the tradition by, for example, Domingo de Soto.[25] Arnauld and Nicole, however, do not discuss supposition nor the appropriate order of ascent and descent from terms, but manage, as we shall see, to capture the relevant scope restrictions nevertheless.

[25] See *Ashworth 1973* and *Priest and Read 1980*.

The intuitions underlying the correct process will be described first informally – if somewhat tediously – and then more succinctly in a formal manner. By an *assertion* let us mean either an affirmation (an affirmative assertion) or a denial (a negative assertion).

Informally, a proposition's proper quantifier scope is captured by the rule that a categorical descends first from the subject. That is, in the first instance, the proposition asserts that, and entails, a second proposition in which the predicate P is (either affirmatively or negatively) asserted to be true of the relevant 'quantity' of subject constants $s_1, \ldots, s_n$. The quantity in this instance is determined by the proposition's quantifier marker, *every* or *some*. If the mark is universal, the categorical descends to a proposition in which P is asserted of every subject constant s_i conjunctively. If it is particular, it descends to a proposition in which P is asserted of them disjunctively.

Further, each occurrence of the common noun P in this entailed proposition must in turn be instantiated. That is, within the entailed proposition each assertion (i.e. affirmation or denial) that the predicate P holds of a subject constant s_i is replaced by a complex proposition. The predicate's quantity in this second instance is again marked by the syntax of the original proposition, in this case either by the quantifier alone or by a combination of the quantifier and the negative marker *not*. For any subject constant s_i, both *every* and *no* mark that every p_j is asserted of s_i; *some* without *not* indicates that some p_j is asserted of s_i; and *some* with *not* that every p_j is asserted of s_i.

Whether these component assertions are affirmations or denials – whether they are affirmative or negative – is determined also by markers, by the presence or absence in the categorical of the negative markers *no* or *not*. If the categorical lacks a negative marker, the proposition entails (i.e. descends further to) the proposition in which each of the proposition's component assertions that p_j is true of s_i is replaced by an identity proposition $s_i = p_j$. If the marker is negative, it descends to one in which each assertion that p_j is true of s_i is replaced by $s_i \neq p_j$.[26]

Describing these descents formally is straightforward. Again the assumption is made that $s_1, \ldots, s_n$ name all the actual supposita of S, and that $p_1, \ldots, p_m$ name those of P.We make use again of the notions defined earlier of positive and negative instances of *S for P* and of *P for S*. Because descent proceeds to an additional step, let us rename what was called earlier a *conjunctive instance of a term*. It will now be a *mediate conjunctive instance*, and what we called a *disjunctive instance* let us now call its *mediate disjunctive instance*. What must be defined is a relevant instantiation of a mediate instance. For this purpose, we first define the instantiation of a proposition for a term when its collateral term is a singular term. It is propositions of this sort that make up the conjuncts and disjuncts of mediate instantiations.

Definitions

> Relative to a subject term s_i and predicate term P a *positive instance* of *P for* s_i is any $s_i = p_j$, and a *negative* instance is any $s_i \neq p_j$.
> Relative to a subject term S and predicate term s_j a *positive instance* of *S for* p_j is any $s_i = p_j$, and a *negative instance* is $s_i \neq p_j$.
> A *conjunctive instance of a term relative to a singular term* is the conjunction of that term's instances for that term, and a *disjunctive instantiation* is the disjunction of these instances.

It remains to define the final step in the descent, the proposition arrived at by substituting complexes of their instances for the constituents of a proposition's mediate instance. Much

[26] As explained in an earlier note, strictly speaking, in syllogistic syntax $s_i = p_j$ is the universal affirmative *every* s_i *is* p_j and $s_i \neq p_j$ is its contradictory, the particular negative *some* s_i *is not* p_j.

like the way a normal form in sentential logic details the possibilities that hold, the final entailment in descent details which facts about identity actually hold. For this reason, it is a kind of 'state description'.[27]

Definition. A categorical proposition's *state description* is any proposition that results from a mediate instance of the proposition's subject for its predicate or of its predicate for its subject either by the replacement of each of its atomic parts by its conjunctive instance or by the replacement of each of its atomic parts by its disjunctive instance.

It follows directly from the standard truth-conditions for categorical propositions that each of the four forms is logically equivalent to a state description.

Theorems

$\textit{every S is P} \dashv\vDash s_1 \ \textit{ is } \ P \wedge \cdots \wedge s_n \ \textit{ is P} \dashv\vDash (s_1 = p_1 \vee \cdots \vee s_1 = p_m) \wedge \cdots \wedge (s_n = p_1 \vee \cdots \vee s_n = p_m)$

$\textit{no S is P} \dashv\vDash s_1 \ \textit{ is not P} \wedge \cdots \wedge s_n \ \textit{ is not P} \dashv\vDash (s_1 \neq p_1 \wedge \cdots \wedge s_1 \neq p_m) \wedge \cdots \wedge (s_n \neq p_1 \wedge \cdots \wedge s_n \neq p_m)$

$\textit{some S is P} \dashv\vDash s_1 \ \textit{ is } \ P \vee \cdots \vee s_n \ \textit{ is P} \dashv\vDash (s_1 = p_1 \vee \cdots \vee s_1 = p_m) \vee \cdots \vee (s_n = p_1 \vee \cdots \vee s_n = p_m)$

$\textit{some S is not P} \dashv\vDash s_1 \ \textit{ is } \ \textit{not P} \vee \cdots \vee s_n \ \textit{ is not P} \dashv\vDash (s_1 \neq p_1 \wedge \cdots \wedge s_1 \neq p_m) \vee \cdots \vee (s_n \neq p_1) \wedge \cdots \wedge s_n \neq p_m)$

Definition. By the *preferred instantiation* of a categorical proposition is meant the state description equivalent to it in the preceding theorem.

It is now possible to simplify supposition theory by defining a term's distributive and non-distributive suppositional properties by appeal to the proposition's preferred instance.

Definitions (Revised)

Relative to a categorical proposition,
a subject has *distributive* supposition relative to the predicate if the proposition is equivalent to the conjunction of the subject's instances for the predicate;
a subject has *a determinate* supposition relative to the predicate if the proposition is equivalent to the disjunction of the subject's instances for the predicate;
a predicate has *distributive* supposition relative to the subject if the proposition's preferred instantiation is a conjunction;
a predicate has *determinate* supposition relative to the subject if the proposition's preferred instantiation is a disjunction.

These revisions correct the analytical flaws of the standard theory. The new semantic notion of distribution remains coextensive with the syntactic concept discussed in Part I. Now, moreover, in both distributive and determinate uses, the containing proposition is equivalent to a conjunction or disjunction of instances that may be fairly said to capture the meaning of the original proposition. Moreover, all four cases of non-distributive supposition now share a single defining property, obviating the need for the special subcategory of a merely confused supposition.

Part IV. Abstraction to a definition of truth

The importance of supposition theory to this discussion is that it points the way to a new analysis of truth. The terms of a true categorical proposition have characteristic suppositional

[27] The terminology is from *Carnap 1947*.

properties that could serve as part of a statement of the propositions truth-conditions if these properties could be defined independently of truth itself. Although, as we have seen in the previous section, the relevant suppositional properties can be defined in terms of characteristic logical equivalents of the proposition as a whole to characteristic conjunctions and disjunctions of instances, these entailments cannot be used directly in an analysis of truth because to do so would be circular. Entailment cannot be used to define truth because, in the semantics of the object language, entailment is defined in terms of truth.

It should be said that although truth is conceptually prior to distributive and non-distributive supposition, this dependency does not constitute a flaw in the medieval theory, because historically it was not the purpose of supposition theory to define truth-conditions. The distinction between distributive and non-distributive supposition, in particular, was part of a broader classification of the way terms stand for things relative to context of use that presupposes an understanding of truth-conditions.

The use by the Port Royal logicians of distributive term for stating truth-conditions is novel. They do so, moreover, in a way that avoids circularity. Their approach is to characterize distributive and non-distributive uses not in terms of a proposition's entailments to conjunctions and disjunctions of instances, but rather to abstract from the conjunctions and disjunctions themselves to the conditions that hold among the referents of the terms that make these conjunctions and disjunctions true. Since term reference is defined prior to truth in object language metatheory, a concept of distributive term defined in terms of term reference can be used to define truth without circularity.

The abstraction will be demonstrated here as follows. First we briefly state the standard reconstruction in set theory of the syntax and semantics for the version of the syllogistic that the authors took as their model. We then define the metatheoretic concepts Arnauld and Nicole require for their abstraction. We conclude with the metatheorem that lays out the new truth-conditions that are equivalent to the old.

The standard theory[28]

By a *syllogistic syntax* let us mean a set of basic expressions called *terms*, the four *quantifiers* A, E, I, and O, and the set of *propositions* (or *sentences*) that result from concatenating any two distinct terms to the right of a quantifier, that is, any ASP, ESP, ISP, or OSP for any term S (called the *subject*) and P (called the *predicate*). We shall let Q range over {A,E,I,O}.

To state the semantic theory in a way that accommodates the *Logic*'s abstraction, it will be convenient to use the framework of general quantification theory. In this framework relative to a domain D, an interpretation $\Im$ assigns to the terms S and P subsets of a domain D, and to the quantifier Q a binary relation $\Im(\mathrm{Q})$ on the power set of D. That is, $\Im(\mathrm{Q})$ is a relation on subsets of D. To aid in exposition, we make use of the notation of restricted quantification, which is defined by eliminative definition:

$$\forall_{\mathrm{A}} vF \;=_{\mathrm{def}} \forall v(Av \rightarrow F)$$

[28] The syntax and semantics of the syllogistic employed here is based on the natural deduction reconstruction of Aristotle's logic developed in *Corcoran 1972*, *Smiley 1962*, and *Martin 1997*. Like Aristotle and medieval logicians, Arnauld and Nicole assume that in standard cases the subject (and hence the predicate) of a true affirmative proposition signifies at least one actual existent. On the existential presuppositions of affirmatives in the *Logic* see *Martin 2011* and *2012*. In the reconstructions by Corcoran and Smiley, which are closer to Aristotle's, existential presupposition is built into the semantics by requiring every term to have a non-empty extension. Arnauld and Nicole, on the other hand, follow the medieval practice of allowing terms to have empty extensions but require as part of the truth-conditions of an affirmative proposition that the subject term be non-empty. (Negatives are then said to be true if the subject term is empty.) Both approaches are equivalent in the sense that they validate the same classical theory (the immediate inferences of 'the square of opposition', the 24 valid moods, the syllogistic reduction rules, etc.). For simplicity of exposition the Corcoran–Smiley truth-conditions are used here. On the equivalence of the approaches see the discussion in *Martin 2004* (p. 6, Note 4).

$$\forall_{AB} vF =_{def} \forall v(Av \wedge Bv \rightarrow F)$$
$$\exists_{A} vF =_{def} \exists v(Av \wedge F)$$
$$\exists_{AB} vF =_{def} \exists v(Av \wedge Bv \wedge F).$$

To distinguish clearly between object and metalanguage usage, we use $\forall$ and $\exists$ for quantifiers in a first-order object language, and $\boldsymbol{\forall}$ and $\boldsymbol{\exists}$ for quantifiers that occur in the semantic theory of the metalanguage. The semantics of the syllogistic is then easily stated:

Definition. A *syllogistic structure* is defined to be any power set algebra $\langle \mathrm{P(D)}, \subseteq, \cap, \emptyset \rangle$. A (*syllogistic*) *interpretation relative to a syllogistic structure* is any function $\Im$ such that

1. $\Im$ assigns to each term T a non-empty subset $\Im(T)$ of D, called *the extension* of T;
2. $\Im$ assigns a two-place relation on D to the quantifiers as follows:

$$\Im(\mathrm{A}) = \{\langle \mathrm{A,B}\rangle | \mathrm{A} \subseteq \mathrm{B}\} = \{\langle \mathrm{A,B}\rangle | \boldsymbol{\forall}_{\mathrm{A}} d \boldsymbol{\exists}_{\mathrm{BA}} d'(d = d')\}$$
$$\Im(\mathrm{E}) = \{\langle \mathrm{A,B}\rangle | \mathrm{A} \cap \mathrm{B} = \emptyset\} = \{\langle \mathrm{A,B}\rangle | \boldsymbol{\forall}_{\mathrm{A}} d \boldsymbol{\forall}_{\mathrm{B}} d'(d \neq d')\}$$
$$\Im(\mathrm{I}) = \{\langle \mathrm{A,B}\rangle | \mathrm{A} \cap \mathrm{B} \neq \emptyset\} = \{\langle \mathrm{A,B}\rangle | \boldsymbol{\exists}_{\mathrm{AB}} d \boldsymbol{\exists}_{\mathrm{BA}} d'(d = d')\}$$
$$\Im(\mathrm{O}) = \{\langle \mathrm{A,B}\rangle | \mathrm{A} - \mathrm{B} \neq \emptyset\} = \{\langle \mathrm{A,B}\rangle | \boldsymbol{\exists}_{\mathrm{A}} d \boldsymbol{\forall}_{\mathrm{AB}} d'(d \neq d')\}$$

3. $\Im$ assigns truth values to propositions as follows:

$$\Im(\mathrm{Q}SP) = \mathrm{T} \text{ iff} \langle \Im(S), \Im(P)\rangle \in \Im(\mathrm{Q}).$$

Definition. An argument from X to F is *syllogistically valid* (briefly $X \vDash_{\mathrm{syl}} F$) relative to a family of structures iff for any syllogistic interpretation $\Im$ for a structure in that family, if for all $G \in X$, $\Im(G) = \mathrm{T}$, then $\Im(F) = \mathrm{T}$.
Theorem. The logical relations of immediate inference (those of the Square of Opposition) hold, and the traditional 24 valid moods are exactly the valid syllogisms.

The standard theory is completed by two further results, which need only be mentioned here. First, from the traditional reduction of the valid moods to Barbara and Celarent, it is possible to reconstruct sound and complete axiomatic and natural deduction systems for not only the valid moods, but also for the set of categorical arguments generally. Second, the *Logic*'s six rules from Part I constitute a decision procedure for not only the valid moods but any finite categorical argument.[29]

The Cartesian Theory. To state the *Logic*'s version of categorical truth-conditions, it is necessary to make use of some terminology that singles out the various parts of a quantifier's definition.

Definitions

1. Relative to a syllogistic syntax and structure $\langle \mathrm{P(D)}, \subseteq, \cap, \emptyset \rangle$, let us call Q a *Cartesian quantifier* iff $\Im(\mathrm{Q}) = \{\langle \mathrm{A,B}\rangle | \mathbb{Q}_{\mathrm{iC}} d \mathbb{Q}_{\mathrm{jC'}} d'(F[d,d'])\}$, where $\mathbb{Q}_{\mathrm{i}}, \mathbb{Q}_{\mathrm{j}}$ range over $\{\boldsymbol{\forall}, \boldsymbol{\exists}\}$; $\mathbb{Q}_{\mathrm{iC}}, \mathbb{Q}_{\mathrm{jC'}}$ are quantifiers restricted to the subsets C and C$'$ of D, respectively; and $F[d,d']$ is either the formula $d = d'$ or $d \neq d'$.
2. If Q is a Cartesian quantifier (as defined above), let us call $F[\Im(S), \Im(P)]$ *the truth-conditions* of QSP, $\mathbb{Q}_{\mathrm{i}}$ the proposition's *subject quantifier (in the metalanguage)*, $\mathbb{Q}_{\mathrm{j}}$ the proposition's *predicate quantifier (in the metalanguage)*; and C and C$'$ *the relevant extensions* of the subject and predicate, respectively.

[29] For a more precise statement of the proof-theoretic system (both in axiomatic or natural deduction form) and the completeness theorem, see *Martin 1997*.

According to these definitions, the traditional four syllogistic quantifiers all count as Cartesian.

Cartesian quantifiers have the nice property that it is possible to read off from a proposition's truth-conditions three important bits of information: a quantifier's relevant extension, the status of a term's quantifier's as universal or existential, and a proposition's status as affirmative or negative. Moreover, the term's quantificational status determines whether the term is distributive. The authors of the *Port Royal Logic* had the important insight that these properties alone determine a proposition's truth-conditions. To see how, we must define each of these properties within the semantics just defined.

Term extension: conservative and non-conservative quantifiers. As defined in general quantification theory, one term in a quantified subject–predicate sentence is said to be 'subject conservative' if the sentence's truth depends only on the extension of that term. More formally, a quantifier Q is said to be subject conservative if the only part of the extension of *P* relevant to the proposition's truth is the part that intersects with the extension of *S*, and it is said to be 'predicate conservative' if the only part of the extension of *S* relevant to the proposition's truth is the part that intersects with the extension of *P*.[30] In terms of restriction, a quantifier is subject conservative if the sentence's truth turns only on that part of the predicate's extension, that is, its restriction by the subject, and is predicate conservative if the only part of the subject's extension that is relevant is that part restricted by the predicate. It is relevant that this notion of 'restriction' is simply the set theoretic intersection of the extensions of the two terms. It is essentially the modern version of the medieval operation of restriction discussed in the introduction.

Definitions. Relative to an interpretation $\Im$ over a domain D,

Q is *subject* (or *left*)*conservative* iff,
$\forall A, B \subseteq D, \langle A, B\rangle \in \Im(Q) \leftrightarrow \langle A, A \cap B\rangle \in \Im(Q)$;
Q is *predicate* (or *right*)*conservative* iff,
$\forall A, B \subseteq D, \langle A, B\rangle \in \Im(Q) \leftrightarrow \langle A \cap B, A\rangle \in \Im(Q)$.

Theorems. The conservative properties terms are stipulated in the table below:

	Subject	Predicate
A	Non-conservative	Conservative
E	Non-conservative	Non-conservative
I	Conservative	Conservative
O	Non-conservative	Conservative

Theorem. A proposition's quantifier is non-conservative with respect to a term iff the term's relevant extension is that term's extension, and is conservative iff its relevant extension is the intersection of the proposition's term extensions.

Distributive and non-distributive terms. It is possible to read a term's distributive status from the quantifier associated with it in a proposition's truth-conditions. If the quantifier

[30] See *Keenan and Westerståhl 1997.*

over a term's relevant extension is universal, the term is distributive. If it is existential, it is non-distributive.

Definition. A syllogistic term is *distributive* or (in the *Logic*'s usage) *universal* in a proposition iff in the proposition's truth-conditions its term quantifier is universal, and is *non-distributive* or *particular* iff its term quantifier is existential.

Theorem. A term is distributive iff its term quantifier quantifies universally over its relevant extension. The table listing the distributive status of terms under the suppositional definition of distribution equally describes their distributive status under the new definition.

A proposition's quality

Syntactically, a proposition's quality is determined by negative markers. However, it is a semantic notion of quality that is relevant to fixing truth-conditions. A categorical is affirmative in a semantic sense if it asserts that identities obtain and negative if it denies them.

Definition. A categorical proposition is (*semantically*) *affirmative* iff its truth-conditions assert that values in the relevant extension of the subject are identical to those in the relevant extension of the predicate, and is (*semantically*) *negative* iff its truth-conditions assert that they are non-identical.

Truth-conditions

It is now possible to state the truth-conditions for categorical propositions in the manner of Arnauld and Nicole. As expressed in the theorem below, the clause characterizing a proposition's truth-conditions appeals only to the concepts of the distributive term, conservative quantifier, and affirmative and the negative proposition. To capture more closely the *Logic*'s wording in the theorem, a distributive term is called *universal* and a non-distributive term *particular*.

Theorem. Relative to a syllogistic syntax and structure $\langle \mathrm{P(D)}, \subseteq, \cap, \emptyset \rangle$, the set of syllogistic interpretations is identical to the set of all functions $\Im$ that assign to each term T a non-empty subset of D and a truth-value to each proposition as follows:

$\Im(\mathrm{A}SP) = \mathrm{T}$ iff A is a Cartesian quantifier, the proposition is affirmative, the quantifier A is subject but not predicate conservative, and S is universal but P is particular;

iff A is a Cartesian quantifier, the relevant extension of S is its entire extension, the relevant extension of P is the restriction of its extension of P by that of S, and every element of the relevant extension of the subject is identical to some element of the relevant extension of P.

$\Im(\mathrm{E}SP) = \mathrm{T}$ iff E is a Cartesian quantifier, the proposition is negative, the quantifier E is neither subject nor predicate conservative, and both S and P are universal;

iff E is a Cartesian quantifier, the relevant extension of S is its entire extension, the relevant extension of P is its entire extension, and every element of the relevant extension of the subject is non-identical to every element of the relevant extension of P.

$\Im(\mathrm{I}SP) = \mathrm{T}$ iff I is a Cartesian quantifier, the proposition is affirmative, the quantifier I is both subject and predicate conservative, and both S and P are particular;

iff I is a Cartesian quantifier, the relevant extension of $S$is the restriction of its extension by that of P, the relevant extension of P is the restriction of its extension by that of S, and some element of the relevant extension of S is identical to some element in the relevant extension of P.

$\Im(\mathrm{O}SP) = \mathrm{T}$ iff O is a Cartesian quantifier, the proposition is negative, the quantifier O is subject but not predicate conservative, and P is universal but S is particular;

iff O is a Cartesian quantifier, the relevant extension of S is its entire extension, the relative extension of P is its extension restricted by that of S, and there is some element in the relevant extension of S that is not identical to any element in the extension of P.

The theorem captures the claim that the Cartesian truth-conditions are fully equivalent to classical truth-conditions. Accordingly, all the metalogical properties of the syllogistic continue to hold under the Cartesian analysis of truth, including the theory of immediate inference, the soundness and completeness of the standard reduction of the valid moods to Barbara and Celarent, and the effectiveness of the Logic's six rule decision procedures for the valid moods and valid categorical arguments generally. It should also be pointed out that, contrary to Parienté's interpretation discussed in the introduction, the notion of restriction that occurs in the various clauses is univocal. It is simply the set theoretic intersection of the extension of two terms, the modern version of medieval restriction.[31]

First-order abstraction

We will complete the exposition of the semantic reconstruction by making use of first-order logic to lay out quite starkly how the Cartesian truth-conditions abstract from medieval supposition theory. The technique makes use of a translation function, called * below, that assigns to each categorical proposition a first-order equivalent. Because the syllogistic incorporates an assumption of existential import (that the extension of every term is non-empty), the first-order translation makes this assumption explicit:

Definition. * is the function from categorical propositions to first-order formulas:

Categorical proposition First-order translation

$\mathrm{A}SP^* = \exists xSx \wedge \forall_S x \exists_{PS} y(x = y)$

$\mathrm{E}SP^* = {\sim}\exists xSx \vee \forall_S x \forall_P y(x \neq y)$

$\mathrm{I}SP^* = \exists xSx \wedge \exists_{PS} x \exists_{SP} y(x = y)$

$\mathrm{O}SP^* = {\sim}\exists xSx \vee \exists_S x \forall_{SP} y(x \neq y)$

[31] Apart from the discussion of truth-conditions in which the wording describing restriction closely follows these formulations, the only passage in which Arnauld and Nicole describe 'indeterminate' quantifier restriction is this:

> Or cette restriction ou resserrement de l'idée générale quant à son étendue, se peut faire en deux manières.
> La première est, par une autre idée distincte & déterminée qu'on y joint, comme lorsqu'à l'idée générale du triangle, qui est le triangle rectangle, je joins celle d'avoir un angle droit : ce qui resserre cette idée à une seule espèce de triangle, qui est le triangle rectangle.
> L'autre en y joignant seulement une idée indistincte & indéterminée de partie; comme quand je dis, quelque triangle : on dit alors que le terme commun devient particulier, parce qu'il ne s'étend plus qu'à une partie des sujets auxquels il s'étendoit auparavant; sans que néanmoins on ait déterminé quelle est cette partie à laquelle on l'a resserré. (*LAP* I,6; *KM* V, p. 145; *B* p. 40)

Parienté interprets the last paragraph of this text as introducing a second and new operation of 'indefinite' restriction. What the text as a whole is saying, however, in the terms just defined, is that although the quantifiers in both universal and particular affirmatives are subject conservative, which is a concept defined in terms of standard restriction alone, the subject of the universal proposition is universal and therefore (the relevant quantity of) the predicate (in this case, at least one) is true of (i.e. is identical to) each of the entire restricted class, but the subject of the particular is particular and therefore (the relevant quantity of) the predicate (in this case at, least one) is true of (i.e. identical to) at least one of the restricted class. The introduction of a second notion of restriction is gratuitous.

A categorical proposition and its translation are equivalent in a precise sense:

Theorem. For any first-order model structure $\langle \mathrm{D}, \Im \rangle$, there is a syllogistic structure $\langle \mathrm{D}, \subseteq, \cap, \emptyset \rangle$ and syllogistic interpretation $\Im'$ such that for any term T, $\Im'(T) = \Im(T)$, and for any categorical formula F, $\Im'(F) = \Im(F^*)$. Conversely, for any syllogistic structure $\langle D, \subseteq, \cap, \emptyset \rangle$ and any syllogistic interpretation $\Im$ over that structure, there is an $\Im'$ such that $\langle \mathrm{D}, \Im' \rangle$ is a first-order model, $\Im$ is the restriction of $\Im'$ to the terms of the syllogistic, and for any categorical formula F, $\Im(F) = \Im'(F^*)$.

Because the quantifiers are in a sense generalized conjunctions and disjunctions, the translations are in effect transformations into first-order notation of a proposition's 'preferred instantiations' as defined in Part II. This fact can be made explicit in first-order model theory by means of suitable substitutional interpretations, which literally interpret the quantifiers as generalized conjunctions and disjunctions. Let $F[c_1, \ldots, c_n]$ be a first-order formula containing the constants $c_1, \ldots, c_n$. and let $F[c/v]$) be the result of substituting the constant c for all free occurrences of v in the first-order formula F.

Definition. A first-order interpretation $\Im$ is *substitutional* iff

$$\Im(\forall v F) = \mathrm{T} \text{ iff, for any constant } c, \Im(F[c/v]) = \mathrm{T};$$
$$\text{and } \Im(\exists v F) = \mathrm{T} \text{ iff, for some constant } c, \Im(F[c/v]) = \mathrm{T}.$$

The first-order translations of categorical propositions accordingly have a substitutional interpretation:

Theorem. In any first-order substitution interpretation $\Im$:

$$\Im(\forall_S x \exists_{PS} y(x = y)) = \mathrm{T} \quad \text{iff} \quad \boldsymbol{\forall}_{\Im(S)} \Im(c) \boldsymbol{\exists}_{\Im(S) \cap \Im(P)} \Im(c')(\Im(c) = \Im(c'))$$
$$\Im(\forall_S x \forall_P y(x \neq y)) = \mathrm{T} \quad \text{iff} \quad \boldsymbol{\forall}_{\Im(S)} \Im(c) \boldsymbol{\forall}_{\Im(P)} \Im(c')(\Im(c) \neq \Im(c'))$$
$$\Im(\exists_{PS} x \exists_{SP} y(x = y)) = \mathrm{T} \quad \text{iff} \quad \boldsymbol{\exists}_{\Im(P) \cap \Im(S)} \Im(c) \boldsymbol{\exists}_{\Im(S) \cap \Im(P)} \Im(c')(\Im(c) = \Im(c'))$$
$$\Im(\exists_{SP} x \forall_P y(x \neq y)) = \mathrm{T} \quad \text{iff} \quad \boldsymbol{\exists}_{\Im(S)} \Im(c) \boldsymbol{\forall}_{\Im(P) \cap \Im(S)} \Im(c')(\Im(c) \neq \Im(c'))$$

If we now add the medieval assumption that there are constants that name all the individuals in the extension of the formula's first-order predicates, these truth-conditions are equivalent to conjunctions and disjunctions of instances. These conjunctions and disjunctions are the first-order versions of a proposition's 'preferred instances':

Theorem. In any first-order model $\langle \Im, D \rangle$ in which $\Im$ is a substitutional interpretation and there are sets of constants $\{s_1, \ldots, s_n\}$ and $\{p_1, \ldots, p_m\}$ such that for every element d of $\Im(D)$, there is some s_i in $\{s_1, \ldots, s_n\}$ such that $\Im(s_i) = d$, and for every element d of $\Im(P)$, there is some p_i in $\{p_1, \ldots, p_m\}$ such that $\Im(p_i) = d$, the following hold:

$$\Im(\forall_S x \exists_{PS} y(x = y)) = \mathrm{T} \quad \text{iff} \quad \Im((s_1 = p_1 \vee \cdots \vee s_1 = p_m) \wedge \cdots \wedge (s_n = p_1 \vee \cdots \vee s_n = p_m)) = \mathrm{T}$$
$$\Im(\forall_S x \forall_P y(x \neq y)) = \mathrm{T} \quad \text{iff} \quad \Im((s_1 \neq p_1 \wedge \cdots \wedge s_1 \neq p_m) \wedge \cdots \wedge (s_n \neq p_1 \wedge \cdots \wedge s_n \neq p_m)) = \mathrm{T}$$
$$\Im(\exists_{PS} x \exists_{SP} y(x = y)) = \mathrm{T} \quad \text{iff} \quad \Im((s_1 = p_1 \vee \cdots \vee s_1 = p_m) \vee \cdots \vee (s_n = p_1 \vee \cdots \vee s_n = p_m) = \mathrm{T}$$
$$\Im(\exists_P x \forall_{SP} y(x \neq y)) = \mathrm{T} \quad \text{iff} \quad \Im((s_1 \neq p_1 \wedge \cdots \wedge s_1 \neq p_m) \vee \cdots \vee (s_n \neq p_1 \wedge \cdots \wedge s_n \neq p_m)) = \mathrm{T}$$

These results entail the theorem below, which captures the Cartesian abstraction of truth-conditions from the entailments of ascent and descent. It lists on the left a proposition's Cartesian truth-conditions and on the right its equivalent, under medieval expressive assumptions, in terms of conjunctions and disjunctions of instances.

Theorem. In any first-order model $\langle\Im, \mathrm{D}\rangle$ in which $\Im$ is a substitutional interpretation and there are sets of constants $\{s_1, \ldots, s_n\}$ and $\{p_1, \ldots, p_{\mathrm{m}}\}$ such that for every element d of $\Im(D)$, there is some s_i in $\{s_1, \ldots, s_n\}$ such that $\Im(s_i) = d$, and for every element d of $\Im(P)$, there is some p_i in $\{p_1, \ldots, p_m\}$ such that $\Im(p_i) = d$, the following hold:

$$\forall_{\Im(S)}\Im(c)\exists_{\Im(S)\cap\Im(P)}\Im(c')(\Im(c) = \Im(c')) \text{ iff } \Im((s_1 = p_1 \vee \cdots \vee s_1 = p_m) \wedge \cdots \wedge (s_n = p_1 \vee \cdots \vee s_n = p_m)) = \mathrm{T}$$

$$\forall_{\Im(S)}\Im(c)\forall_{\Im(P)}\Im(c')(\Im(c) \neq \Im(c')) \text{ iff } \Im((s_1 \neq p_1 \wedge \cdots \wedge s_1 \neq p_m) \wedge \cdots \wedge (s_n \neq p_1 \wedge \cdots \wedge s_n \neq p_m)) = \mathrm{T}$$

$$\exists_{\Im(P)\cap\Im(S)}\Im(c)\exists_{\Im(S)\cap\Im(P)}\Im(c')(\Im(c) = \Im(c')) \text{ iff } \Im((s_1 = p_1 \vee \cdots \vee s_1 = p_m) \vee \cdots \vee (s_n = p_1 \vee \cdots \vee s_n = p_m) = \mathrm{T}$$

$$\exists_{\Im(S)}\Im(c)\forall_{\Im(P)\cap\Im(S)}\Im(c')(\Im(c) \neq \Im(c')) \text{ iff } \Im((s_1 \neq p_1 \wedge \cdots \wedge s_1 \neq p_m) \vee \cdots \vee (s_n \neq p_1 \wedge \cdots \wedge s_n \neq p_m)) = \mathrm{T}$$

Part V. Conclusion. Truth in the *Port Royal Logic*

The actual wording used in the *Logic* to state the truth-conditions of categorical propositions (in Axioms 1–7 and accompanying explanatory passages in Sections 17–20 of Book II) is very similar to that in the theorem above. There are, to be sure, several differences. Some are relatively trivial. Instead, for example, of formulating a single notion of conservative quantifier to cover all cases, the authors explain case by case when a term's relevant extension is restricted by that of its collateral term.

A more important difference is due to the authors' more global Cartesian project. One purpose of the *Logic* is to formulate a theory of truth in terms of relations among ideas rather than among corporeal individuals outside the mind. Accordingly, Arnauld and Nicole define the extension of a term so that it is made up of ideas, not bodies. An idea's extension is the set of all ideas 'inferior' to it. Accordingly, they understand syllogistic quantifiers to range over ideas. For example, A*SP*is true, they say, if the extension of *S* is a subset of that of *P*.[32]

This difference is vitiated to a large extent, however, by the fact that the *Logic* retains a correspondence theory of truth. In particular, there is a 1–1 correspondence between a term's Cartesian extension and its extension in the modern sense. This is not the place to explain the details,[33] but, in brief, a term and its subordinate ideas correspond to things outside the mind through the relation of signification, a mind–world relation which the authors retain from medieval semantics. According to their account – a version of the 'objective being' theory– a term *signifies* all objects that satisfy the term's defining properties (its 'intentional content') or, in the terminology of the *Logic*, that satisfy the modes in its comprehension. (It is the fact that a term has a comprehension that constitutes its having 'objective being'.) Signification determines the 'inferiority' relation, and that relation in turn determines an idea's extension: one idea *is inferior to* another iff everything the first signifies the second also signifies, and an idea's *extension* is the set of its inferiors. Accordingly, the set of individuals that an idea signifies – its 'significance range' – plays a role similar to the term's modern extension because the subordination relation among ideas turns out to be isotonic to the subset relation

32 More precisely, in the terminology of the *Logic,* A*SP* is true iff the extension of the predicate restricted by that of the subject is identical to that of the subject.

33 For a full defense of the existential reading of the semantics of the *Logic,* see *Martin 2011*, *2012*.

among significata sets. Extensions of the one type are subsets iff extensions of the other type are. Thus, a universal affirmative is true due to the subordination of idea extensions iff its term extensions are subordinate in the modern sense of set inclusion. Similar correspondences hold for the other categoricals.

There is a sense, then, in which the *Logic*'s truth theory is doubly abstract. It first abstracts from medieval supposition theory to truth-conditions in terms of distribution understood as a concept that refers to objects outside the mind. This paper describes this first-level abstraction. The authors then go on to abstract from the theory of truth about objects to one about ideas, albeit one that insures a correspondence between relations among ideas and objects. As a result, the preceding theorem, which states truth-conditions in terms of distribution, remains true when read in either sense of extension. It has been the purpose of this paper to explain the first part of this abstractive process.

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References

Arnauld, A. and Nicole, P. 1996. *Logic or the Art of Thinking*, Cambridge: Cambridge University Press. Jill Vance Buroker (translator).

Ashworth, E.J. 1973. 'Existential assumptions in late medieval logic', *American Philosophical Quarterly*, **10**, 141–147.

Auroux, S. 1993. *La Logique des Idées*, Montréal: Bellarmin.

Buridan, J. 2001. *Summulae de Dialectica*, New Haven: Yale University Press. Gyula Klima (translator), based on the ms. Latin text of Hubert Hubien.

Carnap, R. 1947. *Meaning and Necessity*, Chicago: University of Chicago Press.

Corazzon, R. *History of Renaissance and Modern Logic from 1400 to [John] Stuart Mill*. Available online at http://www.ontology.co/logic-modern.htm.

Corcoran, J. 1972. 'Completeness of an ancient logic', *Journal of Symbolic Logic*, **37**, 696–702.

De Rijk, L.M. 1962–1967. *Logica Modernorum; A Contribution to the History of Early Termist Logic*, Assen: Van Gorcum.

Descartes, R. 1897–1909. *Œuvres de Descartes,* Paris, Vrin. C. Adam et P. Tannery (eds.).

Dominicy, M. 1984. *La Naissance de la Grammaire moderne*, Bruxelles: Pierre Mardaga.

Eustace of St. Paul. 1609. *Summa Philosophiae quadripartita, de rebus dialecticis, moralibus, physicis et metaphysicis*. Paris.

Fonseca, P.D. 1964. *Institutionum dialecticarum libri octo*, Coimbra: Universidade de Coimbra.

Keenan, E.L. and Westerståhl, D. 1997. 'Generalized quantifiers in linguistics and logic', in J. Van Benthem and A. Ter Meulen, eds., *Handbook of Language and Logic*, Amsterdam: Elsever.

Kneale, W. and Kneale, M. 1962. *The Development of Logic*, Oxford: Oxford University Press.

Kremer, E. and Moreau, D., eds. 2003. *Œuvres Philosophiques d'Arnauld*, Bristol: Theommes Press.

Lenzen, W. 1990. 'On Leibniz's essay *Mathesis rationis'*, *Topoi*, **9**, 29–59.

Martin, J.N. 1997. 'Aristotle's natural deduction reconsidered', *History and Philosophy of Logic*, **18**, 1–15.

Martin, J.N. 2004. *Themes in Neoplatonic and Aristotelian Logic: Order, Negation, and Abstraction*, London: Ashgate Publishing.

Martin, J.N. 2011. 'Existential import in Cartesian semantics', *History and Philosophy of Logic*, **32**, 211–239.

Martin, J.N. 2012. 'Existential commitment and the Cartesian semantics of the *Port Royal Logic'*, in J.-Y. Beziau and G. Payette, *The Square of Opposition: A General Framework For Cognition*, Bern: Peter Lang.

Morris, C.W. 1939. *Foundations of the Theory of Signs*, Chicago: University of Chicago Press.

Parienté, J.-C. 1985. *L'Analyse du Language à Port-Royal*, Paris: C.N.R.S. Éditions de Minuit.

Peter of Spain. 1990. *Language in Dispute [Summulae logicales]*, Amsterdam: John Benjamins. Francis P. Dinneen, S.J. (translator).

Priest, G. and Read, S. 1980. 'Merely confused supposition', *Franciscan Studies*, **40**, 265–297.

Smiley, T. 1962. 'Syllogism and quantification', *Journal of Symbolic Logic*, **27**, 58–72.

Toletus, F.S.J. 1580. *Commentaria unacum questionibus in universam aristotelis logicam*, Juntas.

William of Sherwood. 1966. *Introduction to Logic*, Minneapolis: University of Minnesota. Norman Kretzmann (translator).