Analysis as a Logic of Induction[[1]](#footnote-1)

Analysis in the Renaissance was understood as a method for acquiring knowledge of causes from effects, and at the same time of “the more general” from “the specific.” Here causation is conflated with generality, a view common in early philosophy. For example, Aristotle thought of a genus as the formal cause of the species, and Neoplatonists though that higher hypotheses, which were more general, generated lower ones. When analysis is conceived process of discovering essential truths, it is understood as taking the investigator from knowledge of species to knowledge of a higher genus.

Analysis on this picture appears to be a species of logical entailment. In claiming to deduce a cause from an effect, or a general law from its instances, it appears, the modern reader, to be claiming that induction is a valid argument. This paper explain how 16th century logicians made sense of this claim.

Analysis and synthesis became the subject of intense discussion in the 15th and 16th centuries, but the distinction is traced to Plato. In the *Phaedrus* Socrates says, “I am myself a great lover of these processes of division [*diairesis*] and generalization [*synagg*].”[[2]](#footnote-2) The classical example of division is the Stranger’s definition of *angler* in the *Sophist* (218e ff.). There he arrives at its characterization of *angler* by progressively dividing larger groups starting with art in general. Generalization was understood as the reverse process.

What seems to be the earliest mention of analysis and synthesis as argument forms is found in a passage inserted into Euclid about Proposition 13 of Book I. Thomas Heath speculates that the interpolation dates from as late as the first century AD:[[3]](#footnote-3)

Analysis is the assumption of that which is sought as if it were admitted <and arrived at> by means of its consequences at something admitted to be true. Synthesis is an assumption of that which is admitted <and arrival> by means of its consequences at something admitted to be true.

A later and more complete statement is found in the geometry of Pappus (c. 340):[[4]](#footnote-4)

Analysis, then, takes that which is sought as if it were admitted and passes from it through its successive consequences [*akolouthn*] to something which is admitted as the result of synthesis; for in analysis we assume that which is sought as if it were already done (γεηονοός)), and we inquire what it is from which this results, and again what is the antecedent [*progoumenon*] cause of the latter, and so on, until by so retracing our steps we come upon something already known or belonging to the class of first principles, and such a method [*ephodos*]we call analysis, as being a solution backwards [*anapalin lysin*].

But in synthesis, reversing the process, we take as already done suppose that which was last arrived at in the analysis and, by arranging in their natural order as consequences [*epomena*] what before were antecedents [*progoumena*], and successively connecting them one with another, we arrive finally at the construction of what was sought; and this we call synthesis.

Now analysis is of two kinds, the one directed to searching for the truth, being called *theoretical*, the other directed to finding what we are told to find and called *problematical*. (1)In the theoretical kind we assume what is sought as if it were existent and true, after which we pass through its successive consequences, [*akolouthn*], as if they too were true and established by virtue of our hypotheses, to something admitted: then (*a*), if that something admitted is true, that which is sought will also be true, but (*b*), if we come upon something admittedly false, that which is sought will also be false. (2) In the *problematical* kind we assume that which is propounded as if it were known, after which we pass through its successive consequences, taking them as true, up to something admitted: if then (*a*) what is admitted is possible and obtainable, that is, what mathematicians call *given*, what was originally proposed will also be possible, and the proof will again correspond in the reverse order to the analysis, but if (*b*) we come upon something admittedly impossible, the problem will also be impossible.”[[5]](#footnote-5)

Here the paradigm is a proof in geometry. Analysis is the process by which the investigator proceeds from “what is to be proven” to the propositions “from which is can be proven.” Synthesis is the process in reverse – the investigator produces the proof from premises understood as given to the desired conclusion, to theorems. Synthesis seems to be an abstraction from the sort of proofs found in the *Elements*, which are roughly similar to modern mathematical proofs. To modern eyes, it is not unreasonable to say these proofs are presentations of what has been discovered or to view them as texts for instruction. Analysis, on the other hand, is rather different. It seems to approximate what in modern times Shiller called “the logic of scientific discovery.”[[6]](#footnote-6) Here an investigator speculates about fundamental a proposition that may be true. He or she then applies analysis in steps that lead to it. We are familiar in modern times with attempts at a logic of discovery – abduction, inference to the best explanation, inductive logic, even probability theory. A major oddity in Pappus’ account is that he clearly seems to have thought that synthesis was deductive, that one can reason deductively from a theorem to be proven to the propositions that entail it.

Pappus’ distinction survived the middle ages although neither analysis nor synthesis themselves were standard topics in medieval logic. [[7]](#footnote-7) They are not mentioned, for example, in Ockham’s *Summa Logic* or Buridan’s *Summulae.*  When they were discussed by name, they were abstracted from the context of geometry and conceived more generally of as applying to all knowledge. Aquinas draws the distinction as follows:[[8]](#footnote-8)

The first is the method of analysis, by which we go from what is complex to what is simple or from a whole to a part, as it is said in Book I of the *Physics* that the first objects of our knowledge are confused wholes. Now our knowledge of the truth is perfected by this method when we attain a distinct knowledge of the particular parts of a whole. The other method is that of synthesis, by which we go from what is simple to what is complex; and we attain knowledge of truth by this method when we succeed in knowing a whole. Thus the fact that man is unable to know perfectly in things a whole and a part shows the difficulty involved in knowing the truth by both of these methods.

His account exhibits two technical features that became standard. First, analysis and synthesis are described abstractly as arguments. Despite the fact that discovery and instruction were understood to be human activities, as behavioral habits and skills, they are represented by formal arguments.

Second, following Pappus, both analysis and synthesis are as deductive. As Pappus puts it, they both proceed from propositions to their “consequences” [*akolouthn*]. The view makes sense for synthesis. The sequence of propositions in a the proof of a theorem in Euclid, which is the paradigm of a synthesis, similar to proofs in modern geometry and clearly deductive. They proceed from premises to a conclusion by steps of logical entailment. The case for analysis is otherwise. In Pappus’ account analysis proceeds backward from a theorem to the propositions from which it is proven. It seems to be a simple mistake to regard the process as deductive. A theorem does not usually logically entail its premises. On the other hand, there is no mistaking Pappus’ intension. He says, “If we come upon something admittedly false, that which is sought will also be false.” A proposition cannot be shown false by its logical relation to a false proposition unless that relation is logical entailment. Pappus’ view would be tenable if he also held that a theorem and its premises were logically equivalent and therefore mutually entail one another. But theorems in geometry are seldom equivalent to the propositions from which they follow.

This backward entailment view persisted in the logical tradition, and is endorsed in the 17th century by Arnauld and Nicole the Port Royal Logic in the their account of analysis:

This is the way to understand the nature of analysis as used by geometers. Here is what it consists in. Suppose a question is presented to them, such as whether it is true or false that something is a theorem, or whether a problem is possible or impossible; they assume what is at issue and examine what follows from that assumption. If in this examination they arrive at some clear truth from which the assumption follows necessarily, they conclude that the assumption is true (*s’ils arrivent dans cet examen à quelque vérité clair dont ce que leur proposé soit une suite nécessaire, ils en concluent, que ce qui leur est proposé est vrai*). Then starting over from the end point, they demonstrate it by the other method which is called *composition.* But if they fall into some absurdity or impossibility as a necessary consequence of their assumption, they conclude from this that the assumption is false and impossible.[[9]](#footnote-9)

Renaissance logicians found a solution to this logical conundrum by appeal to a distinction found in Aristotle. The solution has two steps. The first is illustrated by Aquinas’ text. It is to abstract analysis from geometry and to regard the type of demonstration characteristic of analysis as typical of science generally.

The second step is to apply distinction from *Posterior Analytics* to the type of demonstration characteristic of analysis. Aristotle distinguishes two type of demonstration in syllogistic form. The first takes as premises what is better and immediately known. These are typically propositions describing of observations about sensible particulars. The conclusion of the demonstration is a causal law that explains the observation. In a second sort, the premises consist of a causal law and its conclusion a description of an observation about singular things. The first sort was regarded in medieval logic as reasoning “from effect to cause” and called *demonstratio quia,* and the second was reasoning “from cause to effect” and called *demonstratio propter quid.* Aristotle draws the distinction as follows:[[10]](#footnote-10)

Knowledge of the fact differs from knowledge of the reasoned fact. To begin with, they differ within the same science and in two ways: (1) when the premisses of the syllogism are not immediate (for then the proximate cause is not contained in them – a necessary condition of knowledge of the reasoned fact): (2) when the premisses are immediate, but instead of the cause the better known of the two reciprocals is taken as the middle; for of two reciprocally predicable terms the one which is not the cause [i.e. the observed effect] may quite easily be the better known and so become the middle term of the demonstration [the demonstration *quia*]. Thus (2) (a) you might prove as follows that the planets are near because they do not twinkle [the effect]: let C be the planets, B not twinkling, A proximity [the cause]. Then B is predicable of C; for the planets do not twinkle. But A is also predicable of B, since that which does not twinkle is near – we must take this truth as having been reached by induction or sense-perception. Therefore A is a necessary predicate of C; so that we have demonstrated that the planets are near. This syllogism, then, proves not the reasoned fact but only the fact; since they are not near because they do not twinkle, but, because they are near, do not twinkle.

The major and middle of the proof, however, may be reversed, and then the demonstration will be of the reasoned fact [a demonstration *propter quid*]. Thus: let C be the planets, B proximity [the cause], A not twinkling [the effect]. Then B is an attribute of C, and A – not twinkling – of B. Consequently A is predicable of C, and the syllogism proves the reasoned fact, since its middle term is the proximate cause.

The argument forms are syllogisms:

**Demonstration *Quia* (from observed fact to its reason):**

*Everything that does not twinkle* (effect) *is proximate*(cause) A*MP* effect-cause commutable

*Every planets does not twinkle* (effect) A*SM* Subject-effect

∴*Every planets is proximate* (cause) A*SP* Subject-cause

**Demonstration *Propter Quid* (from reason observed fact):**

*Everything proximate*(cause) *does not tinkle* (effect) A*MP* cause-effect commutable

*Every planets is proximate* (cause) A*SM* Subject-cause

∴*Every planets does non-twinkle* (effect) A*SP* Subject-effect

The major premises of the two syllogisms, if true, are convertible – their subjects and predicates are coextensive. We will return to this dubious assumption in a moment.

To understand the motivation behind the two forms and appreciate their explanatory purpose, it is helpful to reformulate their gist colloquially. The first syllogism says in effect that the reason (“*propter quid*”) planets do not twinkle is that they are close. The second is merely informative. It says that in as much as (“*quia*”) planets are close, it is an interesting fact that they do not twinkle.

Even though analysis and synthesis as such were not widely discussed in medieval logic, Aristotle’s distinction among demonstrations was a standard topic. Aquinas draws it this way:[[11]](#footnote-11)

There are two kinds of demonstration. One kind is through a cause and is called a demonstration *propter quid*—and this sort of demonstration is through things that are prior, absolutely speaking. The second kind is through an effect and is called a demonstration *quia*—and this sort of demonstration is through things that are prior with respect to us. For since an effect is more apparent to us than its cause, we proceed through the effect to a cognition of the cause.

Now from any effect it can be demonstrated that a cause proper to it exists—as long as its effects are more known to us. For since effects depend on a cause, it follows that once an effect is posited, it must be that its cause exists prior to it. Hence, insofar as it is not known to us *per se* there is a God, this is demonstrable though effects that are known to us.

Ockham adds to the terminology by describing a demonstration *quia* as *a priori*, and a demonstration *propter quid* as *a posteriori*:[[12]](#footnote-12)

In light of this, it is important to see that some demonstrations are such that their premises are absolutely prior to the conclusion; and these are called *a priori* or *propter quid* demonstrations. Other demonstrations are such that their premises are not absolutely prior to the conclusion but are nonetheless better known [than the conclusion] to the one who is constructing the syllogism, with the result that it is through these premises that the one constructing the syllogism comes to a cognition of the conclusion; and these are called *quia* or *a posteriori* demonstrations.

John Buridan has chapters on the distinction and its uses in the *Summulae*.[[13]](#footnote-13) It now possible to see how it is applied to explain analysis and synthesis.

Due to a revived interest in method during the 15th and 16th centuries, philosophers again took up the topics of analysis and synthesis. These were understood as a describing the behavior of practicing investigators. It has two parts. First it is the habit or “art” of obtaining knowledge (*inventio*). The second part is the instruction and dissemination of the knowledge obtained. Training in method was understood as a kind of practical logic for use in actual research and instruction. As such it was regarded as superior to the cold formalism of scholastic logic, which was mainly of theoretical interest. Despite being behavioral in principle, however, method continued to be discussed in terms of mathematical paradigms in the manner of Pappus. Directions for investigators and teachers were commonly set out in sets of rules for the formation of syllogism. This approach was later followed by Descartes in his *Regulae* and *Discourse on Method*, and by the authors of the Port Royal Logic.

In his influential *On Methods*, Jacabo Zabarella (1533 –1589) applied Aristotle’s demonstration distinction to Pappus’ analysis and synthesis. In so doing, he hit upon what was regarded as an explanation of how in analysis a theorem can be deduced from its premises:[[14]](#footnote-14)

It happens, therefore, that in every syllogism constructed for the sake of knowing scientifically, it is necessary that progression occurs either from cause to effect or, on the contrary, from effect to cause. I do not find another kind of scientific syllogism also constituted from premises [that are] necessary in this way. For since the middle [term] and the major extreme are included in the major premise, and it is necessary that one be the cause of the other, then, of course, if the middle [term] was the cause of the major, the syllogism would be from cause to effect; and if the major extreme were the cause of the middle [term], it would be from effect to cause. Two scientific methods, therefore, arise -- neither more nor fewer. One, which the Greeks call *kurios apodeixis* (demonstration of the strongest sort) or *apodeixis tou dioti* (demonstration of the on-account-of-which), and we have been accustomed to call demonstration *potissima* or demonstration *propter quid*, is said to be demonstrative method *par excellence*. The other, which progresses from effect to cause, is named resolutive. For a progression of this type is a resolution, just as composition is said to be from cause to effect. The Greeks call this method *syllogismos tou hoti* (syllogism of the that-it-is-the-case) or *dia semeion* ([syllogism] by means of

signs). We [call it] demonstration *quia* or syllogism *a signo* or demonstration of the second degree.

Zabarella conceives of method as a research practice for obtaining knowledge and, once obtained, for its instruction. He calls it the “way of doctrine.”[[15]](#footnote-15) The text above is typical of his practice. He abstracts from human behavior to discuss it in terms of argument forms. The initial stage of scientific inquire, he says, is to reason from what we see to a causal law that explains it. It is represented by a demonstration *quia*. This is a syllogism in which the major premise is a causal law in which the major term names the observed property and the middle term names the causal property on which it depends. In the minor premise the minor term is the predicate, and it names the subject being explained. The conclusion then follows in the mood Barbara. It asserts that the observed subject has the required causal property.

Viewed abstractly, a syllogism of this form can be generated by choosing any property whatever for the middle term. To be plausible as an explanation, however, this open-endedness needs to be narrowed to plausible candidates. What these should be depends on an author’s wider view of science. In the *Port Royal Logic,* for example*,* the authors impose the condition that the causal law must either affirm the content of a clear and distinct idea or be a previously proven.

The second part of science, Zabarella explains, is instruction. It is represented by a demonstration *propter quid*, and he provides directions for its construction from the demonstration *quia*. The major premise of the earlier syllogism is converted, reversing subject and predicate, and the conclusion and minor premise switch places. In the resulting syllogism is in Barbara, and its conclusion affirms of its subject that it possess the observed property. The argument as a whole represents a causal explanation of the observed fact.

To modern eyes, the conversion employed in technique is the resurfacing of the difficulty that afflicts Pappus’ account. For the desired conversion of the major premise to be plausible, it is necessary that the causal property and its effect be co-extensive. Each must determines the other. In Aristotelian terminology the subject and predicate must be convertible. The observed property cannot be merely an accident of the subject – it must be a necessary property, a *proprium,* characteristic of it. The problem is to the one in Pappus’ account of analysis in which a derived theorem is regarded as logically equivalent to the principles from which it was proven. For Aristotle’s technique to work, it seems, an effect must be co-extensive with its cause or, viewed differently, that a species must be co-extensive with its genus.

It is instructive to look at another example of this sort of backwards reasoning in the logic of the period. It occurs in the medieval doctrine of topics or “places” (*topoi*, *loci*). Topics were techniques for constructing from an incomplete enthymeme a valid syllogism that completes an explanation. The general form of the enthymeme is “*S is P*, *because every* *M* is *P*”. The topic explains how to complete the enthymeme by converting it into a valid syllogism, “*every* *M* is *P*, *every S is M, therefore every S is P.*”

Two so-called topics are similar to demonstration *quia* and analysis. The first is called “from the effect to the efficient cause.” The task in this problem to complete the enthymeme, *S is P because P causes M.* The second is the topic called “from the species to the genus.” In it the task is to complete the partial argument, *S is in the genus P because M is a species of P.*

The topics and their completed syllogisms are displayed below on the left. To their right are displayed complementary topics. These reverse the direction of explanation. They too are traditional topics and are called respectively “from the efficient cause to the effect” and “from the genus to the species.” The completed syllogism on the left and right bear the same formal relation to each other as do demonstrations *quia* to demonstrations *propter quid*. These examples are from Peter of Spain:[[16]](#footnote-16)

**From the Effect From the Cause**

Enthymeme:

*The knife is good Everything good is caused by the smith*

*Therefore, the knife is caused by the smithy Therefore, the knife is caused by*

Syllogism:

*Everything good is caused by the smithy Everything caused by the smithy is good*

*The knife is good The knife is caused by the smithy*

*Therefore, the knife is caused by the smithy Therefore, the knife is good*

**From the Species**  **From the Genus**

Enthymeme:

*Socrates is a man Socrates is a an animal*

*Therefore, Socrates is an animal Therefore, Socrates is a man*

Syllogism:

*Every man is an animal Every [rational] animal is a man*

*Socrates is a man Socrates is a [rational] animal*

*Therefore, Socrates is an animal Therefore, Socrates is a man*

The construction of the syllogisms on the right from those on the left exactly follows that of a demonstration *propter quid* from a demonstration *quia.* In the pair of syllogisms “from the effect” and “from the cause” the major premises are converses of each other, and so are those in “from the cause” and “from the species,” and in each pair the conclusion and the minor premise switch places. As in the case of the construction of a demonstration *propter quid* from a demonstration *quia*, if both major premises are true, the effect must be co-extensive with the cause, and the species with the genus. Odd as it may seem, Aristotle does sometimes assume that an efficient cause and its effect are convertible – that they “determine each other” (*allln antia*).*[[17]](#footnote-17)* If the context is right, it may also be appropriate to consider a species as co-extensive with its genus, as when the species is restricted by the relevant difference. Some kind of contextual restriction of this sort seem to be assumed by writers of the period when they think it appropriate to reason to a genus from a species.

The account of analysis in the Port Royal Logic falls squarely in this tradition. The purpose of analysis, its authors say, is knowledge acquisition – to discover the truth (*découvir la vérité*)[[18]](#footnote-18). It proceeds “from what is better known to what is less well known,” and from the particular to the general – from “*vérités connues dans l’examen particulier*” to “c*onaissances générales”* – and from effect to cause.[[19]](#footnote-19) Moreover, as in Pappus and Zabarella, the derivation relation in analysis is logical entailment. The derivation of the general from the particular, and of the cause from the effect, follows necessarily.[[20]](#footnote-20)

In Chapter 2 of Part IV the authors provide two examples analysis. In the first an investigator discovers that a subject has St. Louis as a remote ancestor. St Louis is, in a sense, his “cause.”[[21]](#footnote-21) When spelled out, the logic in question is a series of syllogisms in Barbara. Each has two premises, a minor premise affirming of a subject (the minor term) that he is the descendent of his father (the middle term), and an auxiliary major premise needed to make the inference go through. In this case the needed premise affirms of the subject’s father that he is the descendent of his grandfather (the major term). Auxiliary major premises of this sort are typical of analysis. The authors explain, “… in analysis we introduce clear an evident maxims only to the extent that we need them.”[[22]](#footnote-22) Coming to realize that the extra premise is needed, and that the extra premise is true, is part of the discovery process of analysis. The syllogism’s conclusion affirms that the subject is the descendent of his grandfather.

This pattern is repeated, one syllogism for each subsequent generation, until an ultimate conclusion affirms that the original subject is a descendent of St. Louis. Because increasingly earlier ancestors have increasingly more descendants, each succeeding predicate has a broader extension, and the final predicate *is a descendent of St Louis* is the “most general” of all. The full analysis spelled looks like this:

*every descendant of B is a descendant of C, A is a descendant of B/*∴*A is a descendant of C*

*every descendant of C is a descendant of D, A is a descendant of C/*∴*A is a descendant of D*

*every descendant of D is a descendant of E, A is a descendant of D/*∴*A is a descendant of E*

*every descendant of E is a descendant of St. Louis, A is a descendant of E/*∴*A is a descendant of St Louis*

The corresponding synthesis would be the series in reverse order. As in the earlier examples, the causal premise must be converted, on the assumption evidently that this particular chain of descendants is linear. It looks like this:

*every living descendant of St. Louis is a living descendant of E, A is a living descendant of St. Louis /*∴*A is a living descendant of E*

*every living descendant of E is a living descendant of D, A is a living descendant of E/*∴*A is a living descendant of D*

*every living descendant of D is a living descendant of C, A is a living descendant of D/*∴*A is a descendant of C*

*every living descendant of C is a living descendant of B, A is a living descendant of C/*∴*A is a living descendant of B*

The authors provide a second example reasoning from effect to cause, which is in this case is also reasoning from species to genus.[[23]](#footnote-23) In this series too each syllogism makes use of a major premise necessary to make it complete. This is the discovery that the soul is immortal. Formally the reasoning is as follows:

*Thinking things are immaterial. The soul is a thinking think. /*∴ *The soul is immaterial.*

*Anything that is immaterial is indestructible. The soul is immaterial. /*∴*The soul is indestructible*

*Anything that is indestructible is immortal. The soul is indestructible. /*∴*The soul is immortal.*

Since, as the authors say,[[24]](#footnote-24) “it is impossible to know a species properly without knowing the genus,” is helpful to construct an example like those in the *Logic* but that actually makes explicit reasoning from a genus and a species:

*every human is an animal, Socrates is a human*/∴*Socrates is an animal*

*every animal is a living creature, Socrates is an animal*/∴*Socrates is a living creature*

*every living creature is a body, Socrates is a living creature*/∴S*ocrates is a body*

*every body is a substance,* S*ocrates is a body*/∴*Socrates is a substance*

In the *Logic’*s technical terms this sequence begins by affirming of Socrates in the first syllogism the restricted predicate *human,* which has the comprehension {*rational, self-moving, living, corporeal, being*}. In the subsequent syllogisms species of increasingly less restricted comprehensions are predicated of Socrates. The sequence terminates predicating of Socrates the most general genus.

The corresponding synthesis reasons in the reverse direction. It would require that the containment premise be converted. This assumption in the case of genus-species is plausible only on the assumption that the causal sequence running from genus to species leads in a direct line to a specific individual. It would look like this:

A*ll individuals in (the genus) substance are individuals in (the species) body. Socrates is an individual in (the genus) substance. /∴ Socrates is an individual in (the species) body.*

*All individuals in (the genus) body are individuals in (the species ) living creature. Socrates is an individual in (the genus) body. /∴ Socrates is an individual in (the species) living creature.*

*All individuals in (the genus) living creature is an individual in (the species) animal. Socrates is an individual in (the genus) living creature. /∴ Socrates is an individual in (the species) animal.*

*All individuals in (the genus) animal are individuals in (the species) human. Socrates is an individual in (the genus) animal. /∴ Socrates is an individual in (the species) human.*

The broad understanding of analysis and synthesis in the Renaissance morphed over time into the modern distinction. Hobbes’ version of analysis and synthesis is causal and similar to the *Logic*’s.[[25]](#footnote-25) Although Spinoza advocated a quasi-Neoplatonic causation, he possessed the *Port Royal Logic* in his library,[[26]](#footnote-26) and subscribed to his own version of the thesis that the order of cause to effect was the same as the order in logic of subject to predicate. In various papers Leibniz explores versions of analysis that are essentially more formal versions of the *Logic*’s*.*[[27]](#footnote-27)It was typical of Leibniz to symbolize the predicate of a universal affirmative as a series *P*1…*Pk* of concatenated terms. In his notation the term letters are intended to stand for modes definitive of an idea, much like the *Logic*’s species-comprehensions. In a typical example Leibniz lays down an initial premise *S is P*1…*Pk*. The “analysis,” then, is a deduction that proceeds by the application of a simplifying inference rule that deletes terms from the predicate. The new line’s predicate is therefore more general. The deduction terminates in a line with the most general predicate of all. The inference rule deletes a term: *S is X*1…*Xn*├ *S is X*1…*Xn*-1. An example of a deductive analysis in Leibniz’ style sort would be:

*S is P*1,*P*2,*P*3,*P*4

*S is P*1,*P*2,*P*3

*S is P*1,*P*2

*S is P*1

It is possible to see how terminology evolved. According to the *Logic*, if the premise stated the content of an idea, it would be necessarily true. That premise and the lines deduced from it would be analytic in Kant’s sense. If the soul experienced the idea as clear and distinct, this demonstration is *a priori,* in Ockham’s sense, and would also be known *a priori* in Kant’s sense. In this way medieval *a priori* analysis morphed into Kant’s *analytic a priori*. The *Logic* has a place in the middle.

Logically, of course, the inferential steps using in Leibniz’ rule, and even syllogisms in the mood Barbara, are extremely simple-minded. The *Logic*’s authors were not worried that anybody would make mistakes in logic using these rules. In keeping with the *Logic*’s general emphasis on intentional content, they were much more concerned that people making mistakes about the meaning of words. Accordingly, when they give advice on analysis specifically they give rules, which they borrow from Descartes, that do not talk about logical forms or fallacies – after all, anybody can reason in Barbara. All they say, in effect, is not to be careful in formulating your premises add or to omit content to an idea’s intention.[[28]](#footnote-28)

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1. Arnauld and Nicole, *La Logique ou l’Art de Penser* (abbreviated *LAP*) and Arnauld, *Des vraies et des fausses Idées* (abbreviated *VFI*) are in volumes V and I respectively of Arnauld, Antoine (2003), Œuvres philosophique d’Arnauld, eds. Kremer and Moreau (abbreviated *KM*). *Part x, Chapter y* is abbreviated *x.y*. The English translation of *LAP* is [*Arnauld 1996*](#_ENREF_5) (abbreviated *B*) and that of *VFI* is [*Arnauld 1990 [1683]*](#_ENREF_4) (abbreviated G). References to Descartes are in [*Descartes 1897-1909*](#_ENREF_9) (abbreviated *AT* ). [↑](#footnote-ref-1)
2. 265d, Jowett translation. [↑](#footnote-ref-2)
3. [*Euclid 1956*](#_ENREF_12), p. 442. [↑](#footnote-ref-3)
4. [*Heath 1921*](#_ENREF_14), vol. 2, pp. 399-400. [↑](#footnote-ref-4)
5. Compare the translation in [*Hintikka 1974*](#_ENREF_15), pp. 8-10. [↑](#footnote-ref-5)
6. [*Schiller 1917*](#_ENREF_21). [↑](#footnote-ref-6)
7. [*Diogenes\_Laertius 1970*](#_ENREF_11). III.25, pp. 298-299; [*Proclus 1970*](#_ENREF_20), 211, pp. 163-4. [↑](#footnote-ref-7)
8. Una quidem per modum resolutionis, secundum quam procedimus a compositis ad simplicia, et a toto ad partem, sicut dicitur in primo physicorum, quod confusa sunt prius nobis nota. Et in hac via perficitur cognitio veritatis, quando pervenitur ad singulas partes distincte cognoscendas. Alia est via compositionis, per quam procedimus a simplicibus ad composita, qua perficitur cognitio veritatis cum pervenitur ad totum. Sic igitur hoc ipsum, quod homo non potest in rebus perfecte totum et partem cognoscere, ostendit difficultatem considerandae veritatis secundum utramque viam.

   [*Aquinas 1961*](#_ENREF_2)*, Commentary on the Metaphysics*, Book II, p. 278. See also *Summa Theologicae* Ia.II.q15.a5. [↑](#footnote-ref-8)
9. *LAP* IV.2, *KM* V 367, *B* 238. [↑](#footnote-ref-9)
10. [*Aristotle*](#_ENREF_3) *Posterior Analytics* I.13, trans. G. R. G. Mure. [↑](#footnote-ref-10)
11. *Summa Theologiae* I.q2.a2. Text, [*Aquinas 1888*](#_ENREF_1):

    Respondeo dicendum quod duplex est demonstratio. Una quae est per causam, et dicitur propter quid, et haec est per priora simpliciter. Alia est per effectum, et dicitur demonstratio quia, et haec est per ea quae sunt priora quoad nos, cum enim effectus aliquis nobis est manifestior quam sua causa, per effectum procedimus ad cognitionem causae. Ex quolibet autem effectu potest demonstrari propriam causam eius esse (si tamen eius effectus sint magis noti quoad nos), quia, cum effectus dependeant a causa, posito effectu necesse est causam praeexistere. Unde Deum esse, secundum quod non est per se notum quoad nos, demonstrabile est per effectus nobis notos. [↑](#footnote-ref-11)
12. [*William\_of\_Ockham*](#_ENREF_23) *Summa Logica* III.1, chapter 17:

    Propter quod oportet scire quod quaedam est demonstratio cuius praemissae sunt simpliciter priores conclusione, et illa vocatur demonstratio a priori sive propter quid. Quaedam est demonstratio cuius praemissae non sunt simpliciter priores conclusione, sunt tamen notiores sic syllogizanti, per quas devenit sic syllogizans in notitiam conclusionis, et talis demonstratio vocatur demonstratio quia sive a posteriori. [↑](#footnote-ref-12)
13. [*Buridan 2001*](#_ENREF_7), 8.7.10 to 8.9.3. [↑](#footnote-ref-13)
14. [*Zabarella 2013*](#_ENREF_24), *On Methods*, Book III, Chapt IV, 4, vol 2, pp. 22-25:

    Hinc fit ut in omni syllogismo sciendi gratia constructo necesse sit vel a causa ad effectum vel contra ab effectu ad causam progressum fieri, aliud genus scientifici enim in propositione maiore medium et maior extremitas collocentur et necessum sit alterurn alterius causam esse, medium quidem maioris causa si fuerit, syllogismus est a causa ad effectum; si vero maior extremitas sit causa medii, est ab effectu ad causam. Duae igitur scientificae methodi oriuntur, non plures, nec pauciores, altera per excellentiam demonstrativa methodus dicitur, quam Graeci, κυριως πόδειξιν vel πόδειξιν το διότι vocant, nostri potissimam demonstrationem vel demonstrationem propter quid appelare consueverunt. Altera, quae ab effectu ad causam progreditur, resolutiva nominatur, huiusmodi enim progressus resolutio est, sicuti a causa ad effectum dicitur compositio, methodum hanc vocant Graeci, συλλογισμν το τι vel δι σημείων nostri demonstrationem quia vel syllogismum a signo vel secundi gradus demonstrationem. [↑](#footnote-ref-14)
15. On Zabarella see [*Gilbert 1960*](#_ENREF_13), pp, 167-176. [↑](#footnote-ref-15)
16. [*Peter\_of\_Spain 2014*](#_ENREF_19), pp. 212-213, 218-219. [↑](#footnote-ref-16)
17. See *Metaphysics* D, 1013b9; *Physics* II.3, 195a11. [↑](#footnote-ref-17)
18. *LAP* IV.2, *KM* V 361, *B* 233 [↑](#footnote-ref-18)
19. *LAP* IV.2, *KM* V 362, 366; *B* 234, 237. [↑](#footnote-ref-19)
20. *LAP* IV.2, *KM* V 367; *B* 238. [↑](#footnote-ref-20)
21. *LAP* IV.2, *KM* V 367, *B*238. [↑](#footnote-ref-21)
22. *LAP* IV.2, *KM* V 366, B 237. [↑](#footnote-ref-22)
23. *LAP* IV.2, *KM* V 366, *B* 237 Cf. Descartes, Synopsis of Second Meditation, §13, [*Descartes 1985b*](#_ENREF_10) II.9. [↑](#footnote-ref-23)
24. *LAP* IV.3, *KM* V 368, *B* 238. [↑](#footnote-ref-24)
25. Hobbes, *De Corpore* I.6.1, [*Hobbes 1992*](#_ENREF_16), p. 66. [↑](#footnote-ref-25)
26. The *Logic* was in his library, see [*Offenberg 1973*](#_ENREF_17). [↑](#footnote-ref-26)
27. Leibniz explains his notion of analysis as follows:

    I hold that every true proposition is either *immediate* or *mediate*. An immediate proposition is one that is true by itself, i.e. a proposition whose predicate is explicitly contained in its subject; I call truths of this sort ‘identical’. All other propositions are mediate; a *true* proposition is mediate when its predicate is included virtually in its subject, in such a way that analysis of the subject, or of both predicate and subject, can ultimately reduce the proposition to an identical truth. That’s what Aristotle and the scholastics mean when they say ‘the predicate is in the subject’.

    Leibniz to Arnauld, 14.vii.1686 (unsent draft), [*Bennett 2017*](#_ENREF_6), p. 33. For examples of analysis within the context of a logical system see *De arte combinatoria* in [*Parkinson 1966*](#_ENREF_18), and [*Swoyer 1995*](#_ENREF_22). [↑](#footnote-ref-27)
28. *LAP* IV.2, *KM* V 364-365; *B* 235-236; cf. Rules XIII, *AT* X 435-436, [*Descartes 1985a*](#_ENREF_8), pp. 54-55. [↑](#footnote-ref-28)