All Brutes are Subhuman: Aristotle and Ockham on Privative Negation

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Privation is a species of negation ignored in modern logic. It is a standard topic, however, in the history of logic and linguistics. In this paper I investigate what may be called, broadly speaking, the Aristotelian account to see if it succeeds in explaining the logic of privative negation.

The account will be evaluated as logic. In modern metalogic we would expect the theory to specify a syntax in which logical terms indicate the application of a formation rule to descriptive terms taken as arguments. The theory would provide a semantics with a recursive definition of "possible interpretation." For each formulation rule, there would be a clause in the definition that determines from the values of an expression's immediate parts the value of the whole. An argument would be defined as valid if any interpretation that made its premises true also makes its conclusions true. Each recursive clauses should capture the intuitive meaning of the logical term it corresponds to, and the set of valid arguments should be intuitively acceptable. This review helps situate the objective of the paper. It would be interesting indeed if the historical account provides the basis for a modern metatheory of privative negation, one in which negation is treated as a logical term with an associated semantics and set of valid arguments.

The relevant concept of privation is the second sense discussed by Aristotle in the Metaphysics (1022^b25):

> [Privation] means such a lack in being of class of beings which normally possesses that property; for example a blind man and a mole are in different ways "deprived" of sight: moles as a whole class and of animals are so deprived, whereas only individual men are.

A privation is a dispositional property that fails to hold for a class of individuals that would "normally", elsewhere Aristotle says "naturally", hold for the members of the type. (See *Categories* 11^b15, *Topics* 109^b18, *Metaphysics* 1022^b29.) This formulation is expressed in metaphysical terms – by its referral to classes, properties, and deprivation. More interesting is the rephrasing of the distinction in linguistic terms that became standard in later logic. In this paper I shall be taking William of Ockham as representative of this tradition. Although Ockham's account contains idiosyncratic elements, it is one of the best statements by a logician of the traditional view. His definitions are clear and his examples are ample. Discussing the views of a single theorist also has the advantage of avoiding oversimplified historical generalizations, though it will require that any non-standard features of Ockham's view be clearly flagged.

The paper will consist of several parts. The introduction sets out Ockham's account in his own words and makes initial clarifications. Two subsequent sections set out background material on scalar adjectives and the syllogistic. The paper concludes with a discussion of the adequacy of Ockham's logic.

2. Ockham's View

Ockham hardly ever exploits the idea of privation to develop other views, though he spells it out at length in two sections of the *Summa logicae* and again in his commentary on the *De interpretatione*. The following quotations from the *Logic* capture the main ideas:

> [12] Now not only are propositions in which connotative or relative terms occur equivalent to hypothetical propositions, but also propositions in which negative, privative, and infinite terms occur are also really connotative, since in their nominal definitions there must occur something in the nominative case and something in an oblique case – or in the nominative case with a preceding negation.
>
> For example, the definition of the name 'immaterial' is 'something which does not have matter', and the definition of the term 'blind' is 'something lacking in sight which by nature should have sight'
>
> ….

Now every proposition in which such a term occurs has at least two exponents, and sometimes it has more than two. This can easily be ascertained by looking at the nominal definition of the term in question. Hence, every proposition in which an indefinite term occurs has two exponents. One of them is an affirmative proposition in which 'something' (in the singular or plural) or some other term equipollent to it is the subject or predicate. Hence, 'A donkey is a non-man' is equivalent to 'A donkey is not a man'. Similarly, 'An angel is immaterial' is equivalent to 'An angel is something and an angel does not have matter'. And this should be understood to apply when the negative term in question signifies negatively nothing except what the opposite term signifies affirmatively. I mention this to exclude the following counterinstance: for the conjunctive proposition 'This divine essence is something and it is not generated' is not equivalent to 'The divine essence is ungenerated.'

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[13] Although propositions containing infinite terms or their equivalents have only two exponents, still affirmative propositions that contain privative terms not equivalent to infinite terms have more than two exponents. Hence, the proposition 'He is blind' has these exponents: 'He is something', 'By nature he should have sight', 'He will never be able to see naturally'. But it is not possible to give firm rules for such propositions, for because of the variety of such terms the propositions in which they occur have to be expounded in different ways. Hence, 'Socrates is blind' has the exponents that have been mentioned. But the proposition 'Socrates is foolish' has these exponents: 'Socrates is something' and 'Socrates does not have the wisdom which he ought to have'. Still, this is consistent with its being the case that he is able to have wisdom….even though a privative term occurs in each.[1]

[1]*Summa Logicae,* Part II,12-13, pp119-122, William of Ockham, *Ockham's Theory of Propositions: Part Ii of the Summa Logicae*, trans. Alfred J. Freddoso and Henry Schuurman (Notre Dame, ID: University of Notre Dame Press, 1980). See also Part I, 36, p117, Willam of

The positive thesis is that any affirmative proposition with an infinite or privative predicate may be "set out" as short for a longer hypothetical proposition. Several features of the view should be highlighted at the outset.

1. Mental and Spoken Language. Though the exposition is common among mediaeval logicians, Ockham and conceptualists generally understand it providing an abbreviation (nominal definition) in which a syntactically simpler form in spoken language is declared to stand for ("signify") a more complex form in mental language. It is this complex proposition that one "thinks" when saying the expounded verbal proposition. The complex proposition is also expressible in spoken language, and the two spoken forms are then logically equivalent. Those who do not subscribe to mental language would nevertheless agree that the grammatically simpler exposition abbreviates the expounded proposition, understanding both as parts of spoken language.

2. Exposition. The theoretical function of exposition is to provide a general device for explaining the formal logic of a proposition. The *exponens* reveals a more detailed syntax than that evident in the *exponendum*. This more detailed syntax is then subject to various previously defined logical implications. The technique is common in logic of all periods. Russellian definite descriptions are an example. The logical implications of a proposition that contains a definite description is dentified by Russell with the formulas entailed by its translation into a longer formula written in more primitive logical vocabulary, the logic of which has already been explained. In the mediaeval exposition of privative predication, the background theory is the syllogistic augmented by conjunctive hypotheticals. The logical power of the privative assertion is then captured by the formal manipulations sanctioned in the syllogistic for the expressions in the *exponens*.

3. Contradictory and Infinite Opposition. In this discussion sentence negation is to be understood as the negation operator of modern sentential logic interpreted by its classical bivalent truth-table. In syllogistic logic, which lacks negations of complex sentences, it occurs only with categorical propositions. In

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Ockham, *Ockham's Theroy of Terms: Part I of the Summa Logicae*, trans. Michael J. Loux (Notre Dame, ID: University of Notre Dame, 1974).

the syntax it is place next to (in Latin, in front of) the verb. In the examples below, *not* is used for this sort of negation.

The privative negation of the predicate of an affirmative proposition is like its infinite negation, and unlike sentential negation, in that both presuppose the referent of the subject term and deny that the predicate holds of a relevant type. The two differ in that the type relevant to infinite negation is the entire domain of existing entities, but that of privative negation is a subset of the domain into which the subject would naturally fall. In the discussion below the prefix not_{∞}- will be used for this operator. As will become clear, privative, infinite, and sentential negation are progressively weaker in the sense that the former logically entail the latter but not conversely.

<u>4. Privative Opposition.</u> Though ancient authors and modern linguists point out that in natural language privatives are often complex terms or phrases marked by a negative affix, for Ockham and mediaeval logicians generally, privatives are fully lexicalized nouns that though negative meaning do not display a negative marker in their syntax. It will be useful here to adopt this modern (and ancient) practice and indicate privatives by a negative marking. Below *non-* will stand for this negation, being distinguished as context requires from other relevant operators. The exposition, then, may be broken down as follows:[2]

All S is non-P iff *Some S exists,*
Every S is of sort T that possesses P naturally, and
No S is P

According to this account privations are a special case of infinite negatives, and are a kind of lexicallized relative complementation. Suppose the privative A statement *S is non-P* is true. Then *S exists* because the subject of a true A proposition is non-empty. Further, the sentential negation *S is not P* is true because if the privative *S is non-P* is true, then by exposition so is *No S is P* and hence by subalternation *Some S is not P,* which is logically equivalent

[2] Here for the particular affirmative I am extrapolating somewhat beyond the text and treating the singular as a universal.

to *S is not P*. But {*S exists, S is not P*} is logically equivalent to *S is not*$_\infty$*-P.*

Since strictly speaking *non-P* in the context of the exposition is introduced by a contextual definition, it is a syncategorematic expression and has no interpretation in its own right. It could, however, be introduced into the syntax as a sentence operator in its own right and its semantics defined in its own clause in the definition of an acceptable interpretation R. Let P^τ stand for the natural type associated with *P, i.e.* P^τ stands for the entities of which it is natural that *P* should be true. Privative negation could then be defined directly: R(*non-P*)=R(P^τ)–R(*P*).[3] This is the sense in which *non-* as introduced by contextual definition may be regarded as "effectively" a relative complement operator.

<u>5. Abstraction from Nature.</u> Since the goal of exposition is to exhibit the deeper syntax of a proposition by opening it to formal manipulation, it is relevant to draw attention to the non-formal content of the exposition's second conjunct. This clause states that the subject is of a certain kind and that elements of this kind possess a property normally or naturally. The fact that the property naturally holds of the kind is indeed important in some theoretical applications of privatives, like metaphysics. In Neoplatonism and the later theology of perfection, for example, a major role is ascribed to privative orderings that consist of the removal of *natural* properties. But the naturalness of the property adds little to syntactic structure of the exposition and hence little to exposition's purely formal structure open to logical manipulation. Accordingly I shall here abstract away from the formally irrelevant content. The second condition to the exposition may then be simplified to the bare assertion that the subject is a member of the relevant kind:

[3] Notice that in general there might be some entity in the domain that would fall under the expounded privative but not under the operator just defined. This happens because there might be some individual that falls under the operator but is not named in the language, *i.e.* some *x* such that $x \in$R(*non-P*) but there is no *S* such that

All S is T

It is this syntax that is relevant to the logic. As shall be explained below, this more abstract version has the added advantage of making the idea essentially the same as privative negation as it is studied in linguistics.

6. Redundancy. The exposition can be simplified even more. Its first conjunct, *Some S exists,* says that the subject term is non-empty, but this clause is logical redundant. It is already logically entailed in the syllogistic by the exposition's second conjunct, *All S is T,* because it is part of the truth-conditions of a universal affirmative that its subject picks out a non-empty extension.

Given these introductory remarks it is possible to state the object of the paper succinctly. The question to be investigated is whether the logical implications of the privative assertion are exactly the syllogistic entailments of *All S is non-P* understood as abbreviations for the set {*Every S is* P^{τ}*,No S is P*}. To decide the issue it will be necessary to investigate the logical implications of privative predication, on the one hand, and the syllogistic entailments of the proposed exposition, on the other. These tasks will occupy the next two sections of the paper.

3. Scalar Adjectives and Negations+-

Linguists call *scalar* those adjectives that fall into families that have their interpretation governed in a rule-like way by a common comparative adjective phrase. For example the series *ecstatic, happy, content, so-so* have interpretations that divide a common background scale ranked by the governing comparative adjective *happier than.* Viewed extensionally, the comparative adjective stands for a binary relation on a set of individuals. The set-theoretic "field" of the relation is the set of all relata joined by the relation, and it forms what is sometimes called the *range of significance* of the associated scalar adjectives, *i.e.* the domain over which each individual scalar predicate in the series must be interpreted. A predicate is assigned an extension in the field and is true of objects in its extension, false of objects outside its extension but in the

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x is in R(S) and both *No S is P* and *All P are* P^{τ} are true. To this extent the expounded version fails to meet what is clearly its intended interpretation.

field, and "meaningless" of objects outside the field. (It is irrelevant here how "meaningless" is captured in terms of truth-values). Here are four examples of such families:

is happier than	*ecstatic, happy, content, so-so*
is sadder than	*miserable, sad, down, so-so*
is hotter than	*boiling, hot, warm, tepid*
is colder than	*freezing, cold, cool, tepid*

Laurence Horn has identified what he calls *test frames* that provide criteria of identifying the members of a scalar family and their respective order within it.[4] If the language user finds the following expressions semantically acceptable

x is not only Q, but P

x is P, or at least Q

x is at least Q, if not (downright) P

x is not even Q, {*let alone/much less*} *P*

x is Q, {*or/possibly*} *even P*

x is Q , and is {*in fact/indeed*} *P*

then *P* and *Q* fall in a scalar series in which *P* is higher than *Q* . Horn uses the convention, which I will follow here, of listing a scalar family by writing *x* to the left of *y* iff *x* is higher than *y* is the series.

Semantically the comparative adjective stands for a set theoretic relation, but there are alternative semantic accounts of the associated family of monadic scalar adjectives. Ideally the interpretation of the monadic adjectives themselves will capture the structural properties of the scale, including its order. Let us think extensionally for the moment. If we adopt the standard first-order semantics and assign to the predicates subsets of the domain, we capture some of the relevant

[4]Laurence R. Horn, *A Natural History of Negation* (Chicago: University of Chicago Press, 1989). Horn states a full account of scalars and their negations, including other kinds of evidence for identifying scalars and their negations beyond those employed in this paper. On the logic and semantics of comparative adjectives see Lennart Åqvist, "Predicate Calculi with Adjectives and Nouns," *Journal of Philosophical Logic* 10 (1981). Unlike the account here, Horn favors a pragmatic rather than a model theoretic approach to their interpretation. On the model theory used here for scalars see John N. Martin, "Existence, Negation, and Abstraction in the Neoplatonic Hierarchy," *History and Philosophy of Logic* 16 (1995).

semantic structure, but we do not yet know the order. The natural way to capture this order is to nest these extensions. There are two ways to do so, both corresponding to intuitive readings of the scalars. In the first method predicates are paired with sets so that the “higher” predicate is extensionally included in the “lower”. The intuition here is that its extension may be understood as comprising all objects that have the background property to *at least* a given degree where that degree is characteristic of the predicate in question. On the second reading, the extension embraces all objects that have the background property to *at most* a characteristic degree so that the extension of a predicate lower in the order is a subset of the extension of a higher predicate.[5] This is the interpretation we shall adopt here. Structurally either version will satisfy the desire to represent in the semantics the relevant notion of order. The second has the additional advantage of conforming to the order imposed below by the semantics of the syllogistic.

It should be stressed that the notion of degree in the intuitive explanation of extensions is not intended to imply any commitment to a metric or measurement. All that is needed is a weaker notion of scalar order, which is provided by the subset relation that nests the field of the comparative relation. Indeed, we may abstract from the notion of set or extension entirely and represent the semantics of scalars by an ordering exhibiting relational properties that insures the order is total.

<u>Scalar Negations.</u> Scalar families come in "positive" and "negative" pairs. Paired with the top to bottom relation *happier-than* is the bottom to top relation *sadder-than,* which determines a ranking of predicates that go in the reverse order. Likewise *hotter-than* determines a ranking that reverses the order associated with *colder-than.* More formally, each such relation R+ and its pair R- describe total orders on the field, and $R+ \cup (R-^{\smile})$ is itself a total order on the field.[6] This claim is supported by linguistic intuition. Below is displayed the list for *happier-than* conjoined with that of *sadder-than* in reverse direction, and that for *hotter-than* with that of *colder-than*, again in reverse order. The resulting

[5] Horn, who is not concerned to develop a semantic account, only discusses the former method of nesting.

predicate series meets Horn's test-frame criteria for scalar families relative respectively to their comparatives *happier-than* and *hotter-than*:

ecstatic, happy, content, so-so, down, sad, miserable

boiling, hot, warm, tepid, cool, cold, freezing

The scalar order moreover has a direction that is linguistically important. That is, language presupposes there is a semantic difference between what we call the top and bottom of the scale. Sometimes this is clear from the intended interpretation, and the directionality is grounded in some objective measure of a physical privation process, like loosing one's teeth or hair in the case of Aristotelian privation. But Horn argues that within the semantic properties of the predicate family itself one can find evidence of which end of the order is top or "positive." The evidence is connected with a natural language affix, that I will indicate here by °. The semantics of the affix often presupposes that the semantic scale has a hypothetical *midpoint e* in the sense that any point on the scale is located either above, at, or below *e,* though the family may lack a particular adjective naming *e*. One extreme of the ordering is called the "positive" pole, and the other "negative." The function of ° may be described as converting a positive predicate representing the point at the rank *n* steps above the midpoint into a new predicate synonymous to the predicate assigned to the point at the rank *n* steps below the midpoint. For example, associated with *happy* is *unhappy*, roughly synonymous to *sad*. The role of ° can be motivated by talk of degrees. Let the points on the scalar ordering be numbers (for example, the reals) with the midpoint of the scale fixed at 0. If the points were interpreted extensionally so that for any positive predicate *P,* there is some point *n* in the scale such that *P* is assigned the sets of all elements that have the background property to at most degree *n*, then P° would stand for a subset of the extension of *P*, *viz.* the set of objects that have the background property to at most the degree n°. Abstracting from the notion of measurement, ° may be characterized

[6] For any relation R, R˘ is its converse.

more broadly as an antitonic idempotent operator. The minimal structure necessary is familiar from many-valued logics studied by Kleene: [7]

Definition. < U,≤,°> is a *(strong) Kleene* structure iff

1. ≤ is a total order on U,
2. ° is an antitonic idempotent unary operation on U.[8]

The order relation ≤ on the structure determines operations of meet ∧ and join ∨. (Here $x \wedge y$ is the least upper bound of {*x*,y} under ≤, and $x \vee y$ is its greatest lower bound.) If a Kleene structure contains an element *e* such that $e={}^\circ e$, we shall call *e* the *midpoin*t of the structure. A special case is Kleene's 3-valued matrix for the "strong connectives," with conjunction as ∧, disjunction as ∨, and midpoint at $\frac{1}{2}$.

	–	∧	0	$\frac{1}{2}$	1	∨	0	$\frac{1}{2}$	1
0	1		0	$\frac{1}{2}$	0		0	$\frac{1}{2}$	1
$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
1	0		0	$\frac{1}{2}$	1		1	$\frac{1}{2}$	1

Natural language, moreover, provides indicators pointing out which extreme of the order is "positive" and which "negative." The operator is typically defined (*i.e.* is grammatical and semantically acceptable) for the monadic predicates at one of the extremes, the one we shall call *positive*, but undefined (is ungrammatical or semantically deviant) of the other, which we shall call *negative*. For example, *unsad* is not even grammatical in English. The pole for which it is undefined is "negative". Moreover, double negations of ° are ungrammatical as well, *e.g. ununhappy* and *unimpolite* are not acceptable. For example, in English *immoral* and *impolite* are grammatical and semantically acceptable, but it is not acceptable to negatively mark their lexical synonyms *bad* and *rude.* However, nothing I shall say in this paper turns on whether these

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[7] See S. C. Kleene, "On a Notation for Ordinal Numbers," *Journal of Symbolic Logic* 3 (1938).
Jan Lukasiewicz, "On 3-Valued Logic," in *Jan Lukasiewicz, Selected Works*, ed. I. M. Bochenski (Amsterdam: North-Holland, 1970).

[8] For any *x* in U, $x \leq y$ iff $y^\circ \leq x^\circ$, and $x^{\circ\circ}=x$.

regularities determine a genuine ontological distinction between up/down or positive/negative.

Hyper and Privative Negation. Natural language contains two additional affixes for scalars in addition to the negation we have already identified. Both are "intensifiers," one relative to the positive direction of the scale and the other to the negative. The former is known in classical grammar and among linguists as the *alpha intensivum*, but is more familiar in philosophy as *hypernegation*, as Proclus once called it, and which is familiar in the mediaeval Neoplatonic tradition deriving from Dionysius' divine names in adjectives like *hyper-good*, *hyper-beautiful.* [9] In English it is marked not only by the negative prefix *not* but by *hyper* and *super:*

Its not hot, its boiling.

He's not (merely) active, he's hyperactive.

It's not (merely) a conductor but a superconductor.

The second intensifier is known in grammar as the *alpha privative*, and it is this that shall concern us here. Following the tradition in philosophy and the history of logic I shall call it *privative negation*. It is marked in English by the prefixes *not* and *sub,* and the suffix *less*:

It's not (just) cold, its freezing.

His performance is certainly sub par today; he's not his usual self.

He does not know what's going on; he's (utterly) clueless.

The broad function of this intensifier is to convert a scalar predicate to a complex predicate synonymous to that next lower (to the left) in the scale. The properties of scalar orderings that will be appealed to below may now be summarized.

Definitions. A *scalar structure* is any $<V, \circ, \uparrow, \downarrow>$ such that

a. $\leq$ is a total order on

b. $\uparrow$ and $\downarrow$ are isotonic binary operations on $<V, \leq>$

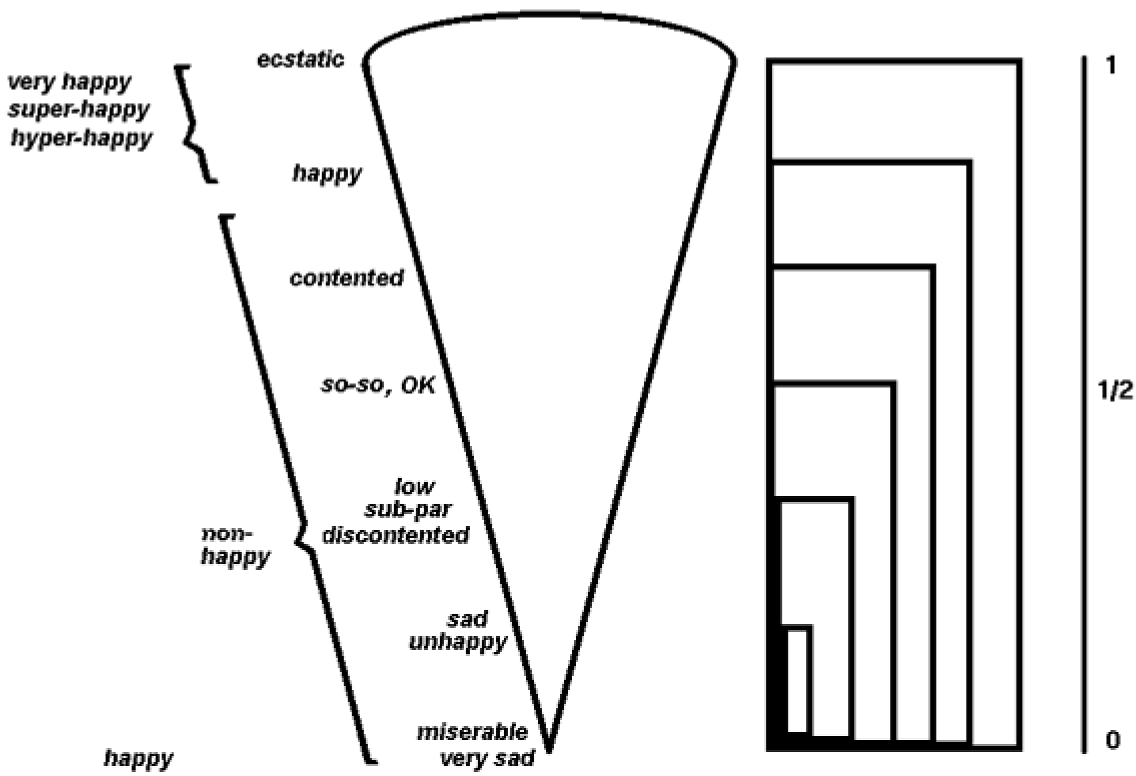
c. for any $x \in V$, $\downarrow x \leq x \leq \uparrow x$

[9] See *hyperapophasesis* at 1172, 35 in Proclus, *Proclus' Commentary on Plato's Parmenides*, trans. John M. Dillon Glenn R. Morrow (Princeton, NJ: Princeton University of Press, 1987).. This use of negation was recognized and introduced into

b. $\circ$ is an antitonic idempotent binary operation on V

Again an element *e* of the structure such that $e=e^{\circ}$, if there is one, is its *midpoint.*

It is helpful to draw pictures of scalar orderings, either as a line or as a series of nested sets growing from the set of all things having at least the background property at most to a minimal degree to those having it to at most the maximal degree. The structure for *happier-than* is illustrated in Figure 1.



Related to scalar adjectives is the concept of privation, and it too is largely algebraic. Classical and mediaeval writers employ a privative negation closely related to scalar negation. When the idea appears within a philosophical discussion, it is usually employed as a tool to describe some physical or metaphysical privation process. But its more formal definition is to be found in the *Categories* where Aristotle distinguishes it as a species of contrary opposition in which what is denied of the subject is a disposition (*hexis*, *Categories* 11^b15,

modern linguistics by Jespersen. See p. 326, Otto Jespersen, *The Philosophy of*

Topics 109^{b}18) that would hold of the subject "naturally" (*pepyke, Metaphysics* 1022^b). As we have seen, Ockham builds these features into his own definition.

Aristotle is himself inconsistent on whether privative negation is marked in the syntax. Sometimes his examples are unmarked like *blind* and *bald* (*typholos* and *phalakros*, *Categories* 11^{b}15, *Metaphysics* 1022^{b}22), other times marked, as in *toothless* (*nōda*, marked from *nē* = *without* and *odous* = *teeth*, *Categories* 12^{a}30). For Ockham and other mediaeval logicians, privative adjectives were regularly understood as unmarked and fully lexicalized.[10]

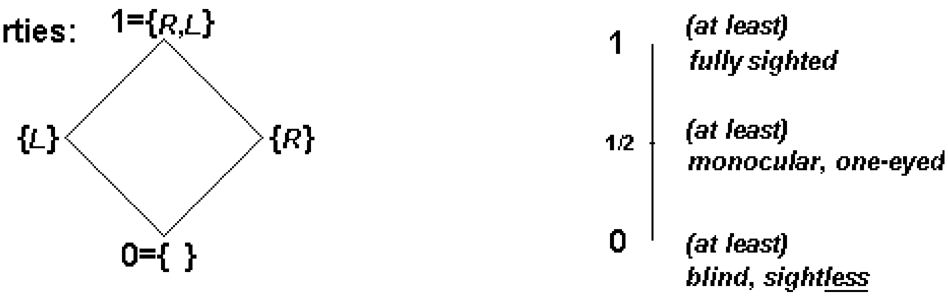
To see the structural properties of privation, let us abstract from Aristotle's paradigms, blindness and toothlessness. Privation of this sort may be described by reference to some background set of what I shall call, with intentional blandness, *features*. It is these that may be used to index the stages of loss in the privative process without attributing to features any properties beyond this use.

For now let us consider an example of a very simple structure for blindness. Let us posit a two element feature set {***R***,***L***} in which ***R*** *represents* sightedness-in-the-right-eye and ***L*** sightedness-in-the-left. There are then four possible feature combinations or "compound features": ∅, {***R***}, {***L***}, and {***R***,***L***}. The power set P of {***R***,***L***} forms a four element Boolean algebra, and an individual's particular sight privation may be thought of as the "representation" in that person of some compound.

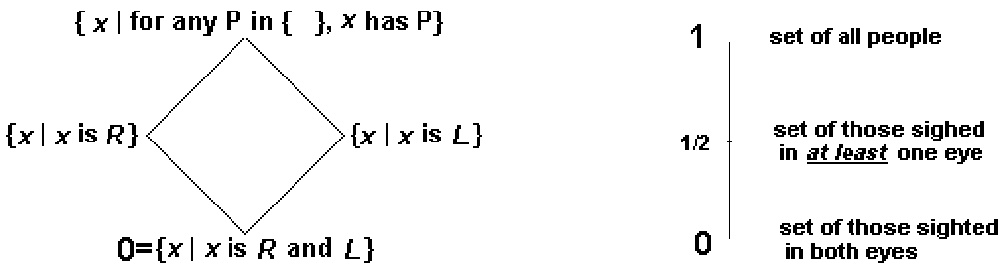
This algebra is illustrated below first in its abstract sense with its points indexed by feature sets, and then more concretely as a structure of extensions. In the lower structure corresponding to a state of privation is the set of all objects that have sightedness to at most its characteristic degree. In both cases the rank structure of the algebra is abstracted to its left as a line, which may then serve as a scalar ordering recording the various degrees of privation as represented by the ranks in the Boolean algebra.

Grammar (London: Allen and Unwin, 1924). and the discussion below.

Properties:



Extensions:



The nodes in the algebra are labeled with feature sets from which a feature is subtracted at each descent as a path moves downward. In the general case, however, the idea of privation need not be limited to finite sets or to discrete feature subtraction. Any Boolean algebra <P,∧,∨,0,1> may be interpreted as a *privation structure* in which the ordering relation ≤ is understood as the privation relation. Under this interpretation I shall call P a *possibility space*. In the special cases, including finite spaces, the set of its *immediate descendants* is well defined, as is the notion of a *complete path* from 1 to 0 (*i.e.* a *path* is any substructure < P ′, ≤| P ′,0,1> such that P ′⊆ P and ≤ is the order relation defined by ∧ and ∨.[11] Below are pictured examples of Boolean algebras with nodes indexed by sets of privation features:

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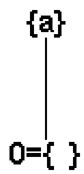
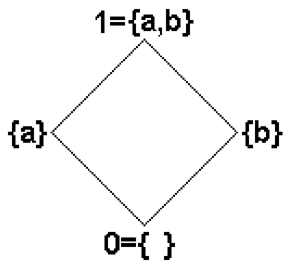
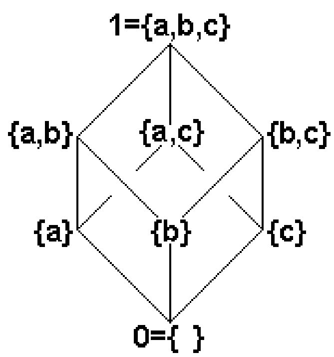
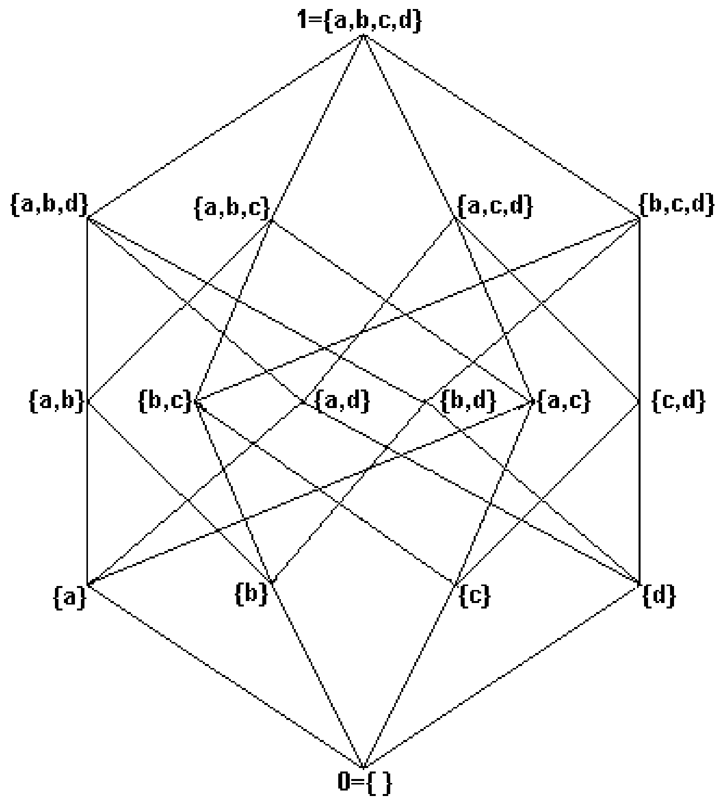
[10] For Ockham's usage see *Summa logica* I.36. On mediaeval usage more generally see D. Paul Henry, *Mediaeval Logic and Metaphysics* (London: Huthchinson, 1972).

[11] R|A is the restriction of the relation R to the set A, *i.e.* the set of all <*x,y*>∈R s.t. *x*∈A.









I turn to applications below, but first let me mention the importance of special "linear" substructures of such algebras. In general a substructure of the privation algebra would indicate various incomplete realizations of possible privational histories. For example, there might be empirical restrictions on the way teeth fall out of humans, so that they only fall out in pairs. In that case, some privative structure representative of actual tooth-privation would be a substructuremade up of the full possibility space. Another type of substructure of privation algebras is the sort already meet, namely scalar structures.

Definition. The scalar structure $\langle V, \leq|V, \circ, \uparrow, \downarrow\rangle$ is a *substructure* of the Boolean algebra $\langle P, \wedge, \vee, 0, 1\rangle$ iff, $V \subseteq P$ and $\leq$ is the Boolean ordering on P defined relative to $\wedge$ and $\vee$.

Prominent substructures of this sort are those represented by any complete downward path of the Boolean algebra. Let us call such a scalar substructure *level complete*. One case of such substructures is provided by the many-valued logics Kleene and Łukasiewicz, which are defined by reference to Kleene structures in which truth-values may be construed as abstractions representing levels of possible privation.[12]

Another example is the linear abstraction from the structure representing the subtractive properties of white light. Light, the celebrated metaphor for Goodness and Truth in the Napoleonic tradition, may be "fractioned" or have part of its reality removed by various filters in a pattern that forms a Boolean algebra. The levels of such subtractions may be abstracted to a total ordering meeting the conditions of a scalar structure. In principle, the ordering is infinite since it reflects the dense ordering appropriate to the measurement of light, but the finite algebra determined by the filters for cyan, magenta, and yellow is familiar and representative of a simple privation structure illustrated in Color Figure 1.

Yet another important variety of substructure of privation algebras is found in the syllogistic and, in particular, in Ockham's application of the syllogistic to the analysis of privatives. Before taking up the case of Ockham, however, we must review some general features of syllogistic semantics.[13]

4. The Syllogistic

Ockham expresses his analysis of privatives in terms of categorical propositions, and structural properties of the syllogistic are important to what he says. It will be useful here to explain this structure by means of the syllogistic's natural deduction reconstruction by Timothy Smiley and John Corcoran.[14] It is appropriate to apply the reconstruction to Ockham's version of the syllogistic because Ockham is perfectly orthodox in his treatment of this core material.

[12] Such is the most natural understanding of Łukasiewicz modal reading of the truth-values as "necessary", "true", and "possible". These are quasi-Neoplatonic representations of "levels" of truth.
[13] For more on the algebra of privation in scalar contexts see John N. Martin, "Lukasiewicz' Many-Valued Logic and Neoplatonic Scalar Modality," forthcoming in *Synthese.*
[14] John Corcoran, "Completeness of an Ancient Logic," *Journal of Symbolic Logic* 37 (1972). Timothy Smiley, "What Is a Syllogism?," *Journal of Philosophical Logic* 2 (1973). The more abstract lattice semantics and completeness result used here are detailed in John N. Martin, "Aristotle's Natural Deduction Reconsidered," *History and Philosophy of Logic* 18 (1997).

Moreover, appealing to the reconstruction will then enable us to apply its general constraints on the form of any semantics for privatives to the theory proposed by Ockham.

The relevant features of the reconstruction may be easily stated. Sentences consist of the A,E,I, and O forms. Deductions with any finite number of premises are allowed. Natural deduction rules are abstracted from the *Prior Analytics* and include Barbara, Celarent, various immediate inferences, and reduction to the impossible. The proof theory may be stated using the prefix operator (***A**, **E**, **I*** and ***O***) notation for the four sentence forms. Let *A* and *B* range over sentences, *X* and *Y* over sets of sentences, *x, y* and *z* over terms, and let **N***A* be a the contradictory opposite of *A* (*e.g.* $\mathbf{N}\boldsymbol{A}xy$ is $\mathbf{O}xy$) :

Definitions

1. A *basic deduction* is any $X \vdash A$ such that $A \in X$.
2. The set of acceptable deduction rules are:

Conversion1: $\dfrac{X \vdash \boldsymbol{E}xy}{X \vdash \boldsymbol{E}yx}$ Conversion2: $\dfrac{X \vdash \boldsymbol{A}xy}{X \vdash \boldsymbol{I}xy}$ Reductio: $\dfrac{X \vdash A \quad Y \vdash \mathbf{N}A}{X \cup Y - \{B\} \vdash \mathbf{N}B}$

Barbara: $\dfrac{X \vdash \boldsymbol{A}zy \quad Y \vdash \boldsymbol{A}xz}{X, Y \vdash \boldsymbol{A}xy}$ Celarent: $\dfrac{X \vdash \boldsymbol{E}zy \quad Y \vdash \boldsymbol{A}xz}{X, Y \vdash \boldsymbol{E}xy}$

3. The set $\vdash_{syl}$ of *provable deductions* is defined as the inductive closure of the basic deductions under the rules. We write $X \vdash_{syl} A$ to mean $X \vdash A$ is provable.

In Smiley and Corcoran's original semantics terms are interpreted over non-empty sets in a Boolean algebra of sets. For Ockham, however, it is not appropriate to interpret terms by sets. Accordingly, I shall use a more abstract model theory that includes the Boolean interpretation as a special case but which more accurately delimits the set of structures that characterize $\vdash_{syl}$. In this interpretation terms are assigned to non-0 points in a meet semi-lattice. Nor is it essential to the Smiley-Corcoran characterization that every term be non-empty, or non-0 in the more abstract version. The soundness and completeness proof continues to hold under the weaker assumption that the subject terms of true A propositions are non-0.[15]

Definitions

1. By *a syllogistic lattice* is meant any <U,≤,∧,0> such that
   a. <U,≤> is a partially ordered structure with least element 0;
   b. <U,∧> is the meet semi-lattice determined by <U,≤>.
2. A *syllogistic interpretation* relative to <U,≤,∧,0> is any function R mapping terms and sentences to U∪{T,F} such that:
   a. For any term x, $R(x) \in U$ and,
   b. There are four cases in defining R's assignment to sentences:
      i. R(***A****xy*)=T iff, $R(x)\neq 0$ and $R(x)\leq R(y)$,
      ii. R(***E****xy*)=T iff $R(x) \wedge R(y)=0$,
      iii. R(***I****xy*)=T iff $R(x)\wedge R(y)\neq 0$,
      iv. R(***O****xy*)=T iff either $R(x)=0$ or not($R(x)\leq R(y)$).

3. An argument from *X* to *A* is *(syllogistically) valid* (briefly $X \models A$) iff for any syllogistic interpretation R of a syllogistic structure for the syllogistic syntax, if for all $B\in X$, R(*B*)=T, then R(*A*)=T.

For comparison to privative negation it will be useful to express sentential negation. It may be introduced by the following eliminative definitions:

i. ¬***A****xy*$=_{def}$ ***O****xy*,
ii. ¬***E****xy*$=_{def}$ ***I****xy*,
iii. ¬***I****xy*$=_{def}$ ***E****xy*
iv. ¬***O****xy*$=_{def}$ ***A****xy*

Theorem. For any syllogistic interpretation R and propositions *P*, R(*P*)=T iff R(¬*P*)=F

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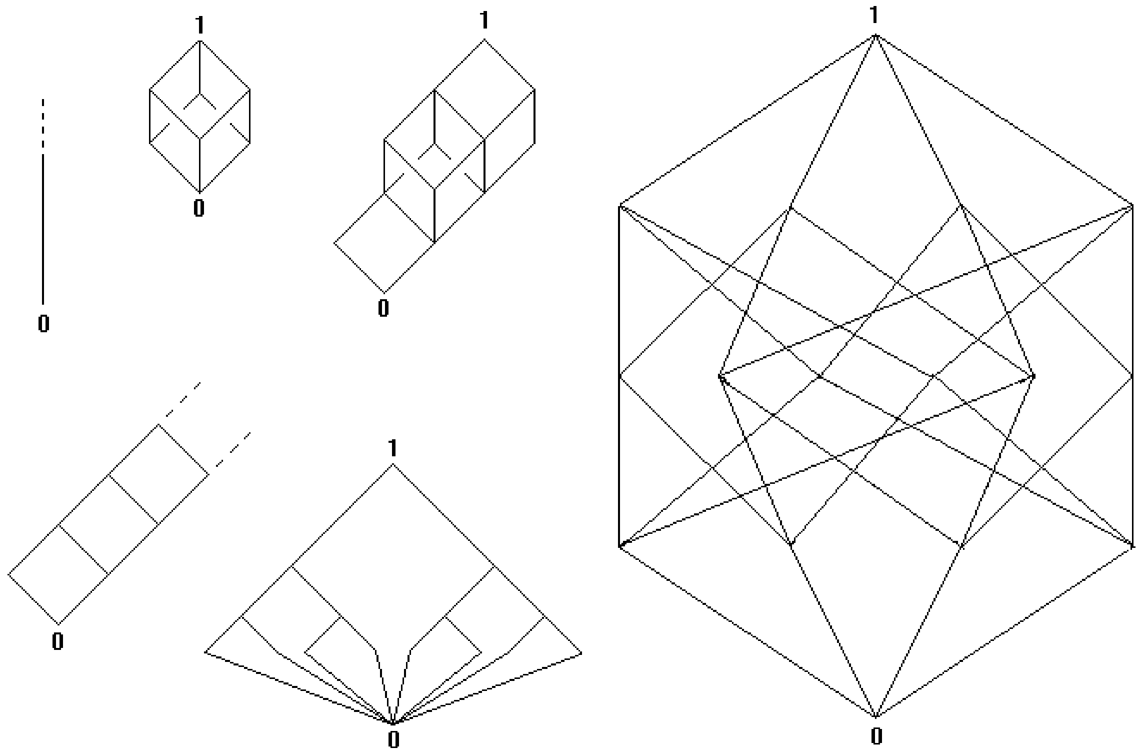
[15] The completeness proof in Ibid. holds if the definition of an acceptable syllogistic interpretation is amended by deleting the condition that for every term *x*, R(*x*) is non-0, by adding a clause to the truth-conditions of A statements that for it subject *x*, $R(x)\neq 0$, and by reading the conditions for

The following are examples of syllogistic lattices:



Especially relevant to the theory of privation will be the example of a binary branching tree with 0 element.

Note that the relative complement of *x* in such a lattice is not in general well defined:

$x\text{–}y$ = the *z* such that

1. l.u.b.$\{z,y\}\leq x$
2. $z\wedge y=0$
3. for any *w* , if l.u.b.$\{w,y\}\leq x$ and $w\wedge y=0$, then $w=z$.

It is, however, well defined for binary branching tree structures with least element 0., a fact that will be used shortly in applications to syllogistic classification and associated privative reasoning.

Clearly Boolean algebras of sets of the sort used by Smiley and Corcoran count as syllogistic structures, but so do other structures, including any Boolean

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E and O statements as the contradictories of those of I and A respectively. I am indebted to Terry Parsons for pointing out the relevance of this abstraction to the mediaeval syllogistic.

algebra, the Tree of Porphyry (with a 0 element added) and Proclus' linear hierarchy of Being. One can reason syllogistically about the points on any such structure, as verified by the soundness and completeness theorem.

Theorem $X \vdash_{syl} A$ iff $X \models A$.

The background discussion may be concluded with the remark that the three sorts of structures mentioned – scalar, syllogistic, and privative (a.k.a. Boolean) – may be found within one another. The idea is made precise in terms of subalgebras.

Definitions

1. A scalar structure $\langle V, \leq|V, \circ, \uparrow, \downarrow\rangle$ is a *substructure* of a syllogistic lattice $\langle U, \leq, \wedge, 0\rangle$ iff $V \subseteq U$.
2. A syllogistic lattice $\langle U, \leq|U, \wedge^*, 0\rangle$ is a *substructure* of a privative structure $\langle P, \wedge, \vee, 0, 1\rangle$ iff $U \subseteq P$, $\leq$ is the Boolean ordering on P defined relative to $\wedge$ and $\vee$, and $\wedge^*$ is the g.l.b. operation relative to $\leq|U$.

With the background prepared, it is now possible to restate the proposed exposition of privative negation. The question, the reader will recall, is the adequacy of the account of the logical properties of privative predicates as provided in the analysis:

S is blind S is a human that naturally sees and no S sees

Or more abstractly:

S is non-P *All S is T and No S is P*

The proposal is an attempt to capture the order properties typical of scalars by using the Aristotelian negations as expressed in traditional categorical sentence forms. This approach assumes that the syllogistic structure over which the terms of the language are interpreted represents the relevant scalar order. How it does so may be described in terms of the structures defined above.

It is clear from his examples and constraints on syllogistic semantics that it is committed to an interpretation of terms over a tree structure. The tree form is dictated by the lattice interpretation of A and E propositions, and his use of these propositions in his analysis of privative assertions. An A statement requires for its truth that the point paired with the subject fall lower in the lattice than that paired with the predicate. Hence the semantics assumes a partial ordering. The E statement requires for its truth that the nodes paired with its terms share no non-0 descendant. Hence their semantics dictates a branching. An account of a complete family of scalar predicates would then require one branching for each predicate in the order. The result is a tree, which together with a 0-element would form a substructure of a syllogistic lattice. Thus, the approach in effect presupposes a background privation structure, within which there exists a syllogistic lattice, which in turn contains a scalar structure. The tree determined by a complete branch of a privation structure may be defined formally by specifying a function *f* on the nodes of the path that picks out for each node one of its non-0 descendants in the structure to serve as its "positive" descendant in the tree.

Definition. An *acceptable abstraction* relative to a complete path $<V, \leq|V,0,1>$ of a privative structure $< P,\wedge,\vee,0,1>$ with order relation $\leq$ is any $<U,\leq|U,\wedge^*,0>$ and a function *f* such that *f* is a 1-1 mapping *f* from V into P such that

1. $f(x)$ is some non-0 $\leq$-descendant of *x* not in V ,
2. U = V$\cup$*Range*(*f*), and
3. $\wedge^*$= is the l.u.b. operation on $\leq|U$.

An explanation of the abstraction process in terms of the earlier concepts then follows:

Theorem

1. An acceptable abstraction is a syllogistic lattice that is a substructure of the privative structure, for which relative complementation is well defined.
2. An abstraction minus the 0 element is a tree,
3. There are operations $\circ$, $\uparrow$, and $\downarrow$ such that $<V, \leq|V,\circ,\uparrow,\downarrow>$ is a scalar substructure of both the syllogistic lattice and the privation structure,
4. for any $x\in$ V, x=l.u.b.$\{f(x), \downarrow x\}= f(x)\wedge^* \downarrow x$.

The process is pictured in Color Figure 3. A tree is highlighted as a substructure within a Boolean privation algebra, the left-most branch of which is a complete path that may be understood as a scalar ordering from 1 to 0. Each node on the path branches into two immediate descendants, one of which is not on the path and one of which is. Note that the nodes in the privation structure are labeled by feature sets. These are intended as heuristics indexing the order of privation and are not to be understood as necessary parts of the theory (e.g. as ontological commitments). The theory need only be understood as committed to the order properties of the privation relation itself.

The highlighted non-0 elements of the Boolean algebra form a tree, which is pulled out and pictured alone beneath the Boolean algebra. For illustration, the nodes in the tree's spine (the scalar order) are there labeled with predicates from the *happier-than* scalar family together with their translations in the manner of Ockham. In green are indicated the nested extensions (sets of things in the world having happiness to at most to the relevant degree) that the predicates would have if they were assigned extensions. Again, depicting these sets is intended to be heuristic. Being a nominalist, Ockham himself would eschew any assignment of extensions to the points in the tree because no mediaeval logicians, realists or nominalist, would regard the nodes of the tree as sets. Realists would regard them in effect as universals outside the mind, and nominalists like Ockham would understand them intentionally.

5. The Logical Problem

It is now possible to state precisely what the goal of a logical theory of privation would be and how that goal fits the syllogistic exposition of privative locutions that Ockham proposes. An adequate logical theory would be able to express the privation operation directly as an operator on terms or indirectly by contextual definition, and would then define the relevant validity relation. Let us say this more precisely. Let the syllogistic syntax be augmented to contain four one place term operators $\downarrow$ (Neoplatonic privative negation) , $\uparrow$

(Neoplatonic hypernegation), **–** (Aristotelian privative negation) , and τ (the "natural" type operator). The semantics for the expanded language is then defined as follows.

Definition. A *privative syllogistic interpretation* relative to the syllogistic lattice $< U,\leq|U,\wedge*,0>$ that has as substructure a scalar structure $<V, (\leq|U)|V,\circ,\uparrow,\downarrow>$ that is an acceptable abstraction from (and hence a substructure of) a Boolean algebra $<P,\wedge,\vee,0,1>$ relative to *f,* is defined as any syllogistic interpretation R for $< U,\leq|U,\wedge*,0>$ of such that [16]

for any term *x,* if $R(x)\in V$, then $R(\downarrow x)= \downarrow R(x)$

for any term *x,* if $R(x)\in V$, then $R(\uparrow x)= \uparrow R(x)$

for any term *x,* if $R(f^{-1}(x))\in V$, then $R(\mathbf{-}x)=R(f^{-1}(x))–R(\downarrow x)$

for any term *x*, if $R(f^{-1}(x))\in V$, then $R(x^{\tau})= f^{-1}(x)$

Note that even though R will be undefined for terms that apply one of the new operators to terms not meeting the relevant conditions, it will remain bivalent because by the truth-conditions any sentence containing a term for which R is undefined will be false. However, for the sake of comparison it will be useful to consider the properties of these operators in the restricted set of interpretations in which the conditions are met. Let a complex term be any term or any term that that results by applying some operator to another term. Then, $\models$ is defined as the relation that preserves truth from premises to conclusion for all interpretations that are well defined for every complex term occurring in the argument.

$X\models A$ iff for any (privative) syllogistic interpretation R of a syllogistic structure for the syllogistic syntax such that R is well defined for all complex terms in $X\cup\{A\}$, if for all $B\in X$, $R(B)=T$, then $R(A)=T$.

Infinite Negation. The varieties of opposition are distinguished by Aristotle in the *Categories* ($11^{b}15$*ff*). Contradictory opposition is what we would call today sentence negation, though traditional logicians usually limit its application to

[16] Here f^{-1} is the inverse of the 1-1 function *f.*

categorical propositions expressed by a negative particle attached to (in Latin, in front of) the copula, indicated here informally by *not* and formally by $\neg$. Its semantic function is to reverse the truth-value of the proposition from true to false and from false to true. Infinite negation, a predicate affix, is unlike sentential negation in that it carries existential import. Semantically it converts a term into one that stands for its non-empty complement in the domain if such exists. If the domain D is itself the top node of a syllogistic structure abstracted from scalar ordering, relative to a function f, infinite negation is the Aristotelian privation of its Neoplatonic privation: $f(D) - \downarrow D$. Let the syntax be augmented to include a designated term $\exists$, the *existence predicate*, and an infinite negation operator $\neg_{\infty}$, and let the definition of a syllogistic interpretation relative to the syllogistic lattice $< U,\leq,\wedge,0>$ be supplemented with the clauses:

R($\exists$) =1 , where 1 is the $\leq$ maximal element in U if there is one.

for any term x, if 1–R(x) is defined and not 0 then R($\neg_{\infty}x$)= D–R(x)

As above, let $\vDash$ be defined for an argument relative to the interpretations that are fully defined for the complex terms it contains. Then Ockham's exposition for infinite terms is a a special case of validity defined relative to interpretations that are fully defined for the complex terms contained in the argument.

Theorem

1. $\mathbf{A}x\neg_{\infty}y \dashv\vDash \{\mathbf{I}x\exists, \mathbf{E}xy\}$
2. $\vDash \mathbf{A}{-}x\neg_{\infty}x$

Thus every brute is non-human. We will now see why it is subhuman as well.

Privative Opposition in Linguistics. It is now possible to explain how privative negation as it appears in the logical tradition relates to the idea as studied in modern linguistics. Let a pair of adjectivdes be called *contrary* if there is no interpretation in which they are both true of the same object. In linguistics *privative negation* is understood as a negative affix, sometimes lexicalized, that meets two conditions. First, it marks one adjective from a pair of contraries.

Secondly, the unmarked adjective is systematically ambiguous between two readings. Under the first reading the two adjectives, marked and unmarked, partition a wider class or type called the *range of significance* of the pair. It is under this reading the two adjectives are contraries because nothing can fall into both halves of the partition. In the current framework, this pair is represented by the terms x and $-x$ because R(x) and R($-x$) partition the type R(x^{τ}) in a privative semantic structure. In linguistics, moreover, unmarked (non-negative) adjective of the pair is ambiguous. Its second meaning is systematically related to the first. Under the second reading the meaning of the marked (negative) adjective $-x$ is unchanged. It continues to stand for R($-x$). The unmarked adjective x, however, may also stand for the entire range of significance R(x^{τ}). The adjective x and its privative $-x$ are then no longer contraries because anything in the extension of the marked adjective is automatically in its range of significance because R($-x$) $\subseteq$ R(x^{τ}). In English the pair *man, woman* qualifies in the linguist's sense as privative opposites because whereas the extension of *woman* is fixed, being the relative complement of males within the set of humans, *man* is systematically ambiguous. It stands in one sense for males. The two adjectives *man* and *woman* partitions the set of humans. In its second sense *man* stands for the entire set of humans. Thus of the different privative negations distinguished, it is clear that what linguists study is a variety of Aristotelian privation.

The Logic of Privation. The various senses of privation are related, however, in ways that are reflected in the arguments they validate. Suppose that *animal* and *human* are a scalar predicates standing for succeeding nodes, that *sub* indicates Neoplatonic privation, and that *brute* is a lexicalized form of the Aristotelian privative *–human.* It follows by the truth-conditions, then, that *All brutes are subhuman* is true:

$$\models \boldsymbol{A}{-x}{\downarrow}x^{\tau}$$

This is just one formal truth relating these operators. In general the structural properties of operators $\uparrow$,$\downarrow$, and $-$ affect the validity of arguments formulated in terms of their operators.

It is especially relevant to the technique of exposition to note that the logic of privation is non-classical in the sense that it is not straightforwardly reducible to sentential negation. Sentential negation for example validates Double Negation:

$\neg\neg P \dashv\vDash P$

But the rule fails for privatives. The following metatheorems, interesting in their own right, are definitely not characteristic of classical negation.

$\nvDash \boldsymbol{A}x{\downarrow}{\downarrow}x$

$\nvDash {\uparrow}{\uparrow}\boldsymbol{A}xx$

$\vDash \boldsymbol{A}{\downarrow}xx$

$\vDash \boldsymbol{A}x{\uparrow}x$

$\boldsymbol{A}{\downarrow}xy \dashv\vDash \boldsymbol{A}xy \dashv\vDash \boldsymbol{A}x{\uparrow}y$

In the history of logic the non-classical features of privative negation that came in for special discussion were failures of the rule that came to be called *obversion*.[17] In fairly extended discussions in the *De Interpretatione* (19^b20-20^b13, 19^b27-29) and the *Prior Analytics* (51^b37-52^b4), Aristotle distinguishes sentence negation, from infinite and privative negations, and remarks on the differences in their logical power. What he says can be easily summarized if we preserve the ambiguity of the text by using *not* for sentential negation and *non-* for infinite or privative negation (in the Aristotelian sense):

All S is non-P	$\vDash$	*All S is not P*
All S is not P	$\nvDash$	*All S is non-P*
All S is P	$\vDash$	*All S is not non-P*
All S is not non-P	$\nvDash$	*All S is P*

Of these the failures of validity are especially important. His examples are of two kinds: those in which the subject is singular term that fails to stand for an existing individual and those in which the subject is a common noun that has an empty extension.

The counter-examples that are formulated in terms of infinite negations are really a red herring. Though Aristotle does not here make the point, Ockham does in his commentary on the *De Interpretatione*[18]. These counter-examples are already precluded by the truth-conditions of A propositions: the subject terms of true affirmatives stand for things that exist. For, as remarked in the introduction, if the predicate is non-empty, as it is in a true A statement, its infinite and sentential negation have the same logical power. This is the very point of Ockham's exposition, quoted in the introduction, of an infinite statement *All S is non-P* as *S exists* and *No S is P*. It makes explicit the truth of *S exist.* In this case *No S is P* is the contradictory of *All S is P* and the equivalent of *All S is non-P.* (Proof: R(*All S is non-P*)=T iff R(*All S is not P*)=T iff R(*S*)–R(*P*)≠0 iff R(*S*)∩R(*P*)=0 iff R(*No S is P*)=T.)

This equivalence of non-empty privative predicates to sentential negations, however, fails for privatives. The *locus classicus* of the counter-example in found in Aristotle himself:

> From *every man is non-just* there follows the statement that *no man is just; not every man is non-just* its opposite, follows from *some men are just.* For there must, indeed, be some just men. (20^a20-23, see also20^a38-40)

Or in modern terms:

All S is non-P ⫤⊨ *No S is P*
Some S is not non-P ⫤⊨ *Some S is P*

Aristotle dismisses the example for infinite terms, remarking this time that the inferences hold even for infinite terms because the predicate may be assumed to be a "referring" term. However, this option is not available for privative terms, though neither Aristotle nor Ockham remark on the fact. These equivalences fail for privative negation. Indeed, these are special instances of the more general rule of Obversion, which fails for privatives. The rule in its general form is formulated by Proclus:

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[17] See p. 82, Lynn E. Rose, *Aristotle's Syllogistic* (Springfield, Ill.: Charles C. Thomas, 1968).
[18]Cap. 2, sections 1, Cap. 3, Sections 4-6, Guillelmi de Ockham, "Expositio in Librum Preihermenias Aristoteles," in *Opera Philosophica Et Theologica*, ed. Angelus Gambatese and Stephanus Brown (1978).

> **The Canon of Proclus.** [19] Two propositions are logically equivalent if they are of the same quantity but reverse the quality of the subject and predicate.

To see why obversion fails for privatives, consider the predicates of a privative syllogistic syntax be interpreted over a single scalar structure <V, ≤,°,↑,↓>. (These could be a part of a fuller syllogistic language interpreted over a syllogistic structure of which <V, ≤,°,↑,↓> is a part.) By the canon the following should be valid:

All **–***S is P*	⫤⊨	*No S is* **–***P*
Some S is **–***P*	⫤⊨	*Some* **–***S is P*

One can easily construct syllogistic structures abstracted from scalar orders in privation trees in which *All* **–***S is P* is true but *No S is* **–***P* false, and cases in which *Some* **–***S is P* is true *Some S is* **–***P* false.

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[19] The rule is attributed by Ammonius to Proclus (his "benevolent benefactor"). See pp. 181:30-186:24, esp. the quotation 182:6-17 in Ammonius, "In Aristotelis De Interpretatione Commentarius," in *Commentaria in Aristotelem Graeca*, ed. Adolfus Busse (Belin: Gregorius Reimerus, 1895). The rules reads:

> …predicates are either simple or transpositional [marked negatively as privatives], and next making their division according to quantity, he says that some are clearly universal and also others particulars, also these are either indefinites or singulars, moreover some according to quality are affirmative also others are negative. These things being thus, he instructs us to look at the proposed proposition of which we wish to find the consequence, what condition any one of these is in, and announce that the consequence from it is that which is the same according to subject and according to quantity but in both the remaining things are different.

For discussion see Allan Bäck, *Aristotle's Theory of Predication* (Leiden: Brill, 2000).. In his commentary on Aristotle Ammonius himself never adopts the privative or hyper-negations of his teacher to the explain the validity of the canon, but rather attempts to interpret it using negatives of a more Aristotelian sort.

Proclus however supplements the syllogistic with two negations, privative negation and its inverse *hyper-negation.* With the new operator ↓ the issue can be reinterpreted. Indeed Proclus uses the privative negation operator to organize ontology into parallel ordered levels and explains knowledge acquisition as inferences involving the privative negation operator from knowledge of one level to that of another. As he put it, "negation generates affirmation." The thesis is captured in the following reinterpretation of his canon in terms of the privative negation operator:

A↓a↓b ⫤⊨ Aab ⫤⊨ A↑a↑b

Proclus' reform of the syllogistic is relatively radical. He continues to use the syllogistic in ontological reasoning, and explicitly makes use of syllogistic figures in laying out his arguments. He accepts a totally ordered scalar syllogistic universe, abandoning the then trivial E and O statements in favor of various scalar negations. This totally ordered universe is at once a syllogistic lattice, a substructure of a Boolean privation space, and a scalar structure. He is able to use the syllogistic only because the set of lattices that characterize standard syllogistic theory is abstract enough to include algebras that are totally ordered. See John N. Martin, "Proclus and the Neoplatonic Syllogistic," *J. of Philosophical Logic* 30 (2001).

These non-classical results pose a problem for classical syllogistic that lacks special expressions needed to formulate them. The counter-examples to obversion, for example, show that there is no simple way to represent privative negation by sentential or infinite negation. The point of Ockham's exposition then is to find some way to translate privative expressions into the more limited classical vocabulary while preserving the correct inferences. Is the traditional exposition sufficient to this task?

It is clear that certain occurrences of privative negation, either Neoplatonic or Aristotelian, are logically equivalent to syllogistic expressions:

$\boldsymbol{A}x\text{–}y \;\dashv\vDash\; \{\boldsymbol{A}xy^{\tau}, \boldsymbol{E}xy\}$

$\boldsymbol{A}x{\downarrow}y^{\tau} \dashv\vDash\; \{\boldsymbol{A}x\text{–}y, \boldsymbol{E}xy\}$

These then could be part of an introduction of **–** or ↓ (with τ) into the syllogistic that lacks them. But there are other positions in which these operators should be expected to occur; they can appear in E, I and O propositions attached to either the subject or predicate or both. The equivalences above address one of these twelve possibilities. In the remainder of this section we shall consider whether it is possible to eliminate privative negation in the remaining positions. For simplicity only the case of Aristotelian privation will be considered because this is the sense that exposition is designed to explain, and to keep the discussion intuitive *non-* will be used to express the object language privative operator in examples from natural language.

As specified in the exposition, *All S is non-P* is true in a scalar structure if there is some scalar point *x* which branches into two immediate descendants, a positive node *y* and its privative negation ↓*x.* The positive form *P* of the term names a node *x*, and the privative *non-P,* if it were a genuine term, would name ↓*x.* Since *non-x* is it not a genuine term like ↓*x* and is introduced by contextual definition, the fact that *S* names a point beneath ↓*x* must be expressed indirectly. This is accomplished by giving a name *T* to x. Then the fact that *All S is non-P* may be expressed, as set out in the introduction, by *All S is T and no P is S,* which is true only when the desired truth-conditions obtain, *viz.* $R(S) \leq {\downarrow}R(T)$.

The semantics for the translation (and for others to be discussed below) pictured in Color Figure 3.

Consider now the translation for the case of particular affirmatives with privative predicates. (This case will cover its converse as well.) There is a way to express the particular proposition in the syllogistic. The idea is essentially the same as that underlying the ecthesis rule used by Aristotle in the reductions of the *Prior Analytics.* This is the inference pattern in which a "name" is assigned to the "something" shared by the two terms of a particular affirmative. If some entity makes *Some S is non-P* true, we name it by stipulating that there is some term *Q* such that *x* falls under *Q* (as well as perhaps other entities that fall under both *S* and *non-P*) in such a way that *All Q are S* and *All Q are non-P* are both true.

There are, however, two versions of ecthesis that are easily confused. The first is correct for the syllogistic but the second is false, and it is the second that the exposition would need. The distinction is illustrated best in terms of two natural deduction rules:

$$\frac{X \vdash \boldsymbol{I}xy \quad Y, \boldsymbol{A}zx, \boldsymbol{A}zy \vdash B}{X, Y \vdash B} \qquad \frac{X \vdash \boldsymbol{I}xy}{\text{for some term } z,\ X \vdash \boldsymbol{A}zx \text{ and } X \vdash \boldsymbol{A}zy}$$

The rule on the left says that if (1) *Some S is P* is provable from some background assumptions and (2) *B* is provable from another set of background assumptions by adding assumptions naming that "something", then (3) *B* follows from the combined set of background assumptions alone. It is easy to show in the syllogistic semantics defined earlier the metatheorem that if $X \models \boldsymbol{I}xy$ and $Y, \boldsymbol{A}zx, \boldsymbol{A}zy \models B$, then $X, Y \models B$.

The incorrect rule says that if *Some S is P* is provable on some set of background assumptions, then there is some term that names that "something" that falls under both *S* and *P.* The rule is incorrect because it is possible to prove that for any syllogistic lattice there is some interpretation R such that R($\boldsymbol{I}xy$)=T and there is no term *z* such that R(*z*) is in both R(*x*) and R(*y*).[20] But to translate out arbitrary particular privatives such a term would be needed.

[20]Though incorrect, the second rule describes exactly the property that makes a maximally consistent set saturated in the Henkin completeness proof for the syllogistic natural deduction theory (*i.e.* X is saturated iff it is maximally consistent and if ($\boldsymbol{I}xy \in X$ iff for some *z*, $\boldsymbol{A}zx \in X$ and

Consider next universal affirmatives with negative subjects, *All non-S are P.* Here the subject is privative. Assume as before that a scalar node *x* divides into *y* and ↓*x,* but that this time R(*S*) is *y.* Then if *non-S* were a predicate, the sentence would be true if R(non-*S*)≤R(*P*). But since it is not, this fact must be expressed indirectly. I see no way to do so without positing the ability to refer a series of nodes z_i that "bars" the lattice below ↓*x*, *i.e.* a series such that every path from ↓*x* down to 0 passes through some member of the series. Any such series would do. If each node z_i in the series has a predicate Q_i assigned to it, then *All non-S are P* would be equivalent to some *All P are T and all* Q_1 *are P and …and all* Q_n *are P.*

But these expressive assumptions are rather strong. To express these propositions, the language would need to have names for lots of nodes. At this level of speculation, moreover, it appears that the particular series intended might vary from speaker to speaker. In addition, the nodes barring a privative node would include at least one node that was also on the privative scale but lower down. This node would have a term naming it. Hence to talk indirectly about a privative node entails being able to talk directly about other privative nodes lower in the order.

If we can ever talk directly about privative nodes, why not just do so? One reply would be that doing so indirectly reveals logical form that simple categorical propositions lack, and that this form is relevant to inferences. But the theory is getting quite complex. Privative reasoning requires translations into sometimes numerous conjuncts. Some of these are truths by nature and others contingent, and the definitional analysis of the same privative sentence may vary among speakers and listeners.

***A**zy*∈*X*)). The situation parallels that in first-order logic in which existential instantiation is valid but the inference from an existential to an instance is not: If *c* is some term not present in any sentence of *X* or *Y*, or in *B*, then if *X*⊨∃x*A*[*x*] and *Y*,*A*[c]⊨*B*, then *X*,*Y*⊨*B.* But there are models M such that M⊨∃x*A*[*x*] but there is no *c* such that M⊨*A*[c]. Moreover it is this second property that makes a maximally consistent set saturated: ∃x*A*[*x*] ∈*X* iff for some *c, A*[c]∈*X.* See Martin (1997).

Consider lastly a universal affirmative with both privative subject and predicate, *All non-S is non-P*.[21] What would be a direct rendering of the fact that ↓x≤↓y holds in the scalar tree? If we had recourse to the names of the type nodes *x* and *y* as we did earlier, say the predicate T_1 for y and T_2 for *x*, then there is indeed a proposition true iff this fact obtains, *viz. All non-P is* T_1 *and all non-Q is* T_2 *and all* T_1 *is* T_2 . The privative predications occurring in this *analysans* would then need to be translated out, requiring even more predicates and conjuncts.

I will stop the exercise at this point. I believe it has been sufficient to show that any theory of syllogistic exposition or, equivalently, of contextual definition for Aristotelian privative negation is somewhat implausible in its complexity. Let me conclude by remarking on a philosophical strength of the theory.

Abstraction. Abstraction is historically closely tied to privation. The Pythagoreans used abstraction (*aphairesis*) to describe the mental process of understanding associated with the physical composition of whole from part.[22] The One generates the Dyad, together they produce the Triad, and the Triad with the One produces the Tetrad. Abstraction is the epistemic converse of the process of physical composition. For Plotinus and his followers like Dionysius abstraction is explicitly epistemological and consists of the mental process of reversion to the One. Ontologically the Chain of Being proceeds downward through the process of causation, but the Understanding remounts backward from the bottom to the top. The process of remotion is called abstraction, and it is accomplished by the mechanism of repeated applications to steps in the hierarchy of hyper-negation, the inverse of Neoplatonic privative negation. The theory is elaborated by Proclus who explicitly introduces privative and hypernegation as syntactic operators in his syllogistic arguments.

Ockham shares with the Platonic tradition a commitment to an order of increasing abstraction that is a scalar ordering on which privation is well defined.

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[21] It is impossible to avoid definitional circles by first translating out the subject and then the predicate or vice versa. The circle leads through various possible translations of E, I and O privative statements and is not worth pursuing here.

[22] Martin, "Existence, Negation, and Abstraction in the Neoplatonic Hierarchy,".

That expositional account of Aristotelian privation needs to posit such a semantic order for privatives. The fact that Neoplatonists appeal to similar orders is a direct reflection not so much of historical influence, though certainly Neoplatonic ideas circulated broadly in the Middle ages, but of the demands of the subject matter. Both traditions study privation, and the semantics of privatives requires such orders. The nodes get "greater" as they "go" up in the precise sense that they are higher in a privative scale. In this sense, Aristotelians and Platonists agree that knowing a more abstract concept is knowing something "greater" or "higher".

Mediaeval philosophers differ on their understanding of what sort of entity an abstract idea is and on the mechanisms of abstraction. They also differ on whether there are universals outside the mind corresponding to abstract ideas. But on what we would call the algebraic structure of abstract ideas there is agreement. They form a hierarchy. Abstract concepts are structural correlates to the Aristotelian universals in the theory of definition. Among nominalists like Ockham abstract concepts in fact function as genera and species, there being no such things outside the mind. Thus, the tree of abstract ideas replicates (for the realist) or constitutes (for the nominalist) the tree of genera and species. The tree of genera and species, which ranks nodes according to their relative abstractness, is simultaneously an ordering of definitional simplification.[23] The standard theory of Aristotelian definition, abstracted from Aristotle's metaphysics, entails that what is more abstract is the definitionally simpler. Animality, for example, is more abstract than the concept of man because it has a simper, pared down definition. Algebraically the simpler concept may be described as the result of applying an operation of "subtracting" (a.k.a. abstracting) to the concept of Rationality.

How can both the scalar ordering of the Platonic tradition and the Aristotelian tree of genera and species ordered by definitional simplification cohere in a theory of privative predication? How could scalar privative order be

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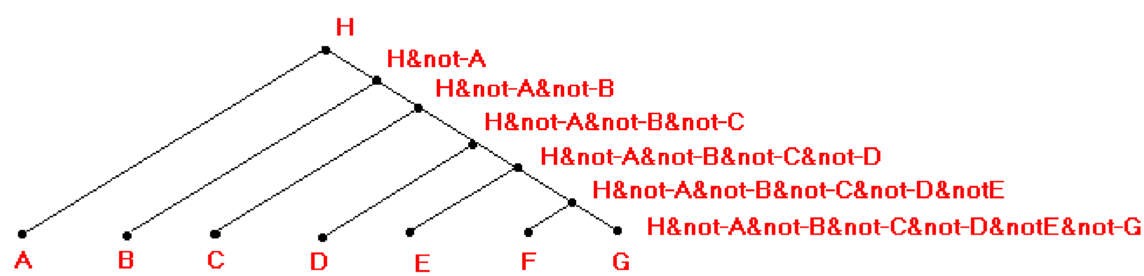
[23]For an account of abstraction in Ockham see Marilyn McCord Adams, *William of Ockham*, 2 vols. (Notre Dame, ID: University of Notre Dame, 1987).

correlated to a ranking of abstract concepts understood in terms of descriptive complexity? The higher nodes of the order of being, for example, are increasingly more perfect. How could divine goodness be characterized by paring off conceptual parts of lesser forms of goodness?

The method of exposition for privatives in the syllogistic, which makes use a semantics of tree structures, offers answers to these questions. Suppose we can understand a class of individuals, say humans, and various facts described in E-propositions that are definitionally equivalent to privative formulations. By some mechanism, the details of which are irrelevant from an algebraic perspective, we then form an abstract idea that encapsulates this information. This node occupies the position at a relatively high level on the spine of the privative tree, in the reverse direction to privation, each stage of which represents a "subtraction" of the negative accretions.



At the top are the less definitionally complex negative concepts that correspond to higher points in the privative scale. The relative complexity of the mental proposition, as reflected in its grammatical structure, decreases with an increase in abstraction. As in the case of abstraction to genera and species, the abstraction here, the "perscinding" of "subtracting", may be understood as subtracting (in an algebraic sense) the clauses in the mental conjunction. (In Neoplatonic philosophy *aphairesis* is used to express both abstraction and subtraction.)

The grammatical complexity provided in the expositional definition of privative sentences thus provides the structure for the relevant sort of conceptual abstraction. Viewed in this way the account is more than an exercise in a variety of linguistic parsimony. It is not just a technique for expressing, and in that sense analyzing, a privative predicate operator in limited primitive vocabulary. It also provides the structure basis for the theory of abstraction. The need for the “abstractive” scalar structure is in fact independent of the expositional account of Aristotelian privation. It would be required by any account of privation, whether the operator is defined contextually or introduced as an explicit operator with its own interpretation. To the degree the process of expounding privative predications can be made to work, it harmonizes on an abstract level two traditions, the Neopythagorean/Neoplatonic in which abstraction is the intellectual converse of privation, and the Aristotelian in which it is a mental process of progressively subtracting definitional clauses. To the extent that the exposition of privative assertions in syllogistic terms succeeds, there is an algebraic sense in which both are true.

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All Brutes are Subhuman:
Aristotle and Ockham on Privative Negation

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Abstract

The mediaeval logic of Aristotelian privation, represented by Ockham's exposition of *All A is non-P* as *All S is of a type T that is naturally P and no S is P,* is critically evaluated as an account of privative negation. It is argued that there are two senses of privative negation: (1) an intensifier (as in *subhuman*), the inverse of Neoplatonic hypernegation (*superhuman*), which is studied in linguistics as an operator on scalar adjectives, and (2) a (often lexicalized) Boolean complement relative to the extension of a privative negation in sense (1) (*e.g. Brute*). This second sense, which is the privative negation discussed in modern linguistics, is shown to be Aristotle's. It is argued that Ockham's exposition fails to capture much of the logic of Aristotelian privation due to limitations in the expressive power of the syllogistic.

Keyterms: privative negation, Ockham, Aristotle, scalar adjectives, Neoplatonism.