Notes on Scotus

Identity, Equivalence Relations and Sameness. In modern logic identity (also called equality), which is symbolized by =, is a two place relation. It is a very strict relation. The only entity that *x* is identical to is *x* itself. If you count the number of things that *x* is identical to, there is only one, namely *x.* It is for this reason that since Aristotle, this relation is sometimes called *numerical identity.* (In arithmetic, example, when we say 4=3+1 we are saying that the numeral 4 names the very same entity named by the expression 3+1, namely the number four.)

Identity obeys two fundamental laws. Let $P(x)$ be a sentence (formula) containing the variable *x*:

$\forall x(x=x)$ (*Self-Identity*, the same as the reflexive property)
$\forall x\forall y((x=y \wedge P(x)) \rightarrow P(y))$ (*Substitutivity of Identity*)

In logic the identity relation is understood as holding both between individuals and sets. That is, both individuals and sets are viewed as really existing entities[1], and true assertions may be made about each. Among the assertions true of both is that they are numerically self-identical. (Duns Scotus claims that common natures stand in a different relation, which he calls *less than numerical identity*.) Sets obey an additional law of identity that says that two sets are identical if they have the same elements (if thy have "the same extensions"):

$\forall x\forall y(x=y \leftrightarrow \forall z(z\in x \leftrightarrow z\in y))$ (*The Axiom of Extensionality*)

It should be remarked that in traditional ontology, properties are viewed as failing to obey this law. It is perfectly possible for two properties, e.g. *redness* and *squareness*, to have the same extensions – to be instantiated in the same individuals – yet remain numerically distinct properties. Properties are thus said to be *non-extensional*. Indeed, the failure of any such law setting out the "identity conditions" for properties is cited as one of the ways in which set theory is superior to traditional property ontology.

Sameness, Equivalence Relations and Partitions. Identity is a special case of a more general concept of sameness or equivalence. In logic a two-place relation $\equiv$ is called an *equivalence relation* if it satisfies three conditions:

$\forall x(x\equiv x)$ (In this case = is said to be *reflexive.*)
$\forall x\forall y(x\equiv y \rightarrow y\equiv x)$ (in this case = is said to be *symmetric.*)
$\forall x\forall y\forall z((x\equiv y \wedge y\equiv z) \rightarrow x\equiv z)$ (in this case = is said to *transitive*.)

[1] Indeed, in advance axiomatic set theory individuals are themselves represented by sets, and the identity relation holds only between sets.

Examples of equivalence relations are *is the same age as* and *has the same price as.* Identity is one of the most important examples.

Note that these relations hold respectively between things that have "the same" age, same price, and "being". It is for this reason that logicians say that the concept of an equivalence relation explains the more informal notion of a s*ameness relation*. Every sameness relation that is well behaved is in fact an equivalence relation and obeys its laws.

One important property of an equivalence relation is that is allows us to classify things into kinds: given an object x and an equivalence relation $\equiv$, it is possible to define the set of all objects that are "equivalent to x under $\equiv$". More formally, *the equivalence class of x*, abbreviated as $[x]_{\equiv}$, is defined as follows: :

$\{y| \, y\equiv x\}$ (The set of all objects y such that $y\equiv x$.)

Moreover, if an equivalence relation is defined for all members of a background set A, then that set may be "partitioned" in to a family of mutually exclusive subsets:

Theorem. If for any x in A there is some y in A such that $x\equiv y$, $\equiv$ is an equivalence relation, then there is a family $\{B_1,\ldots,B_n,,\ldots\}$ of subsets of A such that each B_i is an equivalence class of some member of A, every member of A is in one and only one B_i.

Conversely, any partition allows us to define an equivalence (sameness) relation: entity is the same subset of the partition are "the same":

Theorem. If there is a family $\{B_1,\ldots,B_n,,\ldots\}$ such that every member of A is in one and only one B_i, then the relation $\equiv$ defined as

$x\equiv y$ if and only if for some B_i, $x\in B_i$ and $y\in B_i$

is an equivalence relation defined for every element of *A.*

It follows that every successful classification of a set A into a partition may be reformulated in an equivalent manner in terms of a sameness relation, and conversely any well formulated sameness relation (i.e. any equivalence relation) of elements of a set A may be reformulated as a classification of the elements of A into a series of "natural kinds". Analysis in terms of kinds and sameness thus are equivalent and mutually interdefinable. If you combine this fact with a natural correlation between sets and properties, analysis in terms of properties, kinds and sameness all become equivalent and interdefinable. If partitions are restricted to those in which each subset is the extension of some property (the set of objects that posses that property), then a series of well behaved properties (i.e. those that have extensions that determine a partition) correlates with a partition and thus a sameness relation. Conversely, a sameness relation determines a partition which in turn correlates with properties. Thus as W.V.O. Quine argues in his paper "Natural Kinds", *properties, kinds* and *sameness* *relations* are mutually interdefinable, and essentially contain the same information.

Properties, Particulars and Inherence. In modern metaphysics one standard form for an ontological theory postulates three undefined primitive terms: *particular* (also called *individual*), *property,* and the *inherence relation.* Though undefined, the three are supposed to obey a law: *properties inhere in (are instantiated by) particulars.* From this point theories vary.

A *realist* theory holds that particulars and properties are disjoint (every particular is *bare* in the sense that in itself it does not determine any property). In addition, a numerically identical property can be instantiated in more than one individual simultaneously. Individuals are then explained as being "different" if they are numerical distinct bare particulars, and as "the same" if they instantiate numerically the same property.

A *nominalist* theory holds that numerically distinct individuals must instantiate numerically distinct properties. Numerically the same property cannot therefore be instantiated in numerically distinct individuals, and there is a 1 to 1 correspondence between properties and property instances. Such properties are called *tropes*. Such a theory has no difficulty explaining difference: two individuals are different if they and (hence all their instantiated properties) are numerically distinct. But they have a problem explaining sameness. One account is to do so by families of property instances. Properties fall into families of numerically distinct instances such that no instance inheres in more than one particular. Then, two numerically distinct particulars are the same if they instantiate properties from the same family. An alternative is a sameness theory. A set of property instances is associated with a primitive (undefined) sameness (i.e. equivalence) relation such that each relatum of the relation instantiates one and only one property instance from the associated family. Clearly, the two nominalist accounts are interdefinable, *à la* Quine : any partition in terms of property instances determines a sameness relation and conversely.

A variant theory, which is used in formal semantics (for example, as the intension of singular terms in Montague grammar), augments ontology with concepts from set theory and analyzes an *individual* in terms of properties and sets: an individual is (literally) a set of non-contrary properties such that the intersection of their extensions is a unit set (a set with only a single element). Two individuals are numerically identical if they are sets that contain exactly the same properties (recall the axiom of extensionality); they are similar ("the same") if they share as elements at least one property, and are distinct if the do not contain exactly the same properties. (Such a theory is similar to Abelard's account of an individual as an accretion of advening forms.)

Common Nature, Universal, Singular. The background theory is Aristotelian and not particularly simple. It includes his account of the ten Categories of entities – those such that each counts as a being (*ens*) and that have existence (*esse*): substance, quality, quantity, relation, time, place, action, passion, habit, inclination) – the five Predicables (genus, species, difference, proprium and accident), hyleomorphism (that substance is a combination of matter and form) and definition (that a species is defined in terms of its difference and genus). All entities fall into a finitely branching fine tree hierarchy of genera and species. The leaves of the subtree of substances are *primary substances* and count in the theory as ontological individuals or particulars. Higher nodes on the substance subtree are genera and species, and are called *secondary substances*. In Scotus's terminology the genus *is contracted to* the species by the difference. The genus is also said to be *divided* into its immediate subspecies and each of these to be *separated from* its collateral species by their respective differences.

Let us use the modern technical term *property* to refer broadly to differentiae and accidents from any of the non-substance categories – quality, quantity, relation, time, place, action, passion, habit, and inclination. Note that there is a 1-1 correspondence between species and differentiae: to each species corresponds the difference that distinguishes it from its immediate genus. For this reason, that some authors write as if differentiae are virtual substances, and do not refer to them as falling in one of the nine non-substance categories.

A definition (Scotus calls it a *ratio,* a transliteration of the Greek *logos*) describes a specie's *nature* or *essence*. Scotus use the term *form* to refer to the differentia of a species or to the string of differentiae that distinguish the species and all its higher genera. It is the property or full set of properties characteristic of that species. Thus there is a 1-1 correspondence between a species, its difference, its form, its definition, and its nature. Scotus often uses one of these terms where he might as well used another, and switches back and forth. Because a species could be contracted to more than one primary substance is said by Scotus to be *common.* It is *universal* if it is in fact contracted to more than one individual. Any substance that cannot by definition be contracted to or divided into more than one individual is said to be *singular.* It follows from the theory that all primary substances are singular.

Haecceity. Scotus adopts a realistic theory of common natures but combined with a tropist account, which is typically nominalists, if individual difference. He allows that numerically the same differentia and accidents can be instantiated in numerically distinct primary substances, and hence that common natures defined in terms of them are truly common. If a nature is in fact instantiated in more than one substance it is universal. However Scotus posits the existence of a special class of properties, that apply to infima species in the way that differentia do to higher species but that are at the same time tropes in that each haecciety property is instantiated in at most a single individual. The theory applies the more general account of species division to the special case of infima species by positing that there exits a property, the haecciety, that distinguishes the specie's natural subdivisions. As such the haecciety functions like a difference and determines the form and nature of the primary substance in which it inheres. However, the haecciety is singular and non-universal by definition (*ratio*). Hence it is not common. Hence is can explain how the individual differs numerically from all other individuals.

(Note that the three persons of the Trinity are each distinguished by a special haecciety that though it divides the divine nature into three it does not result in different individuals, only different persons of numerically the same individual.)

Numerical Identity. The concept of numerical identity, which seems to obey all the laws of modern identity, holds among two primary substances or singulars. There are in addition at least four other identity concepts.

Real Identity. Scotus also uses the term *real identity* for entities that must exist together in reality. It follows that if two entities can exist separately from each other in time or place, they are not really identical. Separabilty is thus used by Scotus as a mark that two thing are really distinct.

Less than Numerical Identity. Two species or natures are said to have *less than numerical identity* if they are in fact contracted to more than one singular. They are said to be *not identical in their instances*, a formula that seems to summarize the conjunctive fact that a single numerically identical entity (namely a species or nature with its characteristic difference) is instantiated in numerically distinct individuals.

Formal Identity. Two entities *E* and *E′* are formally identical if and only if the equivalence of *x is E* and *x is E′* hold as a consequence of the definitions of *E* and *E′.* Two entities *E* and *E′* are formally distinct every if really identical if the equivalence of *x is E* and *x is E′* does not old as a consequence of the definitions of *E* and *E′.* For example the *rationality* and *animality* are formally equivalent relative to Socrates in addition to being really identical because given the definitions of rational and *animal*, the fact that Socrates is rational entails he is animal and conversely. However the *rationality* and *risibility* are formally distinct relative to Socrates even though they are inseparable and hence really identical because given the definitions of *rationality* and *risibility*, the fact that Socrates is rational does not entail that he is risible. In general a proprium (one of the five predicables) is a property that is necessarily inseparable from a deference but for reasons due to reasons of natural causation short of definition.

In Scotus's theory an individual substance, its haecciety, and its instances of the differentiae of its various species are all inseparable, but because they each have a different *ratio* they are formally distinct.

Modal Identity. In this context the term *mode* is a synonym for accident. Two entities *E* and *E′* are modally identical if and only if the equivalence of *x is E* and *x is E′* hold as a consequence of facts of nature short of definition. Two entities *E* and *E′* are modally distinct if the equivalence of *x is E* and *x is E′* does not hold as a general fact of nature. .A standard case of modal identity that falls short of formal identity is a difference like *rationality* and a proprium like *risibility* are inseparable. (Note that since modal non-identity of *E* and *E′* seems to entail that nature would allow that one thing be *E* and another not *E′,* modal non-identity seems to entail real difference.)

Distinction of Reason. Two really identical entities may be distinct in reason if they are conceived under distinct concepts. Unlike formal and modal distinctness which turn of facts independent of mind, distinctness of reason presupposes conceptualization and a thinker. It is for this reason that Scotus says that the difference between a primary substance, its haecciety, and the instances of its differentiae are not distinctions of reason.