Lukasiewicz' Many-Valued Logic and Neoplatonic Scalar Modality

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Abstract

This paper explores the modal interpretation of Lukasiewicz' *n*-truth-values, his conditional, and the puzzles they generate by exploring his suggestion that by 'necessity' he intends the concept used in traditional philosophy. Scalar adjectives, used in ancient and mediaeval modality, form families with nested extensions over the left and right fields of an ordering relation described by an associated comparative adjective, *e.g.* increasingly-less-happy orders the sets referred to by

ecstatic, happy, content, so-so, down, sad, miserable

and these are in turn associated with the comparative *is happier than*. Also associated is a privative negation that reverses the "rank" of a predicate within the field, e.g. *unhappy* is synonymous with *sad*. If the scalar semantics is interpreted over a totally ordered domain of cardinality *n* and metric μ, an *n-valued Lukasiewicz algebra* is definable: $<C,\wedge,\vee,\Rightarrow,-,e>$ such that

$$\mu(C \xrightarrow{1\text{-}1} \{\tfrac{i}{n} \mid 0 \le i < n\});$$

$$\mu(e)=1;\ \mu(-a)=1-\mu(a);$$

$$\mu(a\wedge b)=\min\{\mu(a),\mu(b)\};$$

$$\mu(a\vee b)=\max\{\mu(a),\mu(b)\};$$

$$\mu(a\Rightarrow b)=\min\{1,1-\mu(a)+\mu(b)\}.$$

In such a structure the dual $\Leftarrow$ of $\Rightarrow$ is defined:

$$a\Leftarrow b = -(-a\Rightarrow -b).$$

Privation is analyzed in terms of non-scalar adjectives: a scalar of a given rank holds exactly when a family of standard predicates interpreted over a finite Boolean algebra applies. Any Boolean algebra of 2^n "properties" determines an n+1 valued Lukasiewicz algebra, *e.g.* {sighted-in-the-right-eye, sighted-in-the-left-eye} determines a 4-element Boolean algebra of three ranks, and a corresponding 3-

valued Lukasiewicz algebra that interprets scalar predicates for blindness, Aristotle's paradigm privative property:

The Neoplatonic "hierarchy of Being" is essentially the order presupposed by natural language modal scalars in series like *necessary, likely/probable, possible, unlikely/improbable, impossible.* Lukasiewicz' ~ is privative negation, and → proves to stands for ⇒, the extensional (antitonic) dual of an intensional ⇐ that is show to capture the meaning of *if…then* for scalar adjectives, especially modals. Relations to product logics and frequency interpretations of probability are sketched.