The Port Royal Logic

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Introduction

Logic or the Art of Thinking[1] was first published by Antoine Arnauld and Pierre Nicole in 1662 for instruction at the abbey of Port-Royal-des-Champs. Through its many editions, translations, and imitations, it had a major influence on logic for the next two centuries.[2] What could be called its formal logic is limited to a short and somewhat idiosyncratic summary the syllogistic, and a brief review of standard consequences in propositional logic. From the perspective of modern logic the work's main interest lies in its semantics. Because Descartes's dualism with its rejection of body-soul causation was inconsistent with earlier accounts of reference, Cartesianism required a reworking of the foundations of semantics. The most interesting parts of the *Logic* consist of its efforts to preserve as much as possible of earlier logical theory while remaining compatible with Descartes. The *Logic* is supplemented by other works by Arnauld, especially *On True and False Ideas*[3]. The discussion here will follow the four part organization of the *Logic* itself. The first three parts – the logic of terms, propositions and arguments – were standard divisions in earlier logical treatises. The fourth concerns method, a topic with new emphasis in the seventeenth century.

Part I. The Logic of Terms

Ontology. As Cartesians the *Logic*'s authors were impatient with the Aristotelian tradition and frequently dismiss as fruitless much of earlier metaphysics, for example, debates about realism and substantial form[4]. Part I opens, however, by laying down a basic ontology on which its later semantics depends.[5] It accepts Aristotle's basic division of being into substances and accidents. A substance exists in its own right, but an accident, which is called a "mode" in Cartesian terminology, exists only as inhering in substances. This basic ontology is supplemented by dualism: substance is divided into matter and spirit, and modes of the one sort cannot inhere in the other. It follows that the causal transfer of modes from matter to the soul, which in earlier logic had been a standard part of the explanation of sensation and concept formation, is impossible.

Mental and Spoken Language, and Grammar. In the 1960's Chomsky revived interest in the *Logic* by observing that it posits a mental grammar.[6] What Chomsky did not say was that theories of mental language had been a part of logic since the Middle Ages. The *Logic* appropriates large parts of this earlier framework making changes as necessary to fit Cartesianism. Mental language is broken down into various operations of the soul. Ontologically, a mental operation or "act" is a mode of the soul, and any such act is an idea in a loose sense of the term. Mental operations include thinking, doubting, willing, desiring, sensing, and imagining, but three are central to language: conceiving, judging, and reasoning.[7] Each presupposes the one before. The process starts by forming a concept or idea in the narrow sense. Ideas in this sense constitute the terms of mental language. Various types of ideas constitute the parts of mental speech. The next step in the process is the formation of propositions from terms. Again propositions come in a variety of forms. Propositions in turn enable the soul to make judgments, which are either affirmations or denials. The soul can also reason with propositions using

the rules of logic, which include among others the syllogistic, simple propositional logic, and indirect proof.

The *Logic* does not give formal definitions in the modern sense to the grammatical types of mental language. Over the course of the work, however, a general picture emerges of "mental" as similar to that developed in earlier logic. It consists of a stylized version of natural language comprising nouns, adjectives, relative clauses, categorical propositions, with and without tense indicators and modal adverbs, and "hypothetical" propositions, i.e. conjunctions, disjunctions and conditionals.

Grammatical types are explained by a mixture of examples and descriptions of their semantic roles. Simple ideas are divided into *substantives* (*nouns*) and *adjectives* by their semantics, which is described below. Depending on its cause, a simple term is either innate or abstract. Complex nouns and relative clauses are formed ("generated" in the modern sense) by the grammatical operation of restriction in which a simpler noun is modified by an adjective or relative clause. The four categorical propositions are distinguished by both their syntax and truth-conditions. They may also contain tense markers and modal adverbs.

As explained below, the *Logic* carries over from earlier logic the standard view about the relation of spoken to mental language. By convention a spoken word is paired with (or *signifies* in one sense of that term) an idea. The proposition that declares what idea a spoken word signifies is called its *nominal definition*. The *Logic* maintains that equivocation on nominal definitions is the most important source of logical error as well as the the root of much moral evil.

Intentionality, Comprehension and Signification. In the earlier logical tradition truth had usually been explained as a correspondence between language and the world. Its definition was captured in formulas like "the subject and predicate signify the same." Signification in this sense is the medieval notion of reference. Prior to Descartes signification had usually been explained by body-mind causation together with Aristotle's account of sensation. A concept in the mind was literally the intentional instantiation in the soul of a mode that had traveled into the intellect via sensation and abstraction from a substance outside the mind. This conceptualized mode was said to "signify" all those things outside the mind that instantiated the mode non-intentionally. Because Descartes rejected body-soul causation, the Port-Royal logicians needed to propose a new analysis of signification. To this end, the authors made use of the medieval notion of objective being, which had been used by Descartes in the ontological argument of *Meditation III*.

Objective Being. Objective being had been proposed earlier by various medieval logicians as the object of problematic mental states that do not seem to have as their object normal substances outside the mind. William of Ockham at one point posits objective being as the object of knowledge when we know an abstract term, and Peter Aureol posits it as the object of perception when we "see" and illusion.[8] According to the medieval doctrine, which was still current in the logic texts of Arnauld's time, a concept is a single entity that has two kinds of being. Because a concept is a mode of the soul and is in that sense a mental act, it is also, in a sense, part of the soul's form, and for this reason the concept was said to possess *formal being*. But a concept also is about something. It brings something or some kind of thing to mind. Moreover, it brings something to mind as having certain properties. As such the concept possesses *objective being.*[9] According to the version of the doctrine advanced in the *Logic*, every noun by nature possesses a list of defining modes, which the authors call the idea's *comprehension*,[10] and it is because it has a comprehension that it has objective being. These modes are said to constitute the idea's "content." For example, the idea *God* as appealed to in Descartes' ontological proof contains the property of infinity. Likewise, the idea *human* contains rationality.

Comprehension, in turn, determines signification and does so in a way that obviates any appeal to body-soul causation. According to the *Logic*, an idea *signifies* all those actual objects that satisfy all the modes in its comprehension.[11] Although the authors do not single out this set as such, it will be convenient to refer to the set of entities that an idea signifies as its *significance range*. An idea's

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significance range is what is called today its extension, although, as we shall see, that term is reserved in the *Logic* for a special use. Comprehension has an epistemological role. The soul's understanding of an idea clearly and distinctly consists of knowing its comprehension. That is, it understands that the idea signifies those objects that possess the properties in its comprehension.

So far the discussion has concerned nouns, but an adjective too, by nature, is associated with a mode or modes that constitute its intentional content.   Adopting a standard account from medieval logic, the *Logic* classifies adjectives as connotative terms. As the name implies, a connotative term signifies in two ways. It *signifies primarily* the actual objects that possess all the modes in its intentional content, and it *signifies secondarily* those modes. Secondary signification fills the role for an adjective that comprehension serves for a noun. It is the set of modes that determine what the term signifies in its primary sense. Because for most purposes nouns and adjectives are indistinguishable, we will refer here to the secondary signification of an adjective as its comprehension.

The *Logic* makes clear that ideas, both nouns and adjectives, are identified by their content: two ideas are identical if, and only if (hereafter iff), they have the same comprehension.

Moreover, comprehension imposes a minimal structure on ideas. Because a comprehension is essentially an unordered list of modes, it may be viewed as a set of modes. Indeed, some modern interpreter find in the *Logic*'s treatment of ideas an anticipation of Boolean algebra.[12] It is clear that, at a minimum, the set inclusion relation on comprehensions induces the containment relation on ideas: idea *A* is contained in idea *B* iff the comprehension of *A* is a subset of that of *B*.   Thus, at a minimum, ideas form a partially ordered structure, although the authors do not themselves formally define containment or explicitly attribute to it any of the defining properties of a partial ordering.

It is notable that the *Logic* departs from the standard medieval account of signification in two ways. First, although it was standard in the Middle Ages to say that a concept signifies not only actual objects but *possiblia* as well, Arnauld, no doubt as part of his more general rejection of arcane metaphysics, rejects the entire notion of possible object.[13] For him only actual objects exist, and these alone can be signified. Secondly, although it was still current at the time, the *Logic* makes no mention of the doctrine of supposition, a notion closely related to signification in medieval semantics. It was still standard in logic texts of the time to distinguish the variety of ways a proposition's terms may stand or "supposit" for things.[14] The varieties of supposition, however, were notorious as examples of the sometime excessive subtly of medieval logic. Moreover, supposition was problematic if assigned a role in truth theory. Because a proposition's supposition was explained by its inferential roles, and these inferences were in turn defined as truth-preserving, supposition could not without circularity be turned around and given a role in defining truth. One of the novelties of the *Logic* is that its streamlined truth theory does not mention supposition.

Extension. The *Logic* is famous for formulating its definition of truth not in terms of signification, which is its version of reference, but in terms of an extension, an entirely new idea in logic.[15] Unfortunately there is scholar dispute about the exact definition and role of extension in the *Logic*'s truth theory. The problem lies in the obscurity of the text, which is remarked upon by Kneale and Kneale:

> The extension of a term, on the other hand, is the set of things to which it is applicable, or what the older logician called its inferiors. The distinction may perhaps be intended to replace the medieval distinction of *significatio* and *suppositio*; but it does not exactly correspond, since the comprehension and the extension of a term are not properties of it, but rather sets of entities to which it is related in certain ways. Since *compréhension* means in ordinary French the same as 'understanding' does in English, it is natural to assume that Arnauld and Nicole intend to refer here to what we understand by a term, i.e. *significatum* or meaning. But their definition and their example do not support this view. For having interior angles equal to two right angles is said by

them to be included in the comprehension of the idea of triangle, and this character is indeed something which the character implies in a larger sense (*enferme en soi*); but it is certainly not part of the meaning of the word 'triangle'. … Again, according to Arnauld and Nicole, the extension of a general term is the set of its inferiors, but it is not clear whether the inferiors of which they speak are supposed to be species or individuals. When working out their example they say that the idea of triangle in general extends (s'étend) to all various species of triangle, but in the next paragraph they make the point that the extension of a term, unlike its comprehension, might be cut down without destruction of the idea ('*on peut la reserrer quant à son étendue … san que pour cela on la détruise*'), and this is not true of the set of species falling under a genus. … The confusion of their exposition seems to be due to their use of the word 'inferiors', which is itself metaphorical and unclear. It will be remembered that in medieval representations of Porphyry's tree individuals such as Socrates, Plato, and Brunellus were often mentioned at the bottom of the table in which all the other entries were general terms.[16]

As remarked, extension is defined in terms of the inferiority relation among ideas: the *extension* of idea *A* is consists of all the ideas inferior to *A*. Thus, an extension is essentially a set of ideas. As the Kneales observe, its interpretation turns on the meaning of the inferiority relation, which is not defined in the text. Its meaning must be reconstructed from its uses and examples. There are two contending interpretations.

The Intentional Interpretation. On the first interpretation the inferiority relation is defined directly in terms of comprehension containment. Idea *A is inferior to* idea *B* means that the comprehension of *B* is included in that of *A*. It follows that the inferiors of a genus are its species, a standard view in the Middle Ages. The interpretation has several important implications.

The first concerns the structure of ideas. Comprehensions are essentially sets of modes and form a power set algebra. On this reading extensions too are sets ordered by the containment relation. It follows that there is a 1-1 antitonic mapping from the structure of comprehensions to that of extensions, and that extensions are dual ideas in the algebraic sense.   This interpretation, then, provides grounds for claiming, as some interpreters have suggested, that the *Logic* attributes to idea the structure of a Boolean algebra dual to extensions.[17]

The second implication concerns truth. In Part II the *Logic* explains that a universal affirmative is true iff the extension of the subject is contained in that of the predicate. On this interpretation, then, truth reduces to intentional containment. As explained below, the interpretation poses problems for other views in the *Logic*, including the correspondence theory of truth, the distinction between contingent and necessary truth, the truth-conditions for categorical propositions, and the doctrine of false ideas.

The Referential Interpretation and Occasionalism. The second interpretation reads inferiority as defined in terms of signification and truth as consisting of correspondence to facts in the world. On this reading *A* is *inferior* to *B* iff everything that *A* signifies *B* also signifies. It follows that the extension of *A* is a subset of that of *B* iff the significance range of *A* is a subset of that of *B*. The *Logic* grants, in addition, that there are cases in which a mode inheres in a subject contingently. It then follows on this reading that there are contingent truths in which the comprehension of the predicate is not included in that of the subject. For example, when John is hungry, John instantiates all the modes definitive of *hungry* yet not all these modes are in the comprehension of *John.* This reading too has wider implications.

Some of these support the reading. As explained below, it is consistent with the *Logic*'s commitment to a correspondence theory of truth, the distinction between accident and essence, and the existence of contingent truth.

The reading also has structural implications. Because extensions are essentially sets, they are structured in the form of a power set algebra. It follows that there is an onto antitonic homomorphism

from ideas as ordered by containment to their extensions as ordered by set inclusion. Thus, ideas are dual to extensions in the algebraic sense.[18] Order, however, is not preserved in the other direction. If John is hungry, the noun *John* is inferior to the adjective *hungry* because every mode characteristic of hunger is true of John, but *hunger* is only accidentally true of John because it is not definitive or part of the comprehension of *John*.

Extension on this reading also has an epistemological role. Like other Cartesians, the *Logic*'s authors accept a form of occasionalism.[19] They hold that there is providential correspondence between perceptions in the soul and the sensory motions of the brain. On the occasion in which a material substance with its modes causes motions in the brain characteristic of bodily sensation, God causes there to be instantiated in the soul a sensory experience of that substance. Moreover, this experience has "content:" the substance is experienced as having modes. Some of these modes, like those of three-dimensional extension and movement, are in fact possessed by the substance. Others, especially the sensory properties of color, taste, and sound, are really properties of the soul itself. It is the providential correspondence of the sensory movements of the brain to the subjective experience of sensation that insures that the soul has a limited understanding of the properties of the bodies outside the mind that it is sensing. Through this divine causation the soul experiences an understanding of a term's extension on the occasion of bodily sensation.

The interpretation, however, also poses difficulties, which are detailed below. First is the issue of the existential import of affirmative categorical propositions. What is the truth-value of *every S is P* if the subject fails to signify anything? A modern logician would observe that the empty significance range of the subject is trivially included in that of the predicate and that on this view the proposition should be true. But the *Logic*'s doctrine of false ideas maintains that an affirmative proposition with a false idea as subject is false.

The interpretation must also be reconciled with remarks in Part III that suggest that the truth-value of an essential definitions is purely a matter of whether the subject's comprehension contains that of the predicate.

Predicables, Species, and the Tree of Porphyry. The *Logic* accepts Porphyry's "predicables."[20] These make up the five-fold division of predications into genus, species, difference, property and accident. Differences, properties, and accidents are adjectives, and accordingly they signify modes secondarily. Differences signify modes that are definitive of species and necessarily true of their instances. Properties signify necessary properties that are not part of a species' definition. Accidents signify contingent modes. Genera and species are common nouns. Due to the *Logic*'s commitment to dualism and Cartesian physics, the examples it cites of spiritual genera and species are usually virtues and vices. Those of extended substances tend to be geometric figures and varieties of motion.

The text goes on to espouse its version of essential definition. The broader characterization of species comprehension possesses features that a modern logician would recognize as a definition by induction. Species are ordered in a tree. There is a root node or "highest" genus, which is identified as *existence* or *being*. In addition, an idea of rank $n+1$ has a unique parent of rank n. A descendant is called a called *species* relative to its parent, and the parent a *genus* relative to its descendants. Moreover, each species is assigned a unique mode as its *difference*. The essence of *existence* or *being*, the first node or "highest" genus, is presumable a most general mode, one true of everything that exits. The specification of the comprehension of this root node would function as the basis clause of an inductive definition of *species comprehension*. The comprehension of each species is defined as consisting of the modes of its genus plus its assigned difference. This specification would function as the inductive clause is the definition. The proposition that predicates of a species its genus and difference is called its *real definition*. It is assumed that the extensions of a species and its difference are identical.[21] The overall species structure is assumed to be a finite tree. This is the *Logic*'s version of the traditional Tree of Porphyry.[22]

Innate Ideas, Abstraction, and Restriction. Like Descartes, the *Logic* posits three ways in which ideas come to be instantiated in the soul. Some are innate. Others are formed by the soul in one of two ways, either by abstraction from sensation and other ideas, or by the restriction of one idea by another. In each case comprehension plays a role.

The example cited by the *Logic* of an innate idea is the idea of God. It includes in its comprehension the property of infinity.[23] Innate ideas are directly caused by God to be instantiated in the soul.

Abstraction is a mental operation that forms new concepts from sensation or less general concepts. It has been a standard part of logical theory since ancient times. What is new in the *Logic*'s version is that it is conceived of as forming a new idea by operating on intentional content.[24] It forms a new idea in the soul by selecting a subset of modes from the content of either a sensory perception or from the comprehension of another idea that is already present in the soul, and forms a new idea that has as its comprehension the selected modes.

Although some commentators have suggested that the *Logic*'s abstraction operation anticipates a Boolean or matrix join operation, the reading faces several difficulties.[25] It is true that an abstracted idea is contained in the idea it is abstracted from because the comprehension of the one is a subset of the other. But ideas as structured by abstraction falls short of a join semi-lattice in two ways. The *Logic*'s operation, like traditional abstraction, is monadic rather than dyadic. In addition, there is no suggestion that the set of all ideas is closed under abstraction.

Restriction too in a mental operation that forms a new idea. The *Logic* adopts the standard view dating to the Middle Ages that restriction is an operation in mental grammar by which – to use modern terminology – a noun phrase is formed by modifying a head noun phrase by an adjective or relative clause. Again the novelty of the *Logic*'s view lies in the claim that the operation operates on comprehensions. The operation yields an idea that has as it comprehension the intersection of the comprehensions of its two arguments. Adapting a medieval distinction, the *Logic* calls a restriction that has the same signification as its head noun an *explication*, and one that signifies a narrower range a *determination*.

The authors who interpret abstraction as a join operation also see restriction as an early version of a matrix meet operation. They also suggest that there is a minimal idea, a kind of ultimate restriction, the comprehension of which is internally contradictory because it contains all modes.   Although restriction is clearly binary, this reading faces the difficulty that there is no textual evidence to suggest either that ideas are closed under restriction or that the authors envisage the suggested minimal idea.

The Structure of Ideas. An implication of abstraction and restriction for the structure of ideas is that the operations generate non-species, which are ideas that do not fall on the Tree of Porphyry. Two examples of this sort stand out: any idea formed by the abstraction of an accidental mode from a sensory perception, and any restriction of a species by a non-inferior idea. Overall, the structure of ideas is not very suggestive of any standard modern algebra. Ideas in the *Logic* clearly do have some structure. Species form a finite tree; ideas in general are partially ordered by containment; extensions form a power set algebra; and ideas as ordered by containment are dual to extensions ordered by set inclusion. There is, however, no evidence to suggest that ideas have a minimal contradictory element. Although *being* is a highest genus, it cannot serve as a maximal idea in general because it is possible to construct by restriction ideas that do not include the mode *being*, e.g. false ideas that are true of nothing. Nor can abstraction and restriction be understood as matrix operations because there is no reason to think that ideas are closed under them. The structure the *Logic* envisages is rather the traditional Tree of Porphyry augmented by various non-species generated by abstraction and restriction. Novelty lies rather in the explanation of this traditional structure by appeal to relations and operations defined in terms of intentional content.

Privative Negation. The *Logic* recognizes two forms of negation: the sentence negation of the negative categorical propositions and a term operation called privative negation.[26] Privative negation is a grammatical operation that generates a new species from a previously distinguished species within a genus. A species' privative negation signifies everything within its genus that the species does not. It is explained in terms of comprehension, and seems to entail the existence of negative modes.   More formally, the *privative negation* of species *S* of genus *G* with difference *D* is that idea that has as its comprehension the modes of *G* plus the mode *not-D*, where *not-D* is the mode that is true of everything signified by *G* that is not *D*. For example, the idea *beast* is said to be defined as all animals that are irrational. As the example *beast* illustrates, privative negations are generally lexicalized. The doctrine presupposes a variety of Neoplatonic order. Both the relation of genus to species in the Tree of Porphyry and that of a species to its privative negation within their genus is one of formal or eminent cause, and the privative negation of a species is described as "less perfect" than the genus and species.[27]

Part II. The Logic of Propositions

Truth Theory for Categorical Propositions. The *Logic* does not provide a case by case statement of the truth-conditions for the various sorts of propositions envisaged in mental language. The exception is the categorical propositions of the syllogistic. These are explained at length in Part II, in a series of six axioms and remarks.[28]   In these the concept of extension plays a central role. Just as there are two alternative interpretation of extension, one intentional and one referential, so too there are two interpretations of the truth-conditions. One is an intentional reading and understands truth as conceptual containment; the other is referential, and truth corresponds to set theoretic relations among entities in the world.

The Intentional Interpretation. This interpretation understand a term's extension to be the set of ideas that have that term as part of their comprehension. Let Ext be the function mapping ideas to their extensions. As interpreted by Jean-Claude Pariente's the truth-conditions for the categorical propositions are formulated as relations on these sets.[29] Those for A, E and I propositions are:

Every S is P is true	iff	$\text{Ext}(P) \cap \text{Ext}(S) = \text{Ext}(S)$ [equivalent to $\text{Ext}(S) \subseteq \text{Ext}(P)$]
No S is P is true	iff	$\text{Ext}(S) \cap \text{Ext}(P) = \varnothing$
Some S is not P is true	iff	$\text{Ext}(S) \cap \text{Ext}(P) \neq \varnothing$

The conditions for the particular affirmative require special comment. In evaluating the validity of syllogisms Aristotle made use of the method called *ecthesis* or "setting out," which exemplifies the truth of a particular affirmative *some S is P* by assigning a name, call it *Q*, to the elements shared by *S* and *P*. In such cases the proposition is true exactly when both *every Q is S* and *every Q is P* are true. George Boole also makes use of a version of ecthesis to characterize the truth of O propositions. He says,[30]

> If some Xs are Ys, there are some terms common to the classes X and Y. Let those terms constitute a separate class V, to which there shall correspond a separate elective symbol *v*…

He remarks that an O proposition may be symbolized as

$$vx=vy=v$$

and is equivalent to

$$vx=vy$$

As Pariente reads the *Logic*, its authors propose something similar.[31] The *Logic*, he holds, posits in addition to ideas in the normal sense something he calls "indefinite ideas" (*une partie indéterminée*) represented in the notation below by *X*, and a second variety of restriction called "indefinite restriction," represented by ∩*.[32] The truth-conditions for O propositions are then formulated as follows:

Some S is P if true   iff $\text{Ext}(S)∩^*\text{Ext}(X) = \text{Ext}(P)∩^*\text{Ext}(X)$

The analysis is intended to say that the O proposition is true iff the result of indefinite restriction of subject by an indefinite idea has the same extension as the indefinite restriction of the predicate by the same indefinite idea.

It should be remarked, however, that the *Logic* itself does not provide an explicit explanation of indefinite restriction or indefinite ideas. The text's intention is perhaps better captured by a more economical reading that posits only the traditional notion of restriction and that reads the text's reference to of an "indefinite" idea as its idiom for a metalinguistic existential quantification over terms, in a manner similar to Aristotle's original technique:

Some S is P if true   iff   for some *Q*, $\text{Ext}(S)∩\text{Ext}(Q) = \text{Ext}(P)∩\text{Ext}(Q)$
                                             iff   for some *Q*, $\text{Ext}(Q)⊆\text{Ext}(P)$ & $\text{Ext}(Q)⊆\text{Ext}(P)$

Both readings make the assumption that mental language possesses the expressive resources sufficient for ecthesis, i.e. that it possesses a term that has as its extension a subset of those of the subject and predicate.

This analysis, it is claimed, explains the role in truth theory of extension as interpreted intentionally, viz. as comprising all ideas that have that term in their comprehension.   The interpretation is also claimed to establish the historical point that the *Logic*'s truth theory does not depend on medieval semantics. In particular, it is supposed to show the account's independence from medieval notions like signification and supposition. Rather, by its use of set theoretic relations on extensions, it suggests the 19th century analysis of the syllogistic in terms of classes. The truth-conditions above, however, are not uniquely characteristic of the intentional interpretation of extension. As shown below, they follow as well as metatheorems under the referential interpretation.

The Referential Interpretation.

The alternative interpretation of truth is likewise defined in terms of extension.[33] Moreover, an idea's extension continues to be understood as the set of its inferior ideas. The interpretation, however, differs in its definition of inferiority. It is defined in terms of signification: *A* is *inferior* to *B* iff everything that *B* signifies *A* also signifies. Under this interpretation *every S is P* continues to be true iff $\text{Ext}(A)⊆\text{Ext}(B)$, which is a fact about ideas. But because the definition of extension entails that $\text{Sig}(A)⊆\text{Sig}(B)$ iff $\text{Ext}(A)⊆\text{Ext}(B)$, the truth of a universal affirmative also corresponds to a fact about things outside the mind.

The interpretation, however, goes further. It claims that the equivalences listed above which Pariente regards as truth-conditions for the categorical propositions should rather be regarded as metatheorems that follow from a deeper analysis of truth. This analysis, moreover, is not independent of medieval semantics, but rather abstracts from it.   More precisely, the interpretation employs four concepts used in the *Logic*'s axioms that set out truth-condition: a term's relevant extensions, the proposition's quality, a term's quantity, and the universal or particular nature of a term.   The reading holds that it is these concepts, which are abstracted from medieval supposition theory, rather than set theoretic operations, that are the key to the *Logic*'s truth-conditions.

Relevant Extension. The interpretation suggests that authors of the *Logic* were perhaps the first to make a theoretical use of the fact that the terms of the four categorical propositions vary systematically in whether their entire extension is relevant to its truth or only that part it shares with the extension of the other term. A term's *relevant extension* varies depending on the proposition, and is defined as consisting of either the term's entire extension or the intersection of the extensions of the proposition's two terms. Moreover, not only is a term's relevant extension is important to its interpretation, but so is the relevant extension of its collateral term in the proposition. In the terminology of today's general quantification theory, a quantifier term is monotectonic if the quantifier ranges systematically its extension's subsets or supersets. A term is *up monotectonic* the quantifier ranges over its supersets and is *down monotectonic* if it ranges over its subsets. In Arnauld's usage, the *relevant extension* of a term its entire extension if the term is down monotectonic, and if the term is up monotectonic.[34]

Quantity. The quantity of elements in a term's relevant extension that are relevant to a proposition's truth also varies. By a term's *relevant quantity* is meant either all or at least one of the elements of its relevant extension.

Quality. A proposition's quality is affirmative or negative in a semantic sense. Quality determines whether the relevant instances of the subject are required to be identical or non-identical to those of the predicate. A proposition is *affirmative* iff in its truth-conditions that quantity of elements relevant within that term's relevant extension are stipulated to be identical to the quantity of elements relevant to the predicate within its relevant extension. It is *negative* iff they are said to be non-identical.

Universal and Particular Term. The final distinction required is that between a universal and particular term. The *Logic* abstracts the distinction, it is suggested, from the notion of distributive supposition in medieval logic. In supposition theory a proposition's term was said to possess distributive suppositions if the proposition entails and is entailed by the conjunction of the proposition's instances for that term. For example, in *every S is P* the subject *S* possesses *distributive supposition* if the proposition is equivalent to the conjunction $S_1 is P$ & … & $S_n is P,$ where $S_1,…,S_n$ are the names of the individual instances of *S.* Similarly, because a particular affirmative is equivalent to a disjunction of these instances, its subject was said to have *confused supposition.* To the modern reader, these definitions call to mind the substitution interpretation of the quantifier, which stipulates that universal affirmative is true if all its subject instances are true, and a particular affirmative is true if at least one of its subject instances is true. In medieval logic, however, supposition did not occupy the role of a primitive term in an orderly definition of truth in the modern sense. It could not have done so without circularity because supposition itself was explained in terms of logical consequence, and consequence in turn was explain as a truth preserving relation. The *Logic*'s authors made use of the fact that a distributive use can be characterized, not in terms of the consequence relation, which is defined in terms of truth, but directly in terms of identity relations that hold among term instances. The concept is defined below, but due to differences in quantifier scope, it must defined separately for a proposition's subject and predicate.

A proposition's subject is *universal* iff, (1) the relevant quantity of elements in its relevant extension is every element in that set and (2) either the proposition is affirmative and the proposition's truth-conditions stipulate that every element of the subject's relevant extension is identical to the relevant quantity of elements in the predicate's relevant extension, or the proposition is negative and the truth-conditions stipulate that every element of the subject's relevant extension is non-identical to the relevant quantity of elements in the predicates relevant extension.

A proposition's subject term is *particular* (or *non-universal*) iff, (1) the term's relevant quantity is at least one element of its relevant extension and (2) either the proposition is affirmative and the proposition's truth-conditions stipulate that at least one element of the subject's relevant extension is identical to the relevant quantity of elements in the predicate's relevant extension, or the proposition is

negative and the proposition's truth-conditions stipulate that at least one element of the subject's relevant extension is non-identical to the relevant quantity of elements in the predicate's relevant extension.

A proposition's predicate is *universal* iff, (1) the term's relevant quantity is every element in the set of its relevant extension, and (2) the proposition is affirmative and the proposition's truth-conditions stipulate that the relevant quantity of elements of the subject's extension are identical to the all the elements in the predicate's relevant extension, or the proposition is negative and the proposition's truth-conditions stipulate that the relevant quantity of elements of the subject's extension are non-identical to the all the elements in the predicate's relevant extension.

A proposition's predicate is *particular* (or *non-universal*) iff, (1) the term's relevant quantity is at least one element of its relevant extension and (2) either the proposition is affirmative and its truth-conditions stipulate that the relevant quantity of elements of the subject's extension are identical to at least one of the elements in its relevant extension, or the proposition is negative and its truth-conditions stipulate that the relevant quantity of elements of the subject's extension are non-identical to at least one of the elements in its relevant extension.

Truth-Conditions. The truth-conditions for the categorical forms are then stated using these four concepts. The formulations below are claimed to be close approximations to those used in the *Logic*'s seven axioms and explanatory remarks:

Every S is P is true	iff A is a Cartesian quantifier, the proposition is affirmative, the quantifier A is subject down monotectonic but predicate up monotectonic, and *S* is universal but *P* is particular;
	iff every instance of the extension of the subject is identical to some instance of the intersection of the extensions of the subject and predicate.
No S is P is true	iff E is a Cartesian quantifier, the proposition is negative, the quantifier N is subject and predicate down monotectonic, and both *S* and *P* are universal;
	iff every instance of the extension of the subject is non-identical to every instance of the extension of the predicate.
Some S is P is true	iff I is a Cartesian quantifier, the proposition is affirmative, the quantifier E is both subject and predicate up monotectonic, and both S and *P* are particular;
	iff some instance of the intersection of the extensions of the subject and predicate is identical to some instance in the intersection of the extensions of the subject and the predicate.
Some S is not P is true	iff O is a Cartesian quantifier, the proposition is negative, the quantifier O is subject down monotectonic and up predicate monotectonic, and *P* is universal but *S* is particular;
	iff some instance in the intersection of the subject and the predicate is non-identical to every instance of the extension of the predicate.

Recall that *A* is inferior to *B* iff Sig(A)⊆Sig(B), and that Ext(*A*) is the set of all ideas that inferior to it. It follows that there is an isomorphism from the product algebra of significance ranges to that of extensions. Pariente's truth-conditions also follow as metatheorems:

Every S is P is true	iff	Ext(S)⊆Ext(P)	iff	Ext(S)∩Ext(P)=Ext(S)
No S is P is true	iff	Ext(S)∩Ext(P)=∅		
Some S is P is true	iff	Ext(S)∩Ext(P)≠∅		
Some S is not P is true	iff	Ext(S)∩Ext(P)≠Ext(S)		

If mental language contains sufficient expressive power, it also follows that

Some S is P if true	iff	for some *Q*, Ext(*S*)∩Ext(*Q*) = Ext(*P*)∩Ext(*Q*)	

Here the intended interpretation of extension is referential. That is, it is assumed that the extension of *A* is defined as the set of all ideas that signify subjects that possess all the modes in the comprehension of *A*. It follows that the metatheorems remain true if Sig replaces Ext.

As explained below, the interpretation also accommodates essential as well as contingent truths. Let Comp be the function that assigns each idea its comprehension. Briefly, *every S is P* is an essential truth iff Com(*P*)⊆Comp(*S*). But Com(*P*)⊆Comp(*S*) entails Ext(*S*)⊆Ext(*P*). Hence every essential truth is extensionally true. In addition contingent propositions are true but not necessary. For example, John *is hungry* may be true because the person named by *John* possesses all the modes in Comp(*hungry*), but the proposition is contingent because *hungry* is not in the comprehension of *John*.

Existential Import. In both medieval and 15th and 16th century logic it was standard to add to the truth-conditions of affirmative propositions the requirement that the subject term stand for existing things.[35] Likewise, negative propositions were regarded as automatically true if the subject failed to refer. (Thus, preserving the relations of the square of opposition). However, as formulated above, neither the intentional nor referential interpretation imposes existential import on the subject terms of affirmative propositions. That is, it is not required that Sig(*S*) be non-empty. The requirement will be seen to be important in the discussion of essential and contingent truth below.

Essential Truth. The *Logic* is explicit in maintaining the traditional distinction between essential and contingent truth.[36] Essential truths, moreover, retain many of the properties attributed to them in earlier logic. In particular they are universal and necessary.[37] The *Logic* explains these properties in terms of comprehension. Essential truths include any universal affirmative that affirms of an idea the content of its comprehension. These include real definitions, although because it is not always obvious what the real definition of a term is, a real definition may need to be proven.[38] They also include negative universals that describe the incompatibility relations among species imposed by contrary comprehensions, e.g. *no circle is a square*. Although the *Logic* does not directly apply the distinction to "factitious" ideas – to those formed by abstraction and restriction— these too have content, and affirmations of their contents are non-contingent and universal. Because essential truths are universal and have a general terms as subjects, they are said to describe the ultimate "causes" of things in a logical sense. They also have a special status epistemologically because they are known with certainty through clear and distinct ideas: if we perceive the idea *S* as *P* clearly and distinctly, we know with certainty that *every S is P* is true.[39] Essential truths conceived broadly also include any logical consequence of a universal form that can be drawn from other essential truths. This broad body of essential truths constitutes science properly understood.[40] The *Logic* allows that there are non-essential and contingent truths justified by sensation, but that these cannot in general be known with the same degree of certainty as essential truths.[41]

Existential Import. Although it was standard in 15th and 16th century logic to hold that affirmative categorical propositions carry existential import, the *Logic* does not address directly the existential import of essential truths. In Part II it holds that I propositions are subaltern to A propositions.[42] Hence if a universal affirmative has existential import, so does a particular affirmative. The issue arises because on more than one occasion, side by side, the text seems to say that an essential truth both is and is not true independently of what actually exist. For example, in Part IV when laying out axioms for rational inquiry, it says,[43]

> 1st Axiom: *Everything contained in the clear and distinct idea of a thing can be truthfully affirmed of it.*
> 2nd Axiom: *At least possible existence is contained in the idea of everything we conceive clearly and distinctly.*

The first axiom suggests that if *P* is part of the comprehension of *S* and we have a clear and distinct idea of *S* as *P,* this alone is sufficient warrant for knowing the truth of *every S is P,* regardless of whether there are any instances of *S* in the world. One of the advantages of the earlier intentional interpretation of truth is that it would insure that essential truths hold independent of the existential import of its terms. The second axiom, however, seems to say that a clear and distinct idea of *S* as *P* is sufficient only for the proposition's possible but not actual truth.

Earlier logicians were divided on the issue of whether essential truths carry existential import. The majority followed Aristotle – William of Ockham is a prime example – who held that a proposition describing something's nature is true only if its subject term stands for existing things. A minority, however, which included William of Sherwood, John Buridan, and Federico Suarez, maintained that essential truth differs in important ways from contingent truths. A contingent proposition could fail to be true because its subject term failed of its existential import. Essential truths, by contrast, were held to be necessary and eternal because they signify something eternal. Sherwood called this "something" habitual being, but Scotts and Suarez called it objective being. Similarly Descartes seems have thought that objective being is the subject matter of "eternal truths." The *Logic*'s doctrine is similar: an essential truth describes an idea's comprehension, its objective being.[44]

The *Logic* remarks in one edition that when the subject matter concerns essences, possibility is sufficient for truth:

> Thus when a geometer conceives that a line could be described by four or five different motions, he never took the trouble to draw the line, because it was enough for it to be possible in order for him to consider it as true."[45]

There is a modal argument that supports this view. For essential definitions, possibility entails necessity because if it is ever true, it is necessarily true.

On the other hand, contrary to the intent of the intentional interpretation, the *Logic* seems to be committed to a correspondence theory of truth:[46]

> Now suppose the judgments we form in considering these ideas [of things] did not concern things themselves, but only our thoughts. In other words, suppose that from the fact that I see clearly that having three angles equal to two right angles is implied in the idea of a triangle, I did not have the right to conclude that every triangle really has three angles equal to two right angles, but only that I think this way. It is obvious that we would know nothing about things, but only about our thoughts. Consequently, we could know nothing about things except what we were convinced we knew most

certainly, but we would know only that we thought then to be a certain way, which would obviously destroy all the sciences.

One way earlier logicians reconciled the eternal truth of essences with the standard view that affirmatives carry existential import was to propose that an essential truth *every S is P* should be understood as a disguised inference: *if S were to exist, it would follow that S is P*. Like other abstruse issues in earlier logic, the *Logic* does not take up the matter.

Contingent Truth

It is clear that the *Logic* is committed to the existence of contingent truth, and that this poses a serious difficult for the intentional interpretation of extension. If a term's extension is the set of all ideas that have that term in their comprehensions, as the intentional interpretation holds, then a universal affirmative is true iff the extension of the subject is a subject of the extension of the predicate (as both interpretations of truth above hold). It follows that a universal affirmative is true iff the comprehension of the subject is a subset of that of the predicate. Hence, every truth is essential and, therefore, necessary. The authors, however, explicitly draw the distinction between essential and contingent truth. In addition, their commitment to the Aristotelian predicables entails contingent truth. That is, to use the language of the *Logic*, accidents in the formal mode entail contingent truths in the material mode. The reason is that if *S* signifies *B* and *P* signifies *A* secondarily, then *A* is an accident of *B* iff *every S is P* is contingently true.

Auroux has pointed out in addition that the intentional interpretation yields the incorrect truth-conditions for universal negatives.[47] *No cow is horse* would be false on the intentional reading because the subsets of their comprehensions, which would comprise their extensions on the intentional reading, do not intersect.

Examples given in the text of contingent propositions (those that are possibly true and possibly false) include:[48]

> *The king of China has converted to Christianity.*
> *Constantine was baptized by St. Sylvester.*
> *St. Peter was in Rome.*

The fact that these examples have singular terms as subjects is related to their contingency. In Part IV the *Logic* explains that our knowledge of contingent truth is grounded in general in sensation, and much of this is of individuals.   (One import exception is the contingent proposition *I exist* which, it is argued, I know as a result of my clear and distinct idea of myself as a thinking substance.[49]) Although the *Logic* does not cite cases, predications of accidents of general terms would also be contingent, e.g. *every student in the classroom is asleep*, which would also be learned through sensation.

False Ideas and Existential Import. The *Logic* argues for a doctrine that seems to imply that contingent propositions, and possibly essential propositions as well, carry existential import. This is the doctrine of false ideas.

A false idea is any idea that fails of reference or, in the language of the *Logic*, that fails to signify an existing thing. The notion has its roots in Aristotle's *De anima*, and became a standard feature of medieval semantics. It plays a prominent role in Descartes' *Meditation III*. An idea is called false in a derived sense because the idea is generally a combination of two ideas *S* and *P* which are such that the proposition *every S is P* is false in a prior and more basic sense. In the versions espoused in the *Logic* is explained in terms of comprehension. A *false idea* is one that has a comprehension that consist of modes that are not jointly true of any actual object. In conformity with the earlier logical tradition, the idea, or more accurately its objective being, is called a being of reason. Given that God is non-deceiving

and would, therefore, not provide us with an innate idea or raw sensation that was not occasioned by an existing thing, false ideas must be what Descartes calls "factitious." They are constructed by restriction. Examples give include *golden mountain* and *a wealthy man that is happy.* These are composed of modes that are contingently incompatible.   In more serious cases false ideas combine modes that are incompatible by nature, for example, *pain as caused by fire*. The authors attribute many of our moral failings to the tendency to form false ideas.[50]

False ideas have an important logical implication under the referential interpretation of truth. On that interpretation, the *Logic* agrees with remarks of Descartes that a universal affirmative with a false idea must be false. That is, the truth-conditions for the universal affirmative, and hence also of its subaltern, the particular affirmative, must be supplemented with a clause to the effect that its subject term signifies at least one existing thing. The reason is that if the clause were not added, every universal affirmative with a false idea as subject would be trivially true. Suppose S is a false idea. Therefore, Sig(S)=$\varnothing$. Now, $\varnothing \cap$Sig(P)= $\varnothing$. Hence, Sig(S)$\cap$Sig(P)=Sig(S). Hence, on the referential interpretation of extension, Ext(S)$\cap$Ext(P)=Ext(S). Hence, *every S is P* is true.

On the other hand, if existential import is not required, another and quite different problematic results follows for essential truths under the intentional interpretation of extension. Trivial propositions like *every golden mountain is golden* would be true because every idea whose comprehension includes that of *golden mountain* also includes that of *golden*. This result is acceptable if the *Logic* is read as committed to the view of Suarez that essential truths only stand for objective being. Descartes seems to understand eternal truths in that way.[51]

Conclusions. The intentional interpretation of extension together with the intentional analysis of truth provides an analysis of truth for essential propositions that conforms well to the text. In its reconstruction it also makes a case for the claim that the *Logic* rejects medieval semantics, especially the theory of supposition and that it anticipates the 19th century analysis of the syllogistic in terms of classes. But the interpretation faces difficulties. It must grant the truth of propositions that predicate defining modes of false ideas. Moreover, the reading does not allow for the existence of contingent truth. The referential interpretation of truth, on the other hand, endeavors to capture more of the detailed wording of the Logic's of axioms that lay out truth-conditions, and it makes the case for the claim that the *Logic*'s theory of truth is an abstraction from the medieval theory of supposition. In addition, it allows for both contingent and essential truth, and insures the traditional commitment to the existential import of affirmative propositions.

In sum, the two interpretations present rather different views of the *Logic*'s semantics. On the intentional interpretation its authors reject the traditional correspondence theory of truth, holding that the only purpose of language is to describe ideas. As part of this rejection, the *Logic* is read as replacing supposition theory with a more a semantics in terms of classes that anticipates 19th century logic. The referential interpretation, on the other hand, understands the logic more conservatively, as an attempt to reconcile prior logical theory with Descartes dualism. On this reading, truth-conditions abstract from medieval supposition theory, comprehension's role is to explain how ideas signify things, and although truth is a matter of extension-containment, this corresponds to the containment of significance ranges composed of things outside the mind.

Part III. The Logic of Arguments

Part III discusses valid argument forms, and topics related to validity. Most space is devoted to the categorical syllogistic. There are brief, and unoriginal, discussions of several standard consequences in propositional logic, including what we would call today conjunction introduction and elimination, disjunctive syllogism, *modus pones*, *modus tollens*, and proof by cases. There are also brief accounts of enthymemes and topics, and a somewhat longer discussion of fallacies.

Argument forms are described and evaluated in terms of "rules." In the briefer discussions, like those for the propositional logic argument forms, the rules are essentially syntactic, and there is no attempt to formulate or justify them semantically. For the syllogistic basic syntactic distinction are drawn, including *subject* and *predicate*; *major, middle* and *minor term*; *affirmative* and *negative proposition*, *universal* and *particular proposition,* the four categorical proposition forms; and the four figures. Singular affirmatives are classified as a special case of the universal affirmative.[52] Rules for the syllogistic, on the other hand, are formulated party syntactically and partly in semantic terms introduced in Part II. These include *comprehension, extension*, and *universal* and *particular term.* Again, there is no serious attempt to justify the rules beyond simply stating them, perhaps because their validity follows trivially from the definitions of the semantic terms in question, with perhaps the aid of obvious principles like *de omni et nullo*. Syllogistic rules discussed include those for the immediate inferences of the Square of Opposition, including subalternation and conversion. Note that because the *Logic* is committed to the subalternation of I to A propositions,[53] those like Pariente (above) who hold that universal affirmatives lack existential import, must hold as well that particular affirmatives lack existential import. Various valid moods are described individually.

Of special interest are six syllogistic rules that are singled out, not as axioms in the modern sense, but as a kind of decision procedure for the valid moods:[54]

Rule 1: *The middle term cannot be taken particularly twice, but must be taken universally once.*
Rule 2. *The terms of the conclusion cannot be taken more universally in the conclusion than in the premises.*
Rule 3. *No conclusion can be drawn from two negative propositions.*
Rule 4. *A negative conclusion cannot be proved from two affirmative propositions.*
Rule 5. *The conclusion always follows the weaker part. That is, if one of the two propositions is negative, the conclusion must be negative; if one of them is particular, it must be particular.*
Rule 6. *Nothing follows from two particular propositions.*

The set of valid moods is not the closure of the six rules under a set of inference. Rather, a mood is valid iff it violates none of the strictures of the six rules. The rules were not new. The four rules that do not mention universal and particular terms were common in medieval logic, and the two that do, the so-called "process rules," are found in logic texts of the period. The set as a whole appeared earlier in the work of Eustache of St Paul.[55] Leibniz took the same set, dividing one rule into two and interpreting them syntactically, for his own more formal "axiomatization" of the syllogistic.[56] These rules in various forms still appear in textbooks of Aristotelian logic. Again, the authors do not attempt to prove in a modern sense that the rules follow from the definitions of the semantic terms in which they are formulated.

In their first edition the authors also describe the medieval method of assigning to a valid mood mnemonic name that contain code letters indicating the categorical propositions that compose it and the rules appropriate for "reducing" the syllogism to a more evident mood of the first figure – a little "system" that from a modern perspective is really a mini-axiomatization of the valid moods.[57] They, however, prefer the valuation procedure of their own six rules. In particular, they object to the reduction rule *per contradictionem* (if $A, \sim B \vdash \sim C$, then $A, C \vdash B$), no doubt as part of their more general objection expressed in Part IV to indirect proof.[58]

Part IV. Method

Part IV concerns epistemology and the methods by which knowledge is justified. These methods are divided into analysis and synthesis, concept of increasing concern to the logic of the day. In explaining these concepts, the authors make important use of their logical ideas including demonstration, axiom, definition, and comprehension.

Demonstration. As the authors describe demonstration, it sounds quite modern. It is a series of propositions in which each is either an axiom, a definition, a previously demonstrated proposition, description of a construction, or a proposition that follows logically from earlier propositions in the series:[59]

> A true demonstration requires two things: one, that the content include only what is certain and indubitable; the other that there is nothing defective in the form of the argument. Now we will certainly satisfy both of these if we observe the two rules we have laid down.
>
> The content will include only what is true and certain if all the propositions asserted as evidence are:
>
> Either definitions of words that have been explained, which, since they are arbitrary, cannot be disputed;
>
> Or axioms that have already been granted and should not be assumed if they are not clear and evident in themselves, by the 3rd rule;
>
> Or previously demonstrated propositions that have consequently become clear and evident by virtue of the demonstration;
>
> Or the construction of the thing itself in question, whenever there is some operation to be performed. This should also be as indubitable as the rest, since the construction should have been previously shown to be possible, if there had been any doubt about it.

Since the premises of a demonstration are regarded as necessary and the rules of logic as preserving necessity, anything proven by a demonstration is necessary.

Nominal Definitions. The definitions the authors are referring above are nominal definitions. These are discussed at length because the authors attribute most logical errors to equivocation between a term's nominal definitions.[60]

There role of nominal definitions is demonstration derives from their role is explaining synonymy and definitional abbreviation. In principle, there is no synonymy in mental language because ideas are identical if they share the same comprehension. In particular, the terms in a real definition are not synonymous. In a real definition *every S is DG*, a genus G and difference *D* are predicated of a species *S*.   Each of these ideas is distinct because they have different comprehensions. Real definitions, moreover, are matters of scientific inquiry. It is only if you are fortunate enough to have a clear and distinct idea of *S* that you learn with certainty its real definition. Not everybody is so lucky, and, as the authors explain, reasonable people can argue about a real definition.

It may be remarked that the mental grammar of a real definition is a bit puzzling. If *DG* is the restriction of *G* by D, and the definition is true, then *DG* it is the same idea as *S*. But *every DG is DG* is a triviality that repeats the same idea as subject and predicate. In other words, it is difficult to see how a true real definition conveys information to the soul. Perhaps is it better to regard the definition's mental grammar as a conjunction *every DG is G and every DG is D*, which may be contracted to *every DG is D and G*.

A nominal definition, on the other hand, is a rule that relates spoken words to ideas. It stipulates the convention by which a spoken word "signifies" an idea in mental language. This a prior use of "signify" distinct from the reference relation.[61] If an idea is a complex of two idea (i.e. a restriction) and if nominal definitions have preciously determined that each idea separately is signified by a distinct spoken word, then by convention the species is conventionally signified by the spoken word sequence.

For example, if the spoken word "rational" signifies the mental adjective *rational* and "animal" signifies the idea *animal,* then by convention the species *rational animal* is conventionally signified by the spoken phrase "rational animal." Nominal definitions, however, make spoken abbreviation possible because a convention may be established by which a single word signifies a complex idea. For example, by convention "man" signifies *rational animal*. The spoken sentence "man is a rational animal," then, is in a sense ambiguous. Viewed as a real definition it records real information about the essence of the species man. Viewed as a nominal definition it simply informs a speaker what idea the spoken word stands for. Science employs the proposition in the sense as a real definition. Language instruction employs it as a nominal definition.

Nominal definitions, however, are not simply trivial. The authors point out that it is often convenient in science to abbreviate complex ideas, and this role is especially important in geometry. It is for this reason that nominal definitions are included as possible premise in demonstration.[62]

Axioms. The authors allow two sorts of propositions as axioms, those that are established by appeal to clear and distinct ideas and those that have been previously demonstrated. Prominent among these are essential truths grounded in clear and distinct ideas,[63] although there are also some contingent truths grounded in clear and distinct ideas, for example, the proposition *I exist*. The authors also argue that sensation is genuinely reliable and propose that fact as a scientific axiom. They offer as a demonstration of this axiom a version of Descartes' argument based on the premise, confirmed by the idea of God if not accepted on faith, that God is not a deceiver.[64]

Constructions. By construction the authors are referring to essential propositions that lay out the properties of geometrical figures. Because they are necessary, they are true if they are possible.

Inference. Descartes seems to have had a non-standard understanding of logical inference in that he regarded a step in a logical argument not as conveying truth from premise to conclusion, but rather as a technique to make one see that the conclusion is true independently of the premises because it is grounded in its own clear and distinct idea.[65] The authors of the *Logic*, however, regard a demonstration in the more traditional way as conveying truth from premise to conclusion. "Demonstration," they say, "consists of not a single argument, but of a series of several inferences by which some truth is conclusively proved." It justifies "the less well known" because it follows logically from "the better known." On the other hand, the authors do not seem to have regarded spotting an error of logic as very difficult. In particular, despite devoting most of Part III to explaining syllogisms and their rules, they remark, "there is little value in knowing the rules of the syllogism."[66] They regard equivocation as a common mistake, but of other logical errors they say, "… it is almost impossible for a person of average intelligence who has some insight ever to fall into them."[67]

It should be remarked that the authors accept indirect proof as valid, but as in intuitionistic logic, they hold that it should only be used to prove a negative proposition, and then only when there is no direct proof.[68] Induction from the few to the many, however, is rejected as invalid for the obvious reason that a universal cannot be shown to be true without showing that all its instances are true.[69]

Analysis and Synthesis. Analysis and synthesis were increasingly topics of the logic of the day.[70] They fall broadly in epistemology as complementary varieties of scientific method.   As the authors of the *Logic* understand them, analysis it is a method for justifying claims to scientific knowledge and is a method for explaining to others what is already know.

By *analysis* is meant a variety of demonstration that takes as its premises universal truths about individuals or the "less general", and draws conclusions about what is more general. It is said to reason from effects to causes and is also called *resolution*. Because it show the truth of what is less well known by deducing it from what is better known, it is also called the *method of discovery*. Drawing on a "logical" sense of cause, which is found in both the Platonic and Aristotelian traditions, they regard a more general idea as a cause of the less general ideas that contain it. In this sense a genus is the cause

of its species. Recall that a genus is frequently divided into a more perfect species and a less perfect privation. Here the order of "perfection" is the same as that of causation in this sense: genus, more perfect species, privation. For this reason analysis is said to prove causes from effects, and to prove the simpler from the more complex.[71]

Synthesis is the convers of analysis. It reasons from causes to effects, from the general to the specific, and is called *the method of composition*. It starts with causes and ends with effects. It explains what is better known in terms of what is less well known. Hobbs draws exactly the distinctions in his *Logic*.[72]

The paradigm the authors seem to have in mind is a chain of reasoning that consists of syllogisms in the mood Barbara starting with a premise about a particular and finishing with a conclusion about a higher genus.   For example, *Socrates is a human, every human is an animal/∴Socrates is an animal, every animal is a living creature/∴Socrates is a living creature, every living creature is a body/∴Socrates is a body, every body is a substance/∴Socrates is a substance*. This chain of inferences stats with an individual idea, which is complex in the sense that its comprehension consists of multiple modes. It is the set {*rational, self-moving, living, corporeal, being*}. The demonstration proceeds through ideas with comprehensions containing increasingly fewer modes. The example the authors give derives as its conclusion the fact that the subject is the descendant of St Louis and does so from a complex description of his linage. The argument may be reconstructed as a series of Barbaras as follows. The comprehension of S_n is {*descended from A_n, …, descended from A_2, descended from St. Louis*}. The "analysis" would then be: *every S_n is S_{n-1}, every S_{n+1} is S_{n-2}/∴every S_n is S_{n-2}, every S_{n+2} is S_{n-3},/∴ …/∴every S_n is S_1*.[73] Leibniz formalized analysis or resolution in various logic papers in a similar way. He symbolized the predicate as a series $P_1…P_n$ of concatenated terms which are intended to display in the syntax what the *Logic* calls a term's comprehension. He then employs an inference rule *S is $P_1…P_n$ ⊢ S is $P_1…P_{n-1}$*.[74] Although this seems to be the paradigm, the authors also cite Propositions 32 of Book I of Euclid's *Elements* and its tenth corollary as examples of propositions to be proved by derivations "connecting" ideas through an intermediary, as in Barbara. A logician, however, would be hard put to recast Euclid's geometrical proof into syllogisms, even with the wider resources mentioned in the *Logic*.[75]

Epistemology

Clear and Distinct Ideas. The authors make important use of concepts from logic to clarify their Cartesian epistemology. Perhaps the most important of these is that of clear and distinct idea. Examples of clear and distinct ideas are: *ourselves as thinking beings, thinking, judging, reasoning, doubting, willing, desiring, sensing, imagining, shape, motion, rest, extended substance, existence, duration, order, number*, and *god*. The authors explain the concept by appeal to containment, which is in turn explained by comprehension, "Everything contained in the clear and distinct idea of a thing can be truthfully affirmed of that thing."[76] They go on to give as an example the essential definition of *human*, "…because animal is include in the idea of human, I can affirm of humans being that they are animals." The view seems to be that if we have an experience of a clear and distinct idea of *S* as containing in its comprehension the mode *P*, then we are have certain knowledge of the essential truth *every S is P*. These truths make up the core of science. Science, moreover, is enlarged by demonstration, a second logical idea, because any proposition proven by demonstration from what is scientifically known also counts as science although of a less certain sort.

The Cogito. It might seem that clear and distinct ideas warrant only knowledge of necessary truths. The cogito, however, presents an exception. On the one hand, the authors agree with Descartes that the cogito involves clear and distinct ideas:[77]

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there is nothing we conceive more distinctly than our thought itself, nor any proposition clearer to us than this: "I think, therefore I am." Now we could not have any certainty regarding this proposition if we did not distinctly conceive of what *being* is, and what *thinking* is.

The temptation is to understand the argument as claiming that because I have a clear and distinct idea of myself as a thinking being, I also have one of myself as existing. But, if a clear and distinct idea of *S* as *P* entails that *P* is a part of the comprehension of *S*, the absurd conclusion would follow that existence is part my essence and that I am a necessary being. The proper implication, then, is that there are cases of clear and distinct ideas that warrant only contingent knowledge. The authors, however, do not address the issue, nor do they explain how to distinguish between contingent and necessary cases. It is relevant to remark that if the *Logic* is understood as attributing existential import to the essential truths, then the problem is resolved because *I exist* would be part of the truth-conditions of *I am a thinking being*.[78]

Sensation. Sensation has an important place both in the *Logic*'s doctrine of mental language and in its epistemology. Sensation is most fully explained in *On True and False Ideas*. There Arnauld makes clear that he rejects Malebranche's version of representational realism. He rejects the view that the mind apprehends an intermediary or representation – Malebranche's ideas – that stands between the soul and the object of sensation. It is not quite true, however, that Arnauld is a direct realist. His view is that in sensation God instantiates in the mind a mental mode, called a perception. This mode has both formal being because it is a mode of the soul and a kind of objective being because it has a modal content. Simultaneously the mind is self-aware. It is conscious that the perception is occurring and that the perception has the content it does. At the same time, God causes it happen that the object of sensation is present outside the mind, that it is causally affecting the body's organs of sensation, and that it possesses those properties of extended substances that are contained in the content of the soul's perception. The sensation, moreover, is veridical because God causes it to be the case that the object of sensation truly instantiate the material modes in the perception's content. Arnauld writes as if the soul is directly aware of the object of sensation in the sense that it is aware of that object and there is no intermediary. His doctrine, however, is not pure direct realism because in his view there is an additional condition for sensation, namely, that the soul be self-reflectively aware of the content of the perception's objective being.[79] What it means for a sensation to be veridical, then, is explained in terms of its content, which is an attenuated version of the logician's notion of intentional content.

The relevance of logic to sensation is also illustrated by the role of comprehension in abstraction. By paying attention to the content of a sensation, the soul can form an abstract idea that has as its comprehension a selection of modes from the sensation's content. It is clear here that the doctrine presupposes that a sensation in some sense has a "content," and that this content is comprised of multiple modes. Content in this sense is similar to the comprehension of an idea, but it is not the same. A sensation is not an idea in the narrow sense principally because it does not function as a term in mental language. For example, although a sensation has a content, this content cannot be predicated of it so as to form an essential truth, as it can with ideas. Moreover, a sensation is not referred to as a false idea although a sensation normally contains a large number of contrary modes, mixing both modes of the soul and of bodies, for example, sensations of pleasure, pain, color and heat, on the one hand, and sensations motion and shape, on the other.

The authors also hold that logic has an important role in validating knowledge based on sensation. Their view is that sensation can only be trusted as grounding its lesser sort of knowledge if it is supported by a scientific demonstration proving that in general sensation is veridical. Arnauld gives a version of Descartes' demonstration to this effect: if my material body and other people do not exist outside the mind, then God is a deceiver, but God is not a deceiver, therefore my material body and other people do exist outside the mind.[80] It follows that sensations of material modes are in general

veridical. For example, sensations of pain correspond to motions of the body, and sensations of color and touch correspond to motions in the world.[81]

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[1] Volume V, *Œuvres Philosophique d'Arnauld*, *Kremer 2003*. Hereafter *KM*. English translation *Arnauld 1996*, hereafter *B*.

[2] See Appendix I, *Auroux 1993*.

[3] *Des vraies et des fausse Idées*, *KM*:I.   Hereafter *VFI*. English translation *Arnauld 1990*, hereafter *G*.

[4] See *Discours* I, *KM*5:112-113, *B*11-12; IV:6 *KM*5:380, *B*249.

[5] I:1-8.

[6] *Chomsky 1966*.

[7] Introduction, *KM*V:125-126, *B*23-24.

[8] See *Pasnau 1997*.

[9] *VFI*:6, *KM*I:192, *G*60.

[10] I:6.

[11] *Ibid.*

[12] For example, Dominicy reconstructs the structure of ideas as a Boolean algebra of Carnapian properties, i.e. functions from possible worlds to extensions. *Dominicy 1984*. Auroux reconstructs it as a kind of non-complimented lattice. *Auroux 1993*.

[13] Letter Arnauld to Leibniz, May 13, 1686, *KM:*VI:31-32, and *Stencil forthcoming*.

[14] See, for example, *Fonseca 1964 [1575]*, 56.

[15] I:6. In commentaries on Porphyry the adverb *extensive* had been used to explain what Porphyry means by saying that a genus is "more" of a collection than a species. See *Martin 2012*.

[16] *Kneale 1962*, 318-320.

[17] Dominicy defends a duality interpretation in his Carnapian reconstruction of the structure of ideas. *Dominicy 1984*.

[18] *Auroux 1993* accepts the extensional interpretation. He holds in addition that because, in his view, the *Logic*'s idea complementation operation is incoherent, ideas are not dual to extensions. The minimal condition for duality in the algebraic sense, however, is independent of complementation and is satisfied in the extensional interpretation: there is an antitonic homomorphism from ideas as ordered by containment to extensions as ordered by set inclusion.

[19] I:1, *KM*V:132-33, *B*29-30; I:9, *KM*V:157-78, *B*9-50; I:12, *KM*V:168-170, *B*58-60; *VF*I:6, *KM*I:204, G71-71; *VF*I:27, *KM*I:349-50, *G*208. See *Nadler 2011* and *Garber 1993*.

[20] I:7

[21] I:7, *KM*V:147-48, *B*42.

[22] See *Auroux 1992*.

[23] I:1 *KM*V:132, *B*29; *VF*I:27, *KM*I:348-49, *G*207-208.

[24] I:5, *KM*V:143, *B*3; *VF*I:6, *KM*I:207-208, *G*74-75.

[25] This interpretation is advanced in both *Auroux 1993* and *Dominicy 1984*.

[26] Auroux interprets the *Logic*'s idea complementation as a would-be Boolean complementation and remarks that as such it is incoherent. *Auroux 1993*. Dominicy interprets it as an equivocal form of weak and strong negation. *Dominicy 1984*. It is clear however, that the intended operation is traditional privative negation. Privative negation was still standard in the logic of the time. See *Eustachio-De-S.-Paulo 1648*, Logica, Part I, De categoriis, Tract II, Postpredicamenta, Paragraph XI, p. 87; *Raconis 1651*,Physics,SecII:44-45; and *Fonseca 1964 [1575]*,BkI,Chapt 17:128. The relevant texts for the *Logic* are I:7, KMV:148-49, B42-43; *Arnauld 1776*,Livre5,Chapt1:351-2; and Descartes Meditation VI, *Adam 1897-1909*,IXa:62. In these texts a quasi-Neoplatonic order among species is assumed (see Axiom 5, IV:8, *KM*V:382, *B*250) in which the species *human* is more perfect than *beast*, within the genus *animal*, and that *beast* is a lexicalized privative negation of *human*.

[27] *Cf.* the fifth axiom at IV:7, *KM*V:383, *B*250.

[28] II:17-19 *KM*:V:247-252, *B*129-133.

[29] *Pariente 1985*, 264-281.

[30] *Boole 1948 [1847]*, 20-25.

[31] *Pariente 1985*, 269-272.

[32] *Pariente 1985*, 237-238, and *Auroux 1993*,74.

[33] *Martin 2013*.

[34] *Keenan 1984*. If Q is a quantifier and $\Im$ over a domain D that assigns Q a binary relation on extensions in D, then the relevant definitions are:

Q is *subject up monotectonic* iff,

$\forall A,B,C \subseteq D, <A,B> \in \Im(Q) \ \& \ A \subseteq C . \rightarrow <C,B> \in \Im(Q)$;

Q is *subject down monotectonic* iff,

$\forall A,B,C \subseteq D, <A,B> \in \Im(Q) \ \& \ C \subseteq A . \rightarrow <C,B> \in \Im(Q)$;

Q is *predicate up monotectonic* iff,

$\forall A,B,C \subseteq D, <A,B> \in \Im(Q) \ \& \ B \subseteq C . \rightarrow <A,C> \in \Im(Q)$;

Q is *predicate down monotectonic* iff,

$\forall A,B,C \subseteq D, <A,B> \in \Im(Q) \ \& \ C \subseteq B . \rightarrow <A,C> \in \Im(Q)$.

[35] *Ashworth 1973*.

[36] IV:13, *KM*V:398, *B*263. The distinction, moreover, is presuppose by the distinction between accident and the other predicables.

[37] IV:13, *KM*398, *B*263.

[38] I:12, *KM*V:170-174, *B*:60-63.

[39] IV:2 *KM*V:366-367, *B*237-238; IV:6, *KM*V:378, *B*247; IV:7, *KM*V:382, *B*250.

[40] IV:6, *KM*380, *B*249.

[41] IV:I, *KM*V:354-356, *B*227-228; IV:7, *KM*V:383, *B*250.

[42] II:4, *KM*V:201, *B*86; II:18, *KM*V:249-250, *B*131.

[43] IV:7, *KM*V:381-382, *B*250.

[44] See the discussion in *Martin 2012*.

[45] IV:13, *KM*:V 398, *B*263.

[46] IV:6, *KM*V:378-379, *B*248.

[47] *Auroux 1993*, 135.

[48] IV:13, *KM*V:398-400, *B*263-265.

[49] I:1, *KM*V:131-132, *B*29.

[50] See the discussion in *Martin 2012*.

[51] See the discussion in *Martin 2011*.

[52] II:3, *KM*V:199, *B*84; *Pariente 1995*, 246; *Nuchelmans 1998b*, 119.

[53] II:4 *KM*V:201, *B*86.

[54] III,3, *KM*V:258-263, *B*139-142..

[55] For precedents of Rules 3-7 see William of Sherwood, *Intro to Logic* III,ix, p. 67, *William_of_Sherwood 1966*; Peter of Spain, *Tractatus*, IV,4 p. 40 *Peter_of_Spain 1990*; p. 45 *Rijk 1962-1967*; Buridan *Summulae* 5.1.8. pp. 312-313, *Buridan 2001*; Fonseca, *Institytionum dialecticarum* , Liber VI, Caput 18, 1964, p. 382 *Fonseca 1964*. For Rules 1 and 2 see Toletus, *In Lib I posteriorum analyticorum,* Cap XIX, p. 202, *Toletus 1580*; and Fonseca, *Institytionum dialecticarum* VI, Caput 20, 1964, p. 386, *op. cit*. The Logic's group of six rules is used with a similar purpose in Eustachio de S. Paulo, *Summa philosophiae quadripartita, Logia* III.2.I. p. 117, *Eustachio-De-S.-Paulo 1648*.

[56] See *Lensen 1990*.

[57] See *B*, xxxv and 156.

[58] IV:2, *KM*V:367, *B*238, IV:9, *KM*V:388, *B*255.

[59] IV:8, *KM*V:384, *B*251.

[60] I:12-13, IV:3-5.

[61] *Cf*. I:1, *KM*V:129, *B*26.

[62] Arnauld's views on nominal definitions appear to closely follow those of Pascal in *De l'Esprit géométrique et de l'art de persuader.* See *Nuchelmans 1998a* and *Miel 1969*.

[63] IV:6, *KM*V:379-380, *B*248-249.

[64] Axiom 11, IV:7, *KM*V:382, *B*250.

[65] See *Gaukroger 1989*.

[66] IV:introduction, *KM*V:354, *B*227.

[67] IV:8, *KM*V:384-385, *B*252.

[68] IV:2, *KM*V:367, *B*238, IV:9, *KM*V:388, *B*255.

[69] IV:6, *KM*V:377, *B*247.

[70] The literature is large on the topic analysis and synthesis as part of "method." See, for example, *Edwards 1967*.

[71] IV:2, *KM*V:362-366, *B*233-237.

[72] IV:2, *KM*V:362 and 367, *B*233 and 238. Hobbs, *De Corpore* I.6.1, p 66 *Molesworth 1992*.

[73] IV:2, *KM*V:367, *B*238.

[74] See, for example, *De arte combinatoria* in *Parkinson 1966*, and *Swoyer 1995*.

[75] IV:6, *KM*V:379, *B*248.

[76] IV:6, *KM*V:378, *B*247.

[77] I:1 *KM*V:132, *B*29.

[78]*Hintikka 1962* makes a similar point about Descartes.

[79] *VFI*:5-6,28; *KM*I:201-205,357-8; *G*69-72,216.

[80] *VFI*:28, *KM*I:355, *G*213-214.

[81] *VFI*:28, *KM*I:357-358, *G*216-217.