Printed 01/01/02

Proclus and the Neoplatonic SyllogisticJournal of Philosophical Logic, 30

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November 17, 2000

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Journal of Philosophical Logic, **30 (2001), 187-240**

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Abstract

An investigation of Proclus' logic of the syllogistic and of negations in the *Elements of Theology*, *On the Parmenides*, and *Platonic Theology*. It is shown that Proclus employs interpretations over a linear semantic structure with operators for scalar negations (hypernegation/*alpha-intensivum* and privative negation). A natural deduction system for scalar negations and the classical syllogistic (as reconstructed by Corcoran and Smiley) is shown to be sound and complete for the non-Boolean linear structures. It is explained how Proclus' syllogistic presupposes converting the tree of genera and species from Plato's diairesis into the Neoplatonic linear hierarchy of Being by use of scalar hyper and privative negations.

Key words: Proclus, syllogistic, negation, scalar adjective, hypernegation, privative negation, Neoplatonism, *via negativa*, negative theology.

Proclus and the Neoplatonic Syllogistic

Section 1. Proclus as an Aristotelian

It is clear that a central doctrine of the Neoplatonic tradition, starting with Plotinus and certainly including Proclus, is that reality forms a linear (totally ordered) structure emanating from the One. This is the chain of being ordered by the relation of causation.[1] What is more problematic is the extent to which the tradition is also committed to Aristotelian logic.

Plotinus is himself sparing in his use of technical logic, Aristotelian or otherwise. Although he adapts concepts from Aristotle's metaphysics, even these assume rather new meanings in the context of his system.[2] He rarely distinguished use from mention, or refers to the formal properties of language as such, much less to rules of logic. His followers however do make use of the logic of the day.

Probably the most influential Neoplatonic logical work is the *Isagoge* of Porphyry, student and biographer of Plotinus. This introduction sets out the

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[1] This basic interpretive assumption is not controversial and will not be argued for here. See Martin, John N., "Proclus the Logician," *Studies in Neoplatonism*, Ed. R. Baine Harris (S.U.N.Y., forthcoming).

[2] For a discussion of Plotinus' reinterpretation of the relevant ideas of Aristotle's metaphysics see Martin, John N., "Existence, Negation, and Abstraction in the Neoplatonic Hierarchy," *History and Philosophy of Logic*, 16 (1995), pp. 169-196. Below *ET n:m* stands for page *n*, line *m* in the Greek edition with English translation: Proclus, *Proclus: The Elements of Theology*, Trans. E. R. Dodds, Second ed. (Oxford: Clarendon Press, 1963). A citation in the form *IP n:m* stands for page *n*, line *m* in the Greek edition Proclus, "*Procli commentarium in platonis parmenidem*," Ed. and Trans. Victor Cousin, *Procli philosophi platonici opera inedita*, (Paris: Augustus Durand, 1864), pp. 603-1313. Likewise M&D *i* stands for page *i* in the English translation: Proclus, *Proclus' Commentary on Plato's Parmenides*, Trans. Glenn R. Morrow, John M. Dillon (Princeton, NJ: Princeton University of Press, 1987). Lastly, *PT n:m*, S&W *i;j* stands for book *n*, chapter *m*,

form that became standard for the exposition of Aristotle's categories and definition *per genus et differentiam.* What is odd is that though Porphyry espoused a Plotinian metaphysics in his own writing, in the *Isagoge* he lays bare Aristotle's version of ontology and theory of definition with hardly a critical remark*.* Apart from a few ideas of Stoic origin, the doctrines of the *Isagoge* are purely Aristotelian. It contains virtually nothing uniquely from Neoplatonic sources.[3] Indeed, in a famous remark in the opening of the work Porphyry declines to consider whether universals exist independently of the mind, perhaps the main issue that separates Neoplatonists from Aristotelians.   In a short discussion in his *Commentary on the Categories* [91,12-27] Porphyry does consider the reading of Aristotle in which Aristotle appears to defend the anti-Platonic thesis that individuals are ontologically prior to genera and species. Even here, however, Porphyry dodges the issue, choosing rather to read Aristotle as asserting a different priority, that which (he says Aristotle says) holds between the use of expressions standing for individuals and the use of words standing for ideas.[4] For an explicit reconciliation of the two traditions we must wait for Proclus.

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page *i*, line *j* in the Greek edition with French translation: Proclus, *Théologie Platonicienne*, Trans. H.D. Saffrey and L.G. Westerink, Vol. I-VI, (Paris: Les Belles Lettres, 1968-1997).

[3] Paragraph 2, *Isagoge*. Of the *Isagoge* Paul Hadot remarks that the concept of division is traceable to Plato, and his use of accident and individual to the Stoics. De Libera traces to the Stoics Porphyry's treatment of species as arranged beneath genera, the notions of highest genus and least species, the classification of predication into three types and genera into four, and points out a precursor to Porphyry's tree in Seneca. Arguing that the *Isagoge* as a whole should not be viewed as Neoplatonic, he shows how its remaining doctrines are derived from Aristotle. See "Introduction", in Porphyre, *Isagoge: Texte grec, Translatio Boethii*, Trans. Alain de Libera, Alain-Philippe Segonds, *Sic et Non*, Ed. Alain de Libera, (Paris: Librairie Philosophique, J. Vrin, 1998).

[4] Hadot, Paul, "*L'Harmonie des philosophies de Plotin et d'Aristote selon Porphyre dans le commentaire de Dexippe sur les Catégories*," *Atti del convegno internationale sul tema 'Platino e*

As a young man Proclus briefly studied Aristotle and mathematics in Alexandria. Later in Athens he spent almost two years studying under Syrianus, reading among other works of Aristotle all his logical treatises[5]. Logic was part of the curriculum in the Academy under Proclus who urged it upon those who would study more advanced philosophy.[6]

Proclus gives evidence of his general sympathy for logical methods by setting out the doctrines of the *Elements of Theology* in the quasi-mathematical form of propositions followed by scholia, and more clearly by inventing mathematical proofs, some of deep interest, in his commentary on Euclid's *Elements.*[7]

Proclus is a difficult guide to his own views on logic because he does not write about logical theory for its own sake. One source of his logical views, however, is to be found in his general remarks on critical methodology. In a manner modeled on Plato's *Phaedrus*, he assigns discursive thinking and logic in particular to a preliminary stage of understanding. A subject is to be investigated first, he says, by using the tools of logic. By "logic" here Proclus means Platonic *diairesis* and *analysis* combined with the application of syllogistic reasoning. (In Proclus' usage, *analysis* is the converse bottom-up description of the classificatory tree which is given a top-down description in

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il Neoplatonismo in Oriente e in Occidente', Vol. 198/371, Problemi attuali di scienza e di cultura, (Rome: Accademia nazionale dei Lincei, 1974), pp. 31-47.
Strange, Steven K., *Porphyry, On Aristotle's Categories*, (Ithaca, New York: Cornell University Press, 1992).

[5] Marinos, *The Extant Works or the Life of Proclus and the Commentary on the Dedomena of Euclid*, Trans. Al. N. Oikonomides, (Chicago: Ares, 1977). Pages 9 and 13.

[6] See for example *PT* I:2, S&W 10:19-21.

[7] Proclus, *Proclus: A Commentary on the First Book of Euclid's Elements*, Trans. Glenn R. Morrow, (Princeton, NJ: Princeton University Press, 1970).

diairesis.) By *diairesis* and *analysis* Proclus means essentially Porphyry's (Aristotelian) method of species definition in terms of genus and difference. Using these definitions as premises in arguments, he then derives in a perfectly orthodox Aristotelian manner the results of apodictic syllogisms. We shall sketch the details below in Sections 3 and 4. To be sure, he thinks that it is only after logical investigations of this sort that the student may advance to the higher levels of understanding represented by philosophy (Platonic intellection) and theology (religious and mystical understanding).[8] Logical reasoning, however, is an essential first step and is the basic method he uses in setting out his commentary on the *Parmenides.*[9]

A more direct source for Proclus' logical views are those passages in which he actually applies logical paradigms and laws as part of his critical discussion. In such texts we see Proclus' logic at work As he applies logic, he often comments on what he is doing, explaining the relevant rule or concept. An important group of such texts which reveal an essentially Aristotelian sympathy are those in which Proclus subscribes to various principles of Boolean logic, at least as they were known to ancient logicians. These texts fall into two sorts. Neither is an investigation into logical theory as such because there are few passages in which Proclus could be said to be writing on logical issues for their own sake. Rather they are remarks made in passing as he advances arguments on other topics. The first sort of text consists of examples using rules of (what

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[8]Proclus employs this method throughout both *IP* and *PT*. For general descriptions of his method see *IP* 1070-72, M&D 424-5. *PT* I:4, S&W 17:15-25.
[9] See *PT* I:9, S&W 40:3-18.

we today call) sentential logic and the second makes explicit appeal to Aristotle's syllogistic.

Sentential Logic. The Stoics are the main ancient source for explicit rules on sentential logic that fit the modern form. But modern commentators agree that Aristotle's own sentential metalogic is largely "classical" in the modern sense, especially in his use of reduction to the impossible, in conversions, and in appeals to non-contradiction. All these conform to elementary rules of modern Russellian logic. The same is true of Proclus. In the course of one argument, for example, he appeals to *modus ponens*. An example he gives is:

If something is not an animal, it is not a man.

It is not an animal

Therefore, it is not a man

This is one of numerous cases. At other places he cites *modus tollens* or, as he puts it, the syllogism that the denial of the premise follows from the denial of the consequent.[10]

He employs reduction to the absurd in a wide variety of examples. One typical case is a major argument repeated in slightly different versions in all three of his central philosophical treatises. He sets up a fourfold dilemma:

either plurality exists without a unity, or unity exists without plurality, or unity has plurality as parts, or plurality participates in a transcendent unity.

Each but the last leads to (multiple) impossible consequences and is therefore false. Thus the last case must be true.[11] At one point Proclus explains

[10] For *modus ponens* and *modus tollens* see *IP* 1098:2-27, M&D 444-45; *IP* 1055:2 to 1057:4, M&D 413-14; *IP* 1208:11-24, M&D 552-3.

the rationale for *reductio* by citing a principle he attributes to Aristotle, "those hypotheticals are true in which when the antecedent is true, the consequent must of necessity be." A reduction then consists of showing something is not possible by assuming it as a premise and using a (valid) hypothetical to show it leads to the impossible.[12] He sometimes explicitly uses as hypotheses for *reductio* instances of excluded middle or pairs of contraries (*enantia*, the Aristotelian term).[13] He even uses *reductio* to prove (non-intuitionistically) that a proposition is true by reducing its negation (*e.g. the One does not exist*) to the impossible. The impossible conclusion of the reduction is sometimes a conceptual absurdity of some sort, but it is also often a falsity of the form *not-P is P*.[14]

Proclus does not himself explicitly set out a theory of sentential logic. From these and simpler examples it is clear however that Proclus explicitly employs, from time to time, laws shared by Boolean logic, Aristotle, and the Stoics, including non-contradiction, excluded middle, *modus ponens*, *modus tollens*, constructive dilemma, disjunctive syllogism, double negation, and reduction to the absurd. We may summarize this practice and display it as the

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[11] *PT* II is replete with variations of indirect proof. For this argument see *IP* 695:39-697:20, M&D 73-75, *PT* II:1-3 (especially 1), S&W 2-30; *ET* Propositions 1-6, 2-7. See also *IP* 1100:10 to 1101:3 M&D 446-47. In another example Proclus proposes a dilemma of three alternatives $P \vee Q \vee R$, shows that P leads to an absurdity, then shows that R leads to an absurdity, and concludes Q. See *PT* I:11, S&W 48:22-49:5,

[12] See M&D, note 64, p. 74.

[13] For an explicit excluded middle as the dilemma supposed for a *reductio* see *IP* 1055:2 to 1057:4, M&D 413-14. For a pair of contraries as the hypothesis see *PT* II:1,S&W 4.2.

[14] For example, *IP* 1065, M&D 419. At times he also appeals to laws of standard modal logic that appear to use the classical connectives. Two "rules" he uses, in modern symbolization, are:

$$\Box(P \rightarrow Q), \Box P \models \neg\Diamond\neg Q$$

$$\Box(P_1 \wedge \ldots \wedge P_n \rightarrow Q), \Diamond Q \models \Diamond P_1 \vee \ldots \vee \Diamond P_n$$

See *IP* 696:21-27, M&D 73-74 and *IP* 1065:10-16, M&D 419-20. Proclus' modal logic will not be pursued here because Proclus attempts no systematic statement of modal ideas, and the instances in which he uses modal concepts, like those cited, are interpretable as metalinguistic statements about logical inferences of the sort discussed below.

first of a series of interpretive principles that we shall use later in reconstructing the logical theory implicit in Proclus' works:

> **Classical Metalogic.** Sentential metalogic obeys standard rules of Russellian logic

<u>The Tree of Porphyry.</u> Proclus makes use of Aristotle's theory of definition. Prior to Proclus there was no clear attempt to coordinate the Neoplatonic hierarchy with the tree-structures of genera and species. The standard view, which is also that of Proclus, is that Aristotle had correctly followed the teachings of Plato in his views on tree-structures . On this understanding genus-species taxonomy is a direct development of Plato's diairesis as exemplified in the definition of *angler* in the *Statesman*. At one point Plotinus himself employs the tree-like division:

> So much for what is called sensible substances and the one genus. But what species of it should one posit, and how should one divide them? Now the whole must be classed as body, and of bodies some are matterish and some organic; the matterish are fire, earth, water, and air; the organic the bodies of plants and animals, which have their differences according to their shapes.[15]

In the *Isagoge* Platonic division is set out in the so-called Tree of Porphyry which became the standard model for definitional structure:

> Therefore, the most general genera are ten. The most specific species are of a certain number too, surely not infinite. But individuals, which

[15] Plotinus, *Enneads*, Trans. A. H. Armstrong, Loeb Classical Library, 6 vols, (Cambridge, MA: Harvard University Press, 1966-88), Book VI: 3, 9, 1-8, p. 204.

> come after the most specific species are infinite….So going down to the most specific species we must proceed by division through a multitude….Now given what each of the genus and the species is, and given that whereas the genus is one the species are several (for the division of genera is always into several species), the genus is always predicated of the species….[16]

Proclus indeed embraces Porphyry's version of Aristotle's theory of definition. The passage below is an example:

> In demonstrations and definitions the particular (*ton merikon*) must be subordinate to the universal (*tou kathalou*) and the *definition (ton horismon*). Definitions of common features in particular do not take in the particulars as a whole. How, for instance, is the whole of Socrates comprehended by the definition "rational mortal animal," when there exist in him other elements also which make up his so-called 'personal quality'? The reason-principle of Man (*ho tou anthropōu logos*) in us comprehends the whole of each particular, for the particular comprehends unitarily all those potencies which are seen as being involved in the individuals. In the case of "animal" and likewise, the instance of it in particulars is less comprehensive than the particulars themselves or the species; for it does not have in actualized form all the differentiae, but only potentially, wherefore it becomes a sort of "matter" to the specifying differentiae that super-impose themselves upon it. The "animal" inherent in us is greater

[16] See pp. 5-6, in Porphyry, "*Isagoge*," Ed. and Trans. Paul Vincent Spade, *Five Texts on the Mediaeval Problem of Universals*, (Indianapolis, IN: Hackett Publishing Company, 1994), pp. 1-

and more comprehensive than "man," for it contains in unified form all the differentiae, not potentially, like the concept, but actualized. If we are, then, to discover the definition which will serve as the beginning of demonstration, the definition must be of an entity of such a sort as to comprehend everything more particular than itself.[17]

Here Proclus is arguing for a Platonic thesis, that the forms have independent existence. But he does so by appeal to Aristotle's definition of species by genus and difference, and he then uses these definitions as premises for apodictic syllogisms. He even cites the un-Plotinian principle from Aristotle's metaphysics that a species comes to be by the actualization in matter of potencies captured by specific difference. Throughout he is using Aristotelian technical vocabulary.

Elsewhere there are a number of places in which Proclus sets out his own argument in the precise form of a valid mood. For example, he illustrates the apodictic reasoning about the heavens with a case of Barbara:[18]

All great circles bisect one another

Circles in the Heavens are great circles

Therefore, all circles in the heavens bisect one another.

In another example the subject is the higher realm of metaphysics :[19]

The One is not receptive of multiplicity

The unequal is receptive of multiplicity

Therefore, the One is not unequal

19.

[17] *IP* 981:5-27,M&D 335. See also *IP* 767:8-28, M&D 133-34; *PT* IV:3, S&W 13:2-24; *PT* I:10, S&W 43:22-45:18.

[18] *IP* 796:37-797:3, M&D 165-116.

The mood is Baroco if singular propositions are treated as universal (as is Proclus' practice). Though in practice Proclus seems to limit moods to those with universal premises -- a fact we shall remark more upon below -- it is fair to generalize both from his unqualified appeal to syllogistic patterns and his examples that he accepts the Aristotelian argument forms:

The Syllogistic. Proclus accepts as valid the standard moods of Aristotle's Syllogistic

The Boolean Interpretation of the Syllogistic. To explain more clearly what it means to say logic is Aristotelian, it will be useful to sketch the standard interpretation of the metalogic modern readers attribute to the definitional tree and to the syllogistic as understood by Aristotle. On the standard reading the theory is seen as essentially Boolean, even if not fully worked out or stated in modern terms. Definitions of species terms are read as early forms of set definitions taking non-empty sets as extensions. A taxonomic tree T is then understood to be a prototype of what we would call today a subalgebra of the Boolean algebra $<PU,\cap,\cup,-,\varnothing,U>$ of the subsets of a universe U. We might even define *a definitional tree* as any T that is a subalgebra of a Boolean algebra on a universe U such that

1. T is a finitely branching tree,
2. the immediate $\subseteq$-descendants of any node in T form a partition of that node, and
3. each set in the partition is non-empty.

[19] *IP* 1208:11-24, M&D 553. For an explicit Bocardo see *IP* 1208:11-24; M&D 553.

If the branch of a tree terminates in singletons, these would represent "individuals," and the penultimate nodes would then be the *infimae species,* as in Porphyry`s account. The Boolean negation operation – would be well defined for such a tree, and would conform to "classical" laws, *e.g.* non-contradiction ($A\cap -A=\varnothing$), excluded middle ($A\cup -A=U$), double negation ($--A=A$), and contraposition ($-A\subseteq -B$ iff $B\subseteq A$). For example, setting aside troublesome cases like future contingents, Aristotle would agree that there are no human non-humans, that everything is either human or non-human, that what is the contradictory of the contradictory of humans is itself human, and that everything non-animal is non-human iff every human is an animal.

We may summarize the discussion with the observation that in the traditional syllogistic the predicate operator –, representing the prefix *non-*, appears to conform to these Boolean properties:

Boolean Properties of Predicate Complementation

Double Negation. $--A = A$

Antitonicity. $A \leq B$ iff $-B \leq -A$

Given that the standard structure used to interpret the syllogistic contains such a negation operation, the modern assumption has arisen that it is a straightforward matter to extend the syllogistic to contain a negation predicate operator standing for a complementation operation, and that the resulting logic would validate not only the traditional Aristotelian moods but also classical laws like double negation and contraposition. As a recognition of this modern understanding of the syllogistic, one of the points we shall investigate is the extent to which such a predicate operator is to be found in Proclus' texts on the logic of negation and the extent of its independence from the classical syllogistic.[20]

In sum Proclus appears to accept a broad variety of Aristotelian logical views. Like Aristotle's his sentential metalogic conforms to a number of rules recognized in classical logic. In addition, he accepts the syllogistic, which is usually understood by a Boolean interpretation on non-empty sets. Lastly, it is relevant to explore to what extent he might recognize a predicate negation that obeys the laws of Boolean complementation.

Section 2. The Neoplatonic Proclus

Non-Boolean Negation. Proclus is well known for distinguishing three varieties of negation, none of which is clearly Aristotelian, and all of which seem to violate standard Boolean laws. There are two passages in particular in which Proclus attempts to characterize the three operations. Each of the three, he says, is appropriate to its own ontic levels. The first, which he calls at one point *hypernegation*, a terminology we shall follow,[21] is appropriate to the level of the gods, *i.e*. to the henads and with certain important qualifications to the One itself. The second, which for our purposes does not need a technical name, is appropriate to the next lower level, that of Being (of intelligence, ideas and forms). The third, called *privative* negation, is appropriate to levels lower than Being (to Soul, Body, and Matter).

[20] For references to the standard modern reconstruction of the model theory of the syllogistic in terms of non-empty sets see the references in Section 3 to Smiley, Corcoran and Martin.
[21] ὑπεραποφάσεσις, *IP* 1172:35, which is shown to function as the α-*intensivum* of traditional grammar. The term appears in Stoic logic but in the quite different sense of syntactically marked double negation. The

In the first passage below (from *In Parmenidem*) he is commenting on Plato's views in the *Sophist* 258a-b on the topic of Being and Non-Being. Though the account is obscure, it is clear that Proclus' views are linguistic. He holds that negation is a part of language, and discusses the semantics of negations in terms of how they match aspects of reality unpacked in terms of the ontic hierarchy:

> Being, after all, is the classic case of assertion whereas Not-Being is of negation…. So then in every class of Being, assertion in general is superior to negation. But since not-Being has a number of senses, one superior to Being, another which is of the same rank as Being, and yet another which is the privation of Being, it is clear, surely, that we can postulate also three types of negation, one superior to assertion, another inferior to assertion, and another in some way equally balanced by assertion.[22]

In the second passage from the *Platonic Theology* Proclus makes similar points contrasting negation to affirmation, also a linguistic phenomenon:

> En effet, dans les réalités, les négations, à mon avis, présentent trois types particuliers; et tantôt, étant plus apparentée au principe que les affirmations, elles sont génératrices et perfectives de la génération des affirmations; tantôt, elles sont placées sur le même rang que les affirmations, et l'affirmation n'est en rien plus respectable que la négation;

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example given in Diogenes Laertius is *it is not not day.* See VII.69, p. 178 in Hicks, R., ed. *Diogenes Laertius, Lives of Eminent Philosophers, Vol. II, (Cambridge, MA: Harvard University Press, 1970).*
[22] *IP* 1072:28-1073:8, M&D 426

> tantôt enfin elles ont reçu une nature inférieure aux affirmations, et elles ne sont rien d'autre que des privations d'affirmations. [23]

As we shall now see, Proclus makes clear in supplementary texts that the three negations do not conform to classical laws.

Compatible Contrariety. Though Proclus calls an assertion and its negation contraries (*enantiai*), he makes use of a non-standard idea of contrariety that permits contraries to both exist simultaneously. In one telling passage he asserts that man and horse are not contraries, but that the pair made up of an assertion and its privative negation are. That is, he limits contraries to linguistic pairs one of which is marked by the affix for privation. This claim is already un-Aristotelian. Though Aristotle sometimes does illustrate privative opposites by pairs in which one is syntactically marked by a negative affix (e.g. *nōda* = *toothless*, marked, from *nē* = *without* and *odous* = *teeth*)[24], other examples he uses are unmarked and are contraries because of the semantic fact that they are not compossible (e.g. *typhlos* = *blind* and *phalakros* = *bald*)[25]. But Proclus' contrariety is even stranger. He holds that in a sense two contraries can both hold simultaneously. He goes on to say,

> But contraries in the Heavens naturally coexist. The motion of the Same is contrary to the motion of the Other, but the same thing (the heavens) is moved in both ways, and when it is moving in one way, it does not abandon the other motion.
>
> ….

[23] *PT* II:5, S&W 38:18-25.
[24] *Categories* 12a30.

> But the contraries in Intellect, being unified to the highest degree, partless and immaterial, and constituted as a single form, are creative in company with one another….In sum the contraries in Matter flee one another, those in the heavens co-exist. [26]

Proclus' point can be made non-paradoxically. While a given corporeal object cannot be like and unlike, or limited and unlimited, the forms of Likeness and Unlikeness and of Limit and Unlimit coexist and indeed the former of each pair is the cause of the latter. It will suffice for now to note merely for later exploration the fact that non-contradiction fails for Proclus' privative negation, which we shall indicate by $\neg$, That is $A\&\neg A$ is not to be regarded as always interpreted as "false" or the minimal (degenerate) element in a Boolean interpretation. Indeed we shall see that in the appropriate sense $A\&B$ is the greatest lower bound of $\{A,B\}$ relative to the causal order $\leq$, and that various non-Boolean properties hold *e.g*. $\neg A=\text{glb}\{A,\neg A\}$ and $\neg A\leq A$. Similar properties hold for hypernegation, which we shall indicate by $\sim$. That is, $\sim A=\text{lub}\{A,\sim A\}$ and $A\leq\sim A$. Clearly these properties are non-classical in the modern sense, nor are they to be found in Aristotle. For later reference we highlight the basic order property of the two negations

Non-Complementarity of Hyper and Privative Negation

$A\leq\sim A$

$\neg A\leq A$

In a Boolean structure, by contrast, we should expect these to hold only in the trivial case in which A is 0 or 1.

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[25] *Categories* $11^{b}15$, *Metaphysics* $1022^{b}22$.

Privation has other curious properties. Proclus remarks at one point that the following propositions express essentially the same fact in different ways:[27]

If Providence exists, then things will be as they should (all good things exist)

If Providence does not exist, then nothing will be as it should be (evil exists)

Since the conditional form here is really a means to state causal order and the negation is privation, the properties may be recast in term of $\leq$ and $\neg$:

Goodness $\leq$ Providence

$\neg$Goodness $\leq$ $\neg$Providence

The text in which he makes this point goes on to apply the same principle in an interesting way by using it to describe nine hypotheses. Hypotheses 2-5 are familiar from the usual accounts of the Neoplatonic hierarchy. In addition by applying the idiom of privative negation to these, he adds extra hypotheses 6-9.[28]

2. If the One exists, then Being exists.
3. If Being exists, then Soul exists.
4. If the One exists, then Forms-in-matter exist.
5. If the One exists, then matter exists.
6. If the One does not exist, then only sensibles exist.
7. If no Being, then no Knowledge.
8. If no One, then no sensible bodies.
9. If no One, then nothing.

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[26] *IP* 740:6-11and 23-30, M&D 114.
[27] *IP* 1055:1-1056:22, M&D 413.
[28] *IP* 1060:10-26, M&D 416.

Commentators have puzzled about whether the last four are supposed to be further levels of the ontic hierarchy.[29] It is clear, however, that 6-8 are instances of a monotonic (order preserving) property of ¬ and are simply algebraic restatements (equivalents) of 2-5 respectively:

2. Being ≤ One
3. Soul ≤ Being
4. Bodies ≤ One
5. Matter ≤ One
6. ¬Being ≤ ¬One
7. ¬Soul ≤ ¬Being
8. ¬Bodies ≤ ¬One
9. ¬Matter ≤ ¬One

The same property holds for hypernegation. In one passage on privation, for example, Proclus observes:[30]

> If, then, the negations generate the affirmations, it is plain that the first negations generate the first and the second the second.

Hence, if A causes B, A, then the negation of A causes that of B. In sum Proclus accepts the laws:

Isotonic Hyper and Privative Negation

$A \leq B$ iff $\sim A \leq \sim B$

$A \leq B$ iff $\neg A \leq \neg B$

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[29] See M&P p. 387 and Siorvanes, Lucan, *Proclus: Neo-Platonic Philosophy of Science* (New Haven: Yale University Press, 1996).
[30] *IP* 1099:32-35.

Boolean complementation, by contrast, is antitonic and reverses order: $A \leq B$ iff $-B \leq -A$.

<u>The Mutual Generation of Affirmations and Negations</u>. The law just cited is part of a larger picture. Proclus uses the properties of negation, both hyper and privative, to "discover" facts about the ordering. For if we know $A \leq B$, we also know $\sim A \leq \sim B$ and $\neg A \leq \neg B$. Conversely, if we know $\sim A \leq \sim B$ or $\neg A \leq \neg B$ it follows that $A \leq B$. Proclus moreover integrates these facts with the partition structure of the ordering. Proclus makes clear in the *Elements of Theology*, the linear progression is partitioned into equipollent taxa internally ordered by $\leq$ and ordered as taxa by the $\leq$-order of their elements. In addition Proclus understands the negation operators as follows. If A and A′ are in the same taxon and $A \leq A'$, then there are hypernegations $\sim B$ and $\sim B'$ of the next higher taxon such that $\sim B \leq \sim B'$ and privative negations $\neg C$ and $\neg C'$ of the next lower such that $\neg C \leq \neg C'$.[31]

Indeed, as has been often remarked, this method determines the order of exposition and explanation in Proclus' two great metaphysical treatises the *In Parmenidem* and the *Platonic Theology.* In the former he lays out the attributes denied of the One by listing the attributes in causal order of the taxon of the gods. In the latter he investigates the attributes of the gods in the order of those denied of the One. The negation here is hypernegation and the process of discovering negative attributes of the first hypothesis through the positive

[31] These properties are sketched in *ET* (see propositions 11, 14, 21, 100, 103, 108-110, 135, 147, 179) and are described in detail below.

attributes of the second is Proclus' version of the *via negativa*.[32] In describing how the soul comprehends the process he says, "…l'âme …. cordonne aux négations les affirmations,"[33] and again,

> ….these are the only characteristics that pertain to being *qua* being, the ones which are asserted by the Second Hypothesis and are denied by the First. [34]

A more complete general statement of how properties of the first hypothesis are revealed by negation in the second is:

> Seulement cette cause, en tant qu'elle préexiste à tous les êtres, nous ne la célébrons que par des négations, tandis que nous révélons, tout à la fois sous le mode negatif et sous le mode affirmatif, les sommets qui ont procédé d'une manière analogue à cette cause: en tant qu'ils ont une supériorité transcendente sur les êtres inférieurs, nous les révélons sous le mode négatif, mais en tant qu'ils ont part aux êtres qui les précèdent, nous les révélons sous le mode affirmatif. [35]

For example in discussing how Parmenides exemplifies the method, Proclus first states a property that does not hold of the first hypothesis, namely *multiplicity*, and then goes on to describe the general process of finding among the ordered properties of the second hypothesis negative properties of the first, starting with the transcendent (*i.e.* hyper) negation of multiplicity, the monad of the order

[32] See the discussion of the doctrine and its role in the exposition of these two works in H. D. Saffery, L.G. Westerink, "*Introduction*," *Proclus: Théologie Platonicienne*, Vol. I, (Paris: Les Belles Lettres, 1968), pp. ix-lxxxix. For statements of the principle in addition to the texts cited below see *IP* 1133:3-5 and *IP* 1056-1060, M&D 413-16.
[33] *PT* I:12; S&W 58. 2-3.
[34] *IP* 1086:27-29, M&D 435. See also *IP* 1099:32-35, M&D 446, and *IP* 1208:22-24, M&D 553.
[35] *PT* IV:11, S&W 37, 21-27.

under discussion, and proceeding by hypernegating to the properties ranked in order beneath multiplicity:[36]

> *l'Un,* dit Parménide, *s'il est un, ne saurait être multiple.*
>
> …..
>
> Or, le multiple se rencontre pour la première fois au sommet des premiers dieux intellectifs et dans leurs hauteurs intelligibles, comme nous l'apprendra la deuxième des hypothèses. Donc l'Un transcende totalement cette classe, et en est la cause. Car, comme nous le disions, le non multiple n'est pas privation mais cause de multiple.
>
> ….
>
> et à l'aide de ce concept [*i.e.* le multiple] il nie celui qui le suit immédiatement et celui qui vient après celui-ci, à l'aide celui qui le précède, et ainsi de suite toujours de la même manière, et il prend les éléments de ses démonstrations tantôt parmi les concepts immédiatement précédents tantôt parmi les plus éloignés.

In this passage the negation employed is explicitly transcendent (*i.e.* a hypernegation) and is not privative. Privative negation however is equally monotonic, and this property may be used to discover truths about lower orders. In the following passage, for example, Proclus contrasts hypernegation, which he calls "négation par excès" (*kath'hyperochēn*), with privative negation which he calls "négation par défaut" (*kata elleipsin*)[37] The former operates on the ordering of the middle hypotheses (Being, Soul, Bodies) expressed positively to

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[36] *PT* II:12; S&W 66, 7-24.
[37] *PT* I:12, S&W 57:25.

reveal the ordered negative properties of the first hypothesis, the latter to reveal the negative properties in the privative sense of matter, the lowest hypothesis:[38]

> En dernier lieu, vient la procession de la materière, qu'elle soit unique ou qu'elle soit diversifiée, que la cinquième hypothèse montre par le moyen des négations qui portent sur *la similitude de dissimilitude* qu'elle entretient avec le premier principe. Cependant, dans ce dernier cas, les négations sont des privations, tandis que, dans le premier cas [*i.e.* de la première hypothèse], elles sont causes transcendantes de tous leurs effets. Et, chose plus étonnante que tout, les hypothèses extrèmes sont purement négatives, mais la première est négative par excès, la cinquième [*i.e.* de la materière] par défaut.

The general property of negation appealed to here may be generalized as it applies to taxa as follows:

> **Law of Generative Hyper and Privative Negation.** If $T_n=\{A_1,\ldots,A_m\}$ and $A_m\leq\ldots\leq A_1$, then there are $B_1,\ldots,B_m$ and $C_1,\ldots,C_m$ such that
>
> $T_{n-1}=\{\sim B_1,\ldots,\sim B_m\}$ and $\sim B_m\leq\ldots\leq\sim B_1$ in T_{n-1}, and
>
> $T_{n+1}=\{\neg C_1,\ldots,\neg C_m\}$ and $\neg C_m\leq\ldots\leq\neg C_1$ in T_{n+1}.

We shall find that as a rule B_i is arrived at by *m* applications of $\sim$ to A_i and C_i by *m* applications of $\neg$.[39]

The discussion here may be summarized in the observation that ontic structure for Proclus is in several ways non-Boolean. It is an infinite linear ordering partitioned into finite ordered taxa which at the ideal may be triads.

[38] *PT* I:12, S&W 57

Across these taxa non-Boolean hyper and privative negation operations are defined. We may now recall the question posed earlier: how is this linear structure with its negations to be reconciled with the tree structure of diairesis with its syllogisms and Boolean interpretations? Proclus himself indicates the answer by his use of the non-standard negations. As we shall see in Section 5, their role is precisely to transform the tree-structure of diairesis into the linear structure of causation.

Before turning to the details however it will be necessary to review some concepts from modern linguistics. It turns out that Proclus' treatment of causal order and negations is a special case of what is known today as the logic of scalar adjectives and their negative affixes. The semantics for this broad family of expressions closely fits Proclus' account of predication. Moreover, the syllogistic and various Boolean laws of predicate complementation remain valid in these scalar structures. Indeed, the standard natural deduction proof theory for the syllogistic may be extended to embrace Proclus' hyper and privative negations and a predicate operator with some of the properties of complementation, within a system that is sound and complete relative to a family of scalar structures abstracted from Proclus' linear causal ontology. In Section 3 the relevant theory of scalar adjectives and their negations is sketched. In particular the relevant notion of scalar model structure is defined and the completeness proof given for the natural deduction extension of the syllogistic to scalar negations. In Sections 4 and 5 this theory is applied to Proclus. In Section 4 it will be shown how Proclus' inferences and other logical properties

[39] In Section 5 this result is shown to hold relative to a level of analysis in the tree of diairesis.

described in Sections 1 and 2 are implemented in scalar theory. In Section 5 the technique is detailed by which Proclus transforms the tree of diairesis into the line of causation by appeal to scalar negations.

3. Scalar Adjectives and Negation

Scalar Adjectives. Proclus' account of predicates appears to be a special case of what are called in modern linguistics *scalar adjectives*. Linguists have observed that natural language possesses families of monadic adjectives governed by a comparative adjective phrase. For example the series of scalar adjectives *ecstatic, happy, content, so-so* function, as it were, divides a common background scale that is ranked by the governing comparative *happier than.* The comparative takes as its extension a binary relation on a set of individuals. The "field" of the relation understood as the set of its relata then constitutes the "range of significance" of the associated scalar adjectives. That is, the extension of any scalar is a subset of the field of comparison, and the predicate is true of the entities in its extension, false of those in the field but outside the extension, and "meaningless" (however that idea is to be treated in terms of truth-values) for objects outside the field. Some examples of comparative adjectives and their associated scalars are:

is happier than	*ecstatic, happy, content, so-so*
is sadder than	*miserable, sad, down, so-so*
is hotter than	*boiling, hot, warm, tepid*
is colder than	*freezing, cold, cool, tepid*

So-called "test frames" have been identified that provide criteria for identifying members of a scalar series.

Linguistic Criterion for Scalar Families. [40] If the following sentences are semantically acceptable

x is not only Q, but P

x is P, or at least Q

x is at least Q, if not (downright) P

x is not even Q, {*let alone/much less*} *P*

x is Q, {*or/possibly*} *even P*

x is Q , and is {*in fact/indeed*} *P*

then the predicates are members of a scalar series in which *P* is higher in the order than *Q* .

By convention, writing *x* to the left of *y* indicates that *x* is higher than *y* in the series.

The semantic interpretation of the governing comparative adjective is clear. It describes the background relation. This may be treated as a set theoretic relation, a set of ordered pairs. But the semantics of the monadic scalar adjectives associated with the comparative will vary according to which of several methods is chosen for associating the predicates with sets. There are three relevant set concepts that might serve as a predicate's "extension." First is

[40] Horn, Laurence R., *A Natural History of Negation* (Chicago: University of Chicago Press, 1989). Horn states a full account of scalars and their negations, including other kinds of evidence for identifying scalars and their negations beyond those employed in this paper. On the logic and semantics of comparative adjectives see Åqvist, Lennart, "Predicate Calculi with Adjectives and Nouns," *Journal of Philosophical Logic*, 10 (1981), pp. 1-26. Unlike the account here, Horn favors a pragmatic rather than a model theoretic approach to their interpretation. On the model theory

the standard idea of extension, the set of all objects of which the predicate is true. This notion of extension captures only a limited notion of scalar structure, namely that the extensions in this sense partition the field of the comparative relation. It does not capture the more important property that the extensions are ordered.

There are two ways to define extension that captures their order. In each the extensions are nested one in another by the subset relation. In the first each scalar is paired with the set of all objects in the world that have the background predicate to *at least* the degree suggested by the meaning of the scalar predicate.[41] Thus, paired with *happy* would be the set of objects that were *at least* happy, and paired with *content* would be those that were *at least* content. Then, the set paired with a higher predicate would be a subset of the set paired with a lower predicate. The set of at least happy things is, for example, a subset of the set of at least content things. Accordingly, two structures would be distinguishable, one syntactic, the other semantic. The former is the set of monadic scalar adjectives ordered in the scalar rank (indicated by the left to right convention). The latter is the family of extensions in this sense, ordered by the subset relation. In this case the reference relation, call it R, would be an antitonic mapping between the two structures: if P is to the left of Q then $R(Q) \subseteq R(P)$.

The second method of pairing scalars with nested sets reverses the order. Associated with each monadic predicate are the objects that have the background property to a maximal degree. The predicate's extension in other

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used here for scalars see Martin, John N., "Existence, Negation, and Abstraction in the Neoplatonic Hierarchy," *History and Philosophy of Logic*, 16 (1995), pp. 169-196.

words is the set of all objects that have the background property to *at most* the degree suggested by the meaning of the predicate. Paired with *happy*, for example, would be the set of all objects that possesses happiness to a degree less than or equal to those that are truly happy. The set associated with a predicate lower in the scalar order would then be a subset of that associated with a higher predicate. The set of objects that are at most content is a subset of those at most happy. This pairing preserves the order of predicates. Let R* be this sort of interpretation relation: if *P* is to the left of *Q* then $R^*(P) \subseteq R^*(Q)$.

There are several points to be made about these alternatives. The first is that either of the two latter theories which embody the ordering will be more useful than the standard account for explaining the logic of negations that depend on the scalar ordering of the predicate extensions.

The second point to make is algebraic. What will prove to be important about these extensions of scalar predicates is simply their order. The fact that they are sets will be immaterial. We shall find in fact that the scalar theory at its most general is really a kind of many-valued logic that abstracts away from sets. Indeed one of the more interesting features of Proclus' "model theory," a feature that makes it strikingly different from standard Boolean semantics, is that its "points" definitely are not sets.

A third point concerns the limits of algebra. What proves to be important for the limited purpose of capturing the logical inferences of scalars over appropriate model structures is that their interpretations have an order. The

[41] This account is developed in Horn (1989).

direction of the order is unimportant. That is, from a purely structural perspective the set of acceptable interpretations for an adequate scalar semantics may be defined equally well by requiring that interpretations are all isotonic or all antitonic to the order of the left-right predicates.

Algebra however is not the only consideration. A methodological constraint on formal semantics is that its definitions of traditional concepts conform as much as possible to prior usage both in ordinary and technical language. The fact that the two assignments of predicates to semantic values conform to orders that are the converses of one another conforms to the traditional intension/extension distinction. Leibniz and others have observed, for example, that intensions have a definitional structure antitonic to extensions. Animality, it is said, enters into the traditional definition of *man* and is in that sense one of the intensional "parts" of the concept mankind, but the set of men is a subset of that of animals. If scalar adjectives were to be incorporated into a full-blown intensional logic in which expressions were first paired with intensions and then intensions with extensions in an antitonic manner, then values nested in the second way would be appropriate as intensions and those nested in the first as their extensions. In the theory below we shall opt for the account in which values are isotonic to the predicate order. We do so because, as we shall see in Section 4, it is the appropriate abstraction for Proclus' applications.

<u>Scalar Negations: Scalar Opposites</u>. It may be observed in the examples of scalar lists cited above that scalar families come in "positive" and "negative" pairs. Paired with the *happier than* ranking is that of its relational converse

sadder than; paired with that of *hotter than* is that of its converse *colder than.* The pairs *happier-than* and *sadder-than*, and *hotter-than* and *colder-than* share the same fields. If we let $\breve{}$ be the operation that converts a relation into its converse, we see that $(\text{sadder-than})\breve{}$ continues the ranking at the point at which that of happier-than stops, and that $(\text{colder-than})\breve{}$ continues that of hotter-than. That is, scalar comparatives come in pairs R_1 and R_2 that are total orders that share the same fields in such a way that the combination $R_1 \cup (R_2)\breve{}$ is a total order on the field as well. It follows that the two scalar lists may be combined by appending to the list L_1 of predicates (associated with R_1) that formed by reversing the list L_2 (associated with R_2). One would expect then that the longer list so formed would meet the cited text-frame criteria for qualifying as a list of scalar predicates associated with $R_1 \cup (R_2)\breve{}$. This prediction conforms with the linguistic facts. The lists below are respectively those of happier-than $\cup$ (sadder-than)$\breve{}$ and hotter-than $\cup$ (colder-than)$\breve{}$:

ecstatic, happy, content, so-so, down, sad, miserable

boiling, hot, warm, tepid, cool, cold, freezing

It is easy to check that these lists satisfy the test-frame criteria for counting as scalar families relative to *happier-than* and *hotter-than* respectively.

Intuitively there is sometimes "objective" evidence in the form of facts about the comparative relation that allows us to identify which terminus of the ordering is "positive" and which is "negative." The ordering may be of a measurable physical process or some other empirically meaningful measure of quantity in which there is some sort of existential criterion that allows for comparative judgment about one situation having "more stuff" than another. Such would be genuinely privative processes of the sort evidently intended by Aristotle (*e.g.* has-more-teeth, has-more-sight).

Or the ordering may be evaluative in a non-naturalistic or non-measurable way in which one end is "good" or "more valuable" and the other "bad" or "less valuable." (We shall return to the subject of "objective" criteria for "negativity" below in discussing Proclus.) Horn has observed that even in these cases there are criteria for identifying a direction in the list, and that these may be found in properties of the language itself.

Natural language associates with scalar predicates various negative affixes. One standard variety we shall call *(scalar) opposition* and indicate by –. It presupposes that each item in the scalar order falls at a numbered rank, either above, at, or below a "midpoint," and that one end of the ordering has a "positive" pole, and the other a "negative." This scale is then a ranked order with a midpoint and is associated with the combined list in such a way that the midpoint predicate (if there is one) is assigned to the midpoint on the scale and the other predicates are assigned outwardly from that point progressively up and down the order. The function of – is to convert a positive predicate representing the point at the rank at *n* steps above the midpoint into a new predicate synonymous to the predicate assigned to the point at the rank *n* steps below the midpoint. For example, associated with *happy* is its opposite *unhappy* that is (roughly) synonymous with *sad*. If *P* were to stand for the sets of all elements that have a predicate at most to the degree *n* (the first of the notions of extension incorporating scalar order), *–P* would stand for a superset of the extension of *P.*

But in natural languages this operator is not defined for all atomic predicates. For example *unsad* is not grammatical in English. Moreover as a general rule the operator is usually totally undefined for all the lexical predicates on one pole of the ordering. Intuitively this pole is the negative or "bad" half. That is, as a rule, natural languages structure scalar opposition so that a kind of value judgment is built into the grammar. For example English has *immoral* and *impolite*, but there are no acceptable marked opposites of their lexical synonyms *bad* and *rude.* Moreover, double negations of – are ungrammatical as well, *e.g. ununhappy* and *unimpolite* are not acceptable. Accordingly, there is a linguistic test for the directionality of a scalar order. In addition, as in the earlier lists, – is ungrammatical in association with a midpoint predicate. *e.g.* neither *unso-so* nor *untepid* is grammatical. Thus its seems to be a presupposition of the semantics of this sort of negation that the complex predicate –P, for some lexical P, stands in reality for something that is negative in some sense prior to and independent of language. It follows then that one way to tell if something is negative is to see if it is the semantic value of some –P.

> **The Linguistic Criterion for Scalar Directionality.** If scalar predicates form an ordered family within which the operator – is defined (grammatical) for at least some predicates of one of its poles but is undefined (ungrammatical) for all the predicates of the other, then the pole described by the second extreme is negative.

One effect of the grammatical restriction on – then is to indicate the direction of polarity in the scalar order. Unlike classical negations this operator has a semantics that is not metaphysically neutral; it presupposes a difference in "reality" between those properties picked out by atomic predicates and their negations.

The grammatical restriction on double negations of the operator –, however, do not themselves impose limits on the domain of the operation on semantic structure that interprets the operator. The grammatical utility of the restriction against multiple iterations of –, or equivalently the grammatical inutility of allowing multiple iterations, may derive simply from some practical utility, *e.g.* that of communicating the direction of the scalar order. The grammar restricting applications of – to atomic predicates would not prevent the semantic operation – from being defined on all "properties" positive and negative. Indeed the algebraic structure required for the semantics of – suggest that this is the case. The universe of semantic values must be a total ordering with ranks assigned so that one point is distinguished as a midpoint. (A more precise description is given below.) The role of the semantic operation is to pair a point with its "opposite," *viz.* the point that is the same number of steps on the opposite side of the midpoint. This operation is perfectly well defined whether the point is positive or negative. Indeed a scalar opposition operation – defined on an ordering of this type is familiar from many-valued logic. It is the negation of Łukasiewicz' 3-valued logic and of the logics generalized from it. Indeed, the fact that scalar orders have an opposition operation – with a midpoint insures that the other operations of many-valued logic are definable as well. For example, Łukasiewicz' operations for scalar conjunction and disjunction are well defined. Indeed, the logic of scalars with – may be extended to a more complex theory in which predicate conjunction and disjunction are defined. Doing so would require both the setting aside for logical purposes (*i.e.* an abstraction from) any grammatical restriction prohibiting multiple iterations of – and the addition to the grammar of the syntactic operations $\wedge$ and $\vee$. The relevant many-valued metatheory may be sketched briefly.

A (*sentential*) *syntax* is stipulated to be an algebra consisting of a set of expressions which is the closure of a set of atomic expressions under the syntactic operations of conjunction, disjunction, and negation. A *semantic structure* is defined as an algebra of like character to the syntax. An *acceptable valuation* relative to a syntax and semantic structure is then any homomorphism from the former to the latter. The structure becomes a *logical matrix* when it is augmented to include a distinguished subset of the universe called the set of *designated values*. This set is used to define entailment: *X entails P* in matrix M (briefly $X \models_M P$) iff for any acceptable valuation *v*, if *v* assigns every expression in *X* a designated value then it assigns a designated value to *P*.

These ideas may be applied to a syntax made up of scalar predicates and a semantic structure defined on the background ordering. For $\wedge$ and $\vee$ to be defined in the manner originally proposed by Łukasiewicz, it is necessary that the background order meet a strong requirement, one not generally satisfied by scalar orderings, that arithmetic operations + and - be definable for the measurement on the order. Łukasiewicz' uses these operations to define his conditional which he in turn uses to define $\wedge$ and $\vee$. More suitable for scalar logic are Kleene's *strong connectives* which define directly Łukasiewicz' $\wedge$ and $\vee$ using weaker structural assumptions satisfactory for scalar semantics.[42]

Definition. By the (*strong*) 3-*valued Kleene algebra* is meant the structure <{ 0, $\frac{1}{2}$, 1}, max, min, –> such that

1. max and min are respectively the maximum and minimum operations on <{ 0, $\frac{1}{2}$, 1} and
2. – (*scalar opposition*) is a one-place operation such that –1=0, –0=1 and $-\frac{1}{2}=\frac{1}{2}$

By a *Kleene logical matrix* is meant a structure <{ 0, $\frac{1}{2}$, 1}, D_K, max, min, –> such that <{ 0, $\frac{1}{2}$, 1}, max, min, –> is a Kleene algebra and D is a non-empty subset of { 0, $\frac{1}{2}$, 1}.

A Kleene algebra determines the tables for the strong connectives. (These are the tables for conjunction, disjunction, and negation defined by Łukasiewicz in terms of this conditional.)

[42] Łukasiewicz, Jan, "*On 3-Valued Logic*," in *Jan Łukasiewicz, Selected Works*, Ed. I. M. Bochenski, (Amsterdam: North-Holland, 1970). Kleene, S. C., "On a Notation for Ordinal Numbers," *Journal of Symbolic Logic*, 3 (1938), pp. 15-155. The relation of its entailments to classical logic and their syntactic characterizations are well known. See for example Martin, John N., *Elements of Formal Semantics*, (Orlando, FL: Academic Press, 1987).

	$-$	$\wedge$	0	$\frac{1}{2}$	1	$\vee$	0	$\frac{1}{2}$	1
0	1		0	$\frac{1}{2}$	0		0	$\frac{1}{2}$	1
$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
1	0		0	$\frac{1}{2}$	0		1	$\frac{1}{2}$	1

The set of designated values D_K may be {1} or $\{1,\frac{1}{2}\}$ depending on whether entailment is taken to preserve "truth" or "non-falsity." Kleene's three-valued structure is easily generalized to higher values as long as the relevant structural conditions are met. These are essentially those of a scalar order.

Definition. $<\mathrm{U},\leq,-,e>$ is a *(strong) Kleene* structure iff

1. $\leq$ is a total order on U
2. $-$ is an antitonic idempotent unary operation on U
3. $e=-e$

If $<\mathrm{U},\leq>$ is a total order then a subset A of U is said to be *closed upwardly* iff for any $x,y\in A$, if $x\in$ U and $x\leq y$, then $y\in A$. By $|A|$ is meant the cardinality of A.

Lemma. Let $<\mathrm{U},\leq,-,e>$ be a Kleene structure such that $3\leq|\mathrm{U}|$, let D_M be an upwardly closed subset of U such that either all $x\in D_M$ are such that $e<x$ and D_K={1}, or all $x\in D_M$ are such that $e\leq x$ and $D_K=\{1, \frac{1}{2}\}$.

1. There is a onto homomorphism φ from $<\mathrm{U},\max,\min,->$ onto $<\{0,\frac{1}{2},1\},\max,\min,->$ that preserves designation and non-designation ($x\in D_M$ iff $\varphi(x)\in D_K$), and (hence)
2. For any X and P, $X\models_M P$ iff $X\models_K P$

We shall see below that the opposition operation – on a scalar structure is definable within Proclus' causal structure and that this logic is readily extendable into a full "sentential" logic in the manner of Kleene.[43] Note that to this point we have been able to assume merely that the order has a "midpoint" without needing to posit an extra assumption that one extreme is designated "positive" and the other negative."

Scalar Intensifiers and Privatives. Natural language contains two additional kinds of negative affixes for adjectives. Both are "intensifiers," one positive and one negative. The former is known in classical grammar as the *alpha intensivum*. We shall call it *hypernegation* following Proclus and the later Neoplatonists like Dionysius who adopted the technical device of marking it by the prefix *hyper* in place of α, as in *huperagathos* .[44] The second intensifier is called the *alpha privative* in both classical grammar and philosophy, and we shall follow this usage.

Examples of hypernegation in English are

It's not hot, it's boiling.

He's not (merely) active, he's hyperactive.

It's not (merely) a conductor but a superconductor.

As the examples show, English uses not only the negative modifier *not* to mark this negation but also has incorporated the Greek prefix *hyper* as well as *super*, its Latin equivalent. The alpha privative is represented in English by cases like

[43] For a proof theoretic characterization of Kleene's 3-valued logic and discussion see Martin, John N., "A Syntactic Characterization of Kleene's Strong Connectives," *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik*, 21 (1975), pp. 181-184.
[44] See note 21. This use of negation was recognized and introduced into modern linguistics by Jespersen. See p. 326, Jespersen, Otto, *The Philosophy of Grammar* (London: Allen and Unwin, 1924) and the discussion below.

It's not (just) cold, I'ts freezing.

His performance is certainly subpar today; he's not his usual self.

He does not know what's going on; he's (utterly) clueless.

For this negation English uses not only the predicate modifier *not* but also the prefix *sub* (the correlative in Latin to *super*) and the suffix *less*. The broad function of these intensifiers is to convert a scalar predicate to one synonymous to another lower (to its left) in the scalar list.

We shall find that these scalar operators are the most plausible interpretations of Proclus' hyper and privative negations described in the previous section. It is for this reason that we shall refer to the two with the same symbolism, using ~ for scalar hypernegation and ¬ for scalar privative negation. The semantics of the scalar operators is clear. They correspond to operations on the scalar ordering of "properties." Hypernegation pairs a property with its immediate predecessor (one step higher) in the order, and the privative operation with its successor.

Before stating the semantic theory algebraically, it is appropriate to comment more fully on the history of privative negation as a technical concept. Unfortunately, in modern linguistics *privative negation* has a sense that has little to do with its historical usage in philosophy and logic. Linguistics now uses the term to refer to a negative affix that distinguishes the marked from the unmarked item in a pair of adjectives that are contrary and are such that the unmarked item is systematically ambiguous between two senses. In its first sense the unmarked and marked adjectives have extensions that partition a wider set, which may be called the *range of significance* of the pair. The two adjectives, then, are contraries relative to this sense because of this partition. Given that *A* and *not-A* stand for disjoint sets, there is no *x* of which they can both be true. In its second sense the extension of the marked adjective is the same as in the first sense but that of the unmarked adjective is the range of significance itself. In this sense the two are not contraries; rather the fact that *not-A* is true of *x* entails that *A* is true of *x*. An example in English is *woman* understood as the marked form of *man.* Here *man* is systematically ambiguous between the sense in which it names the class of all humans and the sense in which its stands for males, the relative complement of the extension of *woman* within the set of humans.

Among classical and mediaeval writers, on the other hand, *privative negation* is not used to indicate an ambiguous operator. Rather it indicates that there is a distinction in reality, one independent of language, between the properties picked out by a predicate and its negation. There is not complete uniformity on what this difference is supposed to be. Aristotle first introduces the idea into technical logic in the *Categories.* The context in which he does so is that of distinguishing among the various senses in which propositions may be "opposite." The definitions of privative opposition in these passages are not formal in the modern theoretical sense. They are not eliminative abbreviations of longer phrases in an axiomatic theory. Nor are they used as part of a theory in a looser sense. Once Aristotle draws the distinctions among types of opposition the topic is dropped. The context in which the distinctions are made rather suggests that the various senses are part of a conceptual catalogue which

Aristotle is detailing for clarity and its intrinsic interest. The characterization of privation moreover varies from text to text. He always explains it as a species of contrary opposition, but he distinguishes the relevant species of contrariety in two ways. Sometimes he says the non-privative is distinguished by the fact that it indicates a dispositional property (*hexis*, *Categories* 11^b15, *Topics* 109^b18), while the privative is non-dispositional. At other times he says the non-privative holds "naturally" (*pepyke, Metaphysics* 1022^b) and that its privative does not. It is the second of these differences that becomes its distinguishing feature in mediaeval logic.

As we shall see in the discussion of Proclus below, Neoplatonists draw the semantic distinction in terms of a physical or metaphysical process of privative. Privative negation is that which corresponds to a privative process in reality. It is not difficult to see how this idea evolved from Aristotle's original idea. Many of the alpha-privatives in ordinary Greek do indicate genuine privations. Moreover, Aristotle's examples, if not his "definitions," also suggest that a privative process is at work, *e.g. toothless* (*Categories* 12a30) and *blind* (*Categories* 11^b15). Neoplatonists understand causation as a privative process because the effect is like the cause only less so. This Neoplatonic privation is also "natural" because it is the process by which causation unfolds.

That the traditional notion of privative negation is essentially non-syntactic is indicated by the fact that some authors understand the distinction to be independent of language in the sense that the "negation" need not be marked in the syntax at all. Aristotle himself sometimes illustrates "privations" using unmarked terms, like *blind* (*typholos*) and *bald* (*phalakros*). By the time of William of Ockham and other Mediaeval logicians privative adjectives were regularly understood as unmarked and fully lexicalized.[45]

Classifying the privative negation of traditional logic as the scalar operation $\neg$ also helps explain the vacillation among authors on the use of negative markers. The scalar operator $\neg$ does indicate a semantic difference. It is defined relative to a total ordering, one extreme of which is distinguished as "negative." Moreover a physical or metaphysical privation process would satisfy the properties required for a scalar semantic structure. It would determine a total ordering with a direction suitable for $\neg$. Accordingly, the following story can be told. Neoplatonists came to understand Aristotle's privative negation as the same as the negation that was a standard feature in ordinary Greek, namely the α-privative a.k.a. the scalar $\neg$, in which there is a presupposed scalar order indicative of a privation process. They then applied it in their metaphysics to the privation that is a part of causation. The vacillation in Aristotle and later writers between marked and unmarked examples is explained by the linguistic fact that scalar $\neg$ is frequently but not always lexicalized, as *unhappy* is by *sad*. In a curious twist we shall see that Proclus maintains *à la* Aristotle that A and $\neg A$ are "contraries" and hence (at least nominally) meet one of Aristotle's requirements for the distinction, but in doing so Proclus advances a new sense of *contrary*, one in which a contrary and its opposite may both be true together.

[45] For Ockham's usage see *Summa logica* I.36. On mediaeval usage more generally see Henry, D. Paul, *Mediaeval Logic and Metaphysics* (London: Huthchinson, 1972).

We are now in a position to summarize the properties of a scalar syntax and its semantic structure. To facilitate the application later to the logic of Proclus, it will be useful to develop the theory in two stages, the first introducing the operations ~ and ¬, and the second augmenting the language to include the operation – . Without the possibility of confusion we shall use the same expression to name both the operator in the syntax and the operation to which it corresponds in the semantic structure.

Simple and Symmetric Scalar Structures. The syntax for a simple scalar language contains a family of scalar predicates and the operators for hyper and privative negation. Semantically, they are interpreted over a total order organized by two operations appropriate to the intensifiers.

Definitions

1. A *scalar syntax* is any $<\{P_1,\ldots,P_n\},\mathrm{Pred},\sim,\neg>$ in which $\{P_1\ldots,P_n\}$ is a set of expressions, the *atomic monadic predicates*; ~ and ¬ are one-place operators; and Pred is the closure of atomic predicates under ~ and ¬.
2. A *scalar structure* is any $<U,\leq,\sim,\neg>$ such that
    a. $\leq$ is a total order on U
    b. ~ and ¬ are isotonic binary operations on $<U,\leq>$
    c. for any $x\in U$, $\neg x\leq x\leq \sim x$
3. Relative to a scalar syntax $<\{P_1,\ldots,P_n\},\mathrm{Pred},\sim,\neg>$ and structure $<U,\leq,\sim,\neg>$, an *interpretation* is any homomorphism R from $<\mathrm{Pred},\sim,\neg>$ into $<U,\sim,\neg>$ such that

a. for any i and j, if $i \leq j$, then $R(P_j) \leq R(P_i)$

b. for any i, $R(P_i) \in U$,

i. $R(P_i) \leq R(\sim P_i)$

ii. $R(\neg P_i) \leq R(P_i)$

The language is extended by adding an opposition operator $-$. Its semantics presupposes that a "midpoint" for the background order is distinguished.

Definitions

1. A *symmetric scalar syntax* is any $<\{P_n,\ldots,P_0,\ldots,P_{-n}\},\text{Pred},\sim,\neg,->$ in which for some Y, $<\{P_n,\ldots,P_0,\ldots,P_{-n}\},Y,\sim,\neg>$ is a scalar syntax; $-$ is a one-place operator; and Pred is the closure of $\{P_n,\ldots,P_0,\ldots,P_{-n}\}$ under $-,\sim$, and $\neg$.

2. A *symmetric scalar structure* is any $<U, \leq,-,\sim,\neg, e>$ such that

a. $<U, \leq,\sim,\neg>$ is a scalar structure

b. $-$ is an antitonic idempotent binary operation on U

c. $e=-e$

3. Relative to a symmetric scalar syntax $<\{P_n,\ldots,P_0,\ldots,P_{-n}\},\text{Pred},\sim,\neg,->$ and structure $<U, \leq,-,\sim,\neg, e>$, an *interpretation* is any homomorphism R from $<\text{Pred},-,\sim,\neg,P_0>$ into $<U,-,\sim,\neg,e>$ such that

a. R is a scalar interpretation relative to $<\{P_n,\ldots,P_0,\ldots,P_{-n}\},Y,\sim,\neg,->$ and $<U, \leq,\sim,\neg, e>$

b. for any $i \in U$, and $-R(P_i) \leq R(P_i)$,

Theorem. If $<U, \leq,-,\sim,\neg,e>$ is a symmetric scalar structure, then

$<U, \leq, -, e>$ is a Kleene structure.

Accordingly, if in a proof theory the scalar logic appropriate to $\sim$ and $\neg$ is syntactically represented (as shall be done below), the logic could be extended by means of Kleene's methods to a proof theoretic characterization of $-$. If predicate operators with Kleene's logic were also added, their logic could be explained as well.

The Syllogistic. Of special relevance to Proclus is the way scalar adjectives fit the syllogistic. In the subsequent sections of the paper the theory of scalars is applied to Proclus. In preparation scalar theory will now be extended to include the standard syllogistic. As the framework for the syllogistic, we shall employ the natural deduction reconstruction of Timothy Smiley and John Corcoran.[46] Deductions with any finite number of premises are allowed. These will include traditional immediate inferences, syllogisms, and many premised syllogistic arguments traditionally represented by chains of syllogisms. The rules of the natural deduction system are modeled on Aristotle's proto-system of the *Prior Analytics* that "reduces" the valid moods to Barbara and Celarent using various immediate inferences and reduction to the impossible. Since there are a number of equivalent choices for the primitive rule set of the theory, we shall use one that simplifies exposition. The semantics will be non-Boolean in a manner suitable to Proclus. Though it may be surprising to modern readers who are apt to think of a Boolean structure of non-empty sets as the paradigm model for the

[46] Corcoran, John, "Completeness of an Ancient Logic," *Journal of Symbolic Logic*, 37 (1972), pp. 696-702. Smiley, Timothy, "What is a Syllogism?," *Journal of Philosophical Logic*, 2 (1973), pp. 136-154. The metatheory employed here that abstracts away from Boolean algebras of sets

syllogistic, the classical proof theory of the syllogistic actually characterizes a more abstract structure that allows as acceptable models the totally ordered structures of scalars, including those of Proclus' causal ordering.

Syntax. The theory posits a syllogistic syntax in which sentences are made up by attaching one of four sentence operators ***A**, **E**, **I*** and ***O***, to pairs of expressions drawn from a primitive set of terms. A *syllogistic syntax* is any <$\{P_1,\ldots,P_n,\ldots\}$,Sen,***A***,***E***,***I***,***O***> in which $\{P_1,\ldots,P_n,\ldots\}$ (possibly denumerably infinite), called the set of *terms,* is a set of expressions, ***A***,***E***,***I***, and ***O*** are two-place syntactic operators defined on pairs of terms that yield the concatenation of the operator and the ordered terms. The union of their ranges is Sen, the set of *sentences*. We let *x*, *y* and *z* range over terms and follow English syntax in writing the "subject" to the left of the "predicate". Contradictory sentential negations are introduced by eliminative definition: **N*A***xy=***O***xy, **N*E***xy=***I***xy, **N*I***xy =***E***xy, **N*O***xy =***A***xy.

Basic Proof Theory. Any array $X\vdash A$ is called a *deduction* iff X is a finite set of sentences and A is a sentence of the syllogistic syntax.

Definitions. 1. A *basic deduction* is any $X\vdash P$ such that $P\in X$.

2. The set of acceptable deduction rules are:

Conversion1: $\dfrac{X\vdash \boldsymbol{E}xy}{X\vdash \boldsymbol{E}yx}$ Conversion2: $\dfrac{X\vdash \boldsymbol{A}xy}{X\vdash \boldsymbol{I}xy}$ Reductio: $\dfrac{X\vdash A \quad Y\vdash \mathbf{N}A}{X\cup Y-\{B\}\vdash \mathbf{N}B}$

Barbara: $\dfrac{X\vdash \boldsymbol{A}zy \quad Y\vdash \boldsymbol{A}xz}{X,Y\vdash \boldsymbol{A}xy}$ Celarent: $\dfrac{X\vdash \boldsymbol{E}zy \quad Y\vdash \boldsymbol{A}xz}{X,Y\vdash \boldsymbol{E}xy}$

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to lattices is developed in Martin, John N., “Aristotle's Natural Deduction Reconsidered,” *History and Philosophy of Logic*, 18 (1997), pp. 1-15.

3. The set $\vdash_{syl}$ of *provable deductions* is defined as the inductive closure of the basic deductions under the rules. We write $X \vdash_{syl} A$ to mean $X \vdash A$ is provable.

Basic Semantics. The model theory stipulates a structure and set of interpretations defined as mappings from elements of the syntax to those of the structure. Though it is not necessary to assume that the operations are Boolean, it is necessary to assume that there is a least element 0 which determines the "positive/negative" direction of the order and which does not serve as the referent of any term.

Definitions

1. By *a syllogistic structure* is meant any $\langle U,\leq,\wedge,0\rangle$ such that
   a. $\langle U,\leq\rangle$ is a partially ordered structure with least element 0;
   b. $\langle U,\wedge\rangle$ is the meet semi-lattice determined by $\langle U,\leq\rangle$.
2. A *syllogistic interpretation* relative to $\langle U,\leq,\wedge,0\rangle$ is defined as any function R of Syn mapping Terms$\cup$Sen to $U\cup\{T,F\}$ such that:
   a. if $x\in$ Terms, $R(x)\in U$ and $R(x)\neq 0$,
   b. if $A\in$ Sen, then
      i. if A is some ***A****xy*, then $R(A)$=T iff $R(x)\leq R(y)$,
      ii. if A is some ***E****xy*, then $R(A)$=T iff $R(x)\wedge R(y)=0$,
      iii. if A is some ***I****xy*, then $R(A)$=T iff $R(x)\wedge R(y)\neq 0$,
      iv. if A is some ***O****xy*, then $R(A)$=T iff not$(R(x)\leq R(y))$.

3. An argument X to A is *(syllogistically) valid* (briefly $X \models A$) iff for any syllogistic interpretation R of a syllogistic structure for the syllogistic syntax, if for all $B \in X$, R(B)=T, then R(A)=T.

The natural deduction proof theory is characterized by the semantics. Corcoran and Smiley originally showed that a variant of the rules system given above (one interdefinable with it) is sound and complete for a syllogistic structure of non-empty sets. Their result has been abstracted to the semantics given here which, unlike set structures, fits Proclus' causal structure. Soundness and completeness hold for the rules set given here (and for a family of equivalent rule sets) relative to syllogistic structures as just defined.[47]

Theorem (Corcoran and Smiley). $X \vdash_{syl} A$ iff $X \models A$.

<u>Syntax for the Syllogistic with Scalar Predicates.</u> It is a relatively straightforward matter to extend the syntax and restrict the semantics to allow for the predicates in the syllogistic to be scalar. The ***A*** forms, after all, are precisely designed to express the order that underlies a family of monadic scalar predicates. We shall see that they are exploited for just this purpose by Proclus. We pause now to present the extension of the natural deduction theory to include scalar negations.

The syntax is augmented to allow molecular predicates formed by hyper and privative negation. These will serve as grammatically complex syllogistic terms.

Definition. A *scalar syllogistic syntax* is any <$\{P_1,\ldots,P_n\}$,Terms, Sen, ***A***,***E***,***I***,***O***,~,¬> such that

1. $<\{P_1,\ldots,P_n\},\text{Terms},\sim,\neg>$ is a scalar syntax, and

2. <Terms,Sen,***A***,***E***,***I***,***O***> is a syllogistic syntax.

Proof Theory for the Syllogistic with Scalar Predicates. The proof theory is extended by expanding the definition of basic deduction and adding three rules.

Definitions

1. A *basic deduction* is any $X \vdash P$ such that $P \in X$ or $X \vdash P$ is of one of the forms:

   $\varnothing \vdash \mathbf{I}xy$, $\varnothing \vdash \mathbf{A}\neg xx$, or $\varnothing \vdash \mathbf{A}x{\sim}x$

2. The set of *acceptable deduction rules* has as its elements Conversion1, Conversion2, Reductio Barbara, Celarent (defined above) and the following rules:

$$\frac{X \vdash \mathbf{A}xy}{X \vdash \mathbf{A}\neg x \neg y} \qquad \frac{X \vdash \mathbf{A}\neg x \neg y}{X \vdash \mathbf{A}xy} \qquad \frac{X \vdash \mathbf{A}xy}{X \vdash \mathbf{A}{\sim}x{\sim}y} \qquad \frac{X \vdash \mathbf{A}{\sim}x{\sim}y}{X \vdash \mathbf{A}xy} \qquad \frac{X \vdash \mathbf{O}xy}{X \vdash \mathbf{A}yx}$$

Below these shall be called *new deductions* 1-3 and *new rules* 1-5 respectively. Combining the new deductions and rules with old, the set of provable deductions is defined as the inductive closure of the larger set of basic deductions under the larger set of rules. We shall write $X \vdash_{\text{syl+}} A$ to mean that $X \vdash A$ is provable in the new sense.

Model Theory for the Syllogistic with Scalar Predicates. The central semantic notion of an order-theoretic structure for the syllogistic is restricted by adding appropriate operations for the scalar negative affixes.

Definitions

1. A *(simple) scalar syllogistic structure* is a structure $<U,\leq,\wedge,\sim,\neg,0>$ such that

[47] The proof is to be found in Martin(1997). Note that the proof there contains a lacuna. The following text should be inserted in claim 4, p. 23: "*B* is some A_n and ${\sim}B$ is some A_m such that

a. <U, ≤,~,¬> is a scalar structure, and

b. <U,≤,∧,0> is a syllogistic structure.

2. A *(simple) scalar syllogistic interpretation* relative to the scalar syllogistic syntax <{P_1,…,P_n},Terms, Sen, ***A***,***E***,***I***,***O***,~,¬> and scalar syllogistic structure <U,≤,∧,~,¬,0> is any function R such that

   a. R relative to {P_1,…,P_n},Terms,~,¬> and <U, ≤,~,¬> is a scalar interpretation, and

   b. R relative to <Terms,Sen,***A***,***E***,***I***,***O***> and <U,≤,∧,0> is a syllogistic interpretation.

3. An argument *X* to *A* is *valid* (briefly $X \models A$) relative to a scalar syllogistic syntax and structure iff for any scalar syllogistic interpretation R of the scalar syllogistic syntax, if for all $B \in X$, R(B)=T, then R(A)=T.

<u>The Syllogistic with Symmetric Scalar Predicates.</u> The second stage concepts may now be defined which incorporate the symmetric negation –. The procedure will be to add additional details to the concepts just defined.

Definition. A *symmetric scalar syllogistic syntax* is any <{P_n,…,P_0,…,P_{-n}},Terms, Sen, ***A***,***E***,***I***,***O***,~,¬,–> such that

1. <{P_n,…,P_0,…,P_{-n}},Terms,~,¬,–> is a symmetric scalar syntax, and
2. <Terms,Sen,***A***,***E***,***I***,***O***> is a syllogistic syntax.

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without loss of generality it may be assumed that $n<m$."

In the proof theory both the set of basic deductions and the rule set is augmented. By *basic deduction* is meant any basic deduction for the scalar syllogistic plus all instances of:

$\varnothing \vdash \boldsymbol{A}{-}xx$, $\varnothing \vdash \boldsymbol{A}{-}{-}xx$, $\varnothing \vdash \boldsymbol{A}x{-}{-}x$, $\varnothing \vdash \boldsymbol{A}P_0 {-}P_0$, and $\varnothing \vdash \boldsymbol{A} {-}P_0\, P_0$.

The definition of the set rules includes all of those of the scalar syllogistic plus these two new rules:

$$\frac{X \vdash \boldsymbol{A}xy}{X \vdash \boldsymbol{A}{-}y{-}x} \qquad \frac{X \vdash \boldsymbol{A}{-}x{-}y}{X \vdash \boldsymbol{A}yx}$$

In the proof below these shall be called *new deductions* 4-8 and *new rules* 6 and 7. The *set of acceptable deductions* is defined as the augmented set of basic deductions closed under the new set of rules. The semantic structure is a combination of a symmetric scalar and a scalar syllogistic structures. Since Proclus assumes with Aristotle that every term stands for a non-minimal element, combining the two structures must be done in such a way that the privative operation ¬ of the former does not have as a value the 0 point of the latter. To do so we make use of the notion of the restriction of a relation R to a set A, written briefly as R|A. The scalar universe over which terms, negated and otherwise, are interpreted will accordingly be restricted so as not to include 0. Since 0 is not the referent of any term, it is permissible to extend the scalar inverse operation – to the full universe including 0.

Definitions

1. A *symmetric scalar syllogistic structure* is a structure <U,≤,∧–,~,¬,*e*,0> such that

a. $<U-\{0\}, \leq|U-\{0\},\sim,\neg,-|U-\{0\},e>$ is a symmetric scalar structure,

b. $<U,\leq,\wedge,0>$ is a syllogistic structure,

c. -0 is defined and in U.

2. A *symmetric scalar syllogistic interpretation* relative to $<\{P_n,\ldots,P_0,\ldots,P_{-n}\}$,Terms, Sen, ***A***,***E***,***I***,***O***,$\sim,\neg,->$ and $<U,\leq,\wedge-,\sim,\neg,e,0>$ is any function R such that

   a. R is a symmetric scalar interpretation relative to $<\{P_n,\ldots,P_0,\ldots,P_{-n}\}$,Terms,$\sim,\neg,->$ and $<U-\{0\}, \leq|U-\{0\},\sim,\neg,$ $-|U-\{0\},e>$, and

   b. R is a syllogistic interpretation relative to <Terms,Sen,***A***,***E***,***I***,***O***> and $<U,\leq,\wedge,0>$.

3. An argument X to A is *valid* (briefly $X\models A$) relative to a symmetric scalar syllogistic syntax and structure iff for any symmetric scalar syllogistic interpretation R , if for all $B\in X$, R(B)=T, then R(A)=T.

Theorem

1. In a symmetric scalar syllogistic structure $<U,\leq,\wedge-,\sim,\neg,e,0>$, -0 is a unique $\leq$ maximal element 1 in U.

2. **Ineffability**. There is a symmetric scalar syllogistic interpretation R such that there exists a $P\in$Pred such that R(P)=1 or R(P)=0.

3. If $<U,\leq,\wedge-,\sim,\neg,e,0>$ is a symmetric scalar syllogistic structure for a scalar syllogistic syntax, then $<U, \leq,-,\sim,\neg, e>$ is a symmetric scalar structure and $<U, \leq,-,e>$ is a Kleene structure.

Soundness and Completeness. The logic is complete. We may now state and prove the main proof theoretic result for the expanded symmetric syllogistic theory. The proof for simple scalar syllogistic is easily excised from it.

Theorem. $X\vdash_{syl+}A$ iff $X\vDash A$.

Proof. The proof is based on that of $X\vdash_{syl}A$ iff $X\vDash A$ cited above, augmented as follows. Soundness is provable by a simple induction. In the Henkin proof of completeness, the definitions of *consistent*, *maximally consistent* and *saturated sets* remains the same, though these sets will contain new elements as a result of the larger syntax and new deduction rules. The properties of consistent, maximally consistent and saturated sets are needed for the proof to continue to hold. In particular if X is a saturated maximally consistent set and $\boldsymbol{I}xy\in X$, then for some z, $\boldsymbol{A}zx\in X$ and $\boldsymbol{A}zy\in X$. In addition, several new properties hold: if X is maximally consistent, then for any x and y,

1. $\boldsymbol{I}xy\in X$, $\boldsymbol{A}{-}xx\in X$, $\boldsymbol{A}{-}{-}xx$, $\boldsymbol{A}x{-}{-}x$, $\boldsymbol{A}P_0\,{-}P_0$, $\boldsymbol{A}\,{-}P_0\,P_0$, $\boldsymbol{A}\neg xx\in X$, and $\boldsymbol{A}x{\sim}x\in X$
2. $X\vdash\boldsymbol{A}xy$ iff $X\vdash\boldsymbol{A}{-}y{-}x$ iff $X\vdash\boldsymbol{A}\neg x\neg y$ iff $X\vdash\boldsymbol{A}{\sim}x{\sim}y$
3. either $\boldsymbol{A}xy\in X$ or $\boldsymbol{A}yx\in X$
4. $\boldsymbol{A}xy\in X$ iff $\boldsymbol{O}yx\in X$

The construction showing that any consistent set may be extended to a saturated maximally consistent set remains the same. The proof of the lemma showing that any saturated maximally consistent set is the truth-set of some syllogistic interpretation is altered to show it is the truth-set of a symmetric scalar syllogistic interpretation. Let X be a saturated maximally consistent set. We define a function on terms and sentences. R is defined first on terms: If $x \in$ Terms, R(x)={$y\in$ Terms|$\boldsymbol{A}yx\in X$}. A structure <U,≤,∧–,~,¬,e,0> is then defined as follows: U=Range(R)∪{Terms}∪{∅}; ≤ is ⊆|U; ∧ is the greatest lower bound operation determined by < U,≤>; 0 = ∅; e=R(P_0). It is clear that Terms is a ≤-maximal element of U. Operations ~,¬ and – on U are defined as follows,

~R(x) = {$y\in$ Terms | $Ay{\sim}x\in X$} and ~0=Terms

¬R(x) = {$y\in$ Terms | $Ay\neg x\in X$} and ¬0=0

–R(x) = {$y\in$ Terms | $Ay{-}x\in X$} and –0=Terms

We now extend R to sentences:

for $A\in$ Sen, if A is some $\boldsymbol{A}xy$, R(A)=T iff R(x)⊆R(y),

if A is some $\boldsymbol{E}xy$, R(A)=T iff R(x)∩R(y)=∅,

if A is some $\boldsymbol{I}xy$, R(A)=T iff R(x)∩R(y)≠∅,

if A is some $\boldsymbol{O}xy$, R(A)=T iff R(x)⊄R(y).

If <U,≤,∧–,~,¬,e,0> is a symmetric scalar syllogistic structure, it is clear from the definition of R that it is a syllogistic interpretation relative to it. It remains to show that<U,≤,∧–,~,¬,e,0> is a symmetric scalar syllogistic structure. The steps in the original proof are sufficient for showing that <U,≤,∧,0> is a syllogistic structure. That R(x)≠0 follows from the fact that $\neg x\neq 0$ and that the new basic deductions of type 1 insure that for any x and y, $\boldsymbol{I}xy\in X$, and therefore since X is saturated, that for some z, $\boldsymbol{A}zx\in X$ and hence R(x)≠∅; that ≤ is total follows by new rule 5, that ~ is isotonic such that $x\leq{\sim}x$ follows by the definition of ~, the new basic deductions of type 3, and new rules 3 and 4; that ¬ is isotonic and is such that $\neg x\leq x$ follows from the definition of $\neg$, the new basic deductions of type 2, and the new rules 1 and 2; that $-$ is idempotent and antitonic such that $-x \leq x$ follows from its definition, the fact that $\leq$ is total, the new basic deductions 5-6, and the new rules 7 and 8; that $-e=e$ is insured by the fact that $\leq$ is total and the new basic deductions 7 and 8. ■

The proof is adapted to scalar syllogistic syntax and model structure by deleting the steps concerning $-$.

4. Application of the Theory to the Principles of Proclus

The formal results apply to the interpretation of Proclus. In this section we shall argue that the texts support a reconstruction in which the following are true:

1. Proclus employs a logic theory that is a symmetric scalar syllogistic.
2. The structure Proclus attributes to reality is that of a *scalar syllogistic structure* each taxon of which is a *symmetric scalar syllogistic structure*.
3. The language of Proclus' metaphysics is that of the scalar syllogistic.

The argument justifying the interpretation consists, in part, of showing that it validates the various individual algebraic and logical principles abstracted earlier in the paper from Proclus' texts. The key to doing so is Proclus' special use of the syllogistic.

Let us assume that the causal line is determined by a reality independent of language.[48] The logical theory just presented explains how both syllogistic and scalar reasoning may be applied to it.

Intensional and Extensional Logic. Let us begin by remarking on the intensional/extensional distinction. Though there is some reason to apply it to Proclus, doing so risks a serious anachronism. From the perspective of the twentieth century one of Proclus' more interesting semantic claims appears to be that, contrary to the usual view, extensions do not reverse the order of intensions. One of Proclus' structural axioms is that the orders at each level of the hierarchy are isotonic to one another. Thus the order at the level of Being, which includes ideas and forms, is isotonic to that of Body, which includes the bodily manifestations of ideas and forms. Causal order at the level of ideas, moreover, appears to be what would later be called conceptual inclusion. The form of man is more perfect than that of animal and the former causes the latter. In Proclus' theory this fact can be recast in terms of a definition. The idea associated with the term *animal* enters into the definition of the term *man.* Moreover, though Proclus often calls forms genera and species, he also uses genera and species for bodily manifestations of ideas. In doing so the order of ideas is repeated at the level of bodies. The bodily man is more perfect than a bodily animal. Since bodies correlate to the language that describes ideas, they function something like Frege's extensions. Indeed these bodily forms appear to

[48] This linear structure is detailed in Siorvanes, Lucan, *Proclus: Neo-Platonic Philosophy of Science* (New Haven: Yale University Press, 1996). The algebraic interpretation of Proclus' ontology and the logic based upon it that is developed in the present study accords well with

be the only candidates in Proclus’ theory for anything performing the interpretive role that later semantic theory calls "extensions." But if bodily manifestations of ideas are "extensions," then Proclus rejects Leibniz' view that extensions reverse intensions. Given Proclus' clear espousal of Aristotle's theory of definition in explaining the order appropriate for the level of forms (Being) and the centrality of that doctrine in the explication of the notion of conceptual inclusion by figures like Leibniz, there is some ground for saying that Proclus' "logic" is intensional, and that entities at the level of Being serve the role of "intensions." But any such claim must be severely qualified by highlighting differences in terminology and explanatory purpose, and especially in the unorthodox way in which the ordering relations of predicates, ideas and bodies directly mirror one another. Unlike the traditional notion of extension they do not conform to the structural reversal of intensions attributed to extensions by Leibniz. Proclus therefore has his own non-standard views on the structure of explanatory entities that serve in the roles of what later came to be called intensions and extensions.

Necessary Existence and Existential Presupposition. That Proclus' causal structure is a line has some interesting consequences for the syllogistic. The most striking of these is that particular affirmatives turn out to be trivially true, and universal negatives trivially false.

Theorem. In a scalar syllogistic language, $\varnothing \models \boldsymbol{I}xy$ and $\varnothing \models \boldsymbol{NE}xy$.

The result is a consequence of the fact that the semantic order is complete. Hence, for every pair of terms in any interpretation, the semantic

Siorvanes' more traditional philosophical account. He, for example, cites evidence of the scalar nature of the ordering though without describing it in linguistic terms (pp. 73-82).

correlate of one is beneath that of the other. In practice this limits the non-triviality of syllogistic reasoning to arguments that turn on universal affirmatives and particular negatives. Indeed ***A****xy* may be written as $x\leq,y$ and ***O****xy* as $x\nleq y$ or its equivalent $y<x$. The inutility of ***E*** and ***I*** propositions is compensated for by the availability of the various scalar negations. It is in fact a feature of Proclus' use of the syllogism that he restricts his applications to cases composed only of ***A*** and ***O*** sentences.

Conversion *per accidens*. A good example of Proclus' use of the novel negations is in his treatment of conversion. When the negations in the passage below are read classically, Proclus seems to be rejecting syllogistically valid ***E*** to ***O*** *per accidens* conversion. He seems to reject the inference from *No thing is the One* to *the One is no thing*. (Read the text below *without* the glosses). But Proclus cannot reject conversion and also accept the syllogistic, which he clearly does elsewhere. The correct reading of his practice is that he rejects an inference, but the inference in question is not classical conversion. Rather it is a scalar invalidity that argues from $x \nleq y$ to $x \leq \neg y$. The context of the discussion in the text is one in which privative negation is being used to say of non-existence that it is the privative opposite of Being. The intended reading is indicated by the bracketed glosses:

> The One is transcendent over all being [*i.e.* $Being \leq One$], but is not transcendent because it is other [*i.e.* not because $One \leq Other$], lest in being other [*i.e.* lest given $One \leq Other$] it should no longer be transcendent over all being [*i.e.* it would then be that $Being \nleq One$, which

would follow given that *One* $\leq$ *Other* and *Other* < *Being*]; for one must not transpose the negation according to which it [*the One*] is no one of all things [*i.e.* one must not transpose the truth *One* $\nleq$ *Being*] into an assertion arising from negations [into *One* $\leq \neg$*Being*, because the general rule if $x \nleq y$ then $x \leq \neg y$ is invalid].[49]

<u>Denying the Antecedent</u>. There is another good example of how Proclus uses the scalar syllogistic. At various points he remarks that the fact that *A* is causally necessary for *B* entails the fact that there could be no *B* without *A* . In commenting on Plato's reasoning in the *Parmenides*, for example, he observes:[50]

> ….the first mentioned [forms] are more general, while these latter mentioned are more particular. For this reason by eliminating the earlier ones, he eliminates those that follow them in the hypotheses.

In the same text he also remarks that negating the earlier "demolished" the later. In another passage he states the situation more generally:[51]

> If, then, the negations generate the affirmations, it is plain that the first negations generate the first and the second the second.

From a classical perspective, it looks as if Proclus is defending some variety of the fallacy "denying the antecedent:" $P \rightarrow Q$, *not-P* $\models$ *not-Q.*

What he is really saying, however, may be formulated in terms of the causal ordering and the existence or membership in the universe of entities ordered by $\leq$. On this reading Proclus is stating the counter-factual claim that

[49] *IP* 1185:2-8, M&D 534. Note though that the inference from $x \nleq y$ to $x \leq \sim y$ is valid. Proof: $x \nleq y$ entails $y < x$, and since $y \leq \sim y$, it follows that $x \leq \sim y$.
[50] *IP* 1087:2-6, M&D 435.

since $A \leq B$, if A *were not* to exist, then B *would not* exist. That is, the remark may be construed as a remark about a general property of the acceptable structures that accurately represent actual causation. He is saying, on this reading, that there is no such structure that contains B and that does not contain A as one of its predecessors. The point may be put counter-factually: any structure that was like that of the actual world except that it did not contain A would also not contain B or any other causal effect of A. This reading leaves unaddressed, however, whether it is possible to express this fact in the object language using one of its negative operators to represent an appropriate negation operation.

Strictly speaking, in the scalar syllogistic every term has existential presupposition and accordingly non-existence is not expressible . (Though terms stand for points "approaching" 1 or 0, no term actually refers to 1 or 0 themselves.) Indeed, it is a theorem of Proclus' metaphysics that everything exists necessarily, and hence that a name always refers if it ever does. Nevertheless, the language does posses the means for denying that something is a causal consequent of A. Indeed, within the object language there is a perfectly valid argument formulated in terms of ***A*** and ***O*** statements that captures Proclus' intent in the passages above: $A \leq B, A \nleq C \models B \nleq C$. This inference captures what he is saying, *viz.* any object C that is deprived of the being derived from A is also deprived of that derived from B.

[51] *IP* 1099:32-35.

Sentential Logic. Texts were cited earlier in which Proclus appears to be using standard sentential rules like double negation, *modus ponens*, *modus tollens*, contraposition, *reductio*, excluded middle, and non-contradiction. It is possible to explain all these cases within the scalar syllogistic. Double negation holds for all the new negations, and contraposition holds for $-$, satisfying the earlier criterion that $\leq$ and $-$ exhibit the minimal Boolean properties sketched. The properties of $-$ will be explored in more detail in the next section As we have just seen, however, $\sim$ and $\neg$ are isotonic and do not support contraposition. But this too is supported by the earlier discussion of the more Platonic texts in which it is exactly such properties that were required by Proclus' descriptions of hyper and privative negation.

The sentential rules *modus ponens* and *modus tollens* on the other hand are captured by Proclus through syllogisms. In a manner typical of pre-modern logic Proclus treats cases of *modus ponens*, which the modern logician would symbolize $\forall x(Fx \rightarrow Gx), Fc \models Gc$, as cases of Barbara. He does so because he understands singular terms to function semantically as syllogistic terms, as names for points in the ontic order. The point may be made graphically by rewriting the constant in the modern formulation as $\{c\}$ and understanding it "intensionally" so that $\{c\}$, F and G all stand for points in the causal order. *Modus ponens* then becomes $F \leq G, \{c\} \leq F \models \{c\} \leq G$. Likewise *modus tollens* becomes instances of Baroco: $F \leq G, \{c\} \nleq G \models \{c\} \nleq F$.

The form of reduction to the absurd that Proclus uses is most often combined with an argument from cases. If $\perp$ represents an impossible proposition, it may be stated:

$$\frac{X \vdash (P \vee \textit{not-}P) \qquad Y, P \vdash \perp}{X, Y \vdash \textit{not-}P}$$

In Proclus however the negation is question is scalar. The dilemmas he sets up are generally ranked alternatives that are "contrary" in the peculiar sense remarked on earlier in which P and $\sim P$ are called "contrary" yet $P \leq \sim P$. Something which is *hyper-P* contains all the reality and more than does its effect *P;* whatever *P* is, it is so because it falls under $\sim P$. It is in this way that the dilemma setting out possible cases at the start of a *reductio* is to be understood. The possible cases are contraries in Proclus' special sense. In addition, if the dilemma is stated in terms of negation, that negation is scalar. The relevant form of *reductio* remains valid: if $X \vDash P \vee \sim P$ and $Y, P \vDash \perp$, then $X, Y \vDash \sim P$.[52]

A good example is Proclus' use of negation to lay out the hypotheses that (on Proclus' account) are the topics in the Parmenides. The cases are subsumed under one another in a complex tree. First are the cases in which the subject exists and those in which it does not exist. Among the former there are those that are true, those that are not true, and those that are both, and there are yet further subdivisions stated using negations. Viewed classically it might appear that this is an elaborate partition into mutually exclusive alternatives.

[52] A full statement of the theory would require extending the scalar syllogistic to include predicate "connectives" $\wedge$ and $\vee$. Doing so is straightforward given that the scalar structure determines a Kleene algebra and that it is well understood how to interpret sentential logic over such structures. That is, $\vee$ is a maximum operation such that $x \vee y$ is the least upper bound of $\{x,y\}$,

Applying the scalar reading to *not* however suggests that Proclus is sketching a series of ranked hypotheses, as indeed they are later described in his subsequent discussion of the argument in the *Parmenides*. Indeed it is just such a transformation of a tree articulated in terms of scalar negations into a line that forms the basis for transforming the tree of diairesis into the line of causation, as will be explained in Section 5.

The scalar interpretation also affects the understanding of the sort of "absurdity" arrived at by *reductio*. These are scalar falsities or even impossibilities, rather than classical contradictions of the form $P \wedge \textit{not-}P$. Syntactic “contradictions” formed with scalar negations, like $P \wedge {\sim}P$, are not impossibilities. Humans are both humans and animals, but animals are ~human. Hence humans are human and ~human. But Proclus appeals to different absurdities. In the intended interpretation it is clear that a predicate and its hypernegation (or its privative) would never be assigned to the same node. Hence ${\sim}Motion \leq Motion$ would be false. The stronger ${\sim}Motion < Motion$ (*i.e.* the ***O*** statement $Motion \nleq {\sim}Motion$) is invalid (false in all interpretations in any scalar structure). Proclus' *reductio* arguments frequently conclude with just these absurdities, *e.g. the same (i.e. hyper-different) is different*[53] and *rest (hyper-movement) is movement*[54].

<u>The Direction of Ontic Order and its Supremum.</u> The source in the logical theory of the directionality in scalar order may be pinpointed. Curiously, although

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and $\wedge$ is a maximum operation, $x \wedge y$ being the greatest lower bound of $\{x,y\}$. Clearly if $z \leq \mathrm{glb}\{x,y\}$ and $y \leq 0$ then $z \leq x$, and in particular if $y \leq \mathrm{glb}\{x,{\sim}x\}$ and $x \leq 0$, then $y \leq {\sim}x$.

[53] *PT* II:1, S&W 6:3-7:7

scalars have an intuitive direction, which is marked in the ungrammaticality in natural languages of the opposition operator – when applied to predicates that stand for points in the "negative" portion of the order, the semantic theory necessary for the scalar logic of ~, ¬ and – does not need to posit a "distinguished" direction. The need for directional non-symmetry comes rather with the introduction to scalar reasoning of the syllogistic. It derives specifically from the assumption that ***A****xy* statements have semantic direction asserting that the referent of *x* is "lower than" that of *y.* A curious metaphysical consequence follows. The doctrine that there is an "evaluative order" or "directionality" in reality that is typical of Neoplatonic metaphysics, a doctrine that is frequently rejected as a conceptual confusion by logicians in the Russellian tradition, is mandated simply by the application of syllogistic reasoning to scalar predicates. The evaluative order to the scalars derives from the fact that they express true ***A*** statements. Of course, the paradoxes of set theory to one side, it is perfectly reasonable for a Russellian using Boolean algebra to posit a "universal set" in applying the syllogistic to traditional (non-scalar) terms. The linear structure that is necessary for scalar reasoning in its own way carries metaphysical implications. In the semantics of a scalar language there is some supremum to the scalar order, and the metaphysician may ask, what is its nature? If the scale is associated with *is-caused-by* or *is-less-real-than*, if these are legitimate scalar comparatives, then the question becomes, what is the nature of the supremum of the causal order and of existence? It is interesting that the legitimacy of this

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[54] *PT* II; S&W 7:7-8:4.

question follows from the purely logical effort to state the syntax and semantics of scalar predicates within the syllogistic. The existence of such a supremum and the legitimacy of inquiry into it are necessitated by the logical theory of Neoplatonism, without the need for any help from mysticism.

5. Proclus' Transformation of the Tree of Diairesis into the Causal Hierarchy

Neoplatonic Metaphysical Predicates as Scalar. Neoplatonic modality is scalar in the linguist's sense.[55] Plotinus and his followers shared a group of metaphysical theses expressed using scalar ideas. The entities that exist are ranked in an ordering referred to by a variety of comparative adjectives, *e.g*. "more real" (*ousiōteron*), "more general" (*katholikōteron, holikōteron, and merikō-teron*), "more causal" (*aitiōteron*), "more perfect" (*teliōtikon*), "holier" (*timiōteron*), "more powerful" (*dunatōteron)*, and "more infinite" (*apeirōteron*). The following passage from the *Elements of Theology* is typical:

> …the higher cause (*aitiōteron*), being the more efficacious (*drastikōteron*), operates sooner upon the participant (for where the same thing is affected by two causes it is affected first by the more powerful (*dunatōteron*); and in the activity of the secondary the higher is co-operative, because all the effects of the secondary are concomitantly generated by the more determinative cause (*aitiōteron*).
>
> ….

All those characters which in the originative causes have higher (*huperteran*) and more universal (*holikōteron*) rank become in the resultant beings, through the irradiations which proceed from them, a kind of substratum for the gifts of the more specific principles (*merikōteron*).[56]

We have already seen that Proclus uses the syllogistic to reason about entities ranked in the tree of diairesis, and that he uses true affirmations at lower levels of the hierarchy to fashion negations true of higher levels (hypernegation) and lower (privative negation). These orders appear to be the same as that described by the comparatives. Indeed, the texts that use comparatives, and that discuss the syllogistic and negation are the same.[57]

The evidence is therefore overwhelming that Proclus describes his key ontological ideas using scalar vocabulary. There remains, however, a difficulty for the scalar interpretation. How can the scalar order described by the syllogistic which is supposed to form the tree structure of diairesis, also be the linear structure that seems to be presupposed by causation? How can hyper and privative negation that move up and down this causal scale also be negations on the nodes of a tree? How can the same comparatives be used to describe both the hierarchy of the tree and that of causation? Proclus' answer to these

[55] The historical account here is necessarily much simplified. For additional discussion of both historical and logical issues see Martin (1995).

[56] *ET* 66:22-68:2. Such usage of comparatives is frequent. The contexts moreover make it clear that they are meant to refer to the same underlying order. For examples see *ET* 46:19; 58:12; 74:10; 84:14-26; 142:7. In *IP* see 796:14-797:3, M&D 165-166.; 735: 25-29, M&D 110; 892:31-894:34, M&D 253-255; 838:7-14, M&D 211; 1098:3-28, M&D 444-445.

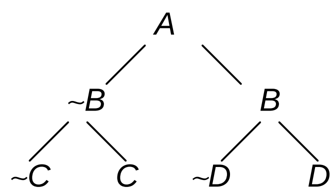
[57] For a text in which comparatives are mentioned in the discussion of the syllogistic and apodictic predications see *IP* 798, M&D 165-166. For their use with hypernegation note that the very distinction between the sense of hyper and privative negation is drawn by referring one to a higher level and the other to a lower. See *PT* II:5, S&W 38 18-25. For a good example of the use of comparatives with negation see *IP* 1098-1110, M&D 444-446.

questions lies in his method of transforming the order of diairesis into the linear progression that ranks all reality. The key to the transformation is negation.

Diairesis and Negation Describe the Ontic Order. The *Elements of Theology* is early and brief. The commentary *In parmenidem* concerns primarily the henads and their relation to the One. It is only in the *Platonic Theology* that Proclus provides an extended discussion of the mutual relations that hold among the full range of taxa that make up the ontic hierarchy. He discusses these both in terms of trees and in terms of linear order. Moreover, he posits the interesting thesis that there is a relation between the two types of structure that allows the former to be transformed into the latter. Indeed in an extension of the method of Plato, he uses diairesis to "discover" causal order. The key idea used in the transformation is negation. The simple case which he generalizes to more complex situations is that of a triad formed from a monad and two subordinate "points," positive and negative. Moreover these points are ordered. The monad comes first. If the negation in question is hypernegation, then the negated element is prior to the un-negated. If the negation is privative, then the non-negated is prior to the negated.[58] The idea may be generalized beyond three, and Proclus often does so. The characterization is in effect inductive: a point *A* is higher than a point *B* if *A* is either the monad of the taxon containing both points, or *A* is the hypernegation of *B*, or *A* is the hypernegation of some point *C* higher than *B*. The order may be generalized equally well using privative negation. Proclus employs the method throughout the *Platonic Theology*. The various books of the work are devoted roughly to key hypotheses or taxa in the ontological hierarchy. He details the linear causal structure of each by reconstructing it from a tree structure provided by diairesis. He characterizes each descent in the tree as a node followed by a set of immediate descendants. He uses negation to describe the members of this set of immediate descendants, and does so in such a way that it is clear that the members of the set are themselves ordered. The linear causal structure of each of the major hypotheses is thereby recovered. Moreover, within a given taxon the process may be repeated for any given point in the taxon. It too is a monad of yet a deeper taxon in the tree.

The method works this way. Suppose we apply diairesis to *A* and learn that it is broken down into ~*B* and *B*. Thus *A* is a node in the analytic tree having points ~*B* and *B* as its immediate descendants. Then this triad of points forms a linearly and causally ordered taxon <*A*,~*B*,*B*> with *A* as its monad. The process may be applied in turn to the subordinate elements in the taxon. That is, diairesis may be applied individually to ~*B* and to *B*. Let us apply it to ~*B*. Suppose we learn that it breaks down into ~C and C. Hence ~B heads the subtaxon <~*B*,~*C*,*C*>. Suppose likewise that *B* heads its own subtaxon <*B*,~*D*,*D*>. The resulting tree structure is:

[58] The triad is reminiscent of Hegel's thesis, antithesis, synthesis. Hegel knew Proclus' work and is often said to have been influenced by him. Proclus' negations are clearer in that they are species of scalar negation and conform to a group of structural axioms.



Moreover the negations order the various taxa. Hence after two stages of analysis we may detail the causal order as a line by nesting taxa within taxa: <*A*,~*B*,~*C*,*C*,*B*,~*D*,*D*>. The process may then be applied to ~*C*,*C*,~*D*, and *D* etc.

In terms of the algebra alone the process invites may questions. Among the more obvious is how long are the tree's branches. Does the analytic division ever stop? Another way to put the question is in terms of a “relative density:” Is the total causal order dense in the sense that between any two immediate descendants *B* and *C* of *A,* there is a non-empty taxon headed by *B* in which all the points of the taxon are higher than *C*? Though Proclus does not address the question squarely, what he says and the way he employs the method in practice suggests his answer would be yes. One of the puzzles this additional structure resolves is how taxa at the same level of analysis can be finite and 1 to 1, yet the full causal hierarchy be infinite. In one place, for example, he describes the initial breakdown of the hypotheses into five: the henads, intellect, soul, cosmic souls, and bodies. But as earlier citations indicate, he often describes these taxa as finite. In addition he maintains that the elements in taxa, which we have just seen are supposed to be finite, stand in one to one correspondences to each other. Moreover, the universe as a whole is infinite. Density relative to a taxon makes these properties jointly possible.

The textual evidence for the conversion of diairesis into total causal order by scalar negations is to be found scattered throughout the *Platonic Theology.* In Book I Proclus summarizes how he will apply it respectively to each of the major hypotheses to exhibit their internal structure, and he then proceeds in the later books to do just that. In Book II he discusses the One, in Book III the Henads (gods) and the Intelligibles (*ta noēta*), in Book IV the Intelligible-Intellectives *(toi noētoi kai noēroi*), in Book V the Intellective (*toi noēroi*). Later books in which he was to have discussed the lower stages (hyper-cosmic, cosmic, higher bodies, bodies, etc.) were never written or are lost. In each of the extant books he describes the tree structure in the manner of Platonic diairesis, but he does so in a manner in which he makes very clear how the immediate descendants of a node are to be linearly ordered among themselves beneath their predecessor node, and that these nodes are to be viewed as higher than the successors of nodes lower than that their predecessor at its rank. He makes the order clear sometimes by just saying what comes first. At other times he uses negations. He more often uses hypernegation than privative, but he uses both.

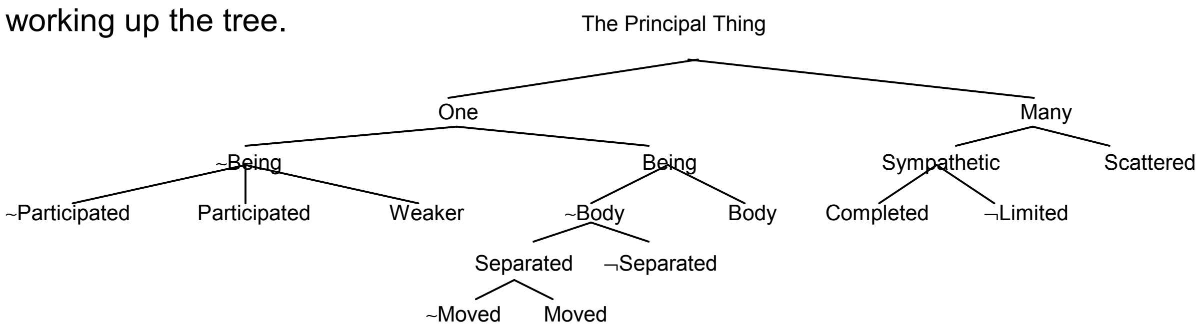
There is a succinct passage in which he summarizes the process at the level of the One. A literal translation will best indicate the use of negation:[59]

> It is necessary that the principal thing [*archēn*] be either one or many, for it is rather from here one must begin. And if many, then sympathetic to one another or scattered from one another, and either completed or limitless [*aperious*, α-privative]. If one, it will be either a not being (*mē*, intensifier)

[59] *PT* II:2; S&W p. 15, author's translation. See note S&W p. 84.

or a being; and if a being, either that in a body or a non-body [*asōmaton*, α-*intensivum*, hypernegation], and if a non-body, then either separated from bodies or non-separated [*axōpiston*, α-privative], and if separated, either moved or unmoved [*akinēton,* α-*intensivum*, hypernegation]; and if not a being [*mē ousian*, intensifier, hypernegation], either weaker with respect to all being or participated beneath being, or unparticipated [*amethekton*, α-*intensivum*, hypernegation].

This structure is displayed below, using ~ for hyper and ¬ for privative negation. The passage progressively eliminates alternatives into which the One may fall by working up the tree.



By moving from any point Proclus indicates ("discovers") what the One is not. It is not the starting node. It is not the next least node, etc. At the end he is at the top. In each case the sense in which the One is *not* is that of hypernegation. The causal order is thus construed as a ranking that subordinates an effect to its cause by hypernegation. At each stage of the ascent up the tree another hypernegation is added to the characterization of the One. Let M be the lowest stage (¬Limited in this tree).

Stage 1. The One is not M, i.e. the One is ~M.

….

Stage *n*+1. The One is not ~….~M (with *n* ~'s), i.e. the One is ~~….~M.

….

It is in this "negative" manner that Proclus characterizes the One, as indicated in the following text:

> ….dans les êtres engendrés après lui [l'Un], à tous les degrés, la cause est totalement différente des effets; et c'est pourquoi la nature est incorporelle [*phusis, asōmatos*], tout en étant cause des corps, l'âme totalement éternelle [*psuchē, aidios*], mais cause de ce qui est engendré, et l'intellect immobile [*nous, akineetos*], parce qu'il est cause de tout ce qui est en mouvement. Si donc, pour chaque procession des êtres, on nie des causes les effets qui en sortent, il faut, je pense, nier de ce qui est cause de tout, tout indifféremment.
>
> ….
>
> je définis au sujet du mode des négations qu'elle ne sont pas privatives de ce sur quoi elles portent, mais productive de ce qui est une sorte de contraire [*antikeimenos*]; car du fait que le premier principe n'est pas multiple, le multiple procède de lui.[60]

Thus, the true predications of the One (though in principle inexpressible) would result from the application of hypernegation ~ to those of Intellect, just as those of Intellect like ~*mobile* are arrived at by applying ~ to those like *mobile* which are true of Soul which moves, those of Soul like *eternal* (the lexicalized form of ~*temporal*) are arrived at by applying ~ those true of nature which is temporal,

[60] *PT* II:10, S&W 62:5-63:17. Note that here and elsewhere (especially *IP,* Book VI) Proclus is careful to make clear that strictly speaking the One is beyond all predication. All references to predications of the One in this reconstruction, even to hypernegations, must be understood as subject to this important Neoplatonic provision.

and those of nature like ~*corporeal* to those of Bodies which possess corporeality.

Indeed, as Saffrey and Westerink have pointed out[61], it is precisely such a linear order of negations that Proclus applies to the One and uses to determine the structure of topics covered in the *Platonic Theology*. Proclus discusses the hypotheses in the order in which they emanate from the One. This order, moreover, is the same as that in which Plato in the *Parmenides* (as that work is explained by Proclus in his commentary) discusses the negative predicates applied to the One. Indeed, on Proclus' view there is a systematic relationship between the order of Plato's negative predications and the ontic hierarchy. As Proclus puts it, *assertion generates negation*. Viewed structurally, the hypernegation of any point yields its immediate superior in the order. Viewed semantically the hypernegation of a predicate true of a point is true of the negation of the point. The accumulation of negations yields a negative characterization of its maximal element. The twelve hypotheses discussed in the *Platonic Theology* with their negative characterizations from the *On the Parmenides* are:

The One = ~ (The One in Being)
The One in Being = ~ (The Intelligible Whole)
The Intelligible Whole = ~ (The Intelligible Many)
The Intelligible Many = ~ (The Intelligible Number)
The Intelligible Number = ~ (The Composite)
The Composite = ~ Shape
Shape = ~ (The In-Itself and In-Others)
The In Itself and In Others = ~ (The In Motion and Rest)
The In Motion and Rest = ~ (The Same and Different)
The Same and Different = ~ (The Like and Unlike)
The Like and Unlike = ~ (The Touching and Not Touching)

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[61] See pp. lx-lxxv in H. D. Saffery, L.G. Westerink, "*Introduction*," Ed. and Tran. , *Proclus: Théologie Platonicienne*, Vol. I, (Paris: Les Belles Lettres, 1968), pp. ix-lxxxix.

The Touching and Not Touching

The same information may be presented as a tree.[62] The following is a trivial corollary that displays the role of hypernegation in "remotion" to the One.

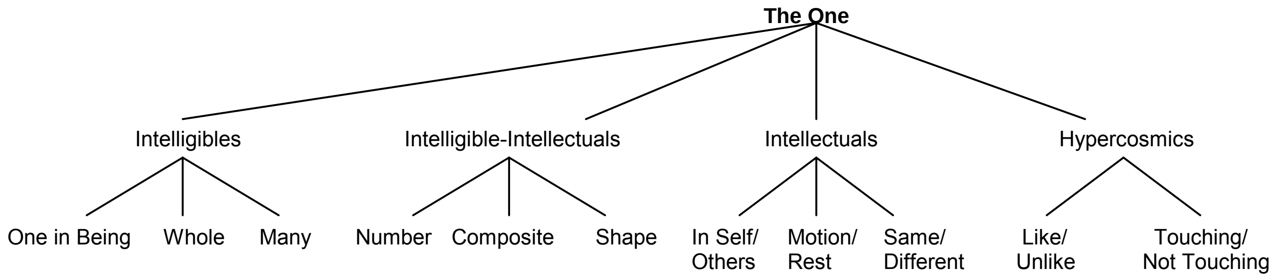
The One = ~ (The One in Being)
= ~~ (The Intelligible Whole)
= ~~~ (The Intelligible Many)
= ~~~~ (The Intelligible Number)
= ~~~~~ (The Composite)
= ~~~~~~ Shape
= ~~~~~~~ (The in Itself and Others
= ~~~~~~~~ (The In Motions and Rest)
= ~~~~~~~~~ (The Same and Different)
= ~~~~~~~~~~ (The Like and Unlike)
= ~~~~~~~~~~~ (The Touching and Not Touching)

This series, however, is just a first approximation of the reflection in negative predicates of ontic structure.

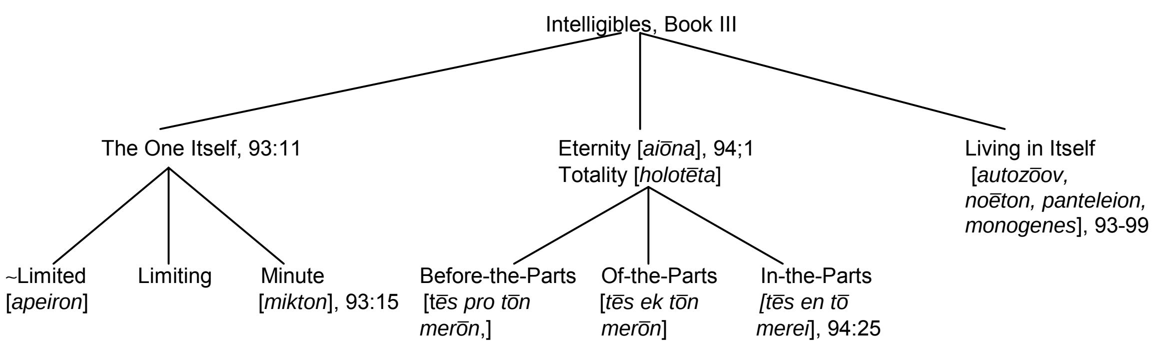
It is instructive to see the analysis in some of its detail. In his detailed analysis (from which Saffrey and Westerink abstract to give the list above) each group of three in the list in fact forms an ordered triad subordinate to a monad, and these monads form a higher level (taxon) of hypotheses subordinated to the One: Intelligibles, Intelligible-Intellectuals, Intellectuals, Hypercosmics, and several lower levels. Each of these nodes has ranged beneath it a taxon, and each of the elements of these taxa in turn is the monad of a taxon ranged

[62] The root of the tree is The One [*to hen on*] = ~ (The One in Being). The taxon of which The One is the monad contains its eleven immediate descendants in a diaretic tree, which given in their causal order (as indicated by their hypernegations) are: 1. The One in Being = ~ (The Intelligible Whole), III,24; S&W 84:27, 85:27; 2. The Intelligible Whole [*hē noētē holotēs*] = ~ (The Intelligible Many), III,25; S&W 87:9, 88:25; 3. The Intelligible Many [*to noēton plēthos*] = ~ (The Intelligible Number), III,26; S&W 92,9; 4. The Intelligible Number [*ho noētos arithmos*] = ~ (The Composite), IV,29; S&W 92:2; 5. The Composite [*hē suvektikē*] = ~ Shape, IV,35; S&W 103:7; 6. Shape [*to skēma*] = ~ (The In Itself and Others), IV,37; S&W 108:20, 109:18; 7. The In Itself and Others [*to en autō kai en allō*] = ~ (The In Motion and Rest), V,37; S&W 134:11; 8. The In Motion and Rest [*to en kineiothai kai estanai*] = ~ (The Same and Different), V,38; S&W 142; 9. The Same and Different [*to tauton kai heteron*] = ~ (The Like and Unlike), V,28; S&W 144:22-145:6; 10. The Like and Unlike [*to omoion kai anomoion*] = ~ (The Touching & Not Touching), VI,14; S&W 70:13; 11. The Touching & Not Touching [*to en aptomenon kai mē aptomenon*], VI,24; S&W 110:3.

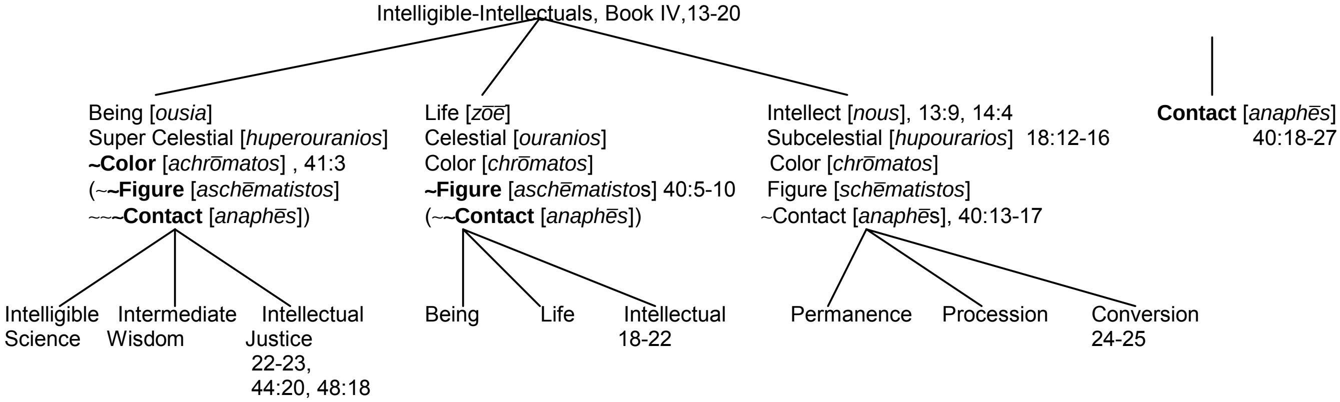
beneath it. The tree of diairesis is the result. Saffery and Westerink break down the initial division using the vocabulary from the *Parmenides* as follows:

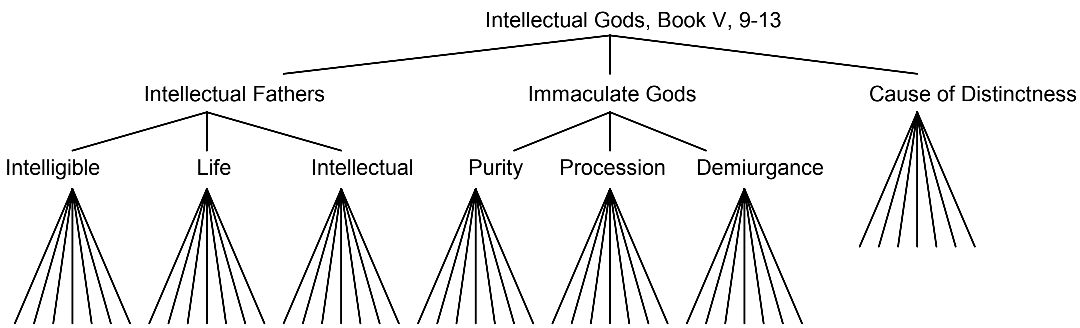


It will be sufficient to set out in greater detail the subtrees of the first three of the principal hypothesis in the tree above. Below are the subtrees for the hypotheses: Intelligibles (Book III), Intelligible-Intellectuals (Book IV), and the Intellectuals (Book V), given in the vocabulary of Proclus' main descriptions (which is somewhat different from that from the *Parmenides*). Of particular interest is the use of negations to characterize the nodes:[63]



[63] The texts describing these details are widely scattered. In this tree and those on following pages, reference in the relevant book of the *PT* (S&W edition) to passages that describe the details of a given "division" are cited to the right of the rightmost node in the taxon.





The tree of the Intelligible-Intellectuals is particularly rich in negations. Each node in its principal triad is characterized by applying hypernegation to the next lower node in the taxon (the node to the immediate right on the same level). This tree may serve as a model from which to generalize Proclus' method. One property that is clear both from the examples and the texts cited earlier is that the tree of diairesis is finitely branching, rooted in the One, and infinitary. In its other properties, however, Proclus varies in his treatment.

Proclus does not always characterize divisions as triadic. Indeed the number of immediate descendants from a parent node varies considerably throughout Proclus' work. In the examples illustrated here the numbers range among two, three, four, seven, and eleven. When he is discussing a triad moreover, it is often obscure whether there are tree nodes consisting of a parent dividing into a dyad, or four nodes with a parent dividing into a ranked triad. In these cases I think it is useful to distinguish between Proclus' theory and his practice, or perhaps a better way to put it is to distinguish among the levels of analysis or precision Proclus is bringing to bear in a discussion.

What he seems to have in mind is that at a limit there is a potentially infinite analysis of nodes into descending ranks of increasingly more detailed levels of the tree. But for the purposes of a given discussion he thinks it is not necessary to go into complete detail, it being sufficient rather to focus on detail up to or at a relevant level of analysis within the tree. It is this practice that explains how it is possible in the first tree above to characterize the One as having a taxon of eleven immediate descendants, yet in subsequent trees to subsume some of these points under prior nodes intermediate between them and the One and simultaneously ignore any of their descendents which are spelled out in the more detailed trees. Indeed Proclus at times offers different accounts of the breakdown of ontic order to its primary hypotheses, e.g. into The One, Intelligibles, Intellectuals, Hypercosmics, Cosmics[64], or into the One, Being, Soul, Matter[65], as well as the positive order of eleven ranks that underlies the eleven negative predications above. In the formal reconstruction below the focus of attention on a given level will be represented by the option of limiting consideration of inference to the nodes on a given level.

The question of whether Proclus intended to view each division as triadic is more difficult. Certainly in many cases he analyzes a node into its triads.

[64] *PT* III:27, S&W 97:9-98:9
[65] *PT* I:12, S&W 57-58

There is, for example, an extended analysis in Book II of the *Platonic Theology* of nodes into their contraries[66]. More importantly Proclus expounds some basic metaphysical doctrines that entail that any "whole" or node in the tree has a three part analysis, as he writes in the *Elements of Theology*:

> Prop. 67 Every whole is either a whole-before-the parts, a whole-of-the parts, or a whole-in-the-part.

He goes on to explain this triad in terms of potency:

> Prop. 81. All that is participated without loss of separateness is present to the participated through an inseparable potency which it implants.
> For if it is itself something from the participant and not contained in it, something which subsists in itself, then they need a mean term to connect them, one which more nearly resembles the participated principle than the participant does, and yet actually resides in the latter…..Accordingly a potency or irradiation, proceeding from the participated to the participant, must link the two; and this medium of participation will be distinct from both.[67]

Thus, any node divides at least in principle into three, and this triple is, as it were, a minimal taxon understood as an ordered set of immediate descendants of a node in the tree. We may then inquire whether the monad is itself one of the members of the triad or a prior point subsuming all three points of the triad as immediate descendants. The theoretical passages are open to either interpretation. It may well be that Proclus is not committed to any precise

[66] *PT* II:4, S&W 34:9-35-9.

number of immediate descendants in diairesis. But when he does provide an analysis in terms of triads he is sometimes careful to distinguish the monad from all three elements of the triad. For example he ranges beneath the gods the triad angels, daimons, and heroes. He could not be more clear that the triadic taxon ranged beneath the demiurge takes the demiurge as its monad and that all three points are distinct from it:

> A nouveau donc nous venons de retrouver, préexistant dans le démiurge d'une manière indivise et uniforme, les propriétés divisées entre les trois pères; et de même que la triade démiurgique participe à l'unité du démiurge grâce à la supériorité incirconscriptible de la monade, de même aussi la monade a embrassé à l'avance en elle-même d'une manière cachée la triade conformément à *la puissance de la cause.*[68]

Because the cardinality of the set of a node's immediate descendants proves to have no bearing on the properties of Proclus' logical relations, we need not here take a position on its precise number, and shall abstract away from its cardinality in the formal reconstruction below. In this reconstruction we shall add a minimum 0 to the linear order of causation. This represents Proclus' state of total privation. Such a point is required by the semantics for the syllogistic, even though the theory requires that both it and the scalar maximum 1 be unnamed (as also Proclus requires of the One and total privation).

Reconstruction. The tree of diairesis together with the negation operations ~ and ¬ determine a family of structures that provide the semantic

[67] *ET* 64 and 76. See also *PT* III:8, S&W 31:20-23, and III:9, S&W 39:20-40:1, and the discussion pp. 102-105 in Siorvanes (1996).

foundation for an increasingly rich logic: first Kleene's strong connectives, next the syllogistic, and lastly scalar negations. That is, the reconstruction is designed to insure that :

1. The causal order determines maximum and minimum operations $\wedge$ and $\vee$ which form a Kleene structure with $-$.
2. The causal order with the three operations $\sim$,$\neg$ and $-$, and a midpoint determine a scalar structure appropriate for the interpretation of scalar predicates over a given level of points in the tree.
3. The causal order together with $\wedge$ and 0 determines a syllogistic structure appropriate for interpreting predicates within the Aristotelian syllogistic.
4. The causal order together with $\wedge$, 0, $\sim$, $\neg$, *e* determine a structure appropriate for interpreting the syllogistic predicates with scalar negations.

The required definitions are straightforward.

Definition. If $<\mathrm{U},\leq>$ is a partial ordering, *x* is *the immediate descendant of y*, briefly $x<<y$, iff $x\leq y$ and for any *z*, if $x\leq z\leq y$ then $x=z$ or $y=z$. By $\{x_{<<}\}$ we mean $\{y|\ y<<x\}$, *the set of immediate descendants of x*.

Definition. $<\mathrm{U},\leq,1>$ is a *tree* iff

1. $<\mathrm{U},\leq>$ is a partial ordering,
2. 1 is the unique maximal element 1 in U*, i.e.* $1\in\mathrm{U}$ and $\forall x\in\mathrm{U}$, $x\leq 1$,

[68] *PT* VI:7, S&W 32:21-26. Emphasis in the translation.

3. $\forall x \in U$, there is a unique $<y_n,\ldots,y_1>$ such that $y_n=x$, $y_1=1$, & for any i=1,…,n, $y_{i+1} << y_i$.

Definitions. If T=<U,≤,1> is a tree, then by a *branch of a tree* <U,≤,1> is any series s of elements of U such that $s_1=1$ and for any $s_{i,}$ $s_{i+1}<<s_i$. By *the finite branch of* T *of length n ending with* y_n is meant any branch with domain {1,…,n}, such that $s_n=x$, $s_1=1$, and for any i, $s_{i+1}<<s_i$.

It is clear from the definition that *branch*, *finite branch*, *the (unique) immediate predecesor of x* (the unique y such that $x<<y$) are well defined. A tree is *finitely branching* iff for any x, $\{x_{<<}\}$ is finite, and it is *finitary* if each branch is finite. It follows that for any branch B of a tree, the restriction of ≤ to B, written ≤|B, is a total ordering on B.

Definition. $<U,\leq,\{\leq_x\}_{x\in U},1>$ is an *ordered tree* iff <U,≤,1> is a *tree*, and for each $x \in U$, $\leq_x$ is a total order on $\{x_{<<}\}$. $T_x=\{x\}\cup\{x_{<<}\}$ is called *the taxon with monad x* in T, and $L_{n,}$ the set of nodes level n of T, is called the *taxonomic level n* of T.

In order for hyper and privative negation to determine a linear order within a taxon, the order must avoid loops.

Definition. A 1-place operation ψ is *regular* in U iff there is no finite series $x_1,\ldots,x_n$ of elements of U such that for all i<n, $\psi(x_i)=x_{i+1}$, and $\psi(x_n)=x_1$. For any operation ψ on a set U, let $<<_\psi$ be a relation on U defined as follows: $x<<_\psi y$ iff $\psi(x)=y$, let $<_\psi$ be the transitive closure of $<<_\psi$ on U, and let $\leq_\psi$ be the reflexive closure of $<_\psi$ on U.

If ψ is a 1-1 regular operation on a finite set U as its domain, then $<_\psi$ is a strict total order on U, $\leq_\psi$ is a total order on U, and ψ is isotonic relative to $\leq_\psi$. Because ~ and ¬ are intended to totally order a taxon, they should be 1 to 1 regular inverses. Proclus' contention that hypernegation rises through the hierarchy to reach the One, is represented by identifying a taxon's monad with the hypernegation of its immediate descendant, *i.e.* the highest element of the taxon formed from its immediate descendants. Likewise, privative negation will be understood as moving down the hierarchy in a manner that approaches a least value, its descendant being the privative negation of the lowest member of a taxon.

Definition. A *diaretic base* is any $<U, \leq, \sim, \neg, 1>$ such that

$<U, \leq, 1>$ is a finitely branching infinitary tree; ~ and ¬ are 1-1 operations on U such that, for any $x \in U$, ~ and ¬ are regular relative to T_x; $\leq_\sim$ and $\leq_\neg$ are inverse relations; for any y in T_x, $\neg y \leq_\sim y \leq_\sim \sim y$; $lub_{\leq_\sim} T_x = x$; and $\neg glb_{\leq_\sim} T_x$ is the $\leq$-immediate descendant of $glb_{\leq_\sim} T_x$.

Theorem. If $<U, \leq, \sim, \neg, 1>$ is a diaretic base, then for any $x \in U$:

1. for any y in T_x, $\sim y \leq_\neg y \leq_\neg \neg y$;
2. $lub_{\leq_\neg} T_x = glb_{\leq_\sim} T_x$;
3. for any $y \in T_x$, $\sim\neg y = y$;
4. $glb T_x = lub T_{glb T_x}$;
5. ~ and ¬ are isotonic wrt $\leq_\neg$ and $\leq_\neg$ on T_x.

Though in general y=~¬y, y need not be ¬~y. Let x be in some T_z, and ~y = x in T_x. Then though ~¬y=y, ¬~y≠y because y is in T_x while ¬~y is the $<<_z$-immediate descendant of ~y=x and is in T_z.

Theorem. If <U, ≤, ~ , ¬,1> is a diaretic base and, if for any $y \in U$, $\leq_x$ is the restriction of $\leq_{\sim}$ to $\{x_{<<}\}$, then $<U,\leq,\{\leq_x\}_{x\in U},1>$ is an ordered tree.

If the antecedent of this theorem is satisfied, we shall say that the base <U, ≤, ~ , ¬,1> *induces* the ordered tree $<U,\leq,\{\leq_x\}_{x\in U},1>$. It is now possible to formulate the key idea in Proclus' transformation of a diaretic tree organized by hyper and privative negation into a linear structure appropriate for scalar and syllogistic reasoning. We introduce an idealized least element, 0.

Definition. The *Procline order* $\underline{\subset}$ *induced by* an ordered tree $<U,\leq,\{\leq_x\}_{x\in U},1>$ is a binary relation on $U \cup\{0\}$ defined : $x \,\underline{\subset}\, y$ iff either

a. $\exists u,v,z\in U$ such that $x\leq u$, $y\leq v$, and $u\leq_z v$, or

b. x=0 and $y\in U\cup\{0\}$,

Since $\underline{\subset}$ does not have a "midpoint" around which a symmetric – could pivot, symmetric scalar reasoning must be relativized to a taxon or "level of analysis" . Let us limit discussion to non-trivial taxa and levels other that {1}. A taxon or a level of analysis perforce determines a symmetric syllogistic structure if it has a "midpoint. " A taxon T_x is totally ordered by $\leq_{\sim}$; its operations ~ and ¬ are isotonic and such that for any y in T_x, $\neg y\leq_{\sim} y\leq_{\sim} {\sim}y$. It contains x as its first element and has a $\lesssim$-least element, called 0_x below. If the cardinality of T_x is odd, there is an antitonic idempotent operation – on T_x and an element e such that $-e=e$.

A level L of rank n (relative to an ordered tree with order $\leq$) is partitioned by $P_L=\{\{x_{<<}\}|x\in L'$ such that L' is the level of rank n-1$\}$. There is a total order on P_L: $\{x_{<<}\}\leq_{P_L}\{y_{<<}\}$ iff $x\leq y$. Let be $<<_{P_L}$ its immediate descendant relation. Define $\sim_L = \sim|L \cup \{<x,y>|$ either $\exists u\exists v$, $(\{u_{<<}\}<<_{P_L}\{v_{<<}\}$ & $(x=\text{lub}\{u_{<<}\}$ & $y=\text{glb}\{v_{<<}\})$, or $x=\text{lub}(\text{lub}P)$ and $y=1)\}$. Likewise, $\neg_L= (\neg|L) \cup \{<x,y>|$ either $\exists u\exists v$, $(\{u_{<<}\}<<_{P_L}\{v_{<<}\}$, $x=\text{glb}\{v_{<<}\}$ & $y=\text{lub}\{u_{<<}\})$ or $(x=\text{glb}(\text{glb}P)$ and $y=0)\}$. Clearly $\underline{C}|L$ is the transitive closure of $\{<x,y>|x,y\in L$ & $x=\neg_{L_n}y\} = \{<x,y>|x,y\in L$ & $y=\neg x\}$ and is a total order. Let $L_{\{0,1\}}$ be $L \cup(0,1\}$, and $\leq_L$ be $\underline{C}|L \cup\{<x,1>| x\in L\}\cup\{<0,x>| x\in L\}$.

These definitions entail that scalar syllogistic reasoning is appropriate to the hierarchy of being relativized to a particular taxon or level.

Theorem. If $\underline{C}$ is the Procline order induced relative to an ordered tree $<U,\leq,\{\leq_x\}_{x\in U},1>$ and its diaretic base $<U, \leq, \sim, \neg,1>$, $x\in U$, the cardinality of T_x is odd, and L is a level of the tree such that the cardinality of $L_{\{0,1\}}$ is odd, then for some $e\in T_x$, a binary operation $-_T$ on T_x, $e'\in L_{\{0,1\}}$ and binary operation $-_L$ on $L_{\{0,1\}}$, $<T_x,\lesssim,-_T,\sim,\neg,e>$ and $<L_{\{0,1\}}, \leq_L,-_L,\sim_L,\neg,e'>$ are symmetric syllogistic structures.

It is syllogistic reasoning on the tree of diairesis relative to such levels of abstraction that Proclus most often employs.

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