On Leibniz's Essay *Mathesis rationis*

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1. Introduction

The text of *Mathesis* (*rationis*) was first edited in 1903 in L. Couturat's collection of *Leibniz – Opuscules et fragments inédits* (C, 193–206).[1] In the sixties German and English translations were given by F. Schmidt and G. H. R. Parkinson (cf. S, 344–357 and P, 95–104). Furthermore, the *Mathesis* has been commented upon in almost every treatise on Leibniz's logic, especially in the works of L. Couturat, K. Dürr, R. Kauppi, and H. Burkhardt.[2] Therefore it will be appropriate to explain why I believe a new edition of the *Mathesis* to be necessary and another commentary on it to be desirable.

First, the state of the art of editing manuscripts has somewhat improved over the past 85 years. This remark is in no way intended as a critique of Couturat's editorial work. As a matter of fact, in view of the presently very unfinished state of the Akademieausgabe of Leibniz's philosophical writings, C still represents a valuable, indeed indispensable resource. Moreover, Couturat himself had emphasized the need of a critical edition of Leibniz's manuscripts, and he had therefore included text-critical marks indicating passages deleted or added by the author. Couturat tried to follow the policy of reproducing at least all those corrections "which offered some theoretical interest" (C, v). The result, however, only seldom attains this goal nor does it come up to the standards of a genuinely text-critical edition like A, AV, or also GI.

As regards the *Mathesis* in particular, Couturat published not much more than the final version of the essay (sheets 1 and 2 of the manuscript LH IV, 6, 14)[3], while a preliminary draft and some related studies (sheets 3–5) were edited only in a very abridged form (cf. C, 203–206). But even the main text is far from complete since, among others, three important paragraphs that Leibniz decided to omit[4] did not find entrance into Couturat's edition. As will be shown below, however, the additional material of these §§ provides the key for a proper understanding of § 24 which – together with the related §§ 3–6 – forms the core of the whole essay.

Second, as regards the previous commentaries and interpretations of the *Mathesis*, I feel that, perhaps due to the lack of a complete and critical text, the real meaning of this fragment has not been recognized so far. Most scholars agreed to Couturat's verdict that Leibniz sketched the theory of the quantification of the predicate – TQP, for short – only in order to refute it.[5] Couturat maintained this view although he was aware of the fact that Leibniz had stressed at several places the importance of TQP for a "foundation of all rules of the figures and moods of syllogistic theory" (*La logique de Leibniz*, o.c., 24). Couturat thought it necessary to close an apparent gap in Leibniz's syllogistic studies by providing a "Précis of classical logic" (o.c., Appendix 1) which basically consisted in a derivation of the theory of the syllogism from TQP. However, a closer analysis of the *Mathesis* reveals that Leibniz was in no need of such help since he not only developed TQP all by himself but also used it in much the same way as Couturat as a tool for deriving the basic laws of the syllogism.

2. Theory of the syllogism and universal calculus

To be somewhat more exact, Leibniz's great aim in logic was to construct a general calculus of concept logic that would enable him to strictly verify the traditional theory of the syllogism. It is not at all easy to chronologize this enterprise but at least the following can be claimed with some degree of certainty. On the one hand, Leibniz dealt with issues in the traditional theory of the syllogism practically throughout his (adult) life, namely from 1665 when he composed the *Dissertatio* until 1715 when we know the *Schedae* were written. The various drafts of a general calculus, on the other hand, date from a much shorter period between 1680 and 1690, approximately.[6]

The validation of the theory of the syllogism by

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means of the *Calculus universalis* involved two tasks which I shall refer to as 'soundness' and 'completeness', respectively. The proof of soundness amounted to showing that both the simple inferences of subalternation, opposition, and conversion and the 24 moods that were generally[7] regarded as valid could indeed be derived as theorems of the general calculus. If, as usual, A, E, I, and O symbolize the categorical forms of a universal affirmative, universal negative, particular affirmative, and particular negative proposition, then the simple consequences may be formalized as:

Opp 1	$\neg A(B,C) = O(B,C)$
Opp 2	$\neg E(B,C) = I(B,C)$
Sub 1	$A(B,C) \rightarrow I(B,C)$
Sub 2	$E(B,C) \rightarrow O(B,C)$
Conv 1	$E(B,C) \leftrightarrow E(C,B)$
Conv 2	$E(B,C) \rightarrow O(C,B)$
Conv 3	$A(B,C) \rightarrow I(C,B)$
Conv 4	$I(B,C) \leftrightarrow I(C,B)$.

The perfect moods of the Ist figure accordingly take the shape:

Barbara	$A(C,D) \wedge A(B,C) \rightarrow A(B,D)$
Celarent	$E(C,D) \wedge A(B,C) \rightarrow E(B,D)$
Darii	$A(C,D) \wedge I(B,C) \rightarrow I(B,D)$
Ferio	$E(C,D) \wedge I(B,C) \rightarrow O(B,D)$.

Actually, the proof of soundness could be simplified to demonstrating only these 4 moods together with the laws of opposition. For Leibniz had shown in *Formis*:

(a) that the laws of subalternation, Sub 1, 2, follow from Darii and Ferio;
(b) that by means of Sub 1 and 2 the remaining two moods of the Ist figure, Barbari and Celaro, can be proved;
(c) that the moods of figures II and III can be reduced to those of the Ist by means of a primitive inference called '*regressus*'; and
(d) that the laws of conversion can be derived from moods of the IInd and IIIrd figure;

Finally in *Mathesis* Leibniz also proved that

(e) the moods of the IVth figure followed from the previous ones by means of the rules of conversion.[8]

Hence {Barbara, Celarent, Darii, Ferio, Opp 1,2} constitutes an axiomatic basis of the theory of the syllogism.

Leibniz who already in 1679 had developed a semantical method for validating these principles by means of "characteristic numbers"[9] started a series of syntactic derivations in *Comprobatione* which was probably written around 1686.[10] At that time, however, the various attempts to prove the axiomatic basis of the theory of syllogism within the general logic of concepts remained without success. It was not before 1690 that Leibniz found a satisfactory proof of the soundness of syllogistic theory in *Principia*[11] and in *Difficultates*.[12]

The proof of completeness, on the other hand, should have

(f) to demonstrate the traditional canon of general rules including the so-called rules of quantity and quality;
(g) to derive from them some more specific rules for the single figures; and
(h) to show that the latter suffice to invalidate all but those syllogisms already proven to be sound.

Before investigating how Leibniz tackled this threefold task in *Mathesis*, we will have to take a closer look at the traditional version of this syllogistic doctrine.

3. Axioms and rules of traditional syllogistics

I refer to the exposition in Arnauld/Nicole's *La Logique ou L'Art de Penser*[13], commonly known as the Logic of Port-Royal. The first "axiom" of their system is nothing but the above mentioned law of subalternation.[14] Three further "axioms" contain the theory of quantity and quality[15], that is:

Quan The subject of a universal proposition is universal. The subject of a particular proposition is particular.

Qual The predicate of an affirmative proposition is particular. The predicate of a negative proposition is universal.

These "axioms" are said to be the basis for the subsequent general rules of the syllogism[16], although Arnauld/Nicole fail to show how the latter might be derived from the former.

GR 1 The middle term may not be particular in both premisses.

GR 2 If a term is universal in the conclusion then it must also be universal in the premiss.

GR 3 At least one of the premisses must be affirmative.

GR 4 If the conclusion is negative, one of the premisses also has to be negative.

Next: "La conclusion suit toûjours la plus foible partie, c'est-à-dire, que s'il y a une des deux propositions negatives, elle doit être negative; & s'il y en a une particuliere, elle doit être particuliere" (o.c., 186). It will be convenient to split this rule up into

GR 5.1 If one of the premisses is particular, then the conclusion must be particular;

GR 5.2 If one of the premisses is negative, then the conclusion must be negative.

Finally one has:

GR 6 At least one of the premisses must be universal.[17]

These general rules in turn are supposed to entail the following special rules for the single figures, although, again, Arnauld/Nicole fail to indicate how the latter might be obtained from the former. The first figure is defined by the fact that the middle term, C, is the subject in the minor-premiss, i.e. the premiss containing the minor-term, B, while C is the predicate in the major-premiss (which contains the major-term D). Here the following restrictions obtain:

I.1 In the first figure the minor-premiss must be affirmative.

I.2 In the first figure the major-premiss must be universal.[18]

In the second figure which is defined by having the middle term both times as a predicate, the corresponding restrictions run as follows:

II.1 In the second figure one of the premisses must be negative.

II.2 In the second figure the major-premiss must be universal.[19]

The third figure is characterized by having the middle term both times as subject. Here the following conditions apply:

III.1 In the third figure the minor-premiss must be affirmative.

III.2 In the third figure the conclusion must be particular.[20]

Finally, with regard to the fourth figure where the middle term is predicate in the major-premiss and subject in the minor-premiss, Arnauld/Nicole mention three conditions: "Quand la majeure est affirmative, la mineure est toujours universelle . . . Quand la mineure est affirmative, la conclusion est toujours particuliere . . . Dans les modes negatives la majeure doit être generale" (o.c., 200). In view of the general rules GR 4 and GR 5.2, a mood is negative if and only if it has a negative conclusion. Hence we can paraphrase the above conditional restrictions as follows:

IV.1 In the fourth figure, if the major-premiss is affirmative, the minor-premiss must be universal.

IV.2 In the fourth figure, if the minor-premiss is affirmative, the conclusion must be particular.

IV.3 In the fourth figure, if the conclusion is negative, the major-premiss must be universal.

4. Early attempts at a proof of completeness

Leibniz appears to have been acquainted with this traditional doctrine, or at least with parts of it, already as a youth. In the *Dissertatio* he does not state the "axioms" Quan and Qual, though, but he mentions in passing the general rules GR 2, 3, 5, 6[21], and he also formulates the special rules in a very condensed way[22]. Only Leibniz's conditions for the IVth figure differ quite considerably from the traditional restrictions: "In IV[ta] Conclusio nunquam est UA. Major nunquam PN. Et si Minor N, Major UA" (o.c., 184).

In *Comprobatione*, probably written 2 decades after the *Dissertatio*, Leibniz gives a riper version of the laws of the syllogism, and he makes some first steps towards a proof of completeness. First he mentions (although does not prove yet) the proper rules of quantity and quality when he points out that "*Terminus distributivus* est idem qui totalis seu universalis; *non distributus*, qui particularis seu partialis. Subjectum est ejusdem quantitatis cujus propositio. . . . Sed praedicatum in omni propositione affirmativa est partiale seu non distributum, et in omni propositione negativa est totale seu distributum" (C, 312).

Second he is now able to demonstrate the validity of the general rules (omitting only GR 4) as follows. As regards GR 1: "*Medius debet esse in alterutra praemissarum distributus seu totalis*; alioqui nulla potest effici coincidentia, si minoris termini aliquid parti medii coincidit aut non coincidit, et majoris termini aliquid rursus parti medii coincidit aut non coincidit, diversae partes medii affici poterunt" (C, 317).

Similarly, we read with respect to GR 2: "... generaliter dici potest terminum non posse [esse] ampliorem in conclusione quam in praemissa, alioqui id quod non venisset in ratiocinationem, ea nempe pars termini, quae in praemissa non afficitur, veniret in conclusionem ... Atque hoc est quod vulgo dicitur *Terminum non distributum ... in praemissa nec posse esse distributum in conclusione*" (C, 316).

Concerning GR 3 Leibniz explains: "Manifestum etiam est ex meris negativis propositionibus nil sequi. Nam sola exclusio ejus quod est in termino extremo ab eo quod est in medio non infert utique ullam coincidentiam, sed ne quidem inferre potest exclusionem ejus quod in uno extremo ab eo quod est in alio extremo." (C, 318).

The proof of the remaining rules GR 5: "Demonstrandum etiam est conclusionem sequi praemissam debiliorem" and GR 6: "... demonstrabimus ex puris negativis nil sequi" (ibid.) is somewhat less satisfactory because Leibniz restricts it to the case of affirmative propositions noting that "omnes syllogismos negativos posse mutari in affirmativos, ex negativa faciendo affirmativam indefiniti [praedicati]" (C, 319).

The special rules for the single figures, however, are not derived very systematically by Leibniz. He just mentions some restrictions that happen to come to his mind as immediate consequences of the general rules. Thus, as a corollary of GR 1, he notes: "Hinc in figuris [?] ubi medius terminus semper est praedicatum [*viz.*, in the IInd figure] conclusio debet esse negativa", i.e. rule II.1, and "ubi semper est subjectum [*viz.*, in the IIIrd] conclusio debet esse particularis", i.e. rule III.2. Furthermore Leibniz infers from GR 2 some conditional restrictions which, however, are much weaker than the traditional rules.[23] Finally, Leibniz promises to derive further rules for the Ist and IVth figure once GR 6 and GR 5 were proven[24], but he fails to make this announcement true.

5. Proving the special rules

By the time of the *Mathesis*, probably around 1705[25], Leibniz has gained a clear knowledge of the logical foundations of the general rules. In what I consider as a preliminary version of the essay, he gives the following summary of the "fundaments of all theorems of the figures and the moods":

> (1) Medius terminus debet esse universalis in alterutra praemissa ...
>
> (2) Alterutra praemissa debet esse affirmativa ...
>
> (3) Terminus particularis in praemissa est particularis in conclusione ...
>
> (4) Si una praemissa sit negativa, etiam conclusio est negativa ...
>
> (5) Subjectum propositionis universalis est universale, particularis particulare
>
> (6) Praedicatum propositionis affirmativae vi formae est particulare, negativae universale.
>
> Ex his [sex] fundamentis omnia Theoremata de Figuris et modis demonstrari possunt. (LH IV, 6, 14, 4 verso)

It is not without interest to note that Leibniz sees no need to distinguish the traditional "axioms" Qual and Quan from the "theorems" GR 1—6; he rather considers them all alike as fundamentals. Actually the above list contains only a part of the traditional rules, *viz.* GR 1, 2, 3, and 5.1. Leibniz evidently forgot to state also GR 4, but in the final version of *Mathesis* he recognizes this omission when he inserts into his formulation of GR 5.2 "Nec minus manifestum est, una praemissa existente negativa, etiam conclusionem esse negativam" (C, 196) the remark "et vicissim".

In contrast, the fact that also GR 5.1 and GR 6 no longer range among the fundamentals should not be taken as another slip of Leibniz but rather as the result of his insight that both principles follow from the remaining ones. Corresponding proofs are provided in §§ 32 and 33 of the main text.

In an admirably clear and strictly deductive way Leibniz shows in §§ 37, 38, 39, 42, 43 that the fundamental principles (in conjunction with the definition of the figures as stated in § 22) entail the following special rules for the first 3 figures. II.1: "... in secunda figura conclusio debet esse negativa"; II.2: "Ibidem major propositio semper est universalis"; III.2: "... in tertia figura conclusio debet esse particularis"; III.1 together with I.1: "In prima et tertia figura, Minor propositio est affirmativa"; I.2: "In prima figura major propositio est universalis". Moreover, the number of special rules for the IVth figure also can be reduced to two. The former IV.1 is stated in § 46 as follows: "In quarta figura non simul est minor particularis et major affirmativa"; and instead of IV.2 and IV.3 Leibniz now formulates: "In quarta figura non simul major prop. particularis, et minor prop. negativa." (§ 46). Hence

Leibniz who in general was fond of symmetries and harmonies concludes: "Quaevis ergo figura accipit duas limitationes" (§ 47).

A careful analysis of the Leibnizian proof of the special rules reveals that each and only each of the 6 fundamentals function as premisses. As will be shown in section 7 below, the special rules in turn are necessary and sufficient to carry out the final step in the proof of completeness by proving "... that there are not more [than the 24 valid moods], but not by an enumeration of the invalid moods, but from the laws of the valid ones" (cf. C, 202). First, however, we will have to describe Leibniz's version of TQP which is the basis for the first step of the completeness proof, *viz.* for validating the six fundamentals.

6. TQP

In order to discuss Leibniz's *Theory of the Quantification of the Predicate* let us consider, e.g., the universal affirmative proposition: "(3) Cum dico: *Omne A est B*, intelligo quemlibet eorum qui dicuntur A, eundem esse cum aliquo eorum qui dicuntur B". What kind of entitites are the informal quantifier-expressions 'quemlibet' and 'aliquo' assumed to refer to, and how is the relation of 'being called' A (or B) to be understood? For a contemporary logician it may be most natural to interpret the quantifiers as referring to individuals which are elements of the set A (or to which the predicate A applies). In this case one arrives at the following version of TQP.

The universal affirmative proposition 'Every A is B' will be paraphrased as: 'Every individual x which is an element of A is identical with some individual y which is an element of B'; formally:

UA 1 $\bigwedge x(x \, \varepsilon \, \mathrm{A} \rightarrow \bigvee y(y \, \varepsilon \, \mathrm{B} \wedge y = x))$.

The particular affirmative proposition 'Some A is B' in the sense of "... aliquem eorum qui dicuntur A, eundem esse cum aliquo eorum qui dicuntur B" accordingly can be formalised thus:

PA 1 $\bigvee x(x \, \varepsilon \, \mathrm{A} \wedge \bigvee y(y \, \varepsilon \, \mathrm{B} \wedge y = x))$.

The universal negative proposition 'No A is B' in the sense of "... quemlibet eorum qui dicuntur A, diversum esse a quolibet eorum qui dicuntur B" amounts to:

UN 1 $\bigwedge x(x \, \varepsilon \, \mathrm{A} \rightarrow \bigwedge y(y \, \varepsilon \, \mathrm{B} \rightarrow y \neq x))$.

Finally, the particular negative proposition 'Some A is not B' in the sense of "... quendam eorum qui dicuntur A, diversum esse a quolibet eorum qui dicuntur B" can be rendered as:

PN 1 $\bigvee x(x \, \varepsilon \, \mathrm{A} \wedge \bigwedge y(y \, \varepsilon \, \mathrm{B} \rightarrow y \neq x))$.

Under the present interpretation the additional propositions mentioned in § 7 make a clear sense, although they are neither very useful ("utilis") nor do they really occur in ordinary language ("non est in usu in nostris linguis"). To say that "*Omne A est omne B*" in the sense of "omnes qui dicuntur A esse eosdem cum omnibus qui dicuntur B" evidently is the same as to state

NC 1 $\bigwedge x(x \, \varepsilon \, \mathrm{A} \rightarrow \bigwedge y(y \, \varepsilon \, \mathrm{B} \rightarrow y = x))$.

But this will never be the case unless the sets A and B are singletons which contain exactly one and the same element.

In the same way the corresponding proposition "*Quoddam A est omne B*" in the more precise sense of "quosdam A esse eosdem cum omnibus B" has to be formalised as

NC 2 $\bigvee x(x \, \varepsilon \, \mathrm{A} \wedge \bigwedge y(y \, \varepsilon \, \mathrm{B} \rightarrow y = x))$.

Again this can't be true unless the set B is a singleton.[26]

The other two propositions which Leibniz obtained by negating NC 1 and NC 2: "... quemlibet eorum qui dicuntur A esse diversum ab aliquo eorum qui dicuntur B" and "... quendam eorum qui dicuntur A diversum esse a quodam eorum qui dicuntur B", i.e.

NC 3 $\bigwedge x(x \, \varepsilon \, \mathrm{A} \rightarrow \bigvee y(y \, \varepsilon \, \mathrm{B} \wedge y \neq x))$

NC 4 $\bigvee x(x \, \varepsilon \, \mathrm{A} \wedge \bigvee y(y \, \varepsilon \, \mathrm{B} \wedge y \neq x))$,

these will in general be tautological statements the truth of which "per se patet" unless, again, B is a singleton: "nisi B sit unicum".

It strikes me as somewhat incomprehensible that not only Couturat but also modern commentators regarded this as a rejection of TQP[27]. Even if Leibniz's remarks about the artificiality ("non est in usu ...") and the redundancy ("inutilis ...") of the non-categorical propositions NC 1–4 (which exhaust all possibilities of a quantification of the predicate) might be interpreted as a rejection of this particular part of TQP, still it could hardly be denied that Leibniz advocated the other, more relevant part of TQP which relates to categorical forms UA 1, PA 1, UN 1, and PN 1. Furthermore, it cannot be overlooked that Leibniz took this very (semi)-formalization of the categorical forms as a conclusive proof of the traditional rules of quantity and quality:

> (9) Itaque . . . patet omnem ac solam propositionem affirmativam habere praedicatum particulare, per art. 3 et 4. (10) Et omnem ac solam propositionem negativam habere praedicatum universale per art. 5 et 6. (11) Porro *Propositio* ipsa a subjecti universalitate vel particularitate *universalis* vel *particularis* denominatur.

As a matter of fact, these counterparts of Qual and Quan follow immediately from the quantification both of the subject and of the predicate as illustrated in UA 1, PA 1, UN 1, and PN 1, provided that the terms A, B are taken to be universal or particular just in case they are modified by a universal or by a particular (i.e., existential) quantifier.

Before discussing a second version of TQP presented in §§ 24, 48–50, let me briefly touch upon Leibniz's proofs of the remaining fundamentals. They basically follow the lines of the corresponding demonstrations in *Comprobatione*. Thus Leibniz immediately infers the fundamental principles GR 3, GR 4 + GR 5.2 from the logical laws for identity stated in §§ 12 and 13[28]:

> (15) Hinc statim colligitur ex duabus propositionibus negativis non posse fieri syllogismum, ita enim pronuntiatur L esse diversum ab M, et [M] etiam esse diversum ab N. . . . (21) Nec minus manifestum est, una praemissa existente negativa, etiam conclusionem esse negativam, et vicissim, quia non alia tunc adhibetur ratiocinatio, quam cujus principium adductum est art. 13.

The proof of the other fundamentals GR 1, 2 resorts in addition to the following definition of a categorical syllogism:

> (12) *Syllogismi quos categoricos simplices vocant* ex duabus propositionibus tertiam eliciunt . . . (16) Patet etiam in syllogismo categorico simplice tres esse terminos, dum tertium aliquid adhibemus, quod dum uni pariter atque alteri extremorum conferimus, modum tentamus conferendi extrema inter se.

This third term, the medius, must be universal in at least one premiss, as Leibniz argues in § 19:

> Nam . . . si medius Terminus utrobique est particularis, non est certum contenta Medii quae adhibentur in una praemissa esse eadem cum contentis medii quae habentur in altera praemissa, atque ideo nec inde colligi aliquid potest de identitate et diversitate extremorum.

And in the subsequent § he shows that if a term is particular in a premiss then it will also be particular in the conclusion:

> Facile etiam intelligi potest Terminum particularem in praemissa non inferri universalem in conclusione, neque enim idem aut diversum in conclusione cognoscitur, nisi de eo quod idem aut diversum medio in praemissa habitum est.[29]

7. The ΨBΨD-formalism

Another version of the TQP is developed in § 24 which is difficult to read in several places since the text is written in very small letters on the margin. The main differences between the present edition and the previous one (in C) are the following:

(1) Leibniz inserted the last sentence of § 24 'propositionis quaecunque . . .' on top of the sentence 'S significabit . . .'. That's why a certain word which Couturat somewhat diffidently interpreted as 'unurarem' seemed to belong to the former sentence while in fact it reads as 'terminum' and belongs to the latter sentence. Accordingly, the crucial passage "S significabit universalem, P particularem, V, Y, Ψ incertam" (C, 196) has to be corrected to 'S significabit terminum universalem, P particularem, V, Y, Ψ incertum'. This is quite important since it conclusively establishes that the symbols 'S' and 'P' characterize the universality and particularity of a term and not, as, e.g., Parkinson assumed[30], of a proposition. Accordingly 'Ψ' symbolizes that it is undetermined whether the subsequent term B is universal or particular; it does not, however, as Burkhardt took it[31], constitute itself an "indefinite term".

(2) The resulting formalisation of the categorical forms is read by Couturat as "Signum itaque SBSD est propositio universalis negativa. SBPD universalis affirmativa. IBSD particularis negativa. IBID, particularis affirmativa." The opening word, however, actually belongs to the preceding sentence ('Propositionis quantitas designabitur per subjecti signum universale, qualitas per praedicati signum'). Furthermore, the textual evidence does not necessarily speak in favor of a letter 'I' within the formulae 'IBSD' and 'IBID'[32], but allows one to read this letter instead as a very slim 'P' (where what at first sight appears as a point above 'I' really is a tiny crook of a 'P').

That Leibniz at any rate meant to write 'P' instead of

'I' is evident from the deleted §§ 48 and 50 where one can read: "Ubi nullus respectus ad praemissas, termini erunt F, G, vel tales. In genere propositio universalis SFΨG propositio particularis PFΨG propositio Affirmativa ΨFPG propositio negativa ΨFSG. In specie Universalis Affirmativa SFPG, Particularis affirmativa PFPG, Universalis negativa SFSG, particularis negativa PFSG."

(3) This unambiguous statement also confirms that the concluding sentence of § 24 ends with 'generaliter exprimitur ΨFΨG' and not, as C has it, with 'generaliter exprimitur unurarem ΨF.ΨS.' Even more misleading is the interpretation of this formula by F. Schmidt and by G. H. R. Parkinson who both suggest 'ΨP.ΨS.'.[33]

Let us now consider in which way Leibniz used this symbolism to complete his proof of completeness. In § 45 he proved the special rule IV.1 indirectly as follows. If one would have at the same time: ". . . major particularis PDΨC, minor negativa [ΨCSB], erit conclusio [particularis] negativa PBSD, sed hoc absurdum, quia (art. 20 [i.e. GR 2]) non potest esse in majore PD et in [conclusione] SD."[34] In § 46 it is similarly shown that one cannot have at the same time ". . . minor particularis et major affirmativa. Existant simul, erit major ΨDPC, minor PCΨB; sed ita medius C utrobique est particularis, quod est contra art. 19 [i.e. GR 1]."

Systematically much more important, however, is the sketch of a proof that Leibniz gives at the very end of *Mathesis* to show that there are not more valid moods than the 24 ones proven elsewhere:

> Contendendum erit, non dari plures, et quidem non per enumerationem illegitimorum, sed ex legibus legitimorum. V.g. in prima praemissae SC.ΨD, ΨB.PD dant:

SCPD	SBPD	AA	A	Barbara	1
			I	Barbari	2
	PBPD	AI	I	Darii	3
SCSD	SBPD	EA	E	Celarent	4
			O	Celaro	5
	PBPD	EI	O	Ferio	6 (C, 202).

In its present form, however, this schema is incomplete and incorrect. As stated in § 22, the position of the terms in the Ist figure is: "Fig. 1. CD. BC. BD". The special rule I.1, according to which the minor-premiss is affirmative, therefore has to be formalized as 'ΨBPC', whereas Leibniz erroneously has 'ΨBPD' which would symbolize an affirmative conclusion. Hence only the following combination of premisses (obtained by substituting 'S' and 'P' successively in the place of 'Ψ') is legitimate:

SCPD	SBPC
	PBPC
SCSD	SBPC
	PBPC.

In the first two cases, in view of GR 4, the conclusion must itself be affirmative: ΨBPD; moreover, in the second subcase it has to be particular according to GR 3: PBPD. In the last two cases, in contrast, the conclusion has to be negative on account of GR 4: ΨBSD; in the second subcase, again, it also must be particular: PBSD. Hence Leibniz's schema for the only valid moods of the Ist figure has to be modified as follows:

SCPD	SBPC	SBPD	Barbara	1
		PBPD	Barbari	2
	PBPC	PBPD	Darii	3
SCSD	SBPC	SBSD	Celarent	4
		PBSD	Celaro	5
	PBPC	PBSD	Ferio	6

This formal method of eliminating the invalid moods "ex legibus legitimorum" can be applied to the other figures as well. E.g., the special rules for the IInd figure (which is characterized by the following position of terms: "Fig. 2 DC BC BD"), state that the major-premiss must be universal (II.2): SDΨC, while according to II.1 one of the premisses and hence (GR 5.2) also the conclusion must be negative: ΨBSD. Thus the following combinations have to be taken into account:

SDSC	SBSD
	PBSD
SDPC	SBSD
	PBSD.

In the former two cases, because of the negativity of the major-premiss, the minor-premiss has to be affirmative on account of GR 3: ΨBPC; furthermore, in the first subcase the minor-term B is universal in the conclusion and hence also has to be universal in the premiss (GR

2): SBPC. Similarly, in the latter two cases the particularity of the middle-term C in the major-premiss entails (GR 1) that C must be universal in the minor-premiss: ΨBSC; and in the first subcase the minor-term again must be universal in the premiss: SBSC. Thus one obtains:

		SBSD	Cesare	1
	SBPC			
SDSC		PBSD	Cesaro	2
	PBPC		Festino	3
		SBSD	Camestres	4
	SBSC			
SBPC		PBSD	Camestros	5
	PBSC		Baroco	6

In the IIIrd figure ("Fig. 3 CD CB BC") the minor-premiss is affirmative (III.1): ΨCPB, and the conclusion is particular (III.2): PBΨD. This yields the following initial combinations:

	PBSD
SCPB	
	PBPD
	PBSD
PCPB	
	PBPD.

In the upper subcases where the conclusion is negative, the major-premiss must be negative (i.e. ΨCSD) according to GR 4, while in the lower subcases it has to be affirmative (ΨCPD) since so is the conclusion. Furthermore GR 1 entails that whenever the middle-term is particular in the minor-premiss, it will be universal in the major-premiss. Thus we arrive at the following schema of the valid moods of the IIIrd figure:

SCSD			Felapton	1
		PBSD		
PCSD			Bocardo	2
	SCPB			
SCSD			Darapti	3
		PBPD		
SCPD			Disamis	4
SCSD		PBSD	Ferison	5
	PCPB			
SCPD		PBPD	Datisi	6

Since the special rules for the fourth figure ("Fig. 4 DC CB BD") are formulated as conditionals, it is somewhat more complicated to define the initial combinations of admissible premisses. First of all, one has to verify that the major-premiss here cannot be a particular negative proposition, or — as Leibniz put it in the *Dissertatio* — "Major nunquam PN" (o.c., 184). If it were a PN, i.e. if one had PDSC, then on the one hand the conclusion would have to be negative on account of GR 5.2: ΨBSD; on the other hand the major-term D which is assumed to be particular in the premiss would also have to be particular in the conclusion (GR 2): ΨBPD. This is impossible. Hence the major-premiss must either be a universal or an affirmative proposition.

If it is both affirmative and universal: SDPC, then in view of rule IV.1 the minor-premiss is universal: SCΨB; if it is affirmative, but not universal: PDPC, then the minor-premiss again is universal on account of IV.1, and it also is affirmative according to Leibniz's second rule for the IVth figure (§ 46): SCPB. If finally the major-premiss is universal but not affirmative: SDSC, then the minor-premiss has to be affirmative because of GR 3: ΨCPB. Thus one obtains the following (5!) initial combinations of admissible premisses for the IVth figure:

	SCPB
SDSC	
	PCPB
	SCSB
SDPC	
	SCPB
PDPC	SCPB.

In the first two cases the conclusion must be negative (GR 5.2) and its subject, being particular in the premiss, has to be particular (GR 2): PBSD. In the third case the conclusion has to be negative on account of GR 5.2: ΨBSD. In the fourth case, the conclusion must be affirmative (GR 4), and its subject, B, again must be particular (GR 2): PBSD. In the last case, both subject and predicate have to be particular on account of GR 2: PBPD. Thus the only valid moods of the IVth figure are:

	SCPB		Fesapo	1
SDSC		PBSD		
	PCPB		Fesiso	2
		SBSD	Calmerens	3
	SCSB			
SDPC		PBSD	Calmerop	4
	SCPB	PBPD	Barmasi	5
PDPC	SCPB	PBPD	Dimaris	6.

This completes the proof of completeness. To round off my discussion of the *Mathesis*, I want to delineate in the following section in which respect the ΨBΨC-formalism may be considered as a second version of TOP. For this sake it will be necessary to describe the general calculus in some detail.

8. Formalisations of the Categorical forms

The most immediate way of expressing the universal affirmative proposition within the general calculus of a logic of concepts is simply to drop the informal quantifier-expression 'omne' in 'Omne A est B'[35], thus keeping only:

UA 2 A est B.

As Leibniz first recognized in the GI, this relation can be reduced to an identity either in the form

UA 3 $A = AB$,

where 'AB' symbolizes conceptual conjunction; or by means of an indefinite concept Y as

UA 4* $A = YB$.

The meaning of the latter formula should better be elucidated by explicitly adding the (existential) quantifier-symbol $\exists Y$: "There is at least one concept Y", so as to yield:

UA 4 $\exists Y(A = BY)$.

In what follows I will similarly use the symbol '$\forall Y$' for the universal quantifier: "For every concept Y".

As Leibniz showed in the *Primaria Calculi Logici Fundamenta*, the quantified formalization UA 4 is provably equivalent to the simpler representation UA 3.[36] Moreover, in *Difficultates* Leibniz recognized that the UA can equivalently be expressed by the generalized statement that every A is B in the sense of $\forall X$(XA est B). Somewhat more exactly: Leibniz first defined the following formal criterion for the universality or non-universality, i.e. particularity, of a term A (within a proposition β): "Generaliter agnoscere poterimus an Terminus A . . . sit universalis, si pro A . . . substitui potest YA . . . , ubi Y potest esse quodcunque cum [A] compatibile" (o.c., 215).

Next he went on to prove that the term A is in fact universal within the proposition 'A est B' (i.e. '$A \infty AB$') by simply pointing out: "In U.A. $AB \infty A$, ergo et [$\forall Y$] $YAB \infty YA$".[37] Hence 'A est B' entails $\forall Y$(AY est B). On the other hand, the universal formula $\forall Y$(AY est B) entails, for arbitrary concepts Y, that 'AY est B', especially for Y = A itself: 'AA est B', i.e. (because of the trivial law "$AA = A$"[38]) 'A est B'. Hence one obtains the further formalisation

UA 5 $\forall X$(XA est B).

Now the remaining 'est' can either be eliminated, as Leibniz did, by means of the equivalence underlying UA 2, or by means of the equivalence underlying UA 3. In this case one arrives at the following formal representation involving two quantifiers:

UA 6 $\forall X \exists Y(XA = YB)$.

The particular affirmative proposition 'Quoddam A est B', on the other hand, was formalized by Leibniz among others as 'XA est B' where the indefinite concept X now of course plays the role of an existential quantifier:

PA 2 $\exists X$(XA est B).[39]

Eliminating, again, the 'est' in the sense of UA 4, one obtains the doubly-quantified version

PA 3 $\exists X \exists Y(XA = YB)$,

which Leibniz expressed elliptically as: "*particularis affirmativa* Qu. C est B sic exprimetur: XB = YC" (C, 302).

In view of the laws of opposition, the universal negative proposition can accordingly be formalized as: "Nullum C est B id est XC non = YB" (C, 303), where both indefinite concepts X, Y now function as universal quantifiers

UN 2 $\forall X \forall Y(XA \neq YB)$.

Finally, for the particular negative proposition one obtains as the negation of UA 6:

PN 2 $\exists X \forall Y(XA \neq YB)$.

Putting these formal representations together into the schema:

UA	$\forall X \exists Y(XA = YB)$	$\forall X \forall Y(XA \neq YB)$	UN
PA	$\exists X \exists Y(XA = YB)$	$\exists X \forall Y(XA \neq YB)$	PN

one obtains — as I would suggest — the real meaning of the ΦBΦC-formalism. All that has to be observed is that the original version of § 24:

UA	SA PB	SA SB	UN
PA	PA PB	PA SB	PN

implicitly contained corresponding '=' and '≠'-symbols as Leibniz explained in the deleted § 49: "Possumus etiam reducere omnia ad principium identitatis et diversitatis per calculum. . . . ut si velim exprimere propositionem negativam fiet ΨFSG, erit ΨF non = SG". Hence the intended meaning of the above schema is better formalised as:

UA	SA = PB	SA ≠ SB	UN
PA	PA = PB	PA ≠ SB	PN.

And here the "signum" S has to be interpreted as an indefinite concept governed by a universal quantifier while P accordingly represents an indefinite concept governed by a particular (or existential) quantifier.

9. Synthesis and conclusion

One of the main theses of Couturat's *La logique de Leibniz* said that the failure of Leibniz's logical efforts was due to his conservativeness with which he followed the traditional, so-called "intensional" point of view while a satisfactory system of logic could only be obtained from the "extensional" standpoint.[40] As I have shown elsewhere,[41] this evaluation is untenable for several reasons. Let me only mention here that the "intensional" point of view can be translated without any restriction into the "extensional" language underlying modern predicate logic. In particular, Leibniz's algebra of concepts, i.e. that part of the *Calculus universalis* which dispenses with indefinite concepts, is provably isomorphic to the ordinary algebra of sets.

The reason why I return to the issue of extensional vs. intensional approach is that the first, "extensional" version of TQP discussed in section 7 is provably equivalent to the second, "intensional" version elaborated in the preceding section. What is particularly remarkable here is that the equivalence can be established exclusively by means of principles of a genuinely Leibnitian logic. A detailed account of the system of Leibniz's logic has been given in a recent monograph, and I must refer the reader to this book (cf. fn. 9) for a deeper understanding of the following sketch which aims at showing how the "extensional" and the "intensional" version of, e.g., the UA can be derived from each other.

In section 8 several laws of the universal calculus have already been cited to show that the "intensional" version of the UA with quantified subject and quantified predicate, $\forall X \exists Y (XA = YB)$, was equivalent to the simple "intensional" formalization of the "*Propositio Affirmativa* A est B sive A continet B" (GI, § 16). Now, as Leibniz observed in an untitled fragment, the UA can also be expressed equivalently as a universal conditional: "A est B, idem est ac dicere si L est A, sequitur quod et L est B" (C, 260).[42] Hence another formalisation of the UA is:

UA 7 $\forall X(X \text{ est } A \rightarrow X \text{ est } B)$.

The next thing that has to be observed is that Leibniz developed several logical criteria as to when a concept A is a complete concept (of an individual substance) or, for short, an individual concept, e.g.: ". . . si duae exhibeantur propositiones *ejusdem praecise subjecti singularis* quarum unius unus terminorum contradictoriorum, alterius alter sit praedicatum, tunc necessario unam propositionem esse veram et alteram falsam".[43] This can be abbreviated as follows:

Def.1 $\text{Ind}(A) := \forall X(A \text{ est } \overline{X} \text{ iff } A \text{ non est } X)$.

(Here '$\overline{X}$' symbolizes the negative concept Non-X). With the help of this definition, one can define the "extensional" quantification over individuals as the following special case of "intensionally" quantifying over individual-concepts:

Def.2 $\wedge X\alpha := \forall X(\text{Ind}(X) \rightarrow \alpha)$
$\vee X\alpha := \exists X(\text{Ind}(X) \wedge \alpha)$.

These new quantifiers allow one to represent the UA also as

UA 8 $\wedge X(X \text{ est } A \rightarrow X \text{ est } B)$.

This formula captures the meaning of Leibniz's example:

> Propositio Universalis affirmativa *Omne b est c* reduci potest ad hanc hypotheticam *Si a est b, a erit c*, verbi gratia: *Omnis homo est animal* id est, *Si quis est homo* (*b*) *is* (*a vel Titius*) *est c* (*animal*) (AV, 79).

The last but one step in the proof of the equivalence between the "extensional" and the "intensional" approach consists in the trivial law according to which the condition $\vee y(y = x \vee \alpha)$ is only a complicated version of $\alpha[x]$. Hence UA 1 may be simplified to

UA 9 $\wedge x(x \, \varepsilon \, A \rightarrow x \, \varepsilon \, B)$.

Now, the intension and the extension of a concept A in general are linked together by the so-called law of reciprocity:

> . . . quando dico omnis homo est animal, volo notionem animalis contineri in idea hominis. Et contraria est methodus per notiones et per individua, scilicet: Si omnes homines sunt pars omnium animalium, . . . vicissim animalis notio erit in notione hominis; et si plura sunt animalia extra homines, addendum est aliquid ad ideam animalis, ut fiat idea hominis. Nempe augendo conditiones, minuitur numerus. (C, 235).

This applies to individual-concepts as well. As captured in Def.1, their intension is maximal. Their extension, therefore, will be minimal consisting of exactly one (possible) individual. In this sense they may properly be called the lowest species: ". . . cuius nomen ad pauciora restringi non potest . . . Species absoluta infima est individuum" (AV, 149).

To sum up: the individual concept X contains the concept A: 'X est A', iff X's extension, i.e. the unit-set $\{x\}$ containing exactly the individual x, is contained in the extension of A, i.e. iff x itself has the property A or is a member of the set of all A's: $x \ \varepsilon$ A![44] In this sense the "extensional" formalisation UA 9 coincides with the "intensional" version UA 8.

Notes

[1] Paris; reprinted Hildesheim 1966. The following additional abbreviations are used:

- A = Akademieausgabe, i.e. *Leibniz Sämtliche Schriften und Briefe*, ed. by the Preußische Akademie der Wissenschaften (Darmstadt, Leipzig, Berlin 1923ff.).
- AV = Vorausedition zur Akademieausgabe, series VI, ed. by the Leibniz-Forschungsstelle, Universität Münster (1983ff.).
- GI = *Generales Inquisitiones de Analysi Notionum et Veritatum*, ed. by F. Schupp (Hamburg, 1982).
- GP = C. J. Gerhardt (ed.) *Leibniz Philosophische Schriften* (Berlin 1890ff.).
- P = G. H. R. Parkinson (ed.) *Leibniz. Logical Papers*, (Oxford, 1966).
- S = F. Schmidt (ed.) *Leibniz Fragmente zur Logik*, (Berlin, 1960).

The most important logical works are abbreviated as follows:

- *Comprobatione* = *De formae logicae comprobatione per linearum ductus* (C, 292—321).
- *Difficultates* = *Difficultates quaedam logicae* (GP 7, 214—218)
- *Dissertatio* = *Dissertatio de Arte Combinatoria* (A VI, 1, 168—230).
- *Formis* = *De formis syllogismorum Mathematice definiendis* (C, 410—416).
- *Principia* = *Principia calculi rationalis* (C, 229—231).
- *Schedae* = *Schedae de novis formis et figuris syllogisticis* (C, 206—210).

[2] H. Burkhardt: *Logik und Semiotik in der Philosophie von Leibniz* (München 1980). L. Couturat: *La Logique de Leibniz* (Paris 1901). K. Dürr: "Leibniz' Forschungen im Gebiet der Syllogistik"; in E. Hochstetter (ed.), *Leibniz zu seinem 300. Geburtstag 1646—1946* (Berlin 1949), pp. 1—40. R. Kauppi: *Über die Leibnizsche Logik* (Helsinki 1966).

[3] The classification of Leibnizian manuscripts (LH) follows the catalogue of E. Bodemann, *Die Leibniz-Handschriften der Königlichen öffentlichen Bibliothek zu Hannover*, Leipzig 1895.

[4] Cf. LH IV, 6, 14, 1 recto: "Omitti possunt 48, 49, 50".

[5] Cf. C, 194, fn.1: "Ici Leibniz conçoit nettement la *quantification du prédicat*, et la rejette."

[6] The first serious attempt is the *Specimen Calculi universalis* (GP 7, 218—227) from around 1678—81 (cf. AV, pp. 94 and 107). The greatest progress towards a really "universal" calculus is achieved in the GI of 1686; and the final elaborations and improvements seem to have been carried out in the (*Primaria*) *Calculi Logici Fundamenta* (C, 235—7 and 421—423) of August 1690.

[7] The word 'generally' should not be taken too literally. I mainly want to say that Leibniz himself in general defended the view that there were exactly 6 valid moods in each of the 4 figures. He put forward this claim already in the *Dissertatio* (o.c., 184: "Ita ignota hactenus figurarum harmonia detegitur, singulae enim modis sunt aequales"), but one may doubt whether at that time he was already entitled to do so. For on the one hand the table of the valid moods contained a 25th syllogism named *Frisesmo* which ". . . ex regulis modorum non sit inutilis" (o.c., 185/6). On the other hand Leibniz mistakenly listed a syllogism *Colanto* among the valid moods of the IVth figure while in fact it had to be replaced by *Calerent*.

[8] Cf. LH IV, 6, 14, 3 recto — 3 verso. Another proof of the IVth figure is given in the *Schedae*; cf. C, 209.

[9] Cf. the essays of April 1679 (C, 42—92 + 245—247). In chapter 1 of my book *Das System der Leibnizschen Logik* (Berlin, 1990) it is examined more closely how far Leibniz was justified in claiming: "Ex hoc calculo omnes modi et figurae derivari possunt per solas regulas Numerorum" (C, 247).

[10] *Comprobatione* also contains another semantic confirmation of the syllogistic laws — announced in the title —, *viz.* by means of linear and circular diagrams. There are several reasons for supposing that this essay was written at about the time of the GI. Cf. my paper "Zur Einbettung der Syllogistik in Leibnizens 'Allgemeinen Kalkül'". *Studia Leibnitiana Sonderheft* 15 (1988), esp. p. 55, fn. 24.

[11] Cf. the marginal note at the top of the fragment: "Hic demonstrantur Modi primae figurae, et regulae oppositionum. Quarum ope (ut alibi jam ostendimus) demonstrantur deinde conversiones et modi reliquarum figurarum." (C, 229).

[12] Cf. the discussion in "Zur Einbettung der Syllogistik . . .", o.c. (fn.10).

[13] 5th edition, 1683, reprinted 1965 (Paris Presses universitaires de France); the 1st edition had appeared in 1662.

[14] "Les propositions particulieres sont enfermées dans les generales de même nature" (o.c., 183).

[15] "Le sujet d'une proposition pris universellement ou particulierement, est ce qui la rend universelle ou particuliere ... L'attribut d'une proposition affirmative ... est toûjours consideré comme pris particulierement ... L'attribut d'une proposition negative est toûjours pris generalement" (o.c., 183).

[16] "Le moyen ne peut être pris deux fois particulierement" (o.c., 183) ... "Les termes de la conclusion ne peuvent point être pris plus universellement dans la conclusion que dans les prémisses" (o.c., 184) ... "On ne peut rien conclure de deux propositions negatives" (o.c., 186) ... "On ne peut prouver une conclusion negative par deux propositions affirmatives" (*ibid.*) ... "On ne peut prouver une conclusion negative par deux propositions affirmatives" (*ibid.*).

[17] "De deux propositions particulieres il ne s'ensuit rien" (o.c., 187).

[18] "Il faut que la mineure soit affirmative ... La majeure doit être universelle" (o.c., 191).

[19] "Il faut qu'il y ait une des deux premieres propositions negatives ... Il faut que la majeure soit universelle" (o.c., 194).

[20] "... la mineure en doit être affirmative ... on n'y peut conclure que particulierement" (o.c., 197/8).

[21] "Ex puris particularibus nihil sequitur ... Conclusio nullam ex praemissis quantitate vincit ... Ex puris negativis nihil sequitur ... Conclusio sequitur partem in qualitate deteriorem" (o.c., 181).

[22] Cf. o.c., 184: "I^{mae} autem et 2^{dae} figurae semper major propositio est U"niversalis (I.2 and II.2); "I^{mae} et III^{tiae} semper minor A"ffirmativa (I.1 and III.1); "In II^{da} semper Conclusio N"egativa (II.1); "In III^{tia} Conclusio semper est P"articularis (III.2).

[23] Cf., e.g., C, 316: "... *si conclusio est universalis, Minorem propositionem esse universalem in figuris ubi terminus minor est praemissae suae subjectum,* scilicet prima et secunda". This condition and 3 similar ones reappear in *Mathesis* as §§ 34–36. Leibniz might have omitted them without any loss, for they are not used at all in the subsequent proofs.

[24] "Colliguntur et quaedam [restrictiones] de figuris ubi [medius] modo subjectum modo praedicatum est, sed consequentiae illae praesupponunt ex puris particularibus propositionibus nil sequi, et conclusionem sequi praemissam debiliorem, quae prius demonstranda" (C, 317).

[25] According to a communication of Prof. Schepers from the Leibniz-Forschungsstelle Münster, the water-sign of the manuscript indicates that the *Mathesis* was written at about that time. The present investigation also suggests that *Mathesis* is a rather late fragment, at any rate later than *Comprobatione* because the TQP-version of the categorical forms given there (cf. C, 311) is clearly inferior to the one presented in *Mathesis*.

Couturat argues that the symbolism of § 24 (which will be analysed below), especially the occurrence of 'S' as alleged symbol for "une proposition singulière, qui équivaut à une universelle" and of 'I' for "une proposition indéfinie, qui équivaut à une particulière" (C, 196/7, fn. 2) would indicate that Leibniz was still working with the same formalism as in the *Dissertatio*. F. Schmidt uncritically adopted Couturat's view and concluded (o.c., 516) that the *Mathesis* was written before 1686. R. Kauppi (o.c., 195 ff.) also suspects that *Mathesis* was quite early, anyhow earlier than *Formis*. Burkhardt (o.c., p. 23) dates *Mathesis* around 1690 without, however, stating any reason why. Dürr, finally, made the guess that "das Schriftstück 'Mathesis rationis' einer ziemlich späten Zeit angehört und um 1710 entstanden ist" (o.c., 4).

[26] It should be noted incidentally that Leibniz commits a fallacy when he believes that NC 2 might as well be expressed by saying "Omnes B esse A". According to UA 1, the latter amounts to the condition $\bigwedge x(x \, \varepsilon \, B \rightarrow \bigvee y(y \, \varepsilon \, A \wedge y = x))$. However, one may not at all interchange the two quantifiers within that formula.

[27] Parkinson remarked in the same vein as Couturat that: "... Leibniz conceives the idea of the quantification of the predicate, only to reject it" (P, liii). Kauppi agrees to Couturat by saying that "... die Quantifikation des Prädikats [wird] als unnötig verworfen" (o.c., 199). Burkhardt shares Couturat's opinion that "[Leibniz hatte] die Quantifizierung des Prädikates ... noch im arithmetischen Kalkül von 1679 abgelehnt". He correctly recognizes, however, that in § 24 "Leibniz noch ein Zeichensystem zur Darstellung der vier kategorischen Satzformen entwickelt [hat], mit dessen Hilfe es möglich ist, Subjekt und Prädikat zu quantifizieren" (o.c., 44, 45). Dürr only remarks concerning §§ 3–7: "daß diese Deutungen der vier aristotelischen Aussageformen den Deutungen, die in modernen Werken ... zu finden sind, durchaus vergleichbar sind" (o.c., 38).

[28] "... si L sit idem ipsi M, et M ipsi N, eadem esse L et N" and "... si L sit idem ipsi M, et M sit diversum ipsi N, etiam L et N diversa esse".

[29] Although these demonstrations do render the general rules GR 1 and GR 2 fully acceptable, they should not necessarily be counted as proper proofs in the strict sense of the word.

[30] Cf. P, 98: "S will stand for a universal, P for a particular, V, Y, Ψ for an indefinite proposition".

[31] Cf. o.c., 47: "Dazu gibt es noch die Zeichen, V, Y, Ψ für den unbestimmten Terminus".

[32] The German and English translations also contain 'I' instead of 'P': cf. S, 349 (where in fact one of the 'I's is even rendered as a 'J') and P, 98.

[33] Cf. S, 349 and P, 98.

[34] Couturat pointed out in C, 202, fn. 1 and 2, that the formula for the negative minor-premiss has to be 'ΨCSB' instead of Leibniz's 'SCΨB', and that the 'in minore' of the manuscript must be read as 'in conclusione'. Leibniz's third inaccuracy of symbolizing the "conclusio negativa" as 'PBSD' instead of 'ΨBSD' is harmless, since under the given premisses the conclusion also has to be particular, hence PBSD.

[35] Cf., e.g., the beginning of the *Specimen Calculi universalis*: "Propositio Universalis affirmativa hoc loco a nobis sic exprimetur: a est b sive (Omnis) homo est animal. Itaque semper intelligimus praefixum signum universale." (GP 7, 218). Leibniz seems to have invented this convention around 1670–80, and he stuck to it ever since.

[36] Cf. C, 235/6: "(19) Si A ∞ AB, assumi potest Y tale ut sit A ∞ YB. ... (31) Si sit A ∞ YB, sequitur A ∞ AB".

[37] GP 7, 215. In the same passage Leibniz also proves all the remaining theorems of quantity and quality.

[38] GI, § 171, *tertio*.

[39] Cf., e.g., GI, § 48: "AY continet B est *Particularis affirmativa*". It should be noted, however, that in view of the trivial law "AB est B" (GP 7, 218) there will always exist at least one Y such that "AY est B". Therefore Leibniz's formalisation of the PA should be modified by requiring in addition that Y is compatible with A. Corresponding remarks apply to the subsequent formulas PA 3 and UN 2.

[40] "L'échec final de son système est donc extrèmement instructif, car

il prouve que la Logique . . . exacte et rigoureuse ne peut pas etre fondée sur la considération confuse et vague de la compréhension; elle n'a réussi à se constituer qu'avec Boole, parcequ'il l'a fait reposer sur la considération exclusive de l'extension, seule susceptible d'un traitement mathématique." (o.c., 387).

[41] Cf. in particular "Zur extensionalen und 'intensionalen' Interpretation der Leibnizschen Logik", *Studia Leibnitiana* **XV** (1983), 129–148, and "Leibniz und die Boolesche Algebra", *Studia Leibnitiana* **XVI** (1984), 187–203.

[42] The subsequent proof of this law is very interesting. It shows that the indefinite term 'L' works as a universal quantifier since "Intelligitur autem L quicunque terminus de quo dici potest L est A".

[43] LH IV, 5, 8d, 17 verso; cf. C, 67. A discussion of this important passage may be found in my paper "'Non est' non est 'est non' – Zu Leibnizens Theorie der Negation". *Studia Leibnitiana* **XVIII** (1986), 1–37, esp. pp. 23–24.

[44] As the formalisations UA 8 and UA 9 make clear, there is always a logical relation between the individual(-concept) x (or X) and the general concept A whether the latter is taken extensionally as a set or intensionally as an idea. Modern predicate logic, however, misleadingly veils this relation behind the functional brackets of 'A(x)'. For a more detailed discussion of this point cf. my paper "Concepts vs. Predicates – Leibniz's challenge to modern Logic", in *The Leibniz-Renaissance*, International Workshop (Firenze, 2–5 June, 1986), Florence, 1989, pp. 153–172.

MATHESIS RATIONIS

(1) Leges syllogismorum categoricorum optime demonstrare licebit per reductionem ad considerationem ejusdem et diversi. Nam in propositione vel pronuntiatione semel id agitur ut duo inter se vel eadem vel diversa pronuntiemus.

(2) Terminus (velut homo) in propositione vel accipitur universaliter de quovis homine, vel particulariter, de quodam homine.

(3) Cum dico: *Omne A est B*, intelligo quemlibet eorum qui dicuntur A, eundem esse cum aliquo eorum qui dicuntur B. Et haec propositio appellatur *Universalis Affirmativa*.

(4) Cum dico: *Quoddam A est B*, intelligo aliquem eorum qui dicuntur A, esse eundem cum aliquo eorum qui dicuntur B, et haec est propositio *Particularis Affirmativa*.

(5) Cum dico: *Nullum A est B*, intelligo quemlibet eorum qui dicuntur A, diversum esse a quolibet eorum qui dicuntur B, et haec est propositio *Universalis Negativa*.

(6) Denique cum dico *Quoddam A non est B*, intelligo quendam eorum qui dicuntur A, diversum esse a quolibet eorum qui dicuntur B, et haec dicitur *Particularis Negativa*. Hinc in affirmativis praedicatum vi formae est particulare, in negativis universale.

(7) Posset quidem Omne A esse omne B, seu omnes qui dicuntur A, esse eosdem cum omnibus qui dicuntur B, seu propositionem esse reciprocam, sed hoc non est in usu in nostris linguis. Quemadmodum nec quosdam A esse eosdem cum omnibus B, id enim exprimimus cum dicimus Omnes B esse A. Inutile autem fuerit dicere Nullum A esse quoddam B, seu quemlibet eorum qui dicuntur A esse diversum ab aliquo eorum qui dicuntur B, hoc enim per se patet nisi B sit unicum; et multo magis Quendam eorum qui dicuntur A diversum esse a quodam eorum qui dicuntur B. Ita videmus perfici doctrinam Logicam rem a praedicatione transferendo ad identitatem.

(8) A in exemplis propositis dicitur *subjectum*, B *praedicatum*. Et propositiones huius modi categoricae appellantur.

(9) Itaque eo quem diximus sensu, patet omnem et solam propositionem affirmativam habere praedicatum particulare, per art. 3 et 4.

(10) Et omnem ac solam propositionem negativam habere praedicatum universale per art. 5 et 6.

(11) Porro *Propositio* ipsa a subjecti universalitate vel particularitate *universalis* vel *particularis* denominatur.

(12) *Syllogismi quos categoricos simplices vocant* ex duabus propositionibus tertiam eliciunt, quod fit utendo duobus principiis quorum unum est, quae sunt eadem uni tertio esse eadem inter se, ut si L sit idem ipsi M, et M ipsi N, eadem esse L et N.

(13) Alterum huc redit, diversa inter se, quorum unum tertio idem est, alterum ei diversum. Ut si L sit idem ipsi M, et M sit diversum ipsi N, etiam L et N diversa esse.

(14) Quod si L sit diversum ipsi M et N sit itidem diversum ipsi M, non potest inde cognosci, utrum L et N sint idem an non; et fieri potest ut L sit idem ipsi N, vel etiam ut L sit diversum ipsi N.

(15) Hinc statim colligitur ex duabus propositionibus negativis non posse fieri syllogismum, ita enim revera pronuntiatur L esse diversum ab M, et N etiam esse diversum ab [M].

Exempli causa si dico Nullus homo est lapis, Nullus canis est homo. Sensus est quemlibet hominem esse diversum a quovis lapide, quemlibet salis, utraque praemissa est universalis. Nam si conclusio est universalis, minor terminus est universalis ubique (coroll. art. 27). Ergo et in minore propositione. Sed quia conclusio etiam est affirmativa ibi est subjectum (art. 31). Ergo (art. 11) minor propositio est universalis, et medius terminus ibidem est praedicatum, ergo medius terminus ibi est particularis (art. 9). Ergo medius terminus [ ] est universalis in prop. majore (art. 19) sed ibi est subjectum (art. 3[1]) ergo (per art. 11) etiam major prop. est universalis. Habemus ergo intentum si conclusio sit universalis affirmativa. Sed si conclusio sit universalis negativa, uterque extremus est universalis (art. 26). Ergo non datur hic praemissa particularis affirmativa (artic. 25). Superest ergo tantum ut si datur particularis detur particularis negativa. Ergo (per art. 15 et 31) altera praemissa est universalis affirmativa. In hac extremus cum sit universalis (ut ostensum est) erit subjectum (art. 9 et 11). Ergo medius in eadem erit praedicatum et particularis (art. 11). Ergo (art. 19) in altera praemissa, nempe particulari negativa, erit universalis. Ergo in ea (art. 10) erit praedicatum. Ergo in ea extremus erit subjectum, sed extremus est universalis (ut ostensum) ergo etiam ipsa propositio est universalis, itaque absurdum etiam est ut detur praemissa particularis negativa, itaque nulla praemissa potest esse particularis sive conclusio sit universalis negativa, sive sit universalis affirmativa. Q.E.D.

Schol. Non sequitur si conclusio sit particularis etiam praemissam esse particularem, nam omnis praemissa universalis simul est tacite particularis. Sed illud sequitur. Si conclusio sit negativa esse et praemissam negativam.

(34) *Ubi Major terminus est subjectum* in praemissa et conclusio negativa, major propositio est universalis. Nam quia conclusio est negativa, ejus praedicatum est universale (art. 11) nempe (art. 17) terminus major. Ergo is etiam est universalis in prop. majore (art. 20). Est autem in ea subjectum (ex hypoth.) Ergo (art. 11) ipsa propositio major est universalis Q.E.D.

Coroll. Hinc ubi major terminus est subjectum in praemissa, majore propositione existente particulari, conclusio est affirmativa.

(35) *Ubi major terminus est praedicatum* in praemissa, conclusione existente negativa, major propositio est negativa. Nam caeteris ut in dem. praecedente repetitis; est in ea praedicatum (ex hyp.). Ergo (art. 10) ipsa propositio est negativa.

Coroll. Hinc ubi major terminus est praedicatum in praemissa, majore propositione existente affirmativa, etiam conclusio est affirmativa.

(36) *Ubi minor terminus est praedicatum* in praemissa, conclusione existente universali, minor propositio est negativa. Nam si conclusio est universalis, minor terminus in ea est universalis (art. 11). Ergo et in praemissa (art. 20) Sed in ea est praedicatum (ex hyp.) Ergo (art. 10) est negativa.

Coroll. Ergo ubi minor terminus est praedicatum in praemissa, minore propositione existente affirmativa, conclusio est particularis.

(37) *Ubi medius terminus semper est praedicatum, seu in secunda* figura conclusio debet esse negativa. Nam medius semel debet esse universalis (art. 19), sed universale praedicatum facit propositionem negativam (art. 10), ergo praemissa alterutra est negativa. Ergo (art. 21) conclusio est negativa.

Coroll. Hinc si conclusio sit affirmativa medius terminus alicubi est subjectum.

(38) Ibidem major propositio semper est universalis. Nam quia conclusio est negativa (art. [37]) major terminus in ea est universalis (art. 10). Ergo et in majore prop. est universalis (art. 20) sed in ea est subjectum (ex hyp). Ergo (art. 11) et ipsam facit universalem.

(39) *Ubi medius terminus semper est subjectum* seu in tertia figura conclusio debet esse particularis. Esto conclusio universalis, ergo minor terminus in ea est universalis, ergo (art. 20) etiam in prop. minore est universalis, sed in minore propositione est praedicatum (ex hyp.) Ergo minor prop. erit negativa (art. 10). Ergo (art. 21) et conclusio est negativa. Ergo et major terminus in conclusione est universalis (art. 10). Ergo major terminus etiam in majore propositione est universalis, (art. 20). Sed in ea est praedicatum (ex hyp.) Ergo (art. 10) et major propositio erit negativa. Itaque ambae praemissae sunt negativae quod est absurdum per art. 15. Itaque ubi medius terminus semper est subjectum, conclusio debet esse particularis Q.E.D.

(40) Ubi medium modo subjectum modo praedi-

catum est, si ea praemissa in qua praedicatum est sit affirmativa, altera praemissa erit universalis. Nam in priore medium erit particulare (art. 9). Ergo in altera universale (art. 19) Sed in ea est subjectum (ex hyp.). Ergo ipsa propositio erit universalis (art. 11).

Coroll. Hinc in quarta Figura si major sit affirmativa minor est universalis; *Schol.* In prima inutile fit corollarium, quod fieri posset, sic enim sonaret, in prima si minor sit affirmativa, major est universalis, quod quidem verum est, sed non satis, cum ibi minor semper sit affirmativa et [*bricht ab*]

(41) Ubi medium modo subjectum modo praedicatum est si ea praemissa ubi subjectum est sit particularis, altera erit negativa. Demonstratur eodem modo.

Coroll. Hinc in quarta figura Si minor sit particularis major erit negativa. Schol. Utraque propositio conjungi potest, cum una sit tantum alterius conversa. Nempe non simul praemissa in qua medius est praedicatum potest esse affirmativa, et in qua est subjectum, universalis.

(42) In prima et tertia figura minor propositio est affirmativa. Nam si minor propositio esset negativa, utique et conclusio foret negativa (art. 21). Jam ubi conclusio est negativa et major terminus est praedicatum in praemissa (ut in prima et tertia fig. art. 22), etiam major propositio est negativa (art. 35). Ergo tam major quam minor praemissa foret negativa, contra art. 15.

(43) In prima figura major propositio est universalis. Nam in ea minor prop. est affirmativa (art. 4[2]) et in ea medius terminus est praedicatum minoris prop. (art. 22) Ergo in ea medius terminus est particularis (art. 11) Ergo medius terminus est universalis in majore propositione. Sed medius terminus in majore propositione est subjectum (art. 22). Ergo (art. 11) major propositio est universalis. Sequitur etiam ex prop. 40 et 42.

(44) Si medius terminus est praedicatum in propositione minore, propositio major est universalis. Nam si medius terminus est praedicatum in propositione minore, figura est prima vel secunda (artic. 22) Sed in fig. 1 major est universalis (artic. 43) et in figura 2 major prop. est etiam universalis (artic. 38). Ergo habetur propositum.

(45) In quarta figura non simul major prop. particularis, et minor prop. negativa. Esto in ea per 24 major particularis PDΨC minor negativa [ΨCSB] erit conclusio [particularis] negativa PBSD, sed hoc absurdum quia (art. 20) non potest esse in majore PD et in [conclusione] SD.

(46) In quarta figura non simul est minor particularis et major affirmativa. Existant simul erit Major ΨDPC minor PCΨB; sed ita medius C utrobique est particularis, quod est contra art. 19. Potest etiam ut corollarium derivari ex prop. 40 vel 41.

(47) Quavis ergo figura accipit duas limitationes, in prima major est universalis, minor affirmativa, in secunda major est universalis, conclusio negativa. In tertia minor est affirmativa et conclusio est particularis. Binae limitationes quartae magis sunt implicatae, ut in artic. 45 et 46.

Die folgenden drei Abschnitte wurden gestrichen:

(48) Cum Figura est data rem ad calculum deduximus, ut in exemplis 45 et 46. Sed cum figura est incerta, non constat, quis nam in praemissis sit C, medius, vel quis extremus. Ergo nota opus erit, quae notet ordinem esse incertum, velut C?D. Ubi nullus respectus ad praemissas, termini erunt F, G, vel tales. In genere propositio universalis SFΨG propositio particularis PFΨG propositio Affirmativa ΨFPG propositio negativa ΨFSG. In specie Universalis Affirmativa SFPG, Particularis affirmativa PFPG, Universalis negativa SFSG particularis negativa PFSG. Duae praemissae non possunt esse negativae sic exprimetur: non simul esse potest ΨS.C?D et ΨS.C?B. Medius non ubique particularis sic exprimetur, non simul esse potest PC?ΨD et PC?ΨB. Term. particularis in praemissa esse part. in conclusione ut exprimatur oportet extremum in genere appellare X ut sit X ≃ B vel D. Si unum extremum sit X, alterum vocetur (X), et fiet non simul esse C?PX et SX?(X).

(49) Possumus etiam reducere omnia ad principium identitatis et diversitatis per calculum. Sed nec sic tamen plures casus evitamus, ut si velim exprimere propositionem negativam fiet sic ΨFSG, erit ΨF non ≃ SG. Sed si velimus exprimere duas praemissas non simul esse negativas, non licet communi ratione exprimere, sit ne medius praedicatum an subjectum propositionis negativae. An forte sic ΨF non ≃ SG et ΨH non ≃ SK. Litera aliqua

prioris coincidat cum aliqua posterioris quae erit medium. Coincidentia vel subordinata erit vel per crucem. Si sit subordinata fiet posterior ΨF non ≈ SK. Sed ideo nil prodit, ut liceat uti hic propositionis A ≈ B, et B ≈ C, ergo A ≈ C, vel hic A ≈ B, B non ≈ C, ergo A non ≈ C. Si coincidentia fit per crucem, fiet posterior ΨH non ≈ SF, vel ΨG non ≈ SK. Nullo horum modos pervenietur ad unam harum duarum fundamentalium propositionum. Affirmativam et negativam licebit sub uno comprehendere per generalem expressionem quantitatis in ⟨subjecti-⟩ non ⟨praesupponend-⟩ ⟨deduct-⟩ ex legibus ejusdam et diversi.

(50) Propositio generaliter sic exprimitur ΨF.ΨG; negativa quaevis ΨFSG, affirmativa quaevis ΨFPG; Universalis quaevis SFΨG, particularis quaevis PFΨG. Si detur ΨFΨG et ΨHΨK, et una litera posterioris sit eadem cum una litera prioris seu uni tertiae C, propositiones dicuntur praemissae syllogismi categorici. C erit medium, caeterae literae sunt extremae quae faciunt conclusionem, ubi subjectum est minor term. B, praedicatum major terminus D, conclusio ΨBΨD; Propositiones fundamentales: non simul ambae praemissae sunt negativae, seu non simul sunt SG et SK in praemissis. Medius non utrobique est particularis, seu non simul in utraque praemissa PC. Non simul datur praemissa negativa et conclusio affirmativa, seu non simul datur SG vel SK et PD. Non simul datur conclusio negativa et utraque praemissa affirmativa, seu non simul datur SD et PG et PK. Si extremum universale in conclusione, erit et universale in praemissa, seu si SB in conclusione, erit et SB in praemissa. Item si SD in conclusione, erit et SD in praemissa. Ex his fundamentalibus caetera probari deberent v.g. ambas praemissas non esse particulares. Sed quamdiu non habetur ratio generaliter exprimendi praemissam quicunque sit situs medii demonstratio non potest commode fieri calculo nisi per casus. Omitti possunt 48, 49, 50.

(48) Conclusio universalis affirmativa non datur nisi in prima figura. Nam excluditur figura secunda et tertia (art. 37 et 3[9]) porro minor terminus est universalis in conclusione (art. 11) ergo et in minore prop. (art. 20). Sed ea est affirmativa (art. 21). Ergo praedicatum ejus est particulare (art. 9) ergo minor universalis non est ejus praedicatum sed subjectum, quod non habet locum in quarta figura (art. 20). Ergo sola superest prima.

Die folgenden zwei Ansätze wurden gestrichen:

(49) Non datur syllogismus cuius major sit particularis affirmativa et minor universalis negativa, seu non datur modus IEO

(49) Modus syllogisticus, constans ex majore universali negativa, minore universali affirmativa, et conclusione particulari negativa (seu modus EAO) est omnium figurarum.

Veniendum jam foret ad modorum enumerationem, demonstranda prima figura in quatuor modis ⟨primariis⟩; hinc demonstrabitur subalternatio assumta identica. Et sic habentur reliqui modi duo primae. Ex sex modis primae per regressum demonstrantur sex modi secundae et sex modi tertiae, et simul demonstratur tot esse modos secundae vel tertiae quot primae. Quartae modi demonstrantur ex prima per conversionem, et demonstrati dant reliquos per regressum. Contendendum erit, non dari plures, et quidem non per enumerationem illegitimorum, sed ex legibus legitimorum. V.g. in prima praemissae SC.ΨD. ΨB.P[C] dant:

SCPD	SBP[C]	A.A.	A	Barbara	1
			I	Barbari	2
	PBP[C]	A.I.	I	Darii	3
SCSD	SBP[C]	E.A.	E	Celarent	4
			O	Celaro	5
	PBP[C]	E.I.	O	Ferio	6

Ex veris non nisi verum sequitur. Hinc quod cum meris veris falsum infert, est falsum. Ope hujus propositionis demonstravi veritatem secundae et tertiae figurae, ut hoc sensu quodammodo indirectae dici possint. Quartam figuram demonstro ex prima accedentibus conversionibus; sed ipsae conversiones prius per figuram secundam et tertiam demonstrantur. In quavis figura inveni sex modos

Fig. 1: CD. BC. BD

AAA	EAE	AII	EIO	AAI	EAO
Barbara	Celarent	Darii	Ferio	Bar*bari*	Ce*laro*

Fig. 2: DC. BC. BD

EAE	AEE	EIO	AOO	EAO	AEO
Cesare	Camestres	Festino	Baroco	Ces*aro*	Cames*tros*

Fig. 3: CD. CB. BD

AAI	EAO	IAI	AII	OAO	EIO
Darapti	Felapton	Disamis	Datisi	Bocardo	Ferison

Fig. 4: DC. CB. BD

AAI	AEE	IAI	EAO	AEO	EIO

Sunt quatuor, senosque modos habet una Figura.
Sponte duo veniunt, satis est effere quaternos.
Barbara, Celarent, Darii, Ferio, bari, laro
Cesare, Camestres, Festino, Baroco, saro, stros
Tertia grande sonans effert

Darapti, Felapton, Disamis, Datisi, Bocardo, Ferison/Barmasi. Calmerens (rop), Fesiso (sapo), Dimaris/Argumentorum quatuor numerata figurata sexque modos dato/qui sat est effere quaternos in tribus, in quarta satis est/edicere ternos. Barbara, Celarent, *primae* Darii Ferioque/Cesare, Camestres, Festino, Baroco *secundae/Tertia vult* Disamis, Datisi, Bocardo, Ferison/Calmerens quarta satis et Fesiso, Dimaris.

Ex genere ad speciem conclusio nascitur. Ultroque ⟨Ast⟩ in ⟨praemiss-⟩ specie genus exit ab ipsa illud prima dabit, sed in hoc se ⟨tertia — versat⟩. Alterutrum secunda, at quarta ⟨exorbet⟩ utrumque. Nam in secunda, secundum priorem collectionem, ex EAE, et AEE, fit EAO, AEO, seu in secundam per posteriorem colligendi rationem ex EIO, et AEE, fit etiam EAO, et AEO. Ex EAE per Celarent fit EAE. Ergo ex AEE fit AEO secundum priorem collectionem et ex EIO fit EAO ⟨-⟩ ex IAI fit AAI secundum posteriorem ex EAE fit EAE; ex Cesare fit Celarent.

Vocabula afficta sic interpretantur, a vocalibus per versus

Asserat A negat E verum generaliter ambae
Asserat I negat O sed particulariter ambae

Sed per literas consonas exprimere volunt modum reducendi ad primam

S vult simpliciter verti, P porro per acci
M vult transponi, C per impossibile duci.

Initiales autem literae ostendunt ad quem quis secundae aut tertiae referatur.

Cesare ad Celarent Cesare est ECD.ABC.EBD. Cesare reducitur ad Celarent. Demonstratio Ex ECD sequitur EDC. Ergo ex ECD.ABC sequitur EDC.ABC, ex EDC.ABC sequitur EBD per Celarent. Ergo ex ECD.ABC sequitur EBD, id est habetur Cesare. Hoc methodo et ad seqq. applicari potest, sed supprimemus.

Camestres ad Celarent. Illud transponitur ob M, simpliciter vertitur ob S. AEE transponendo dat EAE, simpliciter vertendo manet EAE, quod est Celarent.

Festino ad Ferio. Illud simpliciter vertitur ob S, nempe EI[O], fit EI[O] quod est Ferio.

Baroco ad Barbara, sed per impossibile, ob C. Nempe sit Barbara ACD.ABC.ABD. Ejus conclusio ponatur falsa, seu vera est opposita *OBD*. Hinc alterutra praemissarum falsa, ponamus veram esse *ACD*, ergo falsa erit ABC, vel vera opposita *OBC*; itaque ex ACD et OBD sequitur OBC, quod est Baroco.

Cesaro, Camestros, reducuntur ut [Cesare], Camestres.

Darapti ad Darii, ob P,A praecedens convertitur per accidens. Felapton ad Ferio, similiter.

Disamis ad Darii per transpositionem et conversionem simpliciter.

Datisi ad Darii per conversionem simpliciter.

Bocardo ad Barbara, per reductionem ad impossible.

ACD.ABC.ABD, sit conclusio falsa, seu *OBD* vera; ergo alterutra praemissarum falsa, ponamus veram esse *ABC*, erit falsa ACD, seu vera *OCD*. Et OBD.ABC.OCD est Bocardo.

Ferison ad Ferio, nuda conversione simpliciter facta.

Quartae figurae quidam apud Claudium Clementem has numerant: Barmari Calerent Dimaris Firemo[2].

Barmapi ad Barbara. ADC.ACB.IBD transpone fit ACB.ADC unde per Barbara sequitur ADB (vel per Barbari IDB) et convertendo IBD. Patet hic methodus receptas reducendi non succedere nam fit quidem conversio simpliciter, at non in data propositione ut hactenus. Calmerens ad Celarent. ADC.ECB.EBD. Transpone fit ECB.ADC unde per Celarent EDB et convertendo simpliciter EBD.

Dimaris, ad Darii. IDC.ACB.IBD. Transponendo ACB.IDC. unde per Darii IDB. et convertendo simpliciter IBD.

Firemos falsum est, et in nulla figura datur IEO. Certe in quarta sic refuto. Sit IDC.ECB.OBD

<u>C</u> <u>B</u> C impar B par
D D derivativus seu non ⟨primitivus⟩

Ajo id non sequi. Nam ob ECB debent rectae C et B ita collocari, ut nulla pars unius parti alterius respondeat, porro ob IDC, debet D ita collocari, ut pars eius repondeat ipsi C. Videamus jam an hinc sequatur quandam partem ipsius B non respondere ipsi D. Sed patet non sequi possem enim D sic producere ut tota B ipsi respondeat. Claudius Clemens tale exemplum affert Aliquod album est rivisum, Nullum rivisum est lapis. Ergo aliquis lapis non est albus. Sed hoc succedere per accidens ostendam per instantia. Numerum non ⟨primitivum⟩ uno verbo vocabo derivativus. Quidam derivativus est impar. Nullus impar est par. Ergo quidam par non est derivativus, quae conclusio absurda.

Ergo pro Firemos scribemus Ferimos

Fesiso ad Ferio. EDC.ICB.OBD. Tam majorem quam minorem convertendo simpliciter et fiet in Ferio ECD.IBC.OBD D C

B

Supersunt duo adhuc modi quartae figurae aliis neglecti, AEO et EAO, et quidem AEO consequitur ex Calmeres, itaque deduci posset ex Celarent, et quidem universa[lem] negativam convertendo per accidens; nam quicquid simpliciter converti potest, id etiam per accidens converti potest. Itaque scribemus

Calmerop ex Celarent. ADC.ECB.OBD. Ex praemissis transponendo ob M, fiet ECB.ADC, unde per Celarent fit EDB, et conversione per accidens OBD.

Superest EAO, quod reducitur ad Ferio, duplici conversione, una simpliciter altera per accidens, et scribetur

Fesapo ad Ferio. Nempe ex EDC.ACB.OBD, utriusque praemissae conversione, prioris simpliciter, alterius per accidens, fit ECD.IBC, unde in Ferio fit OBD.

Habemus ergo has sex quartae modos ad communem formam expressos Barmasi Calmerens Dimaris Fesiso Calmerop Fesapo/Barmasi, Calmerens, rop, Fesiso, sapo, Dimaris

Hinc versus Barbara, Celarent, Darii, Ferio,
bari, laro/Cesare, Camestres, Festino,
Baroco, saro, stros/Quaeque modos
totidem sic dat Darapti Felapton/
Tertia cum Disamis Datisi Bocardo
Ferison/Barmasi Calmerens, rop,
Fesiso, sapo, Dimaris/Nulli IEO,
cunctis EAO, AAA datur uni.
Aliter modo ext.

Calmentes, Baralimp, Digamis, Fesapo, Fresiso
apud Collium
Cameres, Baramip, Diramis, Fesapo, Fesiso vel
post tertia cum Disamis Datisi Bocardo Ferison
poni potest
Commune est EAO, cui quartae transdata jung.
Celantes Baralip Dabitis Fapesmo Frisesmo

Superest ut consideremus modos quos indirectos vocant, in prima collocant quinque. Hi tales sunt apud Petrum Hispanum. Primus modus indirectus figurae primae est *Baralipton* Omne animal est substantia. Omnis homo est animal. Ergo quaedam substantia est homo. Sed sciendum est, si regulam hanc generalem ponamus, ut minor propositio sit quae continet subjectum, major quae continet praedicatum conclusionis, uti certe aliquid tale assumendum, ut definitio certa habeatur, patet, revera syllogismum esse figurae quartae; si modo transponamus praemissas, seu majorem prop. constanter primo loco, minorem prop. secundo ponamus, nam fiet Omnis homo est animal, omne animal est substantia, ergo quaedam substantia est homo, qui est modus *Barmapi*. ADC.ACB.IBD. Secundus indirectus primae est *Celantes*, [ ] Nullum animal est lapis Omnis homo est animal Ergo Nullus lapis est homo. Hinc similiter transpositis praemissis fiet Omnis homo est animal. Nullum animal est lapis. Ergo nullus lapis est homo ADC. ECB. EBD qui modus est quartae *Calmerens*. Tertius indirectus primae est *Dabitis*: Omne animal est substantia. Quidam homo est animal. Ergo quaedam substantia est homo. Sed transpositis praemissis fit: Quidam homo est animal. Omne animal est substantia. Ergo quaedam substantia est homo. IDC. ACB. IBD qui est modus quartae *Dimarris*. Nota si concludit conclusio ex IDC. ACB. IBD multo magis concludet ADC.ACB.IBD. Itaque ex Dimarris sequitur Barmapi.

Hos tres modos indirectos notat Petrus Hispanus oriri ex directis primae figurae modis per conversionem conclusionis simpliciter. Sed deinde subjicit modos adhuc binos indirectos primae qui reducuntur ad directos, conversione utriusque propositionis. Itaque Quartus indirectus primae est *Fapesmo* Omne animal est substantia. Nullus lapis est animal. Ergo quaedam substantia non est lapis. Sequitur ex Celantes. Caeterum ad quartam reducitur transponendo Nullum lapis est animal, Omne animal est substantia. Ergo quaedam substantia non est lapis. EDC.ACB.OBD est *Fesapo* figurae quartae.

Quintus indirectus primae est *Frisesmo* Quoddam animal est substantia. Nullus lapis est animal. Ergo quaedam substantia non est lapis. Transponendo fit Nullus lapis est animal. Quoddam animal est substantia. Ergo quaedam substantia non est lapis. EDC.ICB.OBD, qui est modus *Fesiso* quartae.

Sed sextum modum obliti sunt logici, *Celantop*, qui sequitur ex Celantes. Nam haec conclusio succedit. Nullum animal est lapis. Omnis homo est animal. Ergo nullus lapis est homo. Succedit etiam Ergo quidam lapis non est homo. Et hoc est modus quartae a me dictus *Calmerop*.

Ex his patet quatuor modos primae indirectos, qui revera quartae sunt, oriri ex conversione conclusionis primae, quod in Barbara fieri potest uno modo, in Celarent duobus modis, quia E potest converti simpliciter et per accidens, in Darii uno modo, in Ferio nullo modo, quia particularis negativa ne per accidens quidem converti potest. Porro Barbari et Celaro non possunt producere quod non dent Barbara et Celarent. Hoc ergo modo ex prima figura sequentes quartae deducentur Barmapi, Calmerens, Calmerop, Dimarris. Sed Fesapo et Fesiso alia ratione deducendi sunt.

Idem est de indirectis primae Fapesmo et Frisesmo. Itaque Fesiso demonstratur per Ferio, dum enim in Fesiso utraque praemissa convertitur simpliciter fit Ferio, et Fesapo similiter, dum enim una praemissa in Fesapo convertitur per accidens, et altera simpliciter, oritur Ferio.

Itaque duobus modis ex prima figura colligimus quartam, unus est, ut quamlibet conclusionem primae convertamus quantum converti potest, et deinde praemissas transponamus; alter modus est ut utramque praemissarum convertamus, ita nulla opus transpositione praemissarum. Prior modus dat modos quatuor, secundus dat modos duos. Pro modis quos vocant primae indirectos priore modo non est opus transpositione praemissarum, sed posteriore est opus.

Ex meris negativis nil sequitur. B —

Ex meris particularibus nil sequitur C —

Conclusio sequitur praemissam debiliorem D —

Hae propositiones generales nondum satis figuris exponi possunt. Terminus velut B, est distributus, si dicere possim omnia B, v.g. omnes homines sunt omnia animalia rationalia. Hic tam subjectum quam praedicatum est distributum, sed hoc non est necesse vi formae: item dici potest terminum est distributum si propositio vera manet quocunque termini substituti. Itaque revera distributus est universalis, non distributus est particularis. Vi formae eadem est quantitas subjecti et propositionis. Praedicatum vi formae est particulare in omni propositione affirmativa, et universale in omni propositione negativa. Quidam homo non est doctus. Substitue quecunque doctum, v.g. Socratem, Platonem, Aristotelem, vera manet propositio, quasi diceres Quidam homo differt ab quovis docto. Omnis homo est sentiens, id est quovis homo idem est cum aliquis [*bricht ab*]

Nullus homo est lapis, id est quivis homo differt a quovis lapide. Omnis homo est animal, id est quivis homo coincidit cum quodam animal Quidam homo est doctus, id est quidam homo coincidit cum quodam docto Nullus homo est lapis, id est quivis homo differt a quovis lapide. Quidam homo non est doctus, id est quidam homo differt a quovis docto

Conclusio est de identitate vel diversitate contenti in termino minore cum contento in termino majore; ea probatur ex eo quod contenta in utroque extremo sunt eadem vel diversa cum aliquo communi contento termini medii. Nam quae sunt eadem uni tertio sunt eadem inter se, quae sunt diversa uni tertio sunt diversa inter se.

Conclusio syllogismi categorici enuntiat identitatem vel diversitatem contenti in termino Minore, cum contento in termino Majore, ut si universalis sit affirmativa enuntiatur quodvis in termino minore, esse idem cum quodam in termino majore. Si particularis sit affirmativa enuntiatur quoddam in termino minore esse idem cum quodam in termino majore. Si sit universalis negativa enuntiatur quodvis in termino minore esse diversum a quovis in termino majore. Si sit particularis negativa, enuntiatur quoddam in termino minore esse diversum a quovis in termino [majore.]

Am Rande:

Qualitas	V vel (V)
Quant.	Y vel (Y)
Aff. F	Neg. G
Univ. S	P part.
A = FS	E = GS
I = FP	O = GP

Quantitas subjecti notat quantitatem propositionis; quantitas praedicati notat qualitatem propositionis, ergo sufficit omnia reduci ad quantitatem.

Sit quantitas X, oppositum (X), erit termini B,C,D

XBZD conclusio Med Y vel Ω

Si Y vel Ω est P, Ω vel Y est S

Si Z est S

(1) Medius terminus debet esse universalis in alterutra praemissarum nam si in utraque est particularis, non habetur idem ejus contentum comparari contentis in termino majori et minori; itaque nec habetur comparatio horum inter se.

(2) Alterutra praemissa debet esse affirmativa, nam si medius terminus a contentis utriusque extremi differt, nulla per ipsum habetur comparatio eorum inter se.

(3) Terminus particularis in praemissa est particularis in conclusione, nec enim plura de extremis conferri possunt, quam quae collata sunt cum termino medio per quem extrema conferuntur.

(4) Si una praemissa sit negativa, etiam conclusio est negativa, nam ex eo quod differt X a Y, et idem est Z et Y, non potest colligi nisi X differre a Z. Non potest autem poni bis diversitas per prop. 2.

(5) Subjectum propositionis universalis est universale, particularis particulare.

(6) Praedicatum propositionis affirmativae vi formae est particulare, negativae universale.

Ex his quinque fundamentis, omnia theoremata de Figuris et modis demonstrari possunt.

(7) Si conclusio sit universalis minor propositio vel est universalis vel negativa. Nam si conclusio sit universalis subjectum ejus nempe minor terminus est universalis per prop. 5. Itaque minor terminus est universalis in minore propositione per prop. 3. Ergo si ejus subjectum est erit ipsa universalis, si ejus praedicatum est erit ipsa negativa, q.e.d.

(7) Si conclusio sit negativa, major propositio vel est universalis vel negativa, nam si conclusio est negativa major terminus est universalis (per prop. 6). Ergo et universalis erit in majore propositione per prop. 3. Ergo si subjectum in ea est, ipsa erit universalis, si in ea praedicatum est, ipsa erit negativa.

Die folgenden zwei Ansätze wurden gestrichen:

(8) Alterutra praemissarum debet esse universalis; nam si utraque est particularis, utriusque subjectum est particulare, et quia alterutra est affirmativa, alterutrius praedicatum est particulare, Medius autem semel est universalis (per prop. 1), ergo non potest esse subjectum utrobique.

(8) Non potest simul esse major propositio particularis et minor propositio negativa. Nam si minor propositio est negativa etiam conclusio erit negativa (per (*bricht ab*)

(8) Si minor prop. sit negativa, major est universalis. Nam si minor prop. est negativa, etiam (per 4) conclusio est negativa. Ergo (per prop. 6) major terminus in conclusione est universalis. Ergo (per prop. 3) etiam in majore propositione major terminus est universalis, sed major propositio est affirmativa per prop. 1 (quia minor negativa ex hypothesi), ergo major terminus ibi non est praedicatum sed subjectum, et cum sit universalis etiam (per 5) major erit universalis. Q.E.D. Corollar.: Itaque non datur modus IEO.

(9) Si major prop. est particularis minor prop. est affirmativa. Nam (praecedente conversa per contrapositionem) si major non est universalis minor non est negativa, seu si major est particularis, minor est affirmativa, quia medium inter universale et parti [culare et] inter aff. et neg. hic non datur.

(10) Si major est negativa (*bricht ab*)

Si propositio sit universalis negativa, uterque ejus terminus est universalis.[4]

Si conclusio est universalis, erit utraque praemissa universalis. Si alterutra praemissa est particularis, conclusio est particularis.

Si conclusio non est particularis erit universalis. Si conclusio est universalis, erit in ea minor terminus universalis. Si minor terminus est universalis in conclusione, erit minor terminus universalis in minore propositione.

Si jam conclusio est affirmativa, erit et minor prop. affirmativa, ergo minor terminus est subjectum prop. minor. Ergo minor prop. est universalis. Rursus his positis, erit medius terminus praedicatum prop. minoris quae cum sit affirmativa, erit med. term. particularis in minore prop. Ergo erit universalis majore. Sed major prop. est aff. Ergo med. term. est in ea subj. Ergo et ipsa est universalis. Rursus si conclusio universalis sit negativa erit uterque terminus extremus universalis. Ergo utraque propositio aut universalis aut negativa;

si utraque universalis, habemus quaesitum. Si utraque sit particularis foret utraque negativa, quod non licet. Ergo una est universalis, altera particularis, quae particularis ea est negativa. Ergo extremum est ejus praedicatum. Ergo medius ejus subjectum, ergo med. particulare. Ergo medium in altera est universale. Sed altera est affirm. Ergo medium in ea subjectum. Ergo et ipsa universalis.

Si conclusio est universalis, utraque ejus praemissa est universalis. Si conclusio est universalis, minor in ea terminus est universalis (art. 11). Itaque minor terminus est universalis in minore prop. (art. 20). Si jam conclusio sit affirmativa erit et minor prop. affirmativa (art. 21). Ergo praedicatum ejus est particulare (art. 9). Ergo minor terminus quippe universalis est ejus subjectum, Ergo (art. 11) minor prop. est universalis. Rursus medius terminus est ejus pradicatum, Ergo (art. 11) medius term. in minore prop. est particularis (art. 9). Ergo (art. 19) medius terminus est universalis in prop. majore. Sed quia conclusio affirmativa est (ex hyp.), Ergo maj. prop. est affirmativa (art. 21). Ergo medius terminus, quia universalis, est subjectum in majore propositione (art. 9). Ergo prop. major est universalis (art. 11). Itaque si conclusio sit universalis affirmativa, utraque praemissa est affirmativa. Quod si jam conclusio sit universalis negativa, erit uterque terminus extremus universalis (art. ...) Ergo quaelibet praemissa est aut universalis aut negativa. Si utraque sit universalis habetur intentum, utraque non potest esse negativa (art. 21). Superest ut una sit universalis, alt particularis. Quae particularis ea erit negati ergo (art. 21) quae universalis erit affirmativa. E ergo praedicatum non potest esse extremum (: 9), id enim universale est, sed medium; e medium in ea praemissa erit particulare; e oportet ut in altera praemissa quae est particul negativa medium sit universale. Sed jam ostens est extremum etiam esse universalem. E uterque terminus est universalis. Ergo ipsa pro sitio est universalis. Itaque demonstratum utramque praemissam esse universalem. In pr particulari affirmativa uterque terminus est part ularis. Si minor terminus sit particularis praemissa, conclusio est particularis.

Duae particulares nihil concludunt.

Nam semper altera praemissarum est affirmat (art. 15). Si ergo duae sunt particulares, erit u particularis affirmativa. Ejus ergo uterque termi est particularis (art. . . .) tum extremus tum et qui ei (art. 18) inest medium. Quia ergo medius hac praemissa est particularis, debet in alt praemissa medius esse universalis; sed hoc f nequit (art. . . .). Si etiam ipsa est particula affirmativa, ergo altera praemissa est particula negativa, et medius, cum universalis esse debeat ea (art. 10) est praedicatum; itaque ipsum e extremum est subjectum, id ergo est particul Habemus ergo ambo extrema particularia praemissis, ergo et minoris extremum, itaque (11). (*bricht ab*)

Apparatus criticus

For the *Apparatus criticus* we have adopted the same conventions as in the critical edition of Leibniz's work: G. W. Leibniz, *Sämtlic Schriften und Briefe*, Hrsg. von der Deutschen Akademie der Wissenschaften zu Berlin, Darmstadt 1923ff, Leipzig, 1938ff, Berlin 1950ff.

41*a*, 1 Proba sunt quae hac plagula et sic satis haberi possunt pro absolutis *am oberen Rande* erg *L.*

2–4 licebit: (1) ex consi *bricht ab*; (2) per reductionem ad (a) Leges (b) considerationem . . .

4–5 diversi: (1) In Enuntiatione (2) Nam . . . ut (a) aliqu *bricht ab*; (b) duo . . .

7 ff. (1) Omnis propositio affirmativa enuntiat (a) subjecti (b) aliquid /idem/ *streicht L.* contentum in subjecto (ba) idem esse cu contento in praedicato, sive omne sive quoddam (bb) idem esse cum quodam contento in praedicato, (bba) nec (bbb) quo omne contentum in subjecto *bricht ab*; (2) Terminus ut homo, significat homines, velut Petrum Paulum. Cum dico Quid homo, Omnis homo, intelligo omnes contentes in hominum numero. (3) Terminus (a) homo (b) (velut homo) propositione/ *erg. L.* vel . . .

12–13 et haec . . . Affirmativa *erg. L.*

41*a–b*, 25–2 Hinc . . . universale *erg. L.*; Et hoc exp *bricht ab, streicht L.*

41*b*, 3–4 quidem: (1) dici (2) notari (3) Omne A . . . seu (a) quemlibet eorum qui /qui/ *streicht L.* dicuntur (b) omnes . . . A, esse (t Omnes (bb) eosdem . . .

5–6 seu . . . reciprocam *am Rande erg. L.*

7 Quemadmodum /nec/ *erg. L.* (1) quoddam A esse omnes B (a) posset etiam (b) seu omnes A esse quosdam B (2) quosd . . . esse (a) quosdam A (b) A.

A: (1) Absurdum autem fuerit dicere quemlibet eorum (2) Inutile . . .

3 Patet: (1) saltem in abstractis abstrahendo ab eo quod case (2) nisi . . . unicum *am Rande erg. L.* (a) quemadm *bricht ab* (b) et . . .

6 Ita . . . identitatem *erg. L.*

9 (8): (1) Ex (2) A . . . categoricae /simplices/ *streicht L.* appellantur *erg. L.*

1 (1) (8); (2) (9) . . . /et solam/ *erg. L.*

3 particulare: (1) Et om *bricht ab* (2) per (a) prop. (b) art. 3 et 4 (ba) Et (bb) (10) Et omnem /ac solam/ *erg. L.*

(11): (1) Patet (2) Porro . . .

9 (1) (12) In syllogismis id agimus ut ex duabus (2) (12) Syllogismi /quos . . . vocant/ *erg. L.* ex . . .

utendo: (1) hoc principio (2) duobus . . .

tertio: (1) sunt (2) esse

si: (1) B sit idem ipsi C, et D ipsi C, eadem esse (a) B et C (b) B et D (2) L . . .

Alterum: (1) si unum (2) quorum unum tertio idem, alterum (3) huc redit (a) quorum unum tertio i *bricht ab* (b) diversa . . .

diversum: (1) Ut si B sit idem ipsi C et C non sit idem ipsi D, nec B et D eadem esse (2) Ut . . .

si: (1) nec B sit idem ipsi C, nec D sit idem ipsi C, nec (2) B non sit idem ipsi C, (a) nec D sit (b) et D non sit idem ipsi C, non potest inde cognosci utrum B et D *bricht ab* (3) L sit . . .

potest: (1) ut B sit idem ipsi D, vel etiam ut B sit diversum ipsi (a) C (b) D (2) ut L . . .

4 syllogismum: (1) veluti (2) Nam si propositio est negativa (3) ita enim (a) nihil (b) revera . . .

5 /esse diversum ab (1) N (2) M *verbessert Hrg.* / *erg. L.*

est: (1) nullum (2) quemlibet . . .

principium: (1) conjungendi (2) comparandi . . .

dicam: (1) Quidam canis non est *bricht ab u. streicht L.* (2) Quidam . . .

est.: (1) (15) (a) Hinc (b) Eodem modo ostendi potest terminum med *bricht ab* (2) (16) Patet . . .

aliquid: (1) invenimus (2) adhibemus . . .

se.: (1) Extremorum priorem a quo propositio universa *bricht ab* (2) Itaque (a) con (b) pro *bricht ab* (3) (17) Hic . . . quam (a) quaerimus (b) ex . . .

conclusionem: (1) conclud *bricht ab* (2) inferimus . . . appellantur (a) ex quibus M (b) in quarum . . .

Majorem /terminum/ *erg. L.* . . . ipsa (1) appell *bricht ab* (2) Propositio . . . Minorem /terminum/ *erg. L.*

his: (1) inferri potest (2) patet . . . Terminum (a) in utraque (aa) propositio (ab) prae *bricht ab* (b) in alterutra . . . /Medii/ *erg. L.*

dicat: (1) Omnis homo est animal (2) Quidam . . .

cum: (1) aliquo fe (2) quodam (3) aliquo (4) quodam felice. /Sed/ *streicht Hrg.* (a) omnis (b) quivis (c) Sed omnis . . .

doctus idem est cum: (1) quodam *streicht L.* (2) quodam . . .

cum: (1) utrobique occu *bricht ab* (2) bis . . . homo (a) non potest (b) potest alius . . .

argumentum: (1) ad comparandu *bricht ab* (2) ad conferendum . . .

potest,: (1) aut (2) ut . . . colligatur (a) aut (b) an . . . vel (ba) non dive *bricht ab* (bb) idem (bba) ⟨con⟩ (bbb) felici (bbc) om *bricht ab* (bbd) alicui . . .

non: (1) posse esse universalem (2) inferri . . .

minus: (1) apparet (2) manifestum est . . .

negativam /et vicissim *erg. L.* (1) quia diversita *bricht ab* (2) quia non . . .

simplicium: /(1) ex dispositio *bricht ab* (2) qu *bricht ab* (3) discriminatione . . . situ/ *am Rande erg. L.*

enim: (1) ter *bricht ab* (2) minor terminus B, (a) major (b) medius . . .

D: (1) Porro (2) Conclusio . . . est (a) BC (b) BD. (ba) Praemissae (bb) In Praemissis . . .

praedicatum in: (1) potes (2) posteriore; vel (a) subjc (b) praedicatum . . .

subjectum: (1) est (2) in *verb. Hrg.* posteriore. (a) Unde (b) /Solemus . . . posteriore/ *erg. L.*

nobis: (1) utrum propositiones sunt (2) propositionum . . .

autem quantitas: (1) termini (2) subjecti . . .

significabit: (1) propositionem (2) terminum . . . Y (a) Ω (b) Ψ . . .

(1) Propositio universalis designabitur per subjectum universale (a) prae *bricht ab* (b) particularis per praedicatum universale (c) ⟨-⟩ (2) Propositionis quantitas . . . signum /universale/ *streicht Hrg.* qualitas . . .

affirmativa,: (1) ⟨IBSD⟩ (2) PBSD *verb. Hrg.*

negativa,: (1) ⟨IBID⟩ (2) PBPD *verb. Hrg.* . . . affirmativa /SB in (a) praedicato (b) conclusione SC medius universalis/ *streicht L.*

Am Rande erg. L.

In: /omni et sola *erg. L.*/ propositione (1) univers (2) particulari . . . particularis. Nam (a) major est (b) minor (c) subjectum est . . .

-2 Coroll. . . . negativa *nachtr.erg.L.*

propositione: (1) affirma (2) universali . . .

7	Ubi: (1) major (2) minor . . .
9	etiam: (1) major (2) minor propositio /est *erg. L./* . . .
13–14	(artic.20): (1) Ergo (2) In qua . . .
15	universalis: (1) Coroll. Hinc ubi minor terminus est subjectum, (a) conclusione existente (b) minore propositione existent particulari, etiam eo *bricht ab* (2) Corollarium quod per hujus *bricht ab* (3) Coroll. . . .
22	haec: (1) ut (2) quod . . .
24	etiam: (1) maj (2) minor . . .
24–25	universalis: (1) sed (2) et sane . . .
30	particularis: (1) quia minor (2) quia terminus . . .
32	conclusione / (art.20) *erg. L.* / Minor: (1) ergo (2) vero . . .
34–36	(art.11): (1) Coroll. Si conclusio sit universalis, minor terminus est universalis ubique. Coroll. 2. *bricht ab und streicht L.* (2 Coroll. Si . . .
39	est: (1) negativa (2) affirmativa . . .
41	Ergo: (1) propositio (2) conclusio . . .
45 ff.	(29): (1) Si conclusio sit universalis, minor prositio est vel universalis vel negativa. (2) Si conclusio sit universalis affirmati minor terminus est subjectum in (a) praemissa (b) prop. minore, et major terminus praedicatum in propositione majore. Na quia conclusio est universalis minor terminus in ea est universalis (art. 10) Ergo universalis est in praemissa (ba) sed (b (art. 20). Sed ea est affirmativa (art.21) ergo (bba) prop (bbb) terminus universalis in ea existens est (per art. 10) ej subjectum. (3) Si . . .
43 *b*, 2–3	(art. 10): (1) Coroll. Si conclusio sit negativa et (2) (30) (a) Si conclusio (b) Si minor . . .
5	(art. 15): (1) ergo ob conclusionem negativam (a) (art.) *bricht ab* (b) (ex hyp.) (ba) conclusio (per art. 29) est universalis (b major terminus in ea est universalis, ergo et in majore prop. (art. 21) (2) porro . . .
9	datur: (1) modus (2) Syllogismus . . .
10–11	minor: (1) affirmativus (2) neg *bricht ab* (3) universalis . . .
12	affirmativa: (1) conclusio cadit in (2) Syllogismus . . .
	affirmativa: (1) quia con *bricht ab* (2) (art. 21) . . .
19–20	praedicatum: (1) Ergo (2) (art. 10) . . . subjectum (a) et cum sit universalis (ut ostensum) facit et ipsam universalem (b) Itaq . . .
24	Universalis: (1) Itaque (2) sed . . .
26–27	esse: (1) subjectum (2) praedicatum, sed subjectum (a) Ergo major terminus in ea erit praedicatum (b) Cum ergo (ba) mi ⟨-⟩ (bb) medius . . .
30	(1) (31) (2) (32) Duae particulares (a) nihil concludunt (b) non . . .
33	/hoc casu/ *nachtr.erg. L.*
35	ergo: (1) et medius terminus (2) extremum . . . ergo /medius/ *erg. L.*
37	(ex hyp.): (1) erit particularis negativa (a) ut medius (b) Medius in ea non est subjectum, (art. 11). Ergo in ea est praedicat Itaque ipsa est negativa (art. 10). (ba) Sed alter extremus (bb) Ergo et alter extremus est particularis *bricht ab* (2) *negative* medius universalis in ea possit esse praedicatum sed alter extremus Ergo *bricht ab* (3) medius . . .
43–44	/(art.20) . . . Ergo/ *nachtr.erg. L.*
45–46	absurdum: (1) (art. ⟨-⟩ (2) quia . . . (art. 21) (a) et *streicht Hrg.* (b) conclusio . . .
44 *a*, 1–2	universalis. /Nam *nachtr.erg. L./* Si . . . universalis: (1) minor in ea terminus est universalis (art. 11) Itaque minor termi est universalis in prop. minore (art. 20) affirmativa. Syllogismus est in prima figura. Itaque cum ambae propositiones affirmativae (art. 21) (a) minor et (b) minor terminus sit universalis (2) minor . . .
9	terminus /terminus *streicht Hrg./* est . . .
14	(art. 26): (1) Ergo (a) utraque propositio (b) quaevis praemissa vel est universalis vel negativa (coroll. art. 25) ergo non da praemissa particularis affirmativa (ba) Si ambae sunt (bb) ambae non possunt esse negativae quia conclusio est affirma (art. 13) nec ambae particulares (art. 31). Quod si ambae sunt universales habemus intentum superest ut una sit affirma altera negativa (art. 21), et ut una sit particularis, altera universalis (bba) ponamus (bbb) Et res redit ut (bbc) quod si una particularis ea erit aut affirmativa aut negativa. Si una propositio sit particularis affirmativa erit utriusque ejus (bbd) e uterque extremus in praemissis est universalis (art. 20) (2) Ergo . . .
15	/(artic. 25) *nachtr.erg.L./*
16	ut: (1) datur praemissa particularis (2) si . . .
20–21	eadem erit: (1) parti (2) praedicatum . . .
22	altera: (1) propositione (2) praemissa . . .
22	particulari: (1) ⟨-⟩ (2) negativa . . .
24	subjectum: (1) ergo (2) sed . . .
26–27	universalis (1) contra (2) itaque absurdum /etiam est *nachtr. erg.L./* . . .
27	negativa: /si conclusio *erg. u.streicht L./* . . .
27–28	praemissa: (1) locum habet (2) potest esse particularis (a) Q.E.D. Conclusione exi *bricht ab* (b) sive . . .

/Schol. . . . negativam *am oberen Rand erg. L./*

len, vermutlich zu **43 b**, 30—**44 a**, 30 *gehörigen Entwürfe wurden gestrichen*

Praemissa Omnis prop. partic. affir. habet utrumque terminum particularem.

Ex meris praemissis particularibus nil sequitur. Si duae praemissae sunt particulares *bricht ab* Si alterutra praemissa sit particularis, etiam conclusio est particularis. Nam esto major particularis, ergo si sit affirmativa, etiam term. major est partic. Ergo conclusio est affirmativa. Ergo et minor prop. est aff. Ergo et minor term. est part. Ergo et conclus. est partic. Sin major partic. sit negativa, ergo et conclusio neg. ergo maj. term. univ. ergo major term. est praedicatum Ergo med. est subjectum. Ergo med. est partic. in maj. prop. Porro minor prop. est aff. et med. in ea debet esse universale. Ergo med. in ea est subjectum. Ergo minor term. in ea est praedic. Ergo minor term. in praem. est partic. Ergo et conclus. part. Esto minor prop. partic. Ergo si sit aff. et ⟨-⟩ term. ⟨-⟩ Omnis praemissa particularis affirmativa et utrumque terminus particularem. Ergo si una praemissa est particularis affirm. debet altera esse negativam et medius ejus praedicatum. Ergo alter terminus ejus subjectum. Ergo uterque extremorum particular ⟨—⟩ Ergo conclusio particularis.

Si alterutra praemissa sit particularis, etiam conclusio est particularis. Nam *bricht ab*

Si conclusio universalis, erit minor term. universalis. Si minor terminus universalis erit minor prop. vel universalis vel negativa. particularis aut nega ⟨-⟩ Si neg. erit conclus. neg. Ergo uterque term. universalis. Si major term. universal. erit major prop. vel universal. vel neg. Sed non est neg. si minor neg. Ergo aff. ergo maj. prop. univ. si minor prop. aff. et sit partic. Si conclusio sit universalis minor propositio est vel universalis vel negativa

(1) (25) (2) (34) . . .

major /propositio *erg. L./* . . .

nempe: (1) terminu *bricht ab* (2) (art. 17) . . .

(1) (26) (2) (35) . . .

Ergo /per *streicht L./* . . .

Coroll. . . . affirmativa *nachtr.erg. L.*

(1) (28) (2) (36) . . .

Ergo: (1) (art. *bricht ab* (2) et . . . (art. 20) (a) Ergo (b) Sed . . .

(art.10) /prop. *streicht L./* . . .

est: (1) subjectum (2) praedicatum . . .

(1) (28) (2) (37) . . .

(art. 19): /universalis *streicht L./* sed (1) terminus (2) universale . . .

(art. 10) /negativam *streicht L./* . . .

Ergo /(art.21) *erg.L./* . . .

negativa: (1) (art. 28) (2) (art. 37) *verb. Hrg.* . . .

(1) (29) (2) (30) (3) (39) . . . subjectum /seu . . . figura *erg. L./* . . .

particularis: (1) Nam (a) ubi (b) si medius semper est subjectum, uterque extremus (ba) est (bb) in praemissis est praedicatum, ergo in praemissa affirmativa extremum est particulare (2) Esto . . . ergo (a) major est un *bricht ab* (b) minor (ba) est (bb) terminus (bba) est (bbb) in ea . . .

est: (1) universalis, ergo (2) universalis, sed . . .

negativa: (1) Ergo et conclusio erit (2) (art. 10). Ergo . . .

(art. 20): (1) Ergo (2) Sed . . .

altera: (1) erit (2) praemissa . . .

prima: (1) absorb *bricht ab* (2) inutile . . .

ubi: (1) praedicatum (2) subjectum . . . negativa (a) Si ea (b) Demonstratur . . .

erit: (1) affirmativa (2) negativa . . .

esse: (1) particularis (2) affirmativa . . .

(1) (30) In prima figura minor propositio debet esse affirmativa. Nam (art. 22) (*bricht ab*) (2) (30) Ibidem semper minor (a) est affirmativa (b) propositio est affirmativa. Nam si minor propositio esset negativa, utique et conclusio foret negativa. itaque et major terminus in conclusione (art. 20) (ba) Sed in ea (bb) jam ubi conclusio est negativa et major terminus est praedicatum /in praemissis *erg. L/* (bba) praedicatum (bbb) etiam major propo (ut hic) (bbc) (ut in prima et (bbca) sec. (bbcb) tertia fig. (art. 22) major propositio est negativa (art. 26). (3) (42) In . . .

Ergo: (1) ergo (2) tam (a) maj quam minor (b) major . . .

(1) (31) In prima figura (a) major propositio debet (b) minor propositio debet esse affirmativa. Nam (ba) in ea (baa) una (bab) major (bb) in ejus praemissis (bba) major terminus est (bbaa) subjectum (bbab) praedicatum (bbb) minor terminus est subjectum. Ergo si minor propositio est negativa, erit et conclusio negativa (art. 21). /conclusio negativa *streicht L./* (bbba) ergo major terminus in conclusione est universalis (art. 10), est universalis, ergo et in majore propositione (bbbaa) sed in ea (art. 20) est praedicatum (bbbab) sed in ea est praedicatum (bbbaba) (prop. 22) (bbbabb) (art. 22) Ergo et major propositio est negativa (bbbb) Sed ubi conclusio est negativa, et major terminus est praedicatum in praemissa (ut hic, per art. 22) etiam major propositio est negativa (art. 26) Ergo tam major quam minor est subjectum in praemissa. (2) (31) In prima et secunda

figura major propositio est universalis. (a) Nam si major propositio esset particularis, utique et conclusio foret particula ergo minor (ba) prop *bricht ab* (bb) terminus est particularis (b) In utraque minor terminus est subjectum. Si jam ma /prop. *nachtr.erg.L.*/ esset particularis, etiam conclusio foret particularis, ergo minor terminus foret particularis conclusione. (3) (a) (31) (b) (43) In . . .

31–32 universalis: (1) Nam in ea minor ter *bricht ab* (2) Nam in ejus praemissis minor terminus est subjectum, et major terminus praedicatum, et minor prop. est affirmativa (3) Nam . . .

32 affirmativa: (1) (art. 20) (2) (art. 40) (3) (art. 42) *verb. Hrg.* (a) Ergo medius terminus in (aa) majore (ab) minore prop. particularis, ergo in majore (aba) est (abb) prop. est universalis (b) et . . .

39 /Sequitur . . . 42 *nachtr. erg.L.*/

40 (1) (33) Ubicunque medius terminus est praedicatum in praemissa, /prop. *streicht L.*/ major est universalis. Huc ubicun minor terminus est subjectum in praemissa, figura est prima vel secunda (artic. 22). (2) (44) Si (a) minor (b) medius . . . subjectum (bb) praedicatum . . .

44–45 universalis: (1) (artic. 32) (2) (artic. 43) . . .

45–46 /etiam *erg.L.*/ . . . universalis /(artic) *streicht L.*/ (artic. 38) . . .

47 ff (1) (45) Ubi medium modo subjectum est modo praedicatum, si ea (a) propositio (b) praemissa in qua est praedicatur affirmativa, altera erit universalis. Nam *bricht ab; am Rande gestr.L.* (2) (45) . . .

48 negativa: (1) Sunto enim (10) DC et *bricht ab* (2) Sunto enim (a) calcu *bricht ab* (b) SDΨC prop *bricht ab* (3) Esto . . .

45*b*, 1–2 negativa: (1) SCΨB (2) ΨCSB *verb.Hrg.* . . . conclusio (a) PB (b) /particularis *erg.Hrg.*/ negativa . . .

3 quia: (1) in majore (per 20) (2) (art. 20) . . .

4 in: (1) minore (2) conclusione *verb.Hrg.* . . .

5 simul: (1) major (2) est minor . . .

7 PCΨB: (1) erit conclusio PBΨD; (2) sed . . .

8 artic. 19: (1) Schol. Quarta fi *bricht ab* (2) Est corollarium (3) Potest . . .

10 (47): (1) Quaecunque (2) Quavis . . .

19 constat: (1) ⟨-⟩ (2) quis . . .

20–21 extremus: (1) ut ergo nihilominus generaliter calculare liceat, poterimus scribere terminos CD et DC, vel CB et BC praemi *bricht ab* (b) Hinc possumus dicere (c) Duas praemissas negativas non simul habere locum, sic exprimemus: simul ΨCDSDC et ΨCBSBC. (ca) Non simul me *bricht ab* (cb) Medium non esse bis (cba) distributum (cbb) particula (2) Ergo . . .

22–23 velut: (1) C ⟨-⟩ (2) C et (3) C ⟨-⟩D (4) Ψ?D (a) Itaque proposition *bricht ab* (b) Ubi (ba) propositio (bb) nullus . . . praem (bba) erit (bbb) termini . . .

23–24 /In genere *erg. L.*/

33 potest: (1) ΨC?SD et (2) ΨS.C?D . . .

36 D: (1) Itaque (2) Si . . .

37 fiet: (1) C?PX (2) non . . .

38 SX?(X): (1) Si conc *bricht ab* (2) praemissa negativa etiam conclusio (3) Non sim *bricht ab* (4) Si una praemissa neg crit et conclusio negativa. Sit praemissa (a) C et X (b) negativa (ba) C et SX (bb) ΨC non ≃ SX *bricht ab und streicht L.*

40–41 calculum: (1) Duae praemissae non simul negativae, erunt (a) ΨC non ≃ SD (b) ΨF non ≃ SG (c) (Ψ?S)F (d) Ψ?S (d et C non ≃ SB et C non ≃ (e) C ≃ non B (f) C non ≃ B (g) C *bricht ab* (2) Sed . . .

43 fiet: (1) ΨB (2) sic . . .

45 negativas: (1) diffi *bricht ab* (2) non . . . communi (a) nota (b) ratione . . .

47–48 sic: (1) ΨF non ≃ SG, ΨL non ≃ SM, (2) ΨC (3) ΨF non ≃ SG et ΨH non ≃ SK: (a) Si coincidunt (b) Ex his (c) I . . .

46*a*, 1 cum: (1) altera (2) aliqua . . .

4 ideo: (1) non apparet qua (2) nil . . .

10 /fundamentalium *nachtr.erg.L.*/

15 (50): (1) Quia (2) Propositio . . . ΨF.ΨG: (a) erit (b) negativa . . .

19 una: (1) ex posterioris et (2) litera . . . litera (a) prioris et ipsi C (b) prioris . . .

21 categorici: (1) nec utrobique potest Ψ posterior esse S, nec quae eadem po *bricht ab* (2) C . . .

22–23 extremae: (1) una X, altera (X) et (a) ⟨-⟩ (b) conclusio est (ba) ΨXΨ(X) quorum *bricht ab* (2) quae

23–24 subjectum: (1) est major (2) est . . . B (a) subjectum (b) praedicatum . . .

25–26 ΨBΨD: (1) ⟨Usu qua⟩ priorum coincidit ipsi (2) Propositiones fundamentales (a) ΨG et Ψ (b) ΨG et ΨK (c) non simu (ca) SK (cb) SG et SK in praemissis (2) non . . .

27 simul: (1) ΨG et ΨK (2) sunt . . .

28–29 non simul: (1) ΨC et (2) in . . . praemissa (a) ΨC (b) PC (ba) sive datur SG sive SK, dabitur (bb) non simul ut praemissa (bc) Non . . .

33 datur: (1) Ψ (2) SD . . .

34 extremum: (1) particulare in eo *bricht ab* (2) universale . . .

universale in: (1) praemissis (2) praemissa (a) ut (b) seu si ΨB et ΨD (bb) SB vel SD, erit et SB vel SD (c) seu si SB /vel SD *streicht L./* in conclusione, erit et SB /vel SD *streicht L./* in (ca) praemissis (cb) praemissa . . .
particulares: (1) Non datur simul (a) SF (b) PF et PH. Erit (2) Sed . . .
Nam: (1) minor term. est universalis et major terminus particularis. Ergo (art. 11) et minor prop. est aff. (art. 21). Ergo minor terminus universalis est ejus subjectum (art. 9). Ergo minor prop. est universalis (art. 11). Ergo et conclusion. (a) Porro etiam (b) Porro medius terminus in ea est particularis (art. 9). Ergo in majore prop. erit universalis. Nam (art. 21, 33) si (ba) am *bricht ab* (bb) praemissae sunt universales affirmativae, ita excl *bricht ab* (2) excluditur . . . tertia: (a) per (b) (art. 37 et (ba) 38 (bb) 39 *verb.Hrg.* . . .
ergo et in: (1) praemissa (2) minore . . .
(art. 9): (1) Sed (2) ergo . . .
(49): (1) Non datur *streicht L.* (2) Non . . . cujus /pro *bricht ab und streicht L./* major . . .
minor: (1) particularis negat (2) universalis . . .
(49): (1) Modus consistens (2) Modus . . .
negativa: (1) majo (2) minore . . .
figura: (1) ex his secunda et tertia per regres *bricht ab* (2) in . . .
ex: (1) legitimis (2) legibus . . .
praemissae: (1) ADC (2) SC.ΨD, (a) ΨBPD (b) ΨBPC *verb. Hrg.*
(1) SBPD (2) SBPC *verb.Hrg.*
(1) ⟨-⟩ (2) SCPD
(1) PBPD (2) PBPC *verb. Hrg.*
(1) SBPD (2) SBPC *verb. Hrg.*
(1) ⟨-⟩CPD (2) SCSD
(1) PBPD (2) PBPC *verb. Hrg.* . . . E.I. (a) ⟨-⟩ (b) O Ferio 6. /Si demonstremus primae Barbara Darii Celarent Ferio quod facile inde demonstremus subalternationem et hinc Barbari Celaro. Deinde ex his secundam et tertiam ⟨—⟩ per regressum Quartam ⟨—⟩ possibilium hic habemus quatuor modos *streicht L.*
sequitur: (1) Quod cum meris veris falsum infert, est falsum. (2) Hinc (3) Hinc quod . . .
possint: (1) Qu (2) Vulgo dant quatuor modos directe concludentes primae figurae, et inve *bricht ab* (3) Quartam . . .
3 /In . . . modos *nachtr.erg.L./*
(1) Vulgo quatuor (a) figur *bricht ab* (b) modis directis primae figurae addunt quinque indirectos. (2) Sunt . . .
(1) ⟨—⟩ dicit (2) Ultima ⟨-⟩ pro (3) Ultima ⟨—⟩ (4) Sufficerent Disamis Datisi Bocardo Ferison (5) Tertia . . .
(1) Sunt quatuor, senosque modos habet una figura. Sponte duo veniunt, satis est efferre quaternos Barbara Celarent *primae* Darii Ferioque Cesare Camestres, Festino, Baroco *secundae* Tertia dat Dis *bricht ab* (2) Cuique modi quatuor ⟨-⟩ bini sponte sequuntur ergo modi sex ⟨-⟩ sunt in quacunque figura (3) sunt ergo in (4) Sufficiat bini primae Darii Ferioque (5) Argumentorum . . .
Ferison: (1) Barmasi Calmerens, quartae (2) Calmerens . . .
Am Rande erg. L.
2 interpretantur: (1) versu vocalibus (2) a vocalibus (a) versu (b) per . . .
E: (1) sed ⟨-⟩ (2) sed (3) verum . . .
literas: (1) consonant (2) consonas . . .
verti: (1) C (2) P (a) vero (b) porro . . .
referatur: /LMN *streicht L./*
(1) Cesare ad Celarent. In CeS est S, ergo E simpliciter verte fit E, et ex (a) Cesare fit (b) Celarent fit Cesare. Camestres ad Celarent. Ob M fit transpositio, et ex (ba) AEE fit EAE (bb) ESE fit AEE, *bricht ab* (2) Cesare . . . Celarent (a) ⟨-⟩ NDC (b) Cesare est (ba) NCD. OBC, NBD (bb) ECD . . .
Celarent: (1) ex NCD fit NDC (2) ex NCD sequitur NDC et habemus NDC.OBC. Ergo ⟨NDC⟩ OBC sequitur *bricht ab* (3) ex NCD sequitur NDC er *bricht ab* (4) Demonstratio . . .
5 Demonstratio Ex: (1) NCD sequitur NDC. Ergo ex NCD. OBC sequitur NDC.OBC, ex NDC. NBC sequitur NBD per Celarent. Ergo ex (a) ND (b) NCD. OBC sequitur NBD (2) ECD . . . EBD
7 /Hoc . . . supprimemus *nachtr.erg. L./*
(1) Camestres ad Celarent. ODC. NBC. NBD. Ob M in Camestres transponendo, fiet NBC. ODC. NBD. Porro ob S. NBC simpliciter vertitur et fi *bricht ab* (2) Camestres ad Celarent. transponitur (3) Camestres . . .
1 simpliciter: (1) vertendo (a) fit (b) manet EAE (2) vertendo . . .
Festino: (1) simpli *bricht ab* (2) ad . . . nempe (a) EII (b) EIO *verb. Hrg.*
fit: (1) EII (2) EIO *verb. Hrg.*
ob C: (1) AAA (2) Nempe (a) AAA (b) sit AAA, porro conclusionem esse falsam. Ergo veram sit OBC et *bricht ab* (3) Barbara.
ABD: (1) Sit ABD falsa, seu opposita OBD vera. Ergo alterutra praemissarum falsa: sit ABC vera. Ergo ACD falsa. Ergo OCD vera. Itaque ex ABC, *bricht ab* (2) Ejus . . . falsa (a) ergo (b) seu . . . est (ba) OBD (bb) opposita . . .

15–16	similiter: (1) Ut Disamis fit (2) Di (3) transpositio (4) Disamis . . .
26	facta: /ipsius I in minore *streicht L.*/
28	(1) Enumerant vulg *bricht ab* (2) Quartae . . .
30–31	Barmapi: (1) ADC (2) ad . . . ADC (a) IBD. Ex IBD sequitur IDB. Ex A *bricht ab* (b) unde . . .
31	Barbara: (1) ACD.ADC.ADB et convertendo conclusionem ACB. A *bricht ab* (2) sequitur . . .
33–34	receptas: (1) non succedere (2) reducendi . . . quidem: (a) trans *bricht ab* (b) conversio . . .
35–36	hactenus: (1) Calerent (2) Calmerens . . . EBD (a) ECB (b) Transpone . . .
37	/per Celarent *nachtr.erg.L.*/
42	Firemos: (1) falsum est, et in nulla figura datur IEO. Nam sit IDC.ECB.OBD, demonstrabo (2) falsum . . .
44	<u>C B</u>: (1) C par B impar (2) C impar . . .
45	D: (1) primii (2) non primi (3) derivativus . . .
47	sequi: (1)⟨-⟩ ob (2) Nam ob ECB (a) posse (b) debent /⟨--⟩ *erg. L.*/ C et B (ba) rectae . . .
49	porro: (1) pars ipsius (2) ob . . .
48*a*, 8	ostendam per: (1) instans. Quiddam ⟨crescens⟩ est lapis. Nullus lapis est animal. Ergo quoddam ⟨--⟩ (2) instantia . . .
14	(1) Ferimos (2) Fesiso . . . Ferio (a) EIO (b) EDC.IBC.OBD (ba) Ex EDC et ICB fit ((bb) Ex (bc)) Tam . . .
15	simpliciter: (1) nam ex (2) et . . .
18	(1) C (2) B
19–20	(1) Super (2) Supersunt . . . neglecti: (a) EAO et AEO (b) AEO . . .
21	ex: (1) Celarent, vel si (2) Calmeres . . .
21–22	Celarent: (1) et si (2) et quidem (a) transponendo per (b) universalitem (c) universalem *verb. Hrg.* . . .
28	et: (1) trans *bricht ab* (2) conversione . . .
30	(1) Superest EAO, nempe (a) ECD (b) EDC.ACB.OBD. (ba) Ex E (bb) Unde fit ECD.IBC.OBD in ter *bricht ab* (2 Superest . . .
31	una: (1) per se (2) simpliciter . . .
34–35	/prioris . . . accidens *erg. L.*/
41	Celarent: (1) Ferio (2) Darii, Ferio (a) ⟨Ba-⟩ (b) bari . . .
42–43	Festino: (1) ⟨Bar-⟩ (2) Baroco . . .
43	stros: (1) Tertia grande sonans addit Darapti Felapton (2) Quaeque . . .
48*b*, 9	quinque: (1) et in quarta etiam quinque. Baralipto (2) Hi . . .
10–11	Hispanum: (1) Baralipton ⟨-⟩ (2) Primus modus (a) primae est (b) indirectus . . .
11	Baralipton: (1) Omnis homo est substantia (2) Omne . . .
17	tale: (1) assumendum est, manifeste (2) assumendum . . .
18–19	quartae: (1) ⟨-⟩ (2) Si . . . majorem /prop. *erg. L.*/ . . .
21–22	homo est: (1) substantia (2) animal . . .
24–25	/indirectus primae *erg. L.*/ . . . Celantes: (1) /qui *streicht Hrg.*/ transpone *bricht ab u. streicht L.* (2) Nullum . . .
30	/quartae *erg.L.*/
36–37	si: (1) sequitur (2) concludit . . .
43	/adhuc . . . primae *erg. L.*/
46	Fapesmo: (1) Omnis homo est (2) Omne . . .
48	Celantes: (1) sed to *bricht ab* (2) Caeterum . . .
49*a*, 2–3	est: (1) Calmerens (2) Fesapo
10	modum: (1) oblitus est (2) obliti . . .
14	/Ergo *erg.L.*/
16	patet: (1) quinque (2) quatuor . . .
17–18	ex: (1) prima (2) conversione /primae *erg.L.*/ . . .
19	modo: (1) ⟨qui⟩ (2) isque non aliud producit quam Barbari; (3) in . . .
20	accidens: (1) nec Celaro aliquid addit (2) in . . .
22–23	potest: (1) Itaque hic non (2) Porro . . .
27–28	sunt: (1) Frisesimo ergo (2) Idem . . .
28–29	Frisesmo: (1) Examinemus primum Frisesmo, nam hoc comprobato multo magis succedit Fapesmo; transponamus nemp Frisesmo, ergo oritur ex prima /Ferio *erg. L.*/ transpositione utriusque praemissae /⟨-⟩ *streicht L.*/ simpliciter, et Fapesm ⟨--⟩ quia transponendo utramque praem. *bricht ab* (2) Itaque (a) Fesa (b) Fesiso oritur ex Ferio (ba) transpon (bl transpositione (bba) utri (bbb) simpliciter utriusque praemi *bricht ab* (3) Itaque (a) Feri (b) Fesiso . . .
29	/enim *erg.L.*/
32	in: (1) Ferio (2) Fesapo (a) demonstr *bricht ab* (b) convertitur . . .
35–36	quamlibet: (1) praemissam (2) conclusionem primae /convertibilem *streicht L.*/ . . .
37	deinde: (1) transp *bricht ab* (2) praemissas . . .
38	praemissarum: (1) transponamus (2) convertamus . . .

33 (1) (6) (2) (7) . . . sit (a) negativa (b) universalis minor (ba) terminus vel (bb) propositio . . .
35–36 ejus: (1) est (2) nempe . . . prop. 5 (a) Porro subjectum ejus est minor Terminus (b) Itaque . . .
37–38 /per prop. 3 *erg. L.* /: (1) Si est universalis in mi *bricht ab* Ergo . . .
42–43 prop.: (1) 5 (2) 6 . . . et (a) major (b) universalis . . . in (ba) sua pr ⟨-⟩ (bb) majore . . .
48–49 esse: (1) particul (2) universalis; (a) nam si utraque est particularis (b) nam quia alterutra praemissa est (ba) nega affirmativa (per prop. 2), alterutrius (bba) subjectum est par (bbb) praedicatum est particulare; quod si jam utraq particularis, utriusque subject (c) nam . . .
50*a*, 49–**50***b*, 1 utriusque: (1) praedicat (2) pro (3) subjectum . . .
2–3 particulare: (1) habemus ergo ⟨-⟩ (2) Potest autem fieri (3) Sit (4) Porro cum medius sit in alterutro universalis (5) Mediu
3–4 /(per prop. 1) *erg. L.*
8 negativa: (1) erit (2) etiam . . .
10 (8): (1) Non potest simul esse major propositio particularis, et minor propositio negativa. (2) Si . . .
12 /(per prop. 6) *erg.L./*
17–18 /ibi *erg.L./* non est (1) subjectum conclusionis, (a) alioqui (b) sed (2) praedicatum . . .
19 universalis: (1) Sin major sit particularis (2) Q.E.D. (a) Itaque sequit ⟨-⟩ IEO nusquam ⟨-⟩ (b) Corollar . . .
21 (9): (1) Ubi major terminus est subjectum, et conclusio (2) Si major /prop. *erg.L./* . . .
22 affirmativa: (1) Nam *streicht Hrg.* Nam si major non est universalis (3) Nam . . .
26 parti/culare et *erg.Hrg./* . . .
28–29 negativa: (1) Si conclusio sit (2) Si . . .
32 universalis: (1) Si conclusio (2) Si . . .
38–39 propositione: (1) Si minor terminus est universalis in minore propositione et minor propositio sit affirmativa, erit terminus subjectum minoris propositionis et minor propositio erit universalis. Et medius terminus erit particularis in minore, ergo medius terminus erit universalis in prop. majore. Ergo (2) Si conclusio est (a) particularis et aff (b) univers affirmativa (3) Si . . .
39 conclusio /universalis *erg.L./* sit negativa (1) ⟨—⟩ (2) erit.
48 universalis: (1) Ergo (2) Porro alterutra praemissa est negativa (3) Ergo . . .
51*a*, 1 quaesitum: (1) Si non universalis, utraque negativa esse non potest, ergo restat ut una sit (a) universalis (b) particularis. erit (2) Si . . .
11 /(art. 11) *erg.L/*
13 (art. 20): (1) Rursus alterutra praemissa est affirmativa, quodsi jam alterutra sit particularis quae affirmativa (2) affirmativa (a) erit et minor propositio affirmativa (art. 21) (b) erit utraque praemissa affirmativa (c) erit . . .
14 Ergo: (1) subje (2) praedicatum . . .
16–17 Ergo: (1) univ (2) (art. 11) . . .
21 Sed: (1) major prop. est affirmativa (quia conclusio affirmativa (art *bricht ab* (2) quia . . .
22 Ergo: (1) (art. 21) (2) maj. . . .
26 utraque: (1) propositio (2) praemissa . . .
29 Ergo: (utraque (2) quaelibet . . .
51*b*, 1 (art. 21): (1) Ergo una est universalis, altera si non particularis (2) Superest . . .
3 ergo: (1) ⟨-⟩ (2) /(art. 21) *erg.L./* . . .
3–4 affirmativa: (1) Ea (2) Ejus ergo (a) subjectum (b) praedicatum . . .
4–5 /(art. 9) *erg.L./*
7 est: (1) uni (2) particularis . . .
8 universale: (1) Ergo haec propositio habet utrumque terminum universalem (2) Ergo et *streicht Hrg.* (3) Sed . . .
11 universalis: (1) habet (2) Itaque . . .
14 Si: (1) medius (2) minor . . .
15–16 particularis: (1) Si praemissa minor sit particularis vel affirm *bricht ab* (2) Duae . . .
17–18 semper: (1) una (2) altera . . . /(art. 15) *erg.L./*
20 est: (1) affirmativus partic. (2) particularis (art. . . .) (a) itaque in ea medius term. (b) continet autem medium terminu 18) in ea ergo medius terminus est par *bricht ab* (c) tum extremus tum (ca) ⟨-⟩ (cb) /nempe *streicht L./* etiam . . .
22–23 debet: (1) ergo (2) in altera (a) propositione (b) medius (c) praemissa (ca) esse *streicht Hrg.* (cb) medius . . .
24 Si: (1) alterutra (2) etiam . . .
30–31 itaque: (1) et conclusio (per 18) (2) (art. 11)

'I' is evident from the deleted §§ 48 and 50 where one can read: "Ubi nullus respectus ad praemissas, termini erunt F, G, vel tales. In genere propositio universalis SFΨG propositio particularis PFΨG propositio Affirmativa ΨFPG propositio negativa ΨFSG. In specie Universalis Affirmativa SFPG, Particularis affirmativa PFPG, Universalis negativa SFSG, particularis negativa PFSG."

(3) This unambiguous statement also confirms that the concluding sentence of § 24 ends with 'generaliter exprimitur ΨFΨG' and not, as C has it, with 'generaliter exprimitur unurarem ΨF.ΨS.' Even more misleading is the interpretation of this formula by F. Schmidt and by G. H. R. Parkinson who both suggest 'ΨP.ΨS.'.[33]

Let us now consider in which way Leibniz used this symbolism to complete his proof of completeness. In § 45 he proved the special rule IV.1 indirectly as follows. If one would have at the same time: ". . . major particularis PDΨC, minor negativa [ΨCSB], erit conclusio [particularis] negativa PBSD, sed hoc absurdum, quia (art. 20 [i.e. GR 2]) non potest esse in majore PD et in [conclusione] SD."[34] In § 46 it is similarly shown that one cannot have at the same time ". . . minor particularis et major affirmativa. Existant simul, erit major ΨDPC, minor PCΨB; sed ita medius C utrobique est particularis, quod est contra art. 19 [i.e. GR 1]."

Systematically much more important, however, is the sketch of a proof that Leibniz gives at the very end of *Mathesis* to show that there are not more valid moods than the 24 ones proven elsewhere:

> Contendendum erit, non dari plures, et quidem non per enumerationem illegitimorum, sed ex legibus legitimorum. V.g. in prima praemissae SC.ΨD, ΨB.PD dant:

SCPD	SBPD	AA	A	Barbara	1
			I	Barbari	2
	PBPD	AI	I	Darii	3
SCSD	SBPD	EA	E	Celarent	4
			O	Celaro	5
	PBPD	EI	O	Ferio	6 (C, 202).

In its present form, however, this schema is incomplete and incorrect. As stated in § 22, the position of the terms in the Ist figure is: "Fig. 1. CD. BC. BD". The special rule I.1, according to which the minor-premiss is affirmative, therefore has to be formalized as 'ΨBPC', whereas Leibniz erroneously has 'ΨBPD' which would symbolize an affirmative conclusion. Hence only the following combination of premisses (obtained by su stituting 'S' and 'P' successively in the place of 'Ψ') legitimate:

SCPD	SBPC
	PBPC
SCSD	SBPC
	PBPC.

In the first two cases, in view of GR 4, the conclusi must itself be affirmative: ΨBPD; moreover, in t second subcase it has to be particular according to G 3: PBPD. In the last two cases, in contrast, the conc sion has to be negative on account of GR 4: ΨBSD; the second subcase, again, it also must be particul PBSD. Hence Leibniz's schema for the only va moods of the Ist figure has to be modified as follows:

SCPD	SBPC	SBPD	Barbara	1
		PBPD	Barbari	2
	PBPC	PBPD	Darii	3
SCSD	SBPC	SBSD	Celarent	4
		PBSD	Celaro	5
	PBPC	PBSD	Ferio	6

This formal method of eliminating the invalid mo "ex legibus legitimorum" can be applied to the ot figures as well. E.g., the special rules for the IInd fig (which is characterized by the following position terms: "Fig. 2 DC BC BD"), state that the major-prem must be universal (II.2): SDΨC, while according II.1 one of the premisses and hence (GR 5.2) also conclusion must be negative: ΨBSD. Thus the follow combinations have to be taken into account:

SDSC	SBSD
	PBSD
SDPC	SBSD
	PBSD.

In the former two cases, because of the negativity o major-premiss, the minor-premiss has to be affirm on account of GR 3: ΨBPC; furthermore, in the subcase the minor-term B is universal in the conclu and hence also has to be universal in the premiss