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Meaning and Necessity

A Study in Semantics and Modal Logic

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Phoenix Books

THE UNIVERSITY OF CHICAGO PRESS

CHICAGO AND LONDON

CHAPTER I

THE METHOD OF EXTENSION AND INTENSION

A method of semantical meaning analysis is developed in this chapter. It is applied to those expressions of a semantical system *S* which we call *designators;* they include (declarative) sentences, individual expressions (i.e., individual constants or individual descriptions) and predicators (i.e., predicate constants or compound predicate expressions, including abstraction expressions). We start with the semantical concepts of *truth* and *L-truth* (logical truth) of sentences (§§ 1, 2). It is seen from the definition of L-truth that it holds for a sentence if its truth follows from the semantical rules alone without reference to (extra-linguistic) facts (§ 2). Two sentences are called (materially) *equivalent* if both are true or both are not true. The use of this concept of equivalence is then extended to designators other than sentences. Two individual expressions are equivalent if they stand for the same individual. Two predicators (of degree one) are equivalent if they hold for the same individuals. *L-equivalence* (logical equivalence) is defined both for sentences and for other designators in such a manner that it holds for two designators if and only if their equivalence follows from the semantical rules alone. The concepts of equivalence and L-equivalence in their extended use are fundamental for our method (§ 3).

If two designators are equivalent, we say also that they have the same *extension*. If they are, moreover, L-equivalent, we say that they have also the same *intension* (§ 5). Then we look around for entities which might be taken as extensions or as intensions for the various kinds of designators. We find that the following choices are in accord with the two identity conditions just stated. We take as the extension of a predicator the class of those individuals to which it applies and, as its intension, the property which it expresses; this is in accord with customary conceptions (§ 4). As the extension of a sentence we take its truth-value (truth or falsity); as its intension, the proposition expressed by it (§ 6). Finally, the extension of an individual expression is the individual to which it refers; its intension is a concept of a new kind expressed by it, which we call an individual concept (§§ 7–9). These conceptions of extensions and intensions are justified by their fruitfulness; further definitions and theorems apply equally to extensions of all types or to intensions of all types.

A sentence is said to be *extensional* with respect to a designator occurring in it if the extension of the sentence is a function of the extension of the designator, that is to say, if the replacement of the designator by an equivalent one transforms the whole sentence into an equivalent one. A sentence is said to be *intensional* with respect to a designator occurring in it if it is not extensional and if its intension is a function of the intension of the designator, that is to say, if the replacement of this designator by an L-equivalent one transforms the whole sentence into an L-equivalent one. A modal sentence (for example, 'it is necessary that . . .') is intensional with respect to its subsentence (§ 11). A psychological sentence like 'John believes that it is raining now' is neither extensional nor intensional with respect to its subsentence (§ 13). The problem of the semantical analysis of these *belief-sentences* is solved with the help of the concept of *intensional structure* (§§ 14, 15).

§ 1. Preliminary Explanations

This section contains explanations of a symbolic language system S_1, which will later serve as an *object language* for the illustrative application of the semantical methods to be discussed in this book. Further, some semantical concepts are explained for later use; they belong to the semantical *metalanguage* M, which is a part of English. Among them are the concepts of *truth*, *falsity*, and (material) *equivalence*, applied to sentences. The term '*designator*' is introduced for all those expressions to which a semantical meaning analysis is applied, the term will be used here especially for sentences, predicators (i.e., predicate expressions), and individual expressions.

The chief task of this book will be to find a suitable method for the semantical analysis of meaning, that is, to find concepts suitable as tools for this analysis. The concepts of the intension and the extension of an expression in language will be proposed for this purpose. They are analogous to the customary concepts of property and class but will be applied in a more general way to various types of expressions, including sentences and individual expressions. The two concepts will be explained and discussed in chapters i and ii.

The customary concept of name-relation and the distinction sometimes made since Frege between the entity named by an expression and the sense of the expression will be discussed in detail in chapter iii. The pair of concepts, extension-intension, is in some respects similar to the pair of Frege's concepts; but it will be shown that the latter pair has serious disadvantages which the former avoids. The chief disadvantage of the method applying the latter pair is that, in order to speak about, say, a property and the corresponding class, two different expressions are used. The method of extension and intension needs only one expression to speak about both the property and the class and, generally, one expression only to speak about an intension and the corresponding extension.

In chapter iv, a metalanguage will be constructed which is neutral with regard to extension and intension, in the sense that it speaks not about a property and the corresponding class as two entities but, instead, about one entity only; and analogously, in general, for any pair of an intension and the corresponding extension. The possibility of this neutral language shows that our distinction between extension and intension does not presuppose a duplication of entities.

In chapter v, some questions concerning modal logic are discussed on the basis of the method of extension and intension.

My interest was first directed toward the problems here discussed when I was working on systems of modal logic and found it necessary to clarify the concepts which will be discussed here under the terms of 'extension'

and 'intension' and related concepts which have to do with what is usually called the values of a variable. Further stimulation came from some recent publications by Quine[1] and Church,[2] whose discussions are valuable contributions to a clarification of the concepts of naming and meaning.

Before we start the discussion of the problems indicated, some explanations will be given in this section concerning the object languages and the metalanguage to be used. We shall take as object languages mostly symbolic languages, chiefly three semantical language systems, S_1, S_2, and S_3, and occasionally also the English word language. For the sake of brevity, not all the rules of these symbolic systems will be given, but only those of their features will be described which are relevant to our discussion. S_1 will now be described; S_2 is an extension of it that will be explained later (§ 41); S_3 will be described in § 18.

The *system* S_1 contains the customary connectives of negation '$\sim$' ('not'), disjunction '$\vee$' ('or'), conjunction '$\bullet$' ('and'), conditional (or material implication) '$\supset$' ('if . . . then . . .'), and biconditional (or material equivalence) '$\equiv$' ('if and only if'). The only variables occurring are individual variables 'x', 'y', 'z', etc. For these variables the customary universal and existential quantifiers are used: '$(x)(\,.\,.\,x\,.\,.)$' ('for every x, . . x . .') and '$(\exists x)(\,.\,.\,x\,.\,.)$' ('there is an x such that . . x . .'). All sentences in S_1 and the other systems are closed (that is, they do not contain free variables). In addition to the two quantifiers, two other kinds of operators occur: the iota-operator for individual descriptions ('$(\imath x)(\,.\,.\,x\,.\,.)$', 'the one individual x such that . . x . .') and the lambda-operator for abstraction expressions ('$(\lambda x)(\,.\,.\,x\,.\,.)$', 'the property (or class) of those x which are such that . . x . .'). If a sentence consists of an abstraction expression followed by an individual constant, it says that the individual has the property in question. Therefore, '$(\lambda x)(\,.\,.\,x\,.\,.)a$' means the same as ' . . a . .', that is, the sentence formed from '. . x . .' by substituting 'a' for 'x'. The rules of our system will permit the transformation of '$(\lambda x)(\,.\,.\,x\,.\,.)a$' into '. . a . .' and vice versa; these transformations are called conversions.

S_1 contains descriptive constants (that is, nonlogical constants) of indi-

[1] [Notes] (see Bibliography at the end of this book). Quine's views concerning the name-relation (designation) will be discussed in chap. iii; and the conclusions which he draws from them for the problem of quantification in modal sentences will be discussed in chap. v.

[2] [Review C.] and [Review Q.]. Church's conceptions will be discussed in chap. iii, in connection with those of Frege. Church's contributions are more important than is indicated by the form of their publication as reviews. It is to be hoped that he will soon find the opportunity for presenting his conception in a more comprehensive and systematic form.

vidual and predicate types. The number of predicates in S_1 is supposed to be finite, that of individual constants may be infinite. For some of these constants, which we shall use in examples, we state here their meanings by semantical rules which translate them into English.

1-1. *Rules of designation for individual constants*
'*s*' is a symbolic translation of 'Walter Scott',
'*w*'—'(the book) Waverley'.

1-2. *Rules of designation for predicates*
'H*x*'—'*x* is human (a human being)',
'RA*x*'—'*x* is a rational animal',
'F*x*'—'*x* is (naturally) featherless',
'B*x*'—'*x* is a biped',
'A*xy*'—'*x* is an author of *y*'.

The English words here used are supposed to be understood in such a way that 'human being' and 'rational animal' mean the same. Further, we shall use 'a', 'b', 'c', as individual constants, and 'P', 'Q', as predicator constants (of level one and degree one); the interpretation of these signs will be specified in each case, or left unspecified if not relevant for the discussion.

In order to speak *about* any *object language*—here the symbolic language systems S_1, etc.—we need a *metalanguage*. We shall use as our metalanguage M a suitable part of the English language which contains translations of the sentences and other expressions of our object languages (for example, the translations stated in 1-1 and 1-2), names (descriptions) of those expressions, and special semantical terms. For the sake of simplicity, we shall usually construct a name of an expression in the customary way by including it in single quotation marks. In order to speak about expressions in a general way, we often use '$\mathfrak{A}_i$', '$\mathfrak{A}_j$', etc., for expressions of any kind and '$\mathfrak{S}_i$', '$\mathfrak{S}_j$', etc., for sentences, sometimes also blanks like '. . .', '- -', etc., and blanks with a variable, e.g., '. . *x* . .', for an expression in which that variable occurs freely. If a German letter occurs in an expression together with symbols of the object language, then the latter ones are used autonymously, i.e., as names for themselves.[3] Thus, we may write in M, for instance, '$\mathfrak{A}_i \equiv \mathfrak{A}_j$'; this is meant as referring to that expression of the object language which consists of the expression $\mathfrak{A}_i$ (whatever this may be, e.g., 'Hs') followed by the sign '$\equiv$', followed by the expression $\mathfrak{A}_j$. (In symbolic formulas both in the object languages and in M, parentheses will often be omitted under the customary conditions.) The term

[3] See [Syntax], § 42.

'***sentence***' will be used in the sense of 'declarative sentence'. The ter 'sentential matrix' or, for short, '***matrix***' will be used for expressio which are either sentences or formed from sentences by replacing ind vidual constants with variables. (If a matrix contains any number of fr occurrences of *n* different variables, it is said to be of degree *n;* for e ample, 'A*xy* V P*x*' is of degree two; the sentences are the matrices of d gree zero). A sentence consisting of a predicate of degree *n* followed by individual constants is called an ***atomic sentence*** (e.g., 'Pa', 'Abc').

A complete construction of the semantical system S_1, which cannot l given here, would consist in laying down the following kinds of rule (1) rules of formation, determining the admitted forms of sentence (2) rules of designation for the descriptive constants (e.g. 1-1 and 1-2 (3) rules of truth, which we shall explain now; (4) rules of ranges, to be e plained in the next section. Of the ***rules of truth*** we shall give here on three examples, for atomic sentences (1-3), for 'V' (1-5), and for '$\equiv$' (1-6

1-3. *Rule of truth* for the simplest atomic sentences. An atomic senten in S_1 consisting of a predicate followed by an individual constant is tr if and only if the individual to which the individual constant refers posse ses the property to which the predicate refers.

This rule presupposes the rules of designation. It yields, together wi rules 1-1 and 1-2, the following result as an example:

1-4. The sentence 'Bs' is true if and only if Scott is a biped.

1-5. *Rule of truth* for 'V'. A sentence $\mathfrak{S}_i \vee \mathfrak{S}_j$ is true in S_1 if and only if least one of the two components is true.

1-6. *Rule of truth* for '$\equiv$'. A sentence $\mathfrak{S}_i \equiv \mathfrak{S}_j$ is true if and only if eith both components are true or both are not true.

There are some further rules of truth for the other connectives, co responding to their truth-tables, and for the quantifiers; another examp of a rule of truth will be given in 3-3. The rules of truth together constitu a recursive *definition for* '*true in* S_1', because they determine, in combir tion with the rules of designation, for every sentence in S_1 a sufficient a necessary condition of its truth (as is given for 'Bs' in 1-4). Thereby th give an *interpretation* for every sentence. Thus, for example, we learn fro the rules that the sentence 'Bs' says that (in other words, expresses t proposition that) Scott is a biped. For the purposes of our discussion it not necessary to give the whole definition of truth.[4] It will suffice to p

[4] The first definition of the semantical concept of truth was given by Tarski [Wahrheits griff]; I have given a slightly different form in [I], § 7. For nontechnical discussions of the ture of the semantical concept of truth see Tarski [Truth] and my [Remarks].

suppose that the term 'true' is defined in such a manner that it has its customary meaning as applied to sentences. More specifically, we presuppose that a statement in M saying that a certain sentence in S_1 is true means the same as the translation of this sentence;[5] for example, 'the sentence 'Hs' is true in S_1' means the same as 'Walter Scott is human'. On the basis of 'true', some further semantical terms are defined as follows, with respect to any semantical system S, e.g., S_1, etc.

1-7. *Definition.* $\mathfrak{S}_i$ is ***false*** (in S) $=_{Df} \sim \mathfrak{S}_i$ is true (in S).

Thus 'false' has here its ordinary meaning.

1-8. *Definition.* $\mathfrak{S}_i$ is ***equivalent*** to $\mathfrak{S}_j$ (in S) $=_{Df}$ $\mathfrak{S}_i \equiv \mathfrak{S}_j$ is true (in S).

This definition, together with the rule of truth for '$\equiv$' (1-6), yields this result:

1-9. Two sentences are equivalent if and only if both have the same truth-value, that is to say, both are true or both are false.

It is to be noticed that the term 'equivalent' is here defined in such a manner that it means merely agreement with respect to truth-value (truth or falsity), a relation which is sometimes called 'material equivalence'. The term is here not used, as in ordinary language, in the sense of agreement in meaning, sometimes called 'logical equivalence'; for the latter concept we shall later introduce the term 'L-equivalent' (2-3c).

I propose to use the term ***'designator'*** for all those expressions to which a semantical analysis of meaning is applied, the class of designators thus being narrower or wider according to the method of analysis used. [The word 'meaning' is here always understood in the sense of 'designative meaning', sometimes also called 'cognitive', 'theoretical', 'referential', or 'informative', as distinguished from other meaning components, e.g., emotive or motivative meaning. Thus here we have to do only with declarative sentences and their parts.] Our method takes as designators at least sentences, ***predicators***[6] (i.e., predicate expressions, in a wide sense,

[5] For detailed discussions of this characteristic of the semantical concept of truth, see Tarski [Truth] and my [Remarks], § 3.

[6] Some terms with the ending '-tor' for kinds of expressions are customary, e.g., 'functor', 'operator'. The terms 'predicator' and 'designator' are formed in analogy to them. A still wider use of the same ending might be taken into consideration with the aim of making the terminology in the metalanguage somewhat more uniform. For this book, only the two terms mentioned are adopted; but the following terms would seem to me quite suitable, too: 'descriptor' (for the customary 'description'), 'abstractor' (for 'abstraction expression'), 'connector' (for 'connective'). Other terms might seem more questionable, but perhaps still worth consideration, e.g., 'individuator' (for 'individual expression'), 'propositor' or 'stator' (for '(declarative) sentence'), 'conceptor' (for 'concept expression,' i.e., 'designator other than sentence'). Morris, [Signs], uses a number of terms with '-tor' (or '-or'), among them some of those mentioned here, for kinds of expressions or, more generally, of signs including non-linguistic signs.

including class expressions), functors (i.e., expressions for functions i the narrower sense, excluding propositional functions), and individual e: pressions; other types may be included if desired (e.g., connectives, bot extensional and modal ones). The term 'designator' is not meant to impl that these expressions are names of some entities (the name-relation wi be discussed in § 24), but merely that they have, so to speak, an ind pendent meaning, at least independent to some degree. Only (declarativ sentences have a (designative) meaning in the strictest sense, a meaning the highest degree of independence. All other expressions derive wh meaning they have from the way in which they contribute to the meanir of the sentences in which they occur. One might perhaps distinguish— in a vague way—different degrees of independence of this derivati meaning. Thus, for instance, I should attribute a very low degree '(', somewhat more independence to 'V', still more to '+' (in an arithme ical language), still more to 'H' ('human') and 's' ('Scott'); I should n know which of the last two to rank higher. This order of rank is, of cours highly subjective. And where to make the cut between expressions with 1 or little independence of meaning ('syncategorematic' in tradition terminology) and those with a high degree of independence, to be taken designators, seems more or less a matter of convention. If a metalangua is decided upon, then it seems convenient to take as designators at lea the expressions of all those types, but not necessarily only those, for whi there are variables in the metalanguage (compare [I], § 12, and referenc to Quine, below, at the beginning of § 10).

§ 2. L-Concepts

By the *explication* of a familiar but vague concept we mean its replaceme by a new exact concept; the former is called explicandum, the latter explicatu The concept of *L-truth* is here defined as an explicatum for what philosoph call logical or necessary or analytic truth. The definition leads to the result th a sentence in a semantical system is L-true if and only if the semantical ru of the system suffice for establishing its truth. The concepts of L-falsity, implication, and *L-equivalence* are defined as explicata for logical falsity, logi implication or entailment, and mutual logical implication, respectively. A se tence is called *L-determinate* if it is either L-true or L-false; otherwise it is call L-indeterminate or *factual*. The latter concept is an explicatum for what Ka called synthetic judgments. A sentence is called *F-true* if it is true but not true; F-truth is an explicatum for what is known as factual or synthetic contingent truth. The concepts of F-falsity, F-implication, and F-equivaler are defined analogously.

The task of making more exact a vague or not quite exact concept us in everyday life or in an earlier stage of scientific or logical developme or rather of replacing it by a newly constructed, more exact concept, belongs among the most important tasks of logical analysis and logical construction. We call this the task of explicating, or of giving an ***explication*** for, the earlier concept; this earlier concept, or sometimes the term used for it, is called the ***explicandum;*** and the new concept, or its term, is called an ***explicatum*** of the old one.[7] Thus, for instance, Frege and, later, Russell took as explicandum the term 'two' in the not quite exact meaning in which it is used in everyday life and in applied mathematics; they proposed as an explicatum for it an exactly defined concept, namely, the class of pair-classes (see below the remark on (i) in § 27); other logicians have proposed other explicata for the same explicandum. Many concepts now defined in semantics are meant as explicata for concepts earlier used in everyday language or in logic. For instance, the semantical concept of truth has as its explicandum the concept of truth as used in everyday language (if applied to declarative sentences) and in all of traditional and modern logic. Further, the various interpretations of descriptions by Frege, Russell, and others, which will be discussed in §§ 7 and 8, may be regarded as so many different explications for phrases of the form 'the so-and-so'; each of these explications consists in laying down rules for the use of corresponding expressions in language systems to be constructed. The interpretation which we shall adopt following a suggestion by Frege (§8, Method IIIb) deviates deliberately from the meaning of descriptions in the ordinary language. Generally speaking, it is not required that an explicatum have, as nearly as possible, the same meaning as the explicandum; it should, however, correspond to the explicandum in such a way that it can be used instead of the latter.

The L-terms ('L-true', etc.) which we shall now introduce are likewise intended as explicata for customary, but not quite exact, concepts. '***L-true***' is meant as an explicatum for what Leibniz called necessary truth and Kant analytic truth. We shall indicate here briefly how this and the other L-terms can be defined. In the further discussions of this book, however, we shall not make use of the technical details of the following definitions but only of the fact that 'L-true' is defined in such a way that the requirement stated in the subsequent convention 2-1 is fulfilled. This is in accord with the purpose of this book, which is intended not so much to carry out exact analyses of exactly constructed systems as to state informally some considerations aimed at the discovery of concepts and methods suitable for semantical analysis.

[7] What is meant here by 'explicandum' and 'explicatum' seems similar to what Langford means by 'analysandum' and 'analysans'; see below, n. 42, p. 63.

We shall introduce the L-concepts with the help of the concepts of state-description and range. Some ideas of Wittgenstein[8] were the starting-point for the development of this method.[9]

A class of sentences in S_1 which contains for every atomic sentence either this sentence or its negation, but not both, and no other sentences is called a ***state-description*** in S_1, because it obviously gives a complete description of a possible state of the universe of individuals with respect to all properties and relations expressed by predicates of the system. Thus the state-descriptions represent Leibniz' possible worlds or Wittgenstein's possible states of affairs.

It is easily possible to lay down semantical rules which determine for every sentence in S_1 whether or not it *holds in a* given *state-description*. That a sentence holds in a state-description means, in nontechnical terms that it would be true if the state-description (that is, all sentences belonging to it) were true. A few examples will suffice to show the nature of these rules: (1) an atomic sentence holds in a given state-description if and only if it belongs to it; (2) $\sim \mathfrak{S}_i$ holds in a given state-description if and only if $\mathfrak{S}_i$ does not hold in it; (3) $\mathfrak{S}_i \vee \mathfrak{S}_j$ holds in a state-description if and only if either $\mathfrak{S}_i$ holds in it or $\mathfrak{S}_j$ or both; (4) $\mathfrak{S}_i \equiv \mathfrak{S}_j$ holds in a state-description if and only if either both $\mathfrak{S}_i$ and $\mathfrak{S}_j$ or neither of them hold in it; (5) a universal sentence (e.g., '$(x)(Px)$') holds in a state-description if and only if all substitution instances of its scope ('Pa', 'Pb', 'Pc' etc.) hold in it. Iota-operators and lambda-operators can be eliminated (for the former, this will be shown later, see 8-2; for the latter, see the explanation of conversion in § 1). Therefore, it is sufficient to lay down a rule to the effect that any sentence containing an operator of one of these kinds holds in the same state-descriptions as the sentence resulting from the elimination of the operator.

The class of all those state-descriptions in which a given sentence $\mathfrak{S}_i$ holds is called the ***range*** of $\mathfrak{S}_i$. All the rules together, of which we have just given five examples, determine the range of any sentence in S_1; therefore they are called *rules of ranges*. By determining the ranges, they give, together with the rules of designation for the predicates and the individual constants (e.g., 1-1 and 1-2), an *interpretation* for all sentences in S_1, since

[8] [Tractatus]; see also [I], p. 107.

[9] The method which I shall use here is similar to, but simpler than, the one I have described in [I], § 19, as procedure E. The simpler form is possible here because S_1 contains atomic sentences for all atomic propositions. The procedure to be used here seems to me the most convenient among those known at present for the semantical construction of a system of deductive logic; I have used it, furthermore, for modal logic in [Modalities] and for inductive logic, that is, the theory of logical probability or degree of confirmation in [Inductive

to know the meaning of a sentence is to know in which of the possible cases it would be true and in which not, as Wittgenstein has pointed out.

The connection between these concepts and that of truth is as follows: There is one and only one state-description which describes the actual state of the universe; it is that which contains all true atomic sentences and the negations of those which are false. Hence it contains only true sentences; therefore, we call it the true state-description. A sentence of any form is true if and only if it holds in the true state-description. These are only incidental remarks for explanatory purposes; the definition of L-truth will not make use of the concept of truth.

The L-concepts now to be defined are meant as explicata for certain concepts which have long been used by philosophers without being defined in a satisfactory way. Our concept of L-truth is, as mentioned above, intended as an explicatum for the familiar but vague concept of logical or necessary or analytic truth as explicandum. This explicandum has sometimes been characterized as truth based on purely logical reasons, on meaning alone, independent of the contingency of facts. Now the meaning of a sentence, its interpretation, is determined by the semantical rules (the rules of designation and the rules of ranges in the method explained above). Therefore, it seems well in accord with the traditional concept which we take as explicandum, if we require of any explicatum that it fulfil the following condition:

2-1. *Convention.* A sentence $\mathfrak{S}_i$ is ***L-true*** in a semantical system S if and only if $\mathfrak{S}_i$ is true in S in such a way that its truth can be established on the basis of the semantical rules of the system S alone, without any reference to (extra-linguistic) facts.

This is not yet a definition of L-truth. It is an informal formulation of a condition which any proposed definition of L-truth must fulfil in order to be adequate as an explication for our explicandum. Thus this convention has merely an explanatory and heuristic function.

How shall we define L-truth so as to fulfil the requirement 2-1? A way is suggested by Leibniz' conception that a necessary truth must hold in all possible worlds. Since our state-descriptions represent the possible worlds, this means that a sentence is logically true if it holds in all state-descriptions. This leads to the following definition:

2-2. *Definition.* A sentence $\mathfrak{S}_i$ is ***L-true*** (in S_1) $=_{Df}$ $\mathfrak{S}_i$ holds in every state-description (in S_1).

The following consideration shows that the concept of L-truth thu defined is in accord with the convention 2-1 and hence is an adequat explicatum for logical truth. If $\mathfrak{S}_i$ holds in every state-description, ther the semantical rules of ranges suffice for establishing this result. [For ex ample, we see from the rules of ranges mentioned above that 'Pa' hold in certain state-descriptions, that '$\sim$Pa' holds in all the other state descriptions, and that therefore the disjunction 'Pa $\vee \sim$Pa' holds i every state-description.] Therefore, the semantical rules establish also th truth of $\mathfrak{S}_i$ because, if $\mathfrak{S}_i$ holds in every state-description, then it hold also in the true state-description and hence is itself true. If, on the othe hand, $\mathfrak{S}_i$ does not hold in every state-description, then there is at leas one state-description in which $\mathfrak{S}_i$ does not hold. If this state-descriptio were the true one, $\mathfrak{S}_i$ would be false. Whether this state-description i true or not depends upon the facts of the universe. Therefore, in thi case, even if $\mathfrak{S}_i$ is true, it is not possible to establish its truth withou reference to facts.

L-falsity is meant as an explicatum for logical or necessary falsity o self-contradiction. *L-implication* is meant as explicatum for logical impli cation or entailment. *L-equivalence* is intended as explicatum for mutua logical implication or entailment. The definitions are as follows:

2-3. *Definitions*

a. $\mathfrak{S}_i$ is ***L-false*** in (S_1) $=_{Df}$ $\sim \mathfrak{S}_i$ is L-true.

b. $\mathfrak{S}_i$ ***L-implies*** $\mathfrak{S}_j$ (in S_1) $=_{Df}$ the sentence $\mathfrak{S}_i \supset \mathfrak{S}_j$ is L-true.

c. $\mathfrak{S}_i$ is ***L-equivalent*** to $\mathfrak{S}_j$ (in S_1) $=_{Df}$ the sentence $\mathfrak{S}_i \equiv \mathfrak{S}_j$ i L-true.

d. $\mathfrak{S}_i$ is ***L-determinate*** (in S_1) $=_{Df}$ $\mathfrak{S}_i$ is either L-true or L-false.

The following results follow easily from these definitions, together witl 2-2:

2-4. $\mathfrak{S}_i$ is L-false if and only if $\mathfrak{S}_i$ does not hold in any state-description

2-5. $\mathfrak{S}_i$ L-implies $\mathfrak{S}_j$ if and only if $\mathfrak{S}_j$ holds in every state-descriptio in which $\mathfrak{S}_i$ holds.

2-6. $\mathfrak{S}_i$ is L-equivalent to $\mathfrak{S}_j$ if and only if $\mathfrak{S}_i$ and $\mathfrak{S}_j$ hold in the sam state-descriptions.

The condition for L-falsity stated in 2-4 means, in effect, that $\mathfrak{S}_i$ can not possibly be true. The condition for L-implication in 2-5 means tha it is not possible for $\mathfrak{S}_i$ to be true and for $\mathfrak{S}_j$ to be false. The conditio for L-equivalence in 2-6 means that it is impossible for one of the tw sentences to be true and the other false. Thus these results show tha L-falsity, L-implication, and L-equivalence as defined by 2-3a, b, c, ma