Readings for class:

The class handout “Square of Oppositions”, also on the class webpage.

Read into Aristotle’s Prior Analytics a bit until you get a flavor for it. We will not be doing a close reading of his text in this course.

Read all of Lukaisiewicz, chapter from Aristotle’s Syllgistic, “Theses of the System.” 20 pages. Lukaiwicz treats syllgisms as conditional sentences.

Read Corcoran, “Aristotelian Syllogisms; Valid Arguments of True Universal Conditionals?” This is short. Here Corcoran argues that syllogisms are not concitional sentences but three line arguments.

Read the first few pages of Corcoran, “Completeness of an Ancient Logic.” Deeper in the paper is a completeness proof.

Read more deeply in to Smile, “What is a Syllogism?”, but you do not need to read the completeness proof. This paper is better written and more elegant than Corcoran’s. Smile too treats syllogisms as arguments.

Read, to the degree possible, Martin, “Aristotle’s Natural Deduction Reconsidered.” I show it doesn’t matter formally whether syllogisms are conditional sentences or arguments, because the two are equivalent. I also give a simpler Henkin-style completeness proof.

Further reading for the future: papers by Landrade-Lotero and Novaes. These generalize results and methods of my paper.

The Issues:

There are two ways to “reconstruct” in modern logic the ancient system of “reduction” of the total set of valid moods of the syllogistic to the two perfect syllogisms, Barbara and Celarent. (Reductions of this sort are in Aristotle but were common as well in medieval and later “traditional” logic.) One way treats a three line syllogism as a conditional sentence with the syllogism’s two premises a conjunction understood as the antecedent of the conditional and its conclusion as the conditional’s consequent. The other way is to read a syllogism as a three line argument, with two premises and a single conclusion. The reading affect the explanation of what a “reduction” is.

Background. The important background idea used in both accounts is that of a constructive set. In the first account the set of valid moods is treated as a set of conditionals, but in the second they are treated as a set of arguments. In both cases the set in question is “constructed.” A set is constructed if it has an inductive definition:

**Definition.** A set *C* is ***constuctive*** relative to a series set *B*1,…*Bn* of ***basic elements*** and a series *R*1,…*Rn* of relations of ***construction rules*** iff *C* is defined as follows:

Basis Clause: Every set *Bi* of basic elements is a subset of *C*;

Inductive Clause: for any *x*1,…*xn* and any *n*-place construction relation *Ri*, if *x*1,…*xn* are all in *C*, then *Ri*(*x*1,…*xn*)∈*C*.

Closure Clause. Nothing else is in *C*.

Axiomatic Reconstruction of the Syllogistic. On this approach the valid moods are treated as a constructive set of “theorems” in an “axiom system.” A single subset of axioms is specified. This set includes Barbara and Celarent, and perhaps a few other sentences trivial sentences. Then a set containing a small number of “inference rules” is specified. These are versions of the traditional rules of reduction from Aristotelian logic. The set of vaid moods is then defined as the set of theorems deducible from the axioms by the rules, i.e. as the closure under the inference rules of the set of axioms.

In this reconstruction we use as the lable (name) for the set of theorems constructed the single turnstyle├. That is, the way to say in set theory that *P* is a theorem is *P*∈├. However it is traditional (a tradition going back to Frege) to rewrite P∈├ in the notation ├*P*. That is, ├*P* says that *P* is a member of the constructive set ├.

To show that the construction in fact captures all the logical truths of the system, one needs to advance a completeness proof. It is traditional to use the double turnstyle ╟ to name the set of all logical truths. This set need to be defined independently is semantics or model theory. Then, to way say in set theory that P is a logical truth is P∈╞. Again, however, it is traditional to write P∈╞ as ╞P.

The construction of ├ then then said to be complete if ├ is the same set as ╞:

**Definition**

├ is ***statement sound*** iff for any P, if├P then ╞P.

├ is ***statement complete*** iff, for any P, if ╞P then ├P.

Natural[[1]](#footnote-1) Deduction Reconstruction of the Syllogistic. On this second approach, syllogisms are viewed as three line arguments. An argument or a “deduction” in set theory are represented as an *n*-tuple of sentences <*P*1,…,*Pn* > with the understanding that are the premises *P*1,…,*Pn-*1 and *Pn* is the conclusion. In the case of the syllogistic, a syllogism is then a three line argument <*P*1, *P*2,*P*3>. The reconstruction then consists of defining a constructive set of such arguments. In this case the basic elements of the construction are Barbara and Celarent understood as arguments, together with perhaps a few other trivial arguments. The traditional reduction rules are then understood to be construction rules that apply to arguments and yield other arguments as values. The set of valid moods is then defined as the closure of the set of basic arguments under the construction rules.

In this reconstruction we again use the single turnstyle ├ as the lable (name) for the constructive set, but this time this set is a set of arguments. That is, the way to say in set theory that the argument <*P*1,…,*Pn* > is an a member of the the is a theorem is <*P*1,…,*Pn* >∈├. However it is traditional (a tradition going back to Gentzen, the inventor of “natural deduction”) to rewrite <*P*1,…,*Pn* >∈├ in the notation *P*1,…,*Pn*-1├ *Pn* . That is, *P*1,…,*Pn*-1├ *Pn* says that <*P*1,…,*Pn* > is a member of the constructive set ├. It is also traditional to call ├ the set of “acceptable deductions.”

It is then an open question whether the set of acceptable deductions, which is defined by construction, captures in fact all the valid arguments of the system. To show that it does, the set of valid argumens has to be defined independently in semantics or model theory. Again, it is traditional to use the double turnstyle ╞ to name the set of valid arguments. Then, the to way say in set theory that <*P*1,…,*Pn* > is avalid arguement is <*P*1,…,*Pn* >∈╞. Again, however, it is traditional to write <*P*1,…,*Pn* >∈╞ as *P*1,…,*Pn-*1╞*Pn* .

The construction of ├ then then said to be complete if ├ is the same set as ╞:

**Definition**

├ is ***argument sound*** iff for any *P*1,…,*Pn*, if *P*1,…,*Pn-*1╞*Pn* then *P*1,…,*Pn-*1╞*Pn*

├ is ***argument complete*** iff, for any *P*1,…,*Pn*, if *P*1,…,*Pn-*1╞*Pn* then *P*1,…,*Pn-*1╞*Pn*

The Issue. Lukaisiewicz reconstructs the syllogistic as an axiom system. Corcoran and Smile, evidently independently but about the same time, treat it as a natural deduction system and provide (rather complex and inelegant) proofs of argument soundess and completeness, for essentially the same systems. In my paper I argue it is trivial to convert an axiom system for the syllogistic into a natural deduction system and vice versa and I give an (elegant) Henkin-style proof for argument completeness.

In a Henkin proof, soundness is shown by an inductive proof. It is shown first that every basic argument in ├ is valid, and then it is shown that for any inference rule, if the arguments that are taken as its inputs are valid, then the argument produced as its output is also valid. It follows by induction that all argumens in ├ are valid. Completeness is proven by first reformulating what is to be proved (namely, for any *P*1,…,*Pn*, if *P*1,…,*Pn-*1╞*Pn* then *P*1,…,*Pn-*1╞*Pn*) into a logically equivalent proposition, a set { *P*1,…, *Pn*} is consistent only if it is satisfiable. The proof has two stages. First it is shown that there is a way to expand a consistent set to one that is maximally consistent. In a second stage, it is shown that the maximally consistent set of sentences can itself be used as a model for the set, thus making the set satisfiable.

1. These systems were called by Gentzen “natural” because in the particular systems he invented for sentential and first-order logic, the rules of inference were thought to correspond to more natural forms of reasoning than did the axioms and inference rules of earlier axiom systems. [↑](#footnote-ref-1)