ARISTOTLE'S SYLLOGISTIC

FROM THE STANDPOINT OF
MODERN FORMAL LOGIC

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CHAPTER II

THESES OF THE SYSTEM

§ 8. *Theses and rules of inference*

THE Aristotelian theory of the syllogism is a system of true propositions concerning the constants *A*, *E*, *I*, and *O*. True propositions of a deductive system I call theses. Almost all theses of the Aristotelian logic are implications, i.e. propositions of the form 'If α, then β'. There are known only two theses of this logic not beginning with 'if', viz. the so-called laws of identity: '*A* belongs to all *A*' or 'All *A* is *A*', and '*A* belongs to some *A*' or 'Some *A* is *A*'. Neither of these laws was explicitly stated by Aristotle, but they were known to the Peripatetics.[1]

The implications belonging to the system are either laws of conversion (and laws of the square of opposition not mentioned in the *Prior Analytics*) or syllogisms. The laws of conversion are simple implications, for instance: 'If *A* belongs to all *B*, then *B* belongs to some *A*.'[2] The antecedent of this implication is the premiss '*A* belongs to all *B*', the consequent is '*B* belongs to some *A*'. This implication is regarded as true for all values of the variables *A* and *B*.

All Aristotelian syllogisms are implications of the type 'If α and β, then γ', where α and β are the two premisses and γ is the conclusion. The conjunction of the premisses 'α and β' is the antecedent, the conclusion γ is the consequent. As an example take the following formulation of the mood Barbara:

If *A* belongs to all *B*
and *B* belongs to all *C*,
then *A* belongs to all *C*.

In this example α means the premiss '*A* belongs to all *B*', β the premiss '*B* belongs to all *C*', and γ the conclusion '*A* belongs to all *C*'. This implication is also regarded as true for all values of the variables *A*, *B*, and *C*.

[1] Cf. p. 9, n. 5, p. 10, n. 1. In the passage quoted in the latter note Alexander says that the proposition '*A* does not belong to some *A*' is absurd. That means that the contradictory proposition '*A* belongs to all *A*' is true.

[2] *An. pr.* i. 2, 25a17 *εἰ δὲ παντὶ τὸ Α τῷ Β, καὶ τὸ Β τινὶ τῷ Α ὑπάρξει.*

It must be said emphatically that no syllogism is formulated by Aristotle as an inference with the word 'therefore' (*ἄρα*), as is done in the traditional logic. Syllogisms of the form:

All *B* is *A*;
all *C* is *B*;
therefore
all *C* is *A*

are not Aristotelian. We do not meet them until Alexander.[1] This transference of the Aristotelian syllogisms from the implicational form into the inferential is probably due to the influence of the Stoics.

The difference between the Aristotelian and the traditional syllogism is fundamental. The Aristotelian syllogism as an implication is a proposition, and as a proposition must be either true or false. The traditional syllogism is not a proposition, but a set of propositions which are not unified so as to form one single proposition. The two premisses written usually in two different lines are stated without a conjunction, and the connexion of these loose premisses with the conclusion by means of 'therefore' does not give a new compound proposition. The famous Cartesian principle, 'Cogito, ergo sum', is not a true principle, because it is not a proposition. It is an inference, or, according to a scholastic terminology, a consequence. Inferences and consequences, not being propositions, are neither true nor false, as truth and falsity belong only to propositions. They may be valid or not. The same has to be said of the traditional syllogism. Not being a proposition the traditional syllogism is neither true nor false; it can be valid or invalid. The traditional syllogism is either an inference, when stated in concrete terms, or a rule of inference, when stated in variables. The sense of such a rule may be explained by the example given above: When you put such values for *A*, *B*, and *C* that the premisses '*A* belongs to all *B*' and '*B* belongs to all *C*' are true, then you must accept as true the conclusion '*A* belongs to all *C*'.

If you find a book or an article where no difference is made between the Aristotelian and the traditional syllogism, you may

[1] In Alexander 47. 9 we find a syllogism in concrete terms with *ἄρα*: *πᾶν ζῷον οὐσία ἐστί, πᾶν ζῷον ἔμψυχόν ἐστι, τὶς ἄρα οὐσία ἔμψυχός ἐστιν.* At 382. 18 we have a complex syllogism in four variable terms with *ἄρα*: *τὸ Α παντὶ τῷ Β, τὸ Β παντὶ τῷ Γ, τὸ Α οὐδενὶ τῷ Δ, τὸ ἄρα Δ οὐδενὶ τῷ Γ.*

be sure that the author is either ignorant of logic or has never seen the Greek text of the *Organon*. Scholars like Waitz, the modern editor and commentator of the *Organon*, Trendelenburg, the compiler of the *Elementa logices Aristoteleae*, Prantl, the historian of logic, all knew the Greek text of the *Organon* well, but nevertheless they did not see the difference between the Aristotelian and the traditional syllogism. Only Maier seems to have felt for a moment that something is wrong here, when he asks for permission to replace the Aristotelian syllogism by the more familiar and more convenient form of the later logic; immediately afterwards he quotes the mood Barbara in its usual traditional form, neglecting differences he has seen between this form and that of Aristotle, and does not even say what differences he has seen.[1] When we realize that the difference between a thesis and a rule of inference is from the standpoint of logic a fundamental one, we must agree that an exposition of Aristotelian logic which disregards it cannot be sound. We have to this day no genuine exposition of Aristotelian logic.

It is always easy to deduce from an implicational thesis the corresponding rule of inference. Let us suppose that an implicational proposition 'If α, then β' is true: if α is true, we can always get β by detachment, so that the rule 'α therefore β' is valid. When the antecedent of an implicational thesis is a conjunction, as in the Aristotelian syllogisms, we must first change the conjunctional form 'If α and β, then γ' into the purely implicational form 'If α, then if β, then γ'. A moment of reflection is sufficient to convince ourselves that this transformation is correct. Supposing now that α and β are true premisses of a syllogism, we get the conclusion γ, applying the rule of detachment twice to the purely implicational form of the syllogism. If, therefore, an Aristotelian syllogism of the form 'If α and β, then γ' is true, the corresponding traditional mood of the form 'α, β, therefore γ' is valid. But conversely, it seems impossible to deduce the corresponding Aristotelian syllogism from a valid traditional mood by known logical rules.

[1] Maier, op. cit., vol. ii *a*, p. 74, n. 2: 'Es ist vielleicht gestattet, hier und im Folgenden die geläufigere Darstellungsform der späteren Logik, die zugleich leichter zu handhaben ist, an die Stelle der aristotelischen zu setzen.' The mood Barbara is quoted ibid., p. 75, thus:

alles B ist A
alles C ist B
alles C ist A

where the stroke replaces the word 'therefore'.

§ 9. *The syllogistic figures*

There are some controversial problems connected with the Aristotelian logic that are of historical interest without having any great logical importance. Among these is the problem of the syllogistic figures. The division of the syllogisms into figures has, in my opinion, only a practical aim: we want to be sure that no true syllogistic mood is omitted.

Aristotle divided the syllogistic moods into three figures. The shortest and clearest description of these figures is to be found not in the systematic part of the *Prior Analytics* but in the later chapters of that work. If we want, Aristotle says, to prove *A* of *B* syllogistically, we must take something common in relation to both, and this is possible in three ways: by predicating either *A* of *C* and *C* of *B*, or *C* of both, or both of *C*. These are the figures of which we have spoken, and it is clear that every syllogism must be made in one or other of these figures.[1]

It follows from this that *A* is the predicate and *B* the subject of the conclusion we have to prove syllogistically. *A* is called, as we shall see later, the major term and *B* the minor; *C* is the middle term. The position of the middle term as subject or predicate of the premisses is the principle by which Aristotle divides the syllogistic moods into figures. Aristotle says explicitly that we shall recognize the figure by the position of the middle term.[2] In the first figure the middle term is the subject of the major term and the predicate of the minor term, in the second figure it is the predicate, and in the last figure the subject, of both the other terms. Aristotle, however, is mistaken when he says that every syllogism must be in one of these three figures. There is a fourth possibility, viz. that the middle term is the predicate of the major term and the subject of the minor term. Moods of this kind are now spoken of as belonging to the fourth figure.

In the above passage Aristotle has overlooked this fourth

[1] *An. pr.* i. 23, 40^{b}30 *εἰ δὴ δέοι τὸ A κατὰ τοῦ B συλλογίσασθαι ἢ ὑπάρχον ἢ μὴ ὑπάρχον, ἀνάγκη λαβεῖν τι κατά τινος.* 41^{a}13 *εἰ οὖν ἀνάγκη μέν τι λαβεῖν πρὸς ἄμφω κοινόν, τοῦτο δ' ἐνδέχεται τριχῶς (ἢ γὰρ τὸ A τοῦ Γ καὶ τὸ Γ τοῦ B κατηγορήσαντας, ἢ τὸ Γ κατ' ἀμφοῖν, ἢ ἄμφω κατὰ τοῦ Γ), ταῦτα δ' ἐστὶ τὰ εἰρημένα σχήματα, φανερὸν ὅτι πάντα συλλογισμὸν ἀνάγκη γίνεσθαι διὰ τούτων τινὸς τῶν σχημάτων.*

[2] Ibid. 32, 47^{b}13 *τῇ τοῦ μέσου θέσει γνωριοῦμεν τὸ σχῆμα.*

possibility, although a few chapters farther on he himself gives a proof by a syllogism in the fourth figure. It is the same problem again: we have to prove *A* of *E* syllogistically, where *A* is the major term and *E* the minor. Aristotle gives practical indications how to solve this problem. We must construct a list of universal propositions having the terms *A* and *E* as subjects or predicates. In this list we shall have four types of universal affirmative proposition (I omit the negative propositions), '*B* belongs to all *A*', '*A* belongs to all *C*', '*Z* belongs to all *E*', and '*E* belongs to all *H*'. Each of the letters *B*, *C*, *Z*, and *H* represents any term fulfilling the above conditions. When we find among the *C*'s a term identical with a term among the *Z*'s, we get two premisses with a common term, say *Z*: '*A* belongs to all *Z*' and '*Z* belongs to all *E*', and the proposition '*A* belongs to all *E*' is proved in the mood Barbara. Let us now suppose that we cannot prove the universal proposition '*A* belongs to all *E*', as the *C*'s and *Z*'s have no common term, but we want at least to prove the particular proposition '*A* belongs to some *E*'. We can prove it in two different ways: if there is a term among the *C*'s identical with a term among the *H*'s, say *H*, we get the mood Darapti of the third figure: '*A* belongs to all *H*', '*E* belongs to all *H*', therefore '*A* must belong to some *E*'. But there is still another way when we find among the *H*'s a term identical with a term among the *B*'s, say *B*; we then get a syllogism with the premisses '*E* belongs to all *B*' and '*B* belongs to all *A*', from which we deduce the proposition '*A* belongs to some *E*' by converting the conclusion '*E* belongs to all *A*' obtained from these premisses by the mood Barbara.[1]

This last syllogism: 'If *E* belongs to all *B* and *B* belongs to all *A*, then *A* belongs to some *E*', is a mood neither of the first figure nor of the second or third. It is a syllogism where the middle term

[1] *An. pr.* i. 28, 44ª12–35 *ἔστω γὰρ τὰ μὲν ἑπόμενα τῷ A ἐφ' ὧν B, οἷς δ' αὐτὸ ἕπεται, ἐφ' ὧν Γ . . . πάλιν δὲ τῷ E τὰ μὲν ὑπάρχοντα, ἐφ' οἷς Z, οἷς δ' αὐτὸ ἕπεται, ἐφ' οἷς H . . . . εἰ μὲν οὖν ταὐτό τι ἔσται τῶν Γ τινὶ τῶν Z, ἀνάγκη τὸ A παντὶ τῷ E ὑπάρχειν· τὸ μὲν γὰρ Z παντὶ τῷ E, τῷ δὲ Γ παντὶ τὸ A, ὥστε παντὶ τῷ E τὸ A. εἰ δὲ τὸ Γ καὶ τὸ H ταὐτόν, ἀνάγκη τινὶ τῶν E τὸ A ὑπάρχειν· τῷ μὲν γὰρ Γ τὸ A, τῷ δὲ H τὸ E παντὶ ἀκολουθεῖ. . . . εἰ δὲ τῷ H τὸ B ταὐτόν, ἀντεστραμμένος ἔσται συλλογισμός· τὸ μὲν γὰρ E τῷ A ὑπάρξει παντί, τὸ γὰρ B τῷ A, τὸ δὲ E τῷ B (ταὐτὸ γὰρ ἦν τῷ H)· τὸ δὲ A τῷ E παντὶ μὲν οὐκ ἀνάγκη ὑπάρχειν, τινὶ δ' ἀνάγκη διὰ τὸ ἀντιστρέφειν τὴν καθόλου κατηγορίαν τῇ κατὰ μέρος.* I read *τὴν καθόλου κατηγορίαν τῇ* with codex B (see Waitz, i. 196; the footnote in Bekker to 44ª34 seems to be a misprint) and Alexander 306. 16 against *τῇ καθόλου κατηγορίᾳ τὴν* in Bekker and Waitz. I am glad to see that this reading is also accepted by Sir David Ross.

B is the predicate of the major term *A* and the subject of the minor term *E*. It is the mood Bramantip of the fourth figure. Nevertheless it is as valid as any other Aristotelian mood. Aristotle calls it a 'converted syllogism' (*ἀντεστραμμένος συλλογισμός*) because he proves this mood by converting the conclusion of the mood Barbara. There are two other moods, Camestres of the second figure and Disamis of the third, which Aristotle proves in the same manner, by converting the conclusion of moods of the first figure. Let us consider the proof of Disamis: 'If *R* belongs to all *S* and *P* belongs to some *S*, then *P* belongs to some *R*'. As the second premiss can be converted into '*S* belongs to some *P*', we get by the mood Darii the conclusion '*R* belongs to some *P*'. By converting this conclusion into '*P* belongs to some *R*' we get the proof of Disamis. Aristotle here applies the conversion to the conclusion of the mood Darii, which gives another syllogism of the fourth figure called Dimaris: 'If *R* belongs to all *S* and *S* belongs to some *P*, then *P* belongs to some *R*.'[1]

All these deductions are logically correct, and so are the moods obtained by their means. Aristotle knows, indeed, that besides the fourteen moods of the first, second, and third figures established by him systematically in the early chapters of the *Prior Analytics* there are still other true syllogisms. Two of them are quoted by him at the end of this systematic exposition. It is evident, he says, that in all the figures, whenever a syllogism does not result, if both the terms are affirmative or negative nothing necessary follows at all, but if one is affirmative, the other negative, and if the negative is stated universally, a syllogism always results linking the minor to the major term, e.g. if *A* belongs to all or some *B*, and *B* belongs to no *C*; for if the premisses are converted it is necessary that *C* does not belong to some *A*.[2] From the second premiss

[1] *An. pr.* i. 6, 28ᵇ7 *εἰ γὰρ τὸ μὲν P παντὶ τῷ Σ τὸ δὲ Π τινί, ἀνάγκη τὸ Π τινὶ τῷ P ὑπάρχειν. ἐπεὶ γὰρ ἀντιστρέφει τὸ καταφατικόν, ὑπάρξει τὸ Σ τινὶ τῷ Π, ὥστ' ἐπεὶ τὸ μὲν P παντὶ τῷ Σ, τὸ δὲ Σ τινὶ τῷ Π, καὶ τὸ P τινὶ τῷ Π ὑπάρξει· ὥστε τὸ Π τινὶ τῷ P.* This passage refutes the assertion of Friedrich Solmsen that Aristotle was not willing to apply the procedure of conversion to the conclusion. *Die Entstehung der aristotelischen Logik und Rhetorik*, Berlin (1929), p. 55: 'Die Umkehrung dringt in die conclusio ein, in der Aristoteles sie nicht kennen wollte.'

[2] *An. pr.* i. 7, 29ª19 *δῆλον δὲ καὶ ὅτι ἐν ἅπασι τοῖς σχήμασιν, ὅταν μὴ γίνηται συλλογισμός, κατηγορικῶν μὲν ἢ στερητικῶν ἀμφοτέρων ὄντων τῶν ὅρων οὐδὲν ὅλως γίνεται ἀναγκαῖον, κατηγορικοῦ δὲ καὶ στερητικοῦ, καθόλου ληφθέντος τοῦ στερητικοῦ, ἀεὶ γίνεται συλλογισμὸς τοῦ ἐλάττονος ἄκρου πρὸς τὸ μεῖζον, οἷον εἰ τὸ μὲν A παντὶ τῷ B ἢ τινί, τὸ δὲ B μηδενὶ τῷ Γ· ἀντιστρεφομένων γὰρ τῶν προτάσεων ἀνάγκη τὸ Γ τινὶ τῷ A μὴ ὑπάρχειν.*

given here by Aristotle we get by conversion the proposition '*C* belongs to no *B*', from the first premiss we get '*B* belongs to some *A*', and from these two propositions results, according to the mood Ferio of the first figure, the conclusion '*C* does not belong to some *A*'. Two new syllogistic moods are thus proved, called later Fesapo and Fresison:

If *A* belongs to all *B*	If *A* belongs to some *B*
and *B* belongs to no *C*,	and *B* belongs to no *C*,
then *C* does not belong to some *A*.	then *C* does not belong to some *A*.

Aristotle calls the minor term *C* and the major term *A* because he treats the premisses from the point of view of the first figure. He says, therefore, that from the given premisses a conclusion results in which the minor term is predicated of the major.

Three other syllogisms belonging to the fourth figure are mentioned by Aristotle at the beginning of Book II of the *Prior Analytics*. Aristotle states here that all universal syllogisms (i.e. syllogisms with a universal conclusion) give more than one result, and of particular syllogisms the affirmative yield more than one, the negative yield only one conclusion. For all premisses are convertible except the particular negative; and the conclusion states something about something. Consequently all syllogisms except the particular negative yield more than one conclusion, e.g. if *A* has been proved to belong to all or to some *B*, then *B* must belong to some *A*; and if *A* has been proved to belong to no *B*, then *B* belongs to no *A*. This is a different conclusion from the former. But if *A* does not belong to some *B*, it is not necessary that *B* should not belong to some *A*, for it may possibly belong to all *A*.[1]

We see from this passage that Aristotle knows the moods of the fourth figure, called later Bramantip, Camenes, and Dimaris, and that he gets them by conversion of the conclusion of the moods Barbara, Celarent, and Darii. The conclusion of a syllogism is a proposition stating something about something, i.e. a premiss, and therefore the laws of conversion can be applied to it.

[1] *An. pr.* ii. 1, 53^{a}4 *οἱ μὲν καθόλου* (scil. *συλλογισμοὶ*) *πάντες ἀεὶ πλείω συλλογίζονται, τῶν δ' ἐν μέρει οἱ μὲν κατηγορικοὶ πλείω, οἱ δ' ἀποφατικοὶ τὸ συμπέρασμα μόνον. αἱ μὲν γὰρ ἄλλαι προτάσεις ἀντιστρέφουσιν, ἡ δὲ στερητικὴ οὐκ ἀντιστρέφει· τὸ δὲ συμπέρασμα τὶ κατά τινός ἐστιν. ὥσθ' οἱ μὲν ἄλλοι συλλογισμοὶ πλείω συλλογίζονται, οἷον εἰ τὸ Α δέδεικται παντὶ τῷ Β ἢ τινί, καὶ τὸ Β τινὶ τῷ Α ἀναγκαῖον ὑπάρχειν· καὶ εἰ μηδενὶ τῷ Β τὸ Α, οὐδὲ τὸ Β οὐδενὶ τῷ Α. τοῦτο δ' ἕτερον τοῦ ἔμπροσθεν. εἰ δὲ τινὶ μὴ ὑπάρχει, οὐκ ἀνάγκη καὶ τὸ Β τινὶ τῷ Α μὴ ὑπάρχειν· ἐνδέχεται γὰρ παντὶ ὑπάρχειν.*

It is important that propositions of the type '*A* belongs to no *B*' and '*B* belongs to no *A*' are regarded by Aristotle as different.

It follows from these facts that Aristotle knows and accepts all the moods of the fourth figure. This must be emphasized against the opinion of some philosophers that he rejected these moods. Such a rejection would be a logical error which cannot be imputed to Aristotle. His only mistake is the omission of these moods in the systematic division of the syllogisms. We do not know why he did so. Philosophical reasons, as we shall see later, must be excluded. The most probable explanation is given, in my opinion, by Bocheński,[1] who supposes that Book I, chapter 7 and Book II, chapter 1 of the *Prior Analytics*, where these new moods are mentioned, were composed by Aristotle later than the systematic exposition of chapters 4–6 of Book I. This hypothesis seems to me the more probable, as there are many other points in the *Prior Analytics* suggesting that the contents of this work grew during its composition. Aristotle did not have time to draw up systematically all the new discoveries he had made, and left the continuation of his logical work to his pupil Theophrastus. Theophrastus, indeed, found for the moods of the fourth figure which are 'homeless' in Aristotle's system a place among the moods of the first figure.[2] For this purpose he had to introduce a slight modification into the Aristotelian definition of the first figure. Instead of saying that in the first figure the middle term is the subject of the major and the predicate of the minor, as Aristotle does,[3] he said generally that in the first figure the middle term is the subject of one premiss and the predicate of another. Alexander repeats this definition, which probably comes from Theophrastus, and seems not to see that it differs from the Aristotelian description of the first figure.[4] The correction of

[1] I. M. Bocheński, O.P., *La Logique de Théophraste*, Collectanea Friburgensia, Nouvelle Série, fasc. xxxii, Fribourg en Suisse (1947), p. 59.

[2] Alexander 69. 27 *Θεόφραστος δὲ προστίθησιν ἄλλους πέντε τοῖς τέσσαρσι τούτοις οὐκέτι τελείους οὐδ' ἀναποδείκτους ὄντας, ὧν μνημονεύει καὶ ὁ Ἀριστοτέλης, τῶν μὲν ἐν τούτῳ τῷ βιβλίῳ προελθών, τῶν δὲ ἐν τῷ μετὰ τοῦτο τῷ δευτέρῳ κατ' ἀρχάς.* Cf. ibid. 110. 12.

[3] Cf. p. 23, n. 1.

[4] Alexander 258. 17 (ad i. 23) *ἡ δὲ τοῦ μέσου σχέσις πρὸς τά, ὧν λαμβάνεται μέσον, τριχῶς γίνεται (ἢ γὰρ ἐν μέσῳ τίθεται αὐτῶν τῷ μὲν ὑποκείμενος αὐτῶν τοῦ δὲ κατηγορούμενος, ἢ ἀμφοτέρων κατηγορεῖται, ἢ ἀμφοτέροις ὑπόκειται).* Ibid. 349. 5 (ad i. 32) *ἂν μὲν γὰρ ὁ μέσος ἐν ἀμφοτέραις ὢν ταῖς προτάσεσιν οὕτως ᾖ ὡς τοῦ μὲν κατηγορεῖσθαι αὐτῶν τῷ δὲ ὑποκεῖσθαι, πρῶτον ἔσται σχῆμα.*

Theophrastus is as good a solution of the problem of the syllogistic figures as the addition of a new figure.

§ 10. *The major, middle, and minor terms*

There is still another error committed by Aristotle in the *Prior Analytics*, with more serious consequences. It concerns the definition of the major, minor, and middle terms as given in his characterization of the first figure. This begins with the words: 'Whenever three terms are so related to one another that the last is contained in the middle and the middle is contained or not in the first, the extremes must form a perfect syllogism.' This is how he begins; in the next sentence he explains what he means by the middle term: 'I call that term the middle which is itself contained in another and contains another in itself, which by position also becomes the middle.'[1] Aristotle then investigates the syllogistic forms of the first figure with universal premisses without using the expressions 'major term' and 'minor term'. These expressions occur for the first time when he comes to the moods of the first figure with particular premisses. Here we find the following explanations: 'I call that term the major in which the middle term is contained and that term the minor which comes under the middle.'[2] These explanations of the major and the minor term, like that of the middle term, are expressed quite generally. It would seem that Aristotle intends to apply them to all moods of the first figure.[3] If he thought, however, that they are capable of covering all cases, he was mistaken.

In fact these explanations can be applied only to syllogisms of the mood Barbara with concrete terms and true premisses, e.g.:

(1) If all birds are animals
and all crows are birds,
then all crows are animals.

In this syllogism there is a term, 'bird', which is itself contained in another term, 'animal', and contains in itself a third term,

[1] *An. pr.* i. 4, 25^{b}32 ὅταν οὖν ὅροι τρεῖς οὕτως ἔχωσι πρὸς ἀλλήλους ὥστε τὸν ἔσχατον ἐν ὅλῳ εἶναι τῷ μέσῳ καὶ τὸν μέσον ἐν ὅλῳ τῷ πρώτῳ ἢ εἶναι ἢ μὴ εἶναι, ἀνάγκη τῶν ἄκρων εἶναι συλλογισμὸν τέλειον. καλῶ δὲ μέσον μὲν ὃ καὶ αὐτὸ ἐν ἄλλῳ καὶ ἄλλο ἐν τούτῳ ἐστίν, ὃ καὶ τῇ θέσει γίνεται μέσον.

[2] Ibid., 26^{a}21 λέγω δὲ μεῖζον μὲν ἄκρον ἐν ᾧ τὸ μέσον ἐστίν, ἔλαττον δὲ τὸ ὑπὸ τὸ μέσον ὄν.

[3] Maier, op. cit., vol. ii *a*, pp. 49, 55, really treats them as definitions valid for all the moods of the first figure.

'crow'. According to the given explanation 'bird' would be the middle term. Consequently 'animal' would be the major term and 'crow' the minor term. It is evident that the major term is so called because it is the largest in extent, as the minor term is the smallest.

We know, however, that syllogisms with concrete terms are only applications of logical laws, but do not belong to logic themselves. The mood Barbara as a logical law must be stated with variables:

(2) If all *B* is *A*
and all *C* is *B*,
then all *C* is *A*.

To this logical law the given explanations are not applicable, because it is not possible to determine extensional relations between variables. It may be said that *B* is the subject in the first premiss and the predicate in the second, but it cannot be stated that *B* is contained in *A* or that it contains *C*; for the syllogism (2) is true for all values of the variables *A*, *B*, and *C*, even for those which do not verify its premisses. Take 'bird' for *A*, 'crow' for *B*, and 'animal' for *C*: you get a true syllogism:

(3) If all crows are birds
and all animals are crows,
then all animals are birds.

The extensional relations of the terms 'crow', 'bird', and 'animal' are of course independent of syllogistic moods and remain the same in syllogism (3) as they were in (1). But the term 'bird' is no longer the middle term in (3) as it was in (1); 'crow' is the middle term in (3) because it occurs in both premisses, and the middle term must be common to both premisses. This is the definition of the middle term accepted by Aristotle for all figures.[1] This general definition is incompatible with the special explanation given by Aristotle for the first figure. The special explanation of the middle term is obviously wrong. It is evident also that the explanations of the major and minor terms which Aristotle gives for the first figure are wrong, too.

Aristotle does not give a definition of the major and minor terms valid for all figures; but practically he treats the predicate

[1] *An. pr.* i. 32, 47^{a}38 μέσον δὲ θετέον τῶν ὅρων τὸν ἐν ἀμφοτέραις ταῖς προτάσεσι λεγόμενον· ἀνάγκη γὰρ τὸ μέσον ἐν ἀμφοτέραις ὑπάρχειν ἐν ἅπασι τοῖς σχήμασιν.

of the conclusion as the major term and the subject of the conclusion as the minor term. It is easy to see how misleading this terminology is: in syllogism (3) the major term 'bird' is smaller in extension than the minor term 'animal'. If the reader feels a difficulty in accepting syllogism (3) because of its false minor, he may read 'some animals' instead of 'all animals'. The syllogism:

(4) If all crows are birds
and some animals are crows,
then some animals are birds

is a valid syllogism of the mood Darii with true premisses. And here again, as in syllogism (3), the largest term 'animal' is the minor term; 'bird', middle in extension, is the major term; and the smallest term, 'crow', is the middle term.

The difficulties we have already met are still greater when we take as examples syllogisms with negative premisses, e.g. the mood Celarent:

If no *B* is *A*
and all *C* is *B*,
then no *C* is *A*.

B is the middle term; but does it fulfil the conditions laid down by Aristotle for the middle term of the first figure? Certainly not. And which of the terms, *C* or *A*, is the major and which is the minor? How can we compare these terms with respect to their extension? There is no positive answer to these last questions, as they spring from a mistaken origin.[1]

§ 11. *The history of an error*

The faulty definition of the major and the minor terms, given by Aristotle for the first figure, and the misleading terminology he adopts, were already in antiquity a source of difficulty. The problem arose in the case of the second figure. All the moods of this figure have a negative conclusion and the first two moods, called later Cesare and Camestres, yield a universal negative conclusion. From the premisses '*M* belongs to all *N*' and '*M* belongs to no *X*' follows the conclusion '*X* belongs to no *N*', and by conversion of this result we get a second conclusion, '*N* belongs to no *X*'. In both syllogisms *M* is the middle term; but how are we to decide which of the two remaining terms, *N* and *X*, is the major term and which is the minor? Do major and minor terms exist 'by nature' (*φύσει*) or only 'by convention' (*θέσει*)?[1]

Such problems, according to Alexander, were raised by the later Peripatetics. They saw that in universal affirmative premisses there can be a major term by nature, because in such premisses the predicate is larger in extension (*ἐπὶ πλέον*) than the subject, but the same is not true in universal negative premisses.[2] We cannot know, for instance, which of the terms 'bird' or 'man' is major, because it is equally true that 'no bird is a man' and that 'no man is a bird'. Herminus, the teacher of Alexander, tried to answer this question by modifying the meaning of the expression 'major term'. He says that of two such terms, 'bird' and 'man', that is the major which in a systematic classification of the animals is nearer to the common genus 'animal'. In our example it is the term 'bird'.[3] Alexander is right when he rejects this theory and its further elaboration given by Herminus, but he also rejects the opinion that the major term is the predicate of the conclusion. The major term, he says, would not be fixed in this case, as the universal negative premiss is convertible, and what till now has been a major term instantly becomes a minor, and it would depend upon us to make the same term major and minor.[4] His own solution is based on the assumption that when we are forming a syllogism we are choosing premisses for a given problem

[1] We have no guarantee, as Keynes (op. cit., p. 286) justly remarks, that the major term will be the largest in extension and the minor the smallest, when one of the premisses is negative or particular. Thus, Keynes continues, 'the syllogism—No *M* is *P*, All *S* is *M*, therefore, No *S* is *P*—yields as one case [here there follows a diagram representing three circles *M*, *P*, and *S*, a large *S* included in a larger *M*, outside of them a small *P*] where the major term may be the smallest in extent, and the middle the largest.' Keynes forgets that it is not the same to draw a small circle *P* outside of a large circle *S* and to maintain that the term *P* is smaller in extent than the term *S*. Terms can be compared with respect to their extent only in the case when one of them is contained in the other.

[1] Alexander 72. 17 *ζητεῖται, εἰ φύσει ἐν δευτέρῳ σχήματι μείζων τίς ἐστι καὶ ἐλάττων ἄκρος, καὶ τίνι οὗτος κριθήσεται.*

[2] Ibid. 72. 24 *ἐπὶ μὲν γὰρ τῶν καταφατικῶν μείζων ὁ κατηγορούμενος καθόλου, ὅτι καὶ ἐπὶ πλέον· διὰ τούτου γὰρ οὐδὲ ἀντιστρέφει· ὥστε φύσει αὐτῷ τὸ μείζονα εἶναι ὑπάρχει. ἐπὶ δὲ τῶν καθόλου ἀποφατικῶν οὐκέτι τοῦτο ἀληθές.*

[3] Ibid. 27 *Ἑρμῖνος οἴεται, ἐν δευτέρῳ σχήματι τὸν μείζονα ἄκρον εἶναι . . . τὸν ἐγγύτερον τοῦ κοινοῦ γένους αὐτῶν (ἂν γὰρ ὦσιν οἱ ἄκροι ὄρνεον καὶ ἄνθρωπος, ἐγγυτέρω τοῦ κοινοῦ γένους αὐτῶν, τοῦ ζῴου, τὸ ὄρνεον τοῦ ἀνθρώπου καὶ ἐν τῇ πρώτῃ διαιρέσει, διὸ καὶ μείζων ἄκρος τὸ ὄρνεον).*

[4] Ibid. 75. 10 *ἀλλ' οὐδὲ ἁπλῶς πάλιν ῥητέον μείζονα τὸν ἐν τῷ συμπεράσματι τοῦ συλλογισμοῦ κατηγορούμενον, ὡς δοκεῖ τισιν· οὐδὲ γὰρ οὗτος δῆλος· ἄλλοτε γὰρ ἄλλος ἔσται καὶ οὐχ ὡρισμένος τῷ ἀντιστρέφειν τὴν καθόλου ἀποφατικήν, καὶ ὁ τέως μείζων αὖθις ἐλάττων, καὶ ἐφ' ἡμῖν ἔσται τὸν αὐτὸν καὶ μείζω καὶ ἐλάττω ποιεῖν.*

conceived as the conclusion. The predicate of this conclusion is the major term, and it does not matter whether we afterwards convert this conclusion or not: in the problem as first given the major term was and remains the predicate.[1] Alexander forgets that when we are forming a syllogism we are not always choosing premisses for a given conclusion, but sometimes we are deducing new conclusions from given premisses.

The problem was settled only after Alexander. What John Philoponus writes on the subject deserves to be regarded as classic. According to him we may define the major and the minor term either for the first figure alone or for all the three figures together. In the first figure the major term is the predicate of the middle and the minor is the subject of the middle. Such a definition cannot be given for the other two figures because the relations of the extremes to the middle term are in the other figures the same. We must therefore accept as a common rule for all figures that the major term is the predicate of the conclusion and the minor term is the subject of the conclusion.[2] That this rule is only a convention follows from another passage of Philoponus, where we read that the universal moods of the second figure have a major and a minor term only by convention, but not by nature.[3]

§ 12. *The order of the premisses*

Around the Aristotelian logic arose some queer philosophical prejudices which cannot be explained rationally. One of them is directed against the fourth figure, disclosing sometimes a strange aversion to it, another is the odd opinion that in all syllogisms the major premiss should be stated first.

[1] Alexander 75. 26 *τὸν δὴ ἐν τῷ προκειμένῳ προβλήματι εἰς τὴν δεῖξιν κατηγορούμενον τοῦτο θετέον μείζονα· καὶ γὰρ εἰ ἀντιστρέφει καὶ διὰ τοῦτο γίνεται ὁ αὐτὸς καὶ ὑποκείμενος, ἀλλ' ἔν γε τῷ ἡμῖν εἰς τὸ δεῖξαι προκειμένῳ κατηγορούμενος ἦν τε καὶ μένει.*

[2] Philoponus 67. 19 *ἴδωμεν πρότερον καὶ τίς ἐστι μείζων ὅρος καὶ τίς ἐλάττων. τοῦτο δὲ δυνατὸν μὲν καὶ κοινῶς ἐπὶ τῶν τριῶν σχημάτων διορίσασθαι καὶ ἰδίᾳ ἐπὶ τοῦ πρώτου. καὶ ἰδίᾳ μὲν ἐπὶ τοῦ πρώτου σχήματος μείζων ὅρος ἐστὶν ὁ τοῦ μέσου κατηγορούμενος, ἐλάττων δὲ ὁ τῷ μέσῳ ὑποκείμενος. καὶ τοῦτο μὲν ἰδιαζόντως ἐπὶ τοῦ πρώτου λέγομεν, ἐπειδὴ ὁ μέσος ἐν τῷ πρώτῳ τοῦ μὲν κατηγορεῖται τῷ δὲ ὑπόκειται. ἀλλ' ἐπειδὴ κατ' οὐδέτερον τῶν ἄλλων σχημάτων διάφορον ἔχουσι σχέσιν οἱ ἄκροι πρὸς τὸν μέσον, δῆλον ὅτι οὐκέτι ἁρμόσει ἡμῖν οὗτος ὁ προσδιορισμὸς ἐπ' ἐκείνων. χρηστέον οὖν κοινῷ κανόνι ἐπὶ τῶν τριῶν σχημάτων τούτῳ, ὅτι μείζων ἐστὶν ὅρος ὁ ἐν τῷ συμπεράσματι κατηγορούμενος, ἐλάττων δὲ ὁ ἐν τῷ συμπεράσματι ὑποκείμενος.*

[3] Ibid. 87. 10 *τὸ δὲ μεῖζον ἄκρον ἐν τούτῳ τῷ σχήματι τῶν δύο προτάσεων καθόλου οὐσῶν οὐκ ἔστι φύσει ἀλλὰ θέσει.*

From the standpoint of logic the order of the premisses in the Aristotelian syllogisms is arbitrary, because the premisses of the syllogism form a conjunction and the members of a conjunction are commutable. It is only a convention that the major premiss is stated first. Nevertheless, some philosophers, like Waitz or Maier, maintain that the order of the premisses is fixed. Waitz censures Apuleius for having changed this order,[1] and Maier rejects Trendelenburg's opinion that Aristotle does not tie it down.[2] No arguments are given in either case.

I do not know who is the author of the opinion that the order of the premisses is fixed. Certainly it is not Aristotle. Although Aristotle has not given a definition of the major and minor terms valid for all the three figures, it is always easy to determine which term and which premiss are regarded by him as the major and which as the minor. Aristotle, in his systematic exposition of the syllogistic, uses different letters to denote different terms; for each figure he puts them in alphabetical order (*θέσις*) and says explicitly which term is denoted by a given letter. We have thus for the first figure the letters *A*, *B*, *C*; *A* is the major term, *B* the middle, and *C* the minor.[3] For the second figure we have the letters *M*, *N*, *X*, where *M* is the middle term, *N* the major, and *X* the minor.[4] For the third figure we have the letters *P*, *R*, *S*, where *P* is the major term, *R* the minor, and *S* the middle.[5]

[1] Waitz, op. cit., vol. i, p. 380: 'Appuleius in hunc errorem se induci passus est, ut propositionum ordinem immutaverit.'

[2] Maier, op. cit., vol. ii *a*, p. 63: 'Darnach is Trendelenburg's Auffassung, dass Aristoteles die Folge der Prämissen frei lasse, falsch. Die Folge der Prämissen ist vielmehr festgelegt.' It is not clear to me what reasons he refers to by *darnach*.

[3] This follows from the definition given by Aristotle for the first figure; see p. 28, n. 1. Cf. Alexander 54. 12 *ἔστω γὰρ μείζων μὲν ἄκρος τὸ Α, μέσος δὲ ὅρος τὸ Β, ἐλάττων δὲ ἄκρος τὸ Γ.*

[4] *An. pr.* i. 5, 26^{b}34 *ὅταν δὲ τὸ αὐτὸ τῷ μὲν παντὶ τῷ δὲ μηδενὶ ὑπάρχῃ, ἢ ἑκατέρῳ παντὶ ἢ μηδενί, τὸ μὲν σχῆμα τὸ τοιοῦτον καλῶ δεύτερον, μέσον δὲ ἐν αὐτῷ λέγω τὸ κατηγορούμενον ἀμφοῖν, ἄκρα δὲ καθ' ὧν λέγεται τοῦτο, μεῖζον δὲ ἄκρον τὸ πρὸς τῷ μέσῳ κείμενον, ἔλαττον δὲ τὸ πορρωτέρω τοῦ μέσου. τίθεται δὲ τὸ μέσον ἔξω μὲν τῶν ἄκρων, πρῶτον δὲ τῇ θέσει.* Cf. Alexander 78. 1 *χρῆται γὰρ στοιχείοις οὐ τοῖς Α, Β, Γ, οἷς ἐν τῷ πρώτῳ σχήματι, ἀλλὰ τοῖς Μ, Ν, Ξ, μέσον μὲν λαμβάνων τὸ Μ τὸ ἀμφοτέρων κατηγορούμενον καὶ τὴν πρώτην ἔχον τάξιν ἐν τῇ καταγραφῇ, μείζονα δὲ ἄκρον τὸ Ν ἐφεξῆς κείμενον μετὰ τὸν μέσον, ἔσχατον δὲ καὶ ἐλάττονα τὸ Ξ.*

[5] *An. pr.* i. 6, 28^{a}10 *ἐὰν δὲ τῷ αὐτῷ τὸ μὲν παντὶ τὸ δὲ μηδενὶ ὑπάρχῃ, ἢ ἄμφω παντὶ ἢ μηδενί, τὸ μὲν σχῆμα τὸ τοιοῦτον καλῶ τρίτον, μέσον δ' ἐν αὐτῷ λέγω καθ' οὗ ἄμφω τὰ κατηγορούμενα, ἄκρα δὲ τὰ κατηγορούμενα, μεῖζον δ' ἄκρον τὸ πορρώτερον τοῦ μέσου, ἔλαττον δὲ τὸ ἐγγύτερον. τίθεται δὲ τὸ μέσον ἔξω μὲν τῶν ἄκρων, ἔσχατον δὲ τῇ θέσει.* Cf. Alexander 98. 20 *ἐπὶ τούτου τοῦ σχήματος πάλιν χρῆται στοιχείοις*

Aristotle states the major premiss first in all the moods of the first and the second figure, and in two moods of the third figure, Darapti and Ferison.[1] In the remaining moods of the third figure, Felapton, Disamis, Datisi, and Bocardo, the minor premiss is stated first.[2] The most conspicuous example is the mood Datisi. This mood is formulated in the same chapter twice; in both formulations the letters are the same, but the premisses are inverted. The first formulation runs: 'If *R* belongs to some *S*, and *P* to all *S*, *P* must belong to some *R*.'[3] The first premiss of this syllogism is the minor premiss, for it contains the minor term *R*. The second formulation reads: 'If *P* belongs to all *S*, and *R* to some *S*, then *P* will belong to some *R*.'[4] The first premiss of this second syllogism is the major premiss, as it contains the major term *P*. Attention must be called to the fact that this second formulation is given only occasionally, while the standard formula of this mood, belonging to the systematic exposition, is enunciated with transposed premisses.

In Book II of the *Prior Analytics* we meet other moods with transposed premisses, as Darii,[5] Camestres,[6] Baroco.[7] Even Barbara, the main syllogism, is occasionally quoted by Aristotle with the minor premiss first.[8] I can hardly understand, in view of these examples, how some philosophers knowing the Greek text of the *Organon* could have formed and maintained the opinion that the order of the premisses is fixed and the major premiss must be stated first. It seems that philosophical prejudices may sometimes destroy not only common sense but also the faculty of seeing facts as they are.

§ 13. *Errors of some modern commentators*

The story of the fourth figure may serve as another example to show how strange philosophical prejudices sometimes are. Carl Prantl, the well-known historian of logic, begins his consideration of this figure with the following words: 'The question why silly playthings, as, for instance, the so-called Galenian fourth figure, are not to be found in Aristotle, is one we do not put at all; it plainly cannot be our task to declare at every step of the Aristotelian logic that this or that nonsense does not occur in it.'[1] Prantl does not see that Aristotle knows and accepts the moods of the so-called Galenian fourth figure and that it would be a logical error not to regard these moods as valid. But let us go farther. Commenting upon the passage where Aristotle speaks of the two moods later called Fesapo and Fresison,[2] Prantl first states these moods as rules of inference:

All *B* is *A* No *C* is *B* Some *A* is not *C*	Some *B* is *A* No *C* is *B* Some *A* is not *C*

—he does not, of course, see the difference between the Aristotelian and the traditional syllogism—and then he says: 'By transposition of the major premiss and the minor it becomes possible for the act of reasoning to begin'; and further: 'Such kinds of reasoning are, of course, not properly valid, because the premisses ordered as they were before the transposition are simply nothing for the syllogism.'[3] This passage reveals, in my opinion, Prantl's entire ignorance of logic. He seems not to understand that Aristotle proves the validity of these moods not by transposing the premisses, i.e. by inverting their order, but by converting them, i.e. by changing the places of their subjects and predicates.

τοῖς *Π*, *P*, *Σ*, καὶ ἔστιν αὐτῷ τοῦ μὲν μείζονος ἄκρου σημαντικὸν τὸ *Π*, τοῦ δὲ ἐλάττονος καὶ ὀφείλοντος ὑποκεῖσθαι ἐν τῷ γινομένῳ συμπεράσματι τὸ *P*, τοῦ δὲ μέσου τὸ *Σ*.

[1] See, for instance, p. 3, n. 2 (Barbara) and p. 10, n. 2 (Ferio).

[2] See p. 9, n. 4 (Felapton), and p. 7, n. (Disamis).

[3] *An. pr.* i. 6, 28^{b}12 εἰ τὸ μὲν *P* τινὶ τῷ *Σ* τὸ δὲ *Π* παντὶ ὑπάρχει, ἀνάγκη τὸ *Π* τινὶ τῷ *P* ὑπάρχειν.

[4] Ibid. 28^{b}26 εἰ γὰρ παντὶ τὸ *Π* τῷ *Σ* ὑπάρχει, τὸ δὲ *P* τινὶ τῷ *Σ*, καὶ τὸ *Π* τινὶ τῷ *P* ὑπάρξει.

[5] Ibid. ii. 11, 61^{b}41 εἰ γὰρ τὸ *A* τινὶ τῷ *B*, τὸ δὲ *Γ* παντὶ τῷ *A*, τινὶ τῷ *B* τὸ *Γ* ὑπάρξει.

[6] Ibid. ii. 8, 60^{a}3 εἰ τὸ *A* μηδενὶ τῷ *Γ*, τῷ δὲ *B* παντί, οὐδενὶ τῷ *Γ* τὸ *B*.

[7] Ibid. 60^{a}5 εἰ γὰρ τὸ *A* τινὶ τῷ *Γ* μὴ ὑπάρχει, τῷ δὲ *B* παντί, τὸ *B* τινὶ τῷ *Γ* οὐχ ὑπάρξει.

[8] See p. 10, n. 5.

[1] Carl Prantl, *Geschichte der Logik im Abendlande*, vol. i, p. 272: 'Die Frage aber, warum einfältige Spielereien, wie z. B. die sog. Galenische vierte Figur, sich bei Aristoteles nicht finden, werfen wir natürlich gar nicht auf; . . . wir können selbstverständlicher Weise nicht die Aufgabe haben, bei jedem Schritte der aristotelischen Logik eigens anzugeben, dass dieser oder jener Unsinn sich bei Aristoteles nicht finde.'

[2] See p. 25, n. 2.

[3] Prantl, op. cit., vol. i, p. 276:

'Alles B ist A Kein C ist B Einiges A ist nicht C	Einiges B ist A Kein C ist B Einiges A ist nicht C

woselbst durch Vertauschung des Untersatzes mit dem Obersatze es möglich wird, dass die Thätigkeit des Schliessens beginne; . . . natürlich aber sind solches keine eigenen berechtigten Schlussweisen, denn in solcher Anordnung vor der Vornahme der Vertauschung sind die Prämissen eben einfach nichts für den Syllogismus.'

Moreover, it is out of place to say that, two premisses being given, the act of reasoning begins when one premiss is stated first, but no syllogism results when the other precedes. From the standpoint of logic Prantl's work is useless.

The same may be said of Heinrich Maier's work. His treatise on the syllogistic figures generally and the fourth figure in particular is in my opinion one of the most obscure chapters of his laborious but unfortunate book.[1] Maier writes that two opinions of the criterion for the syllogistic figures stand opposed to each other: one (especially Ueberweg) sees this criterion in the position of the middle term as subject or predicate, the other (especially Trendelenburg) sees it in the extensional relations of the middle term to the extremes. It is not yet settled, Maier says, which of these opinions is right.[2] He adopts the second as his own, relying on Aristotle's characterization of the first figure. We know already that this characterization is logically untenable. Maier not only accepts it, but modifies the Aristotelian characterizations of the two other figures according to the first. Aristotle describes the second figure somewhat carelessly as follows: 'Whenever the same term belongs to all of one subject and to none of the other, or to all of each subject, or to none of either, I call such a figure the second; by "middle term" in it I mean that which is predicated of both subjects, by "extremes" the terms of which this is said.'[3] Maier remarks: 'When we reflect that the expressions "*B* is included in *A*", "*A* belongs to *B*", and "*A* is predicated of *B*" are interchangeable, then we may put this characterization according to the description of the first figure in the following words.'[4] Maier commits here his first error: it is not true that the three expressions he quotes can be exchanged for each other. Aristotle states explicitly: 'To say that one term is included in another is the same as to say that the other is predicated of all of the first.'[5] The expression '*B* is included in *A*' means, therefore,

[1] See Maier, op. cit., vol. ii *a*, 'Die drei Figuren', pp. 47–71, and vol. ii *b*, 'Ergänzung durch eine 4. Figur mit zwei Formen', pp. 261–9.

[2] Op. cit., vol. ii *a*, p. 48, n. 1.

[3] See the Greek text on p. 33, n. 4.

[4] Op. cit., vol. ii *a*, p. 49: 'Erwägt man nämlich, dass die Ausdrücke "B liegt im Umfang von A", "A kommt dem Begriff B zu" und "A wird von B ausgesagt" mit einander vertauscht werden können, so lässt sich die Charakteristik der zweiten Figur, welche der Beschreibung der ersten parallel gedacht ist, auch so fassen.'

[5] *An. pr.* i. 1, 24^{b}26 *τὸ δὲ ἐν ὅλῳ εἶναι ἕτερον ἑτέρῳ καὶ τὸ κατὰ παντὸς κατηγορεῖσθαι θατέρου θάτερον ταὐτόν ἐστιν.*

the same as '*A* is predicated of all *B*' or '*A* belongs to all *B*', but does not mean '*A* is predicated of *B*' or '*A* belongs to *B*'. With this first error is connected a second: Maier maintains that the negative premiss also has the external form of subordination of one term to another, like the affirmative universal premiss.[1] What is here meant by 'external form'? When *A* belongs to all *B*, then *B* is subordinated to *A*, and the external form of this relation is just the proposition '*A* belongs to all *B*'. But in a negative premiss, e.g. '*A* belongs to no *B*', the subordination of terms does not exist, nor does its form. Maier's assertion is logically nonsense.

Let us now quote Maier's description of the second figure. It runs thus: 'Whenever of two terms one is included, and the other is not included, in the same third term, or both are included in it, or neither of them, we have the second figure before us. The middle term is that which includes both remaining terms, and the extremes are the terms which are included in the middle.'[2] This would-be characterization of the second figure is again logically nonsense. Take the following example: Two premisses are given: '*A* belongs to all *B*' and '*C* belongs to no *A*'. If *A* belongs to all *B*, then *B* is included in *A*, and if *C* belongs to no *A*, it is not included in *A*. We have therefore two terms, *B* and *C*, one of which, *B*, is included, and the other, *C*, is not included in the same third term *A*. According to Maier's description we should have the second figure before us. What we have, however, is not the second figure, but only two premisses '*A* belongs to all *B*' and '*C* belongs to no *A*', from which we can get by the mood Celarent of the first figure the conclusion '*C* belongs to no *B*', and by the mood Camenes of the fourth figure the conclusion '*B* belongs to no *C*'.

The peak, however, of logical absurdity Maier attains by his assertion that there exists a fourth syllogistic figure consisting of only two moods, Fesapo and Fresison. He supports this assertion by the following argument: 'The Aristotelian doctrine overlooks one possible position of the middle term. This term may be less

[1] Op. cit., vol. ii *a*, p. 60, n. 1: 'auch der negative syllogistische Satz hat wenigstens die äussere Form der Subordination.' Cf. also ibid., p. 50.

[2] Ibid., p. 49: 'Wenn im Umfang eines und desselben Begriffes der eine der beiden übrigen Begriffe liegt, der andere nicht liegt, oder aber beide liegen oder endlich beide nicht liegen, so haben wir die zweite Figur vor uns. Mittelbegriff ist derjenige Begriff, in dessen Umfang die beiden übrigen, äußere Begriffe aber diejenigen, die im Umfang des mittleren liegen.'

general than the major and more general than the minor, it may secondly be more general, and thirdly less general, than the extremes, but it may be also more general than the major term and at the same time less general than the minor.'[1] When we remind ourselves that according to Maier the major term is always more general than the minor,[2] and that the relation 'more general than' is transitive, we cannot avoid the strange consequence of his argument that the middle term of his fourth figure should be at the same time more and less general than the minor term. From the standpoint of logic Maier's work is useless.

§ 14. *The four Galenian figures*

In almost every text-book of logic you may find the remark that the inventor of the fourth figure was Galen, a Greek physician and philosopher living in Rome in the second century A.D. The source of this remark is suspect. We do not find it either in the extant works of Galen or in the works of the Greek commentators (including Philoponus). According to Prantl the medieval logicians received the information from Averroes, who says that the fourth figure was mentioned by Galen.[3] To this vague information we may add two late Greek fragments found in the nineteenth century, and also very vague. One of them was published in 1844 by Mynas in the preface to his edition of Galen's *Introduction to Dialectic*, and republished by Kalbfleisch in 1897. This fragment of unknown authorship tells us that some later scholars transformed the moods added by Theophrastus and Eudemus to the first figure into a new fourth figure, referring to Galen as the father of this doctrine.[4] The other Greek fragment was found by Prantl in a logical work

[1] Op. cit., vol. ii *b*, p. 264: 'Die aristotelische Lehre läßt eine mögliche Stellung des Mittelbegriffs unbeachtet. Dieser kann specieller als der Ober- und allgemeiner als der Unterbegriff, er kann ferner allgemeiner, er kann drittens specieller als die beiden äußeren Begriffe: aber er kann auch allgemeiner als der Ober- und zugleich specieller als der Unterbegriff sein.'

[2] Ibid., vol. ii *a*, p. 56: 'Oberbegriff ist stets, wie in der 1. Figur ausdrücklich festgestellt ist, der allgemeinere, Unterbegriff der weniger allgemeine.'

[3] Prantl, i. 571, n. 99, quotes Averroes in a Latin translation edited in Venice (1553): 'Et ex hoc planum, quod figura quarta, de qua meminit Galenus, non est syllogismus super quem cadat naturaliter cogitatio.' Cf. also Prantl, ii. 390, n. 322.

[4] K. Kalbfleisch, *Über Galens Einleitung in die Logik*, 23. Supplementband der Jahrbücher für klassische Philologie, Leipzig (1897), p. 707: *Θεόφραστος δὲ καὶ Εὔδημος καί τινας ἑτέρας συζυγίας παρὰ τὰς ἐκτεθείσας τῷ Ἀριστοτέλει προστεθήκασι τῷ πρώτῳ σχήματι . . ., ἃς καὶ τέταρτον ἀποτελεῖν σχῆμα τῶν νεωτέρων ᾠήθησάν τινες ὡς πρὸς πατέρα τὴν δόξαν τὸν Γαληνὸν ἀναφέροντες.*

of Ioannes Italus (eleventh century A.D.). This author says sarcastically that Galen maintained the existence of a fourth figure in opposition to Aristotle, and, thinking that he would appear cleverer than the old logical commentators, fell very far short.[1] That is all. In view of such a weak basis of sources, Ueberweg suspected a misunderstanding in the matter, and Heinrich Scholz writes in his *History of Logic* that Galen is probably not responsible for the fourth figure.[2]

For fifty years there has existed a Greek scholium in print which clears up the whole matter in an entirely unexpected way. Although printed, it seems to be unknown. Maximilian Wallies, one of the Berlin editors of the Greek commentaries on Aristotle, published in 1899 the extant fragments of Ammonius' commentary on the *Prior Analytics*, and has inserted in the preface a scholium of an unknown author found in the same codex as that in which the fragments of Ammonius are preserved. The scholium is entitled 'On all the kinds of syllogism', and begins thus:

'There are three kinds of syllogism: the categorical, the hypothetical, and the syllogism *κατὰ πρόσληψιν*. Of the categorical there are two kinds: the simple and the compound. Of the simple syllogism there are three kinds: the first, the second, and the third figure. Of the compound syllogism there are four kinds: the first, the second, the third, and the fourth figure. For Aristotle says that there are only three figures, because he looks at the simple syllogisms, consisting of three terms. Galen, however, says in his *Apodeictic* that there are four figures, because he looks at the compound syllogisms consisting of four terms, as he has found many such syllogisms in Plato's dialogues.'[3]

The unknown scholiast further gives us some explanations, from

[1] Prantl, ii. 302, n. 112: *τὰ δὲ σχήματα τῶν συλλογισμῶν ταῦτα· ὁ Γαληνὸς δὲ καὶ τέταρτον ἐπὶ τούτοις ἔφασκεν εἶναι, ἐναντίως πρὸς τὸν Σταγειρίτην φερόμενος, ὃς λαμπρότερον ἀναφανῆναι οἰόμενος τῶν τὴν λογικὴν πραγματείαν ἐξηγουμένων παλαιῶν ὡς πορρωτάτω εὐθέως ἐκπέπτωκε.*

[2] Fr. Ueberweg, *System der Logik*, Bonn (1882), 341. Cf. also Kalbfleisch, op. cit., p. 699; H. Scholz, *Geschichte der Logik*, Berlin (1931), p. 36.

[3] M. Wallies, *Ammonii in Aristotelis Analyticorum Priorum librum I Commentarium*, Berlin (1899), p. ix: *Περὶ τῶν εἰδῶν πάντων τοῦ συλλογισμοῦ. τρία εἴδη ἐστὶ τοῦ [ἁπλοῦ] συλλογισμοῦ· τὸ κατηγορικόν, τὸ ὑποθετικόν, τὸ κατὰ πρόσληψιν. τοῦ δὲ κατηγορικοῦ δύο ἐστὶν εἴδη· ἁπλοῦν, σύνθετον. καὶ τοῦ μὲν ἁπλοῦ τρία ἐστὶν εἴδη· πρῶτον σχῆμα, δεύτερον σχῆμα, τρίτον σχῆμα. τοῦ δὲ συνθέτου τέσσαρά ἐστιν εἴδη· πρῶτον σχῆμα, δεύτερον σχῆμα, τρίτον, τέταρτον σχῆμα. Ἀριστοτέλης μὲν γὰρ τρία τὰ σχήματά φησιν πρὸς τοὺς ἁπλοῦς συλλογισμοὺς ἀποβλέπων τοὺς ἐκ τριῶν ὅρων συγκειμένους. Γαληνὸς δ' ἐν τῇ οἰκείᾳ Ἀποδεικτικῇ δ τὰ σχήματα λέγει πρὸς τοὺς συνθέτους συλλογισμοὺς ἀποβλέπων τοὺς ἐκ δ ὅρων συγκειμένους πολλοὺς τοιούτους εὑρὼν ἐν τοῖς Πλάτωνος διαλόγοις.*

which we can gather how Galen may have found these four figures. Compound syllogisms consisting of four terms may be formed by combinations of the three figures I, II, and III of simple syllogisms in nine different ways: I to I, I to II, I to III, II to II, II to I, II to III, III to III, III to I, III to II. Two of these combinations, viz. II to II and III to III, do not give syllogisms at all, and of the remaining combinations II to I gives the same figure as I to II, III to I the same as I to III, and III to II the same as II to III. We get thus only four figures, I to I, I to II, I to III, and II to III.[1] Examples are given, of which three are taken from Plato's dialogues, two from the *Alcibiades*, and one from the *Republic*.

This precise and minute account must be explained and examined. Compound syllogisms of four terms have three premisses and two middle terms, say *B* and *C*, which form the premiss *B–C* or *C–B*. Let us call this the middle premiss. *B* forms together with *A*, the subject of the conclusion, the minor premiss, and *C* forms together with *D*, the predicate of the conclusion, the major premiss. We thus obtain the following eight combinations (in all the premisses the first term is the subject, the second the predicate):

Figure	*Minor Premiss*	*Middle Premiss*	*Major Premiss*	*Conclusion*	
F1	*A–B*	*B–C*	*C–D*	*A–D*	I to I
F2	*A–B*	*B–C*	*D–C*	*A–D*	I to II
F3	*A–B*	*C–B*	*C–D*	*A–D*	II to III
F4	*A–B*	*C–B*	*D–C*	*A–D*	II to I
F5	*B–A*	*B–C*	*C–D*	*A–D*	III to I
F6	*B–A*	*B–C*	*D–C*	*A–D*	III to II
F7	*B–A*	*C–B*	*C–D*	*A–D*	I to III
F8	*B–A*	*C–B*	*D–C*	*A–D*	I to I

If we adopt the principle of Theophrastus that in the first Aristotelian figure the middle term is the subject of one premiss—it does not matter of which, the major or the minor—and the predicate of another, and define by this principle which figure is formed by the minor and middle premisses on the one hand, and by the middle and major premisses on the other, we get the combinations of figures shown in the last column. Thus, for instance, in the compound figure F2 the minor premiss together with the middle forms the figure I, as the middle term *B* is the predicate of the first premiss and the subject of the second, and the middle premiss together with the major forms the figure II, as the middle term *C* is the predicate of both premisses. This was probably how Galen has got his four figures. Looking at the last column we see at once that, as Galen held, the combinations II to II and III to III do not exist, not for the reason, as the scholiast mistakenly says, that no conclusion results either from two negative or two particular premisses, but because no term can occur in the premisses three times. It is obvious also that if we extend the principle of Theophrastus to compound syllogisms and include in the same figure all the moods that from the same combination of premisses yield either the conclusion *A–D* or the conclusion *D–A*, we get as Galen does the same figure from the combination I to II as from the combination II to I. For, interchanging in figure F4 the letters *B* and *C* as well as the letters *A* and *D*, we get the scheme:

F4 *D–C* *B–C* *A–B* *D–A*,

and as the order of the premisses is irrelevant we see that the conclusion *D–A* results in F4 from the same premisses as *A–D* in F2. For the same reason figure F1 does not differ from figure F8, F3 from F6, or F5 from F7. It is possible, therefore, to divide the compound syllogisms of four terms into four figures.

The scholium edited by Wallies explains all historical problems connected with the alleged invention of the fourth figure by Galen. Galen divided syllogisms into four figures, but these were the compound syllogisms of four terms, not the simple syllogisms of Aristotle. The fourth figure of the Aristotelian syllogisms was invented by someone else, probably very late, perhaps not before the sixth century A.D. This unknown scholar must have heard something about the four figures of Galen, but he either did not understand them or did not have Galen's text at hand. Being in

[1] Wallies, op. cit., pp. ix–x: *ὁ κατηγορικὸς συλλογισμὸς ἁπλοῦς, ὡς Ἀριστοτέλης· σχῆμα Α Β Γ. σύνθετος, ὡς Γαληνός· Α πρὸς Α, Α πρὸς Β, Α πρὸς Γ, Β πρὸς Β, Β πρὸς Α, Β πρὸς Γ, Γ πρὸς Γ, Γ πρὸς Α, Γ πρὸς Β.*

συλλογιστικόν· Α πρὸς Α, Α πρὸς Β, Α πρὸς Γ, Β πρὸς Γ.
A B Γ Δ

ἀσυλλόγιστον· Β πρὸς Β, Γ πρὸς Γ, (οὐ γὰρ γίνεται συλλογισμὸς οὔτε ἐκ δύο ἀποφατικῶν οὔτε ἐκ δύο μερικῶν)·

Β πρὸς Α, Γ πρὸς Α, Γ πρὸς Β,
B Γ Δ

οἱ αὐτοί εἰσιν τοῖς συλλογισμοῖς ὡς ὑπογέγραπται.

opposition to Aristotle and to the whole school of the Peripatetics, he eagerly seized the occasion to back up his opinion by the authority of an illustrious name.

REMARK. The problem of compound syllogisms raised by Galen has considerable interest from the systematic point of view. Investigating the number of valid moods of the syllogisms consisting of three premisses, I have found that there are forty-four valid moods, the figures F1, F2, F4, F5, F6, and F7 having six moods each, and figure F8 eight. Figure F3 is empty. It has no valid moods, for it is not possible to find premisses of the form *A–B*, *C–B*, *C–D* such that a conclusion of the form *A–D* would follow from them. This result, if known, would certainly be startling for students of the traditional logic. Mr. C. A. Meredith, who attended my lectures delivered on this subject in 1949 at University College, Dublin, has found some general formulae concerning the number of figures and valid moods for syllogisms of n terms, including expressions of 1 and 2 terms. I publish these formulae here with his kind permission:

Number of terms	n
Number of figures	2^{n-1}
Number of figures with valid moods .	$\frac{1}{2}(n^2-n+2)$
Number of valid moods . . .	$n(3n-1)$

For all n every non-empty figure has 6 valid moods, except one that has $2n$ valid moods.

Examples:

Number of terms	1, 2, 3, 4,..., 10
Number of figures	1, 2, 4, 8,..., 512
Number of figures with valid moods .	1, 2, 4, 7,..., 46
Number of valid moods . . .	2, 10, 24, 44,..., 290

It is obvious that for large n's the number of figures with valid moods is comparatively small against the number of all figures. For $n = 10$ we have 46 against 512 respectively, i.e. 466 figures are empty. For $n = 1$ there is only 1 figure, *A–A*, with 2 valid moods, i.e. the laws of identity. For $n = 2$ there are 2 figures:

	Premiss	*Conclusion*
F1	*A–B*	*A–B*
F2	*B–A*	*A–B*

with 10 valid moods, 6 in F1 (viz. four substitutions of the propositional law of identity, e.g. 'if all *A* is *B*, then all *A* is *B*', and two laws of subordination), and 4 moods in F2 (viz. four laws of conversion).

CHAPTER III

THE SYSTEM

§ 15. *Perfect and imperfect syllogisms*

IN the introductory chapter to the syllogistic Aristotle divides all syllogisms into perfect and imperfect. 'I call that a perfect syllogism', he says, 'which needs nothing other than what has been stated to make the necessity evident; a syllogism is imperfect, if it needs either one or more components which are necessary by the terms set down, but have not been stated by the premisses.'[1] This passage needs translation into logical terminology. Every Aristotelian syllogism is a true implication, the antecedent of which is the joint premisses and the consequent the conclusion. What Aristotle says means, therefore, that in a perfect syllogism the connexion between the antecedent and the consequent is evident of itself without an additional proposition. Perfect syllogisms are self-evident statements which do not possess and do not need a demonstration; they are indemonstrable, *ἀναπόδεικτοι*.[2] Indemonstrable true statements of a deductive system are now called axioms. The perfect syllogisms, therefore, are the axioms of the syllogistic. On the other hand, the imperfect syllogisms are not self-evident; they must be proved by means of one or more propositions which result from the premisses, but are different from them.

Aristotle knows that not all true propositions are demonstrable.[3] He says that a proposition of the form '*A* belongs to *B*' is demonstrable if there exists a middle term, i.e. a term which forms with *A* and *B* true premisses of a valid syllogism having the above proposition as the conclusion. If such a middle term does

[1] *An. pr.* i. 1, 24^{b}22 *τέλειον μὲν οὖν καλῶ συλλογισμὸν τὸν μηδενὸς ἄλλου προσδεόμενον παρὰ τὰ εἰλημμένα πρὸς τὸ φανῆναι τὸ ἀναγκαῖον, ἀτελῆ δὲ τὸν προσδεόμενον ἢ ἑνὸς ἢ πλειόνων, ἃ ἔστι μὲν ἀναγκαῖα διὰ τῶν ὑποκειμένων ὅρων, οὐ μὴν εἴληπται διὰ πρότασεων.*

[2] Commenting upon the above passage Alexander uses the expression *ἀναπόδεικτος*, 24. 2: *ἑνὸς μὲν οὖν προσδέονται οἱ ἀτελεῖς συλλογισμοὶ οἱ μιᾶς ἀντιστροφῆς δεόμενοι πρὸς τὸ ἀναχθῆναι εἴς τινα τῶν ἐν τῷ πρώτῳ σχήματι τῶν τελείων καὶ ἀναποδείκτων, πλειόνων δὲ ὅσοι διὰ δύο ἀντιστροφῶν εἰς ἐκείνων τινὰ ἀνάγονται.* Cf. also p. 27, n. 2.

[3] *An. post.* i. 3, 72^{b}18 *ἡμεῖς δέ φαμεν οὔτε πᾶσαν ἐπιστήμην ἀποδεικτικὴν εἶναι, ἀλλὰ τὴν τῶν ἀμέσων ἀναπόδεικτον.*