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# Modal Ecthesis

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My semantics for McCall's syntactic presentation of Aristotle's assertoric and apodeictic syllogistic is altered to free it from Thom's objections that it is unAristotelian. The altered semantics rejects Baroco-XLL and Bocardo-LXL, which Thom says Aristotle should have accepted. Aristotle's proofs that use ecthesis are formalized by using singular sentences. With one exception the (acceptance) axioms for McCall's system L-X-M are derivable. Formal proofs are shown to be sound.

## 1. Motivation

Thom *1991* makes three objections to the semantics in Johnson *1989* for McCall's *1963* system L-X-M, which attempts to formalize Aristotle's reasoning involving assertoric and apodeictic sentences: (1) For Aristotle, a general term may designate a property such that no object necessarily has this property; (2) For Aristotle, if some object necessarily has a property designated by a general term then every object that has this property necessarily has it; and (3) A good semantics for Aristotle's logic must be 'intuitively graspable'. Thom also points out that Johnson's deductive apparatus, taken from McCall, does not give proofs that use ecthesis, though Aristotle certainly gave such proofs. My purpose is to develop a system that responds to the above objections. The system below is more fully Aristotelian than Thom's, which accepts both Baroco-XLL and Bocardo-LXL, though they are rejected by Aristotle. Thom (*1991*, 144) suggests that an adequate account of ecthesis must conflict with Aristotle's rejection of Baroco-XLL and Bocardo-LXL. Below we show that this is false.

## 2. Sentences

Sentences are built from
- Names: m, n, o, $m_1$, ...
- General terms: a, b, c, $a_1$, ...
- Copulas: $\in$, $\in_n$, $\notin$, $\notin_n$
- Quantifiers: A, E, I, O
- Modal operator: L

(1) x *c* p is a *singular sentence* iff x is a name, *c* is a copula, and p is a general term. (2) Qpq is a *quantified sentence* iff Q is a quantifier and both p and q are general terms. (3) LX is a *necessity quantified sentence* iff X is a quantified sentence. (4) Singular sentences, quantified sentences, and necessity quantified sentences are *sentences* and are the only *sentences*.

So, for example, m $\in$ a, m $\notin$ a, m $\in_n$ a, and m $\notin_n$ a are singular sentences. Read them respectively as 'm is one of the a', 'm is not one of the a', 'm is necessarily one of the a', and 'm is necessarily not one of the a'. Read LAab, LEab, LIab, and LOab respectively as 'Necessarily all a are b', 'Necessarily no a are b', 'Necessarily some a are b', and 'Necessarily some a are not b'.

Since our language contains singular sentences, it is more inclusive than McCall's system L-X-M and Thom's language in *1991*. Our language, like Thom's, is less inclusive than McCall's, since the latter includes a possibility operator M. Given the semantics that follows, there is no sentence in our language that contradicts LAab. We could add the operator M and add truth conditions for sentences formed with this operator so that corresponding to every sentence in the language there is a sentence that contradicts it.

In the following discussion x and y are metalinguistic variables that range over names; p, q, and r are metalinguistic variables that range over general terms.

## 3. Substitution semantics

A valuation is a function $\nu$ that assigns either True (t) or False (f) to each sentence in the language and meets these conditions:

(1) For every p there is an x such that $\nu(x \in p) = t$.
(2) For every x, p, and y, if $\nu(x \in_n p) = t$ and $\nu(y \in p) = t$ then $\nu(y \in_n p) = t$.
(3) For every x and p, $\nu(x \in p) = t$ iff $\nu(x \notin p) = f$.
(4) For every x and p, if $\nu(x \in_n p) = t$ then $\nu(x \in p) = t$.
(5) For every x and p, if $\nu(x \notin_n p) = t$ then $\nu(x \notin p) = t$.
(6) $\nu(Apq) = t$ iff for every x if $\nu(x \in p) = t$ then $\nu(x \in q) = t$.
(7) $\nu(Epq) = t$ iff no x is such that $\nu(x \in p) = t$ and $\nu(x \in q) = t$.
(8) $\nu(Ipq) = t$ iff $\nu(Epq) = f$.
(9) $\nu(Opq) = t$ iff $\nu(Apq) = f$.
(10) $\nu(LApq) = t$ iff
 (i) for every x, if $\nu(x \in p) = t$ then $\nu(x \in_n q) = t$, and
 (ii) for every x and r, if $\nu(x \notin_n q) = t$ and $\nu(Arp) = t$ then $\nu(x \notin_n r) = t$.
(11) $\nu(LEpq) = t$ iff
 (i) for every x and r, if $\nu(x \in p) = t$ and $\nu(Arq) = t$ then $\nu(x \notin_n r) = t$,
 (ii) for every x and r, if $\nu(x \in q) = t$ and $\nu(Arp) = t$ then $\nu(x \notin_n r) = t$,
 (iii) for every r, if for some x $\nu(x \in p) = t$ and $\nu(x \in r) = t$ then for some y $\nu(y \notin_n q) = t$ and $\nu(y \in_n r)$, and
 (iv) for every r, if for some x $\nu(x \in q) = t$ and $\nu(x \in r) = t$ then for some y $\nu(y \notin_n p) = t$ and $\nu(y \in_n r)$.
(12) $\nu(LIpq) = t$ iff either
 (i) for some x, $\nu(x \in p) = t$ and $\nu(x \in_n q) = t$, or
 (ii) for some x, $\nu(x \in_n p) = t$ and $\nu(x \in q) = t$.
(13) $\nu(LOpq) = t$ iff for some x $\nu(x \in_n p) = t$ and $\nu(x \notin_n q) = t$.

If we added a condition requiring that for each p there is an x such that $\nu(x \in_n p) = t$, or if we omitted condition 2, then the semantics would be objectionable to Thom *1991*, as indicated in section 1 above. Note also that the above semantics is unlike Johnson's *1989* semantics since valuations do not assign subsets of a universe to general terms. Valuations only make assignments to sentences.

Definition: Y is a *logical consequence* of $X_1, \ldots X_n$ ($X_1, \ldots X_n \models Y$) iff there is no valuation that assigns t to each $X_i$ and assigns f to Y. ($X_1, \ldots X_n \not\models Y$ is short for not ($X_1, \ldots X_n \models Y$).)

## 4. Rejections

All arguments that are 'clearly rejected' by Aristotle are rejected according to the above semantics. (The rejected arguments on Ross's table (*1949*, 286) are among those that are clearly rejected.) There are arguments that are neither clearly accepted nor clearly rejected by Aristotle. For example, consider <Ø, LIaa>. Johnson *1989*, following McCall *1963*, accepts it, though as noted below McCall accepts if for the sake of convenience. This inference is rejected by Thom *1991*. And, according to the above semantics, it is rejected.

Consider the sentences $m \in a$, $m \in_n a$, $m \notin a$, and $m \notin_n a$. For a function to be a valuation it must include one of the functions $f_1$-$f_4$ indicated by table 1. Note, for example, that functions that include functions $f_5$, $f_6$, or $f_7$ referred to in table 2, are not valuations. $f_5$, $f_6$ and $f_7$ violate conditions 3, 4, and 5, respectively.

Table 1

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ |
|---|---|---|---|---|
| $m \in a$ | t | t | f | f |
| $m \in_n a$ | f | t | f | f |
| $m \notin a$ | f | f | t | t |
| $m \notin_n a$ | f | f | f | t |

Table 2

| | $f_5$ | $f_6$ | $f_7$ |
|---|---|---|---|
| $m \in a$ | f | f | t |
| $m \in_n a$ | f | t | t |
| $m \notin a$ | f | t | f |
| $m \notin_n a$ | f | t | t |

To specify valuations we will use tables of the following form:

| | a | b | etc. | | |
|---|---|---|---|---|---|
| m | | | | | |
| n | | | | | etc. |
| etc. | | | | | |

etc.

We view these tables as having infinitely many rows and infinitely many columns. We make rows correspond one-to-one to names and columns correspond one-to-one to general terms. And we fill each cell in exactly one of four ways: put '$\in$', '$\in_n$', no mark, or '$\notin_n$' in each cell. A mark '$\in$' in cell m/a means that the function *g*, specified by the table, makes $f_1$'s assignments (in table 1) to the sentences $m \in a$, $m \in_n a$, $m \notin a$, and $m \notin_n a$. Marks '$\in_n$', no mark, and '$\notin_n$' in cell m/a mean that *g* makes $f_2$'s, $f_3$'s, and $f_4$'s assignments, respectively, to the four sentences. The marks will function in the same way for any cell x/p. So, for example, if '$\in$' occurs in cell x/p then $g(x \in p) = t$, $g(x \in_n p) = f$, $g(x \notin p) = f$, and $g(x \notin_n p) = f$. If '$\notin_n$' occurs in cell x/p then $g(x \in p) = f$, $g(x \in_n p) = f$, $g(x \notin p) = t$, and $g(x \notin_n p) = t$. If a table specifies a function that assigns a truth value to each sentence, we will indicate this by putting a function symbol in the upper-left corner.

Consider

| $g_1$ | a | b | c | . . . |
|---|---|---|---|---|
| m | | | | |
| n | | | | |
| o | | | | |
| ⋮ | | | | |

The dots on the table remind us that the table has infinitely many rows and infinitely many columns. Since there is no mark in cell m/a, $g_1(\text{m} \in \text{a}) = \text{f}$. Since there is no mark in cell m/b, $g_1(\text{m} \in_n \text{b}) = \text{f}$. Since there is no mark in cell n/b, $g_1(\text{n} \notin \text{b}) = \text{t}$ and $g_1(\text{n} \notin_n \text{b}) = \text{f}$. Though $g_1$ is a function that assigns either t or f to each sentence, $g_1$ is not a valuation. Condition 1 is violated. Condition 1 says in effect that '$\in$' or '$\in_n$' must occur in each column of the table. (The set-theoretic semantics version of condition 1 amounts to this: every general term must be assigned a non-empty set.)

Consider

| $g_2$ | a | b | c | . . . |
|---|---|---|---|---|
| m | $\in$ | $\in$ | $\in$ | . . . |
| n | | | | |
| o | | | | |
| ⋮ | | | | |

The expression '$\in$ . . .' to the right of the m/b cell indicates that '$\in$' is in the m/c cell and in every cell to the right of it in the m row. Each of conditions 1–5 is satisfied. So $g_2$ is a valuation. Since for any row an occurrence of '$\in$' or '$\in_n$' in the a-column is matched by an occurrence of '$\in$' or '$\in_n$' in the b-column, $g_2(\text{Aab}) = \text{t}$. $g_2(\text{LAab}) = \text{f}$, since '$\in$' occurs in cell m/a but '$\in_n$' does not occur in cell m/b. Since $g_2(\text{LIaa}) = \text{f}$, $\varnothing \not\models$ LIaa.

Consider

| $g_3$ | a | b | c | $a_1$ | . . . |
|---|---|---|---|---|---|
| m | $\in_n$ | $\in$ | | $\in$ | . . . |
| n | $\notin_n$ | | $\in$ | | |
| o | $\in_n$ | $\in$ | $\notin_n$ | | |
| ⋮ | | | | | |

If '$\in$', instead of '$\in_n$', had occurred in cell m/a, keeping the other cells unchanged, then $g_3$ would not have been a valuation, given condition 2. $g_3(\text{Aab}) = \text{t}$, but $g_3(\text{LAab}) = \text{f}$. $g_3(\text{Ebc}) = \text{t}$, but $g_3(\text{LEbc}) = \text{f}$. (Since '$\in$' is in cell m/b, '$\notin_n$' would have to occur in cell m/c for LEbc to be true.) $g_3(\text{Iba}_1) = \text{t}$, but $g_3(\text{LIba}_1) = \text{f}$. $g_3(\text{Obc}) = \text{t}$, but $g_3(\text{LObc}) = \text{f}$.

The truth conditions for LApq and LEpq sentences are complex and deserve special attention.

Consider

| $g_4$ | a | b | c | $a_1$ | $a_2$ | $a_3$ | ... |
|---|---|---|---|---|---|---|---|
| m | $\in$ | $\in_n$ | $\in_n$ | | | $\in$ | ... |
| n | | $\notin_n$ | $\notin_n$ | | | | |
| o | | | | $\in$ | $\in$ | | |
| $m_1$ | | | | $\notin_n$ | $\notin_n$ | | |
| ⋮ | | | | | | | |

$g_4$(LAab) = t. $g_4$(LAbc) = f. (Condition 10(i) is satisfied, but condition 10(ii) is not. $g_4(\text{n} \notin_n \text{c})$ = t and $g_4$(Aab) = t, but $g_4(\text{n} \notin_n \text{a})$ = f.) $g_4$(LA$a_1a_2$) = f. (Condition 10(ii) is satisfied, but condition 10(i) is not. $g_4(\text{o} \in a_1)$ = t, but $g_4(\text{o} \in_n a_2)$ = f.)

Consider

| $g_5$ | a | b | c | ... |
|---|---|---|---|---|
| m | | | $\in$ | ... |
| n | $\in_n$ | $\notin_n$ | | |
| o | $\notin_n$ | $\in_n$ | | |
| ⋮ | | | | |

$g_5$(LEab) = t. To verify that conditions 11(i) and 11(ii) are satisfied, note that though $g_5$ assigns t to Aaa and t to Abb it does not assign t to any other sentence of form Ara or Arb. To verify that conditions 11(iii) and 11(iv) are satisfied note that though $g_5$ assigns t to Iaa and t to Ibb it does not assign t to any other sentences of form Ira or Irb.

Consider

| $g_6$ | a | b | c | ... |
|---|---|---|---|---|
| m | | | $\in$ | ... |
| n | $\in_n$ | $\notin_n$ | | |
| o | $\notin_n$ | $\in$ | | |
| ⋮ | | | | |

$g_6$(LEab) = f. Though conditions 11(i), 11(ii) and 11(iii) are satisfied, condition 11(iv) is violated. By interchanging the entries in cells n/a and o/b we form a function that violates 11(iii) but satisfies the remaining conditions.

Consider

| $g_7$ | a | b | c | . . . |
|---|---|---|---|---|
| m | | | $\in$ | . . . |
| n | $\in$ | | | |
| o | $\in_n$ | $\notin_n$ | | |
| $m_1$ | $\notin_n$ | $\in_n$ | | |
| ⋮ | | | | |

$g_7$(LEab) = f. Though conditions 11(ii), 11(iii) and 11(iv) are satisfied, condition 11(i) is violated. If the entries in cells n/a and n/b are interchanged, then condition 11(ii) is violated but the remaining conditions are satisfied.

The following valuations formally reject arguments that Aristotle clearly rejected.

Baroco-LXL: LAcb, Oab $\not\models$ LOac

| $h_1$ | a | b | c | . . . |
|---|---|---|---|---|
| m | | $\in_n$ | $\in$ | . . . |
| n | $\in_n$ | | | |
| o | | | | |
| ⋮ | | | | |

($h_1$(LAcb) = t, $h_1$(Oab) = t, and $h_1$(LOac) = f.)

Baroco-XLL: Acb, LOab $\not\models$ LOac

| $h_2$ | a | b | c | . . . |
|---|---|---|---|---|
| m | | $\in_n$ | $\in$ | . . . |
| n | $\in_n$ | $\notin_n$ | | |
| o | | | | |
| ⋮ | | | | |

Bocardo-LXL: LObc, Aba $\not\models$ LOac

| $h_3$ | a | b | c | $a_1$ | . . . |
|---|---|---|---|---|---|
| m | $\in$ | $\in_n$ | $\notin_n$ | $\in$ | . . . |
| n | | | | | |
| o | | | | | |
| ⋮ | | | | | |

Bocardo-XLL: Obc, LAba $\not\models$ LOac

| $h_4$ | a | b | c | $a_1$ | . . . |
|---|---|---|---|---|---|
| m | $\in_n$ | $\in$ | | $\in$ | . . . |
| n | | | $\in$ | | |
| o | | | | | |
| ⋮ | | | | | |

Barbara-XLL: Abc, LAab $\not\models$ LAac

| $h_5$ | a | b | c | . . . |
|---|---|---|---|---|
| m | $\in$ | $\in_n$ | $\in$ | . . . |
| n | | | | |
| o | | | | |
| ⋮ | | | | |

Similar counterexamples can be given for the other inferences that Aristotle clearly rejected. All of the above tables would specify counterexamples if condition 2 were dropped. Condition 2 is included to accommodate Thom's *1991* claim that for Aristotle $\frac{\text{LIaa}}{\text{LAaa}}$ is valid, which is invalid according to Johnson's *1989* semantics. Thom says '. . . if a term belongs essentially to some individual, there is according to Aristotle no individual to which it belongs non-essentially' (p. 137). Thom's semantics makes $\frac{\text{LIaa}}{\text{LAaa}}$ valid. (Suppose LIaa is true. Given Thom's 1.5 and 5.3, $f_1 = e_1$. So given 5.1, LAaa is true.) And it is valid according to the above semantics. Note that if '$\epsilon_n$' occurs in a cell in the a-column, then '$\epsilon$' cannot occur in any cell in the a-column without violating condition 2.

The invalidating functions $h_1$–$h_5$ are specified by tables in which there are no more than two rows in which either '$\epsilon$' or '$\epsilon_n$' occurs. It is reasonable to conjecture that there is some finite natural number n such that if $\{X_1, \ldots X_m\} \not\models Y$ (for any finite m) there is an invalidating function specified by a table that requires no more than n rows in which either '$\epsilon$' or '$\epsilon_n$' occurs. Johnson *1991* shows that for the assertoric syllogistic 3 is such a number.

## 5. Deductions

To define deducibility we use these *rules of inference* (designed to enable us to deduce ecthetically any conclusion of form Ipq, LIpq, Opq, or LOpq if the argument in which it occurs is considered valid by Aristotle):

1 $\frac{}{\text{App}}$ ID(entity)

2 $\frac{\text{Epq}}{\text{Eqp}} \quad \frac{\text{LEpq}}{\text{LEqp}}$ CON(version)

$\frac{\text{LApq}}{\text{Apq}} \quad \frac{\text{LEpq}}{\text{Epq}}$ Q-SUB(ordination)

4 $\frac{\text{Aqr} \quad \text{Apq}}{\text{Apr}} \quad \frac{\text{LAqr} \quad \text{Apq}}{\text{LApr}} \quad \frac{\text{Eqr} \quad \text{Apq}}{\text{Epr}} \quad \frac{\text{LEqr} \quad \text{Apq}}{\text{LEpr}}$ Q-SYL(logism)

5 $\frac{\text{Apq}}{x \epsilon p} \quad \frac{\text{Epq}}{x \epsilon p} \quad \frac{\text{Ipq}}{x \epsilon p \;\; x \epsilon q} \quad \frac{\text{LIpq}}{x \epsilon p \;\; x \epsilon_n q \;\;\Big|\;\; x \epsilon_n p \;\; x \epsilon q} \quad \frac{\text{Opq}}{x \epsilon p \;\; x \notin q} \quad \frac{\text{LOpq}}{x \epsilon_n p \;\; x \notin_n q}$ EC(thesis)

6 $\frac{x \epsilon_n p}{x \epsilon p} \quad \frac{x \notin_n p}{x \notin p}$ S-SUB

$$7 \qquad \frac{\begin{array}{c}Apq\\ x\in p\end{array}}{x\in q} \quad \frac{\begin{array}{c}Apq\\ x\notin q\end{array}}{x\notin p} \quad \frac{\begin{array}{c}LApq\\ x\in p\end{array}}{x\in_n q} \quad \frac{\begin{array}{c}LApq\\ x\notin_n q\end{array}}{x\notin_n p} \quad \frac{\begin{array}{c}Epq\\ x\in p\end{array}}{x\notin q} \quad \frac{\begin{array}{c}LEpq\\ x\in p\\ x\in r\end{array}}{\begin{array}{c}y\in_n r\\ y\notin_n q\end{array}}, \text{for } y\neq x \qquad \text{S-SYL}$$

$$8 \qquad \frac{\begin{array}{c}x\in p\\ x\in q\end{array}}{Ipq} \quad \frac{\begin{array}{c}x\in p\\ x\in_n q\end{array}}{LIpq} \quad \frac{\begin{array}{c}x\in p\\ x\in_n q\end{array}}{LIqp} \quad \frac{\begin{array}{c}x\in p\\ x\notin q\end{array}}{Opq} \quad \frac{\begin{array}{c}x\in_n p\\ x\notin_n q\end{array}}{LOpq} \qquad \text{GEN(eralization)}$$

Y is *deducible from* $\{X_1, \ldots X_m\}$ ($\{X_1, \ldots X_m\} \vdash Y$) iff there is a tree T of sentences such that: (i) each $X_i$ ($1 \leq i \leq m$) and Y are non-singular sentences; (ii) each line in the tree is either an $X_i$ or is entered by a rule of inference from members of T that precede it in its path; (iii) EC and GEN are used at most once; (iv) if any line is entered by EC, then there is no singular sentence prior to it that is entered by a rule other than EC; and (v) either EC is not used or the last line in each path is entered by GEN.

## 6. Ecthetic deductions

Each of the following deductions proves a conclusion that Aristotle said could be proven by ecthesis. The passages that follow deductions are taken from Smith's translation *1989* of the *Prior Analytics*.

I-conversion: Iab $\vdash$ Iba

| | | |
|---|---|---|
| 1 | Iab | |
| 2 | $m \in a$, $m \in b$ | 1, EC |
| 3 | Iba | 2, GEN |

'For if it [a] does belong to some (for instance to C [m]), it will not be true that A [a] belongs to none of the Bs [b's], since C [m] is one of the Bs' (25a16–17, pp. 2–3).

Bocardo-XXX: Obc, Aba $\vdash$ Oac

| | | |
|---|---|---|
| 1 | Obc | |
| 2 | Aba | |
| 3 | $m \in b$, $m \notin c$ | 1, EC |
| 4 | $m \in a$ | 2, 3, S-SYL |
| 5 | Oac | 4, 3, GEN |

Darapti-XXX: Abc, Aba $\vdash$ Iac

| | | |
|---|---|---|
| 1 | Abc | |
| 2 | Aba | |
| 3 | $m \in b$ | 1, EC |
| 4 | $m \in c$ | 1, 3, S-SYL |
| 5 | $m \in a$ | 2, 3, S-SYL |
| 6 | Iac | 4, 5, GEN |

'When they [terms] are universal, then when both P [c] and R [a] belong to every S [b], it results of necessity that P will belong to some R. [. . .] It is [. . .] possible to carry out the demonstration through [. . .] the setting-out. For if both terms belong to every S, then if some one of the S's is chosen (for instance N [m]), then both P and R will belong to this; consequently, P will belong to some R' (28a18–26, p. 9).

Baroco-LLL: LAcb, LOab $\vdash$ LOac

1 LAcb

2 LOab

3 $\left.\begin{array}{l} m \in_n a \\ m \notin_n b \end{array}\right\}$ 2, EC

4 $m \notin_n c$ 1, 3, S-SYL

5 LOac 3, 4, GEN

'[. . .] it is necessary for us to set out that part [m] to which each term [b and c] does not belong and produce the deduction about this [m]. For it will be necessary in application to each of these; and if it is necessary of what is set out, then it will be necessary of some part [a] of the former term (for what is set out is just a certain "that")' (30a9–15, p. 13).

Bocardo-LLL: LObc, LAba $\vdash$ LOac

1 LObc

2 LAba

3 $\left.\begin{array}{l} m \in_n b \\ m \notin_n c \end{array}\right\}$ 1, EC

4 $m \in b$ 3, S-SUB

5 $m \in_n a$ 2, 4, S-SYL

6 LOac 5, 3, GEN

## 7. Illicit 'deductions' of Baroco-XLL and Bocardo-LXL

Though Aristotle rejected both Baroco-XLL and Bocardo-LXL, the semantics in Thom *1991* accepts them. We attempt ecthetic deductions of both to see where the deduction breaks down.

Baroco-XLL: Acb, LOab; so LOac

1 Acb

2 LOab

3 $\left.\begin{array}{l} m \in_n a \\ m \notin_n b \end{array}\right\}$ 2, EC

4 $m \notin_n c$ Illicit (from 1 and 3)

5 LOac 3, 4, GEN

(If everything that is chewing is a bear, and Max (a dog) is necessarily not a bear, it does not follow that Max is necessarily not chewing.)

Bocardo-LXL: Obc, LAba; so LOac

1 Obc

2 LAba

3 $m \in b$, $m \notin c$ } 1, EC

4 $m \in_n a$ 2, 3, S-SYL

5 LOac Illicit (from 4 and 3)

(If Meg is necessarily an armadillo, and Meg does not eat cicadas, it does not follow that some armadillos necessarily do not eat cicadas.)

## 8. McCall's L-X-M

If we formulate McCall's axioms (*1963*, 37–38) for L-X-M as rules of inference in the above language, then all but one of his (accepted) 14 basic rules of inference are rules (basic or derived) in our system. The exception is $\overline{\text{LIpp}}$, (Rule 2), shown above to be rejected. His rules 1, 3, 5, and 12 are basic rules in our system, and we showed how to derive his rules 9 and 10. We derive his remaining seven rules.

Datisi-XXX: Aqr, Iqp ⊢ Ipr

1 Aqr

2 Iqp

3 $m \in q$, $m \in p$ } 2, EC

4 $m \in r$ 1, 3, S-SYL

5 Ipr 3, 4, GEN

Cesare-LXL: LErq, Apq ⊢ LEpr

1 LErq

2 Apq

3 LEqr 1, CON

4 LEpr 3, 2, Q-SYL

Darii-LXL: LAqr, Ipq ⊢ LIpr

1 LAqr

2 Ipq

3 $m \in p$, $m \in q$ } 2, EC

4 $m \in_n r$ 1, 3, S-SYL

5 LIpr 3, 4, GEN

Ferio-LXL: LEqr, Ipq ⊢ LOpr

1 LEqr

2 Ipq

3 $m \in p$, $m \in q$ } 2, EC

4 $n \in_n p$, $n \notin_n r$ } 1, 3, S-SYL

5 LOpr 4, GEN

LIpq $\vdash$ LIqp

| | | | |
|---|---|---|---|
| 1 | LIpq | | |
| 2 | $m \in p$, $m \in_n q$ | $m \in_n p$, $m \in q$ | 1, EC |
| 3 | LIqp | LIqp | 2, GEN (twice) |

LIpq $\vdash$ Ipq

| | | | |
|---|---|---|---|
| 1 | LIpq | | |
| 2 | $m \in p$, $m \in_n q$ | $m \in_n p$, $m \in q$ | 1, EC |
| 3 | $m \in q$ | $m \in p$ | 2, S-SUB (twice) |
| 4 | Ipq | Ipq | 2, 3, GEN (twice) |

LOpq $\vdash$ Opq

| | | |
|---|---|---|
| 1 | LOpq | |
| 2 | $m \in_n p$, $m \notin_n q$ | 1, EC |
| 3 | $m \in p$ | 2, S-SUB |
| 4 | $m \notin q$ | 2, S-SUB |
| 5 | Opq | 3, 4, GEN |

Though we have abandoned McCall's $\overline{\text{LIpp}}$, this is not a great loss. We no longer can deduce LIaa from LAbc, for example. But Aristotle would object to such an inference since it has a superfluous premise. McCall indicates that he thought about dropping $\overline{\text{LIpp}}$ and putting $\overline{\text{Ipp}}$ in its place, recognizing that any inference Aristotle 'clearly accepted' as valid can be proven with the weaker rule. (Note that $\overline{\text{Ipp}}$ is a derived rule in our system.) His reason for not dropping $\overline{\text{LIpp}}$ is that 'no awkward consequences follow from assuming [. . .] [$\overline{\text{LIpp}}$] to be true, and so for convenience we assume it' (McCall *1963*, 50). The semantics in Johnson's *1989* accommodates McCall's inclusion of $\overline{\text{LIpp}}$.

## 9. Soundness

Theorem: If $X_1, \ldots X_m \vdash Y$ then $X_1, \ldots X_m \models Y$.

*Proof.* Assume the antecedent and suppose there is a valuation $\nu$ that assigns t to each $X_i$ ($1 \leq i \leq m$). All of the rules, with the exception of EC, the ecthesis rules, are sound. That is, for any of these rules if a valuation assigns t to the premises it assigns t to its conclusions. (This is easily verified. For example, consider $\overline{\text{App}}$. For any valuation $\nu$ if $\nu(x \in p) = t$ then $\nu(x \in p) = t$.) So, if Y has form Apq, LApq, Epq, or LEpq then $\nu(Y) = t$ since Y could only be generated from sentences of these forms and no sentence of these forms can be generated by using rules 5 to 8. Suppose Y has one of the other forms. Let us say that a pair of singular sentences $<x_1\, c_1\, r_1, x_2\, c_2\, r_2>$ has *property* $\mathfrak{F}$ *relative to a valuation* $\nu$ iff for any name y either $\nu(y\, c_1\, r_1) = f$ or $\nu(y\, c_2\, r_2) = f$. We use two lemmas.

Lemma 1: If a pair of singular sentences $<\alpha, \beta>$ has property $\mathfrak{F}$ relative to valuation $\nu$ then: (i) if $\alpha$ is entered by EC and $\beta$ is entered by one of rules 5 or 6 from singular sentence β′, then <α, β′> has 𝔉 relative to ν; (ii) if α is entered by one of rules 5 or 6 from a singular sentence α′ and β is entered by EC then <α′, β> has 𝔉 relative to ν; and (iii) if α is entered by one of rules 5 or 6 from a singular sentence α′ and β is entered by one of rules 5 or 6 from singular sentence β′, then <α′, β′> has 𝔉 relative to ν.

*Proof.* Use the soundness of rules 5 and 6 and the fact that ν assigns t to all sentences of form Apq, LApq, Epq, LEpq in the deduction.

Lemma 2: For each EC rule there is at least one conclusion branch of the rule such that the sentences in this branch are members of a pair of sentences that does not have property 𝔉.

*Proof.* Suppose x ∈ p is entered by $\frac{\text{Apq}}{\text{x} \in \text{p}}$. Then for some y ν(y ∈ p) = t, otherwise condition 1 used to give the semantics would be violated. Similarly, for other EC rules conditions used to give the semantics would be violated if Lemma 2 were false.

Suppose Y is introduced by GEN and ν(Y) = f. Then in each path there is a pair of singular sentences that has 𝔉 relative to ν (given the soundness of GEN). If both members of this pair are introduced by EC, then there is a contradiction, given Lemma 2. If it is not true that both members of this pair are introduced by EC, then by (perhaps repeated) use of Lemma 1 there is a pair of singular sentences with property 𝔉 relative to ν, where each member of the pair is introduced by EC. By Lemma 2, this is impossible.

That soundness obtains even though the ecthesis rules are unsound finds its parallel in contemporary accounts of deduction for the predicate calculus that prove soundness using an unsound existential instantiation rule (see, for example, Jeffrey *1991*, 44–56).

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