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SINGULAR TERMS, TRUTH-VALUE GAPS,
AND FREE LOGIC

IN Strawson's paper "On Referring" [1] the idea was advanced
that a simple, syntactically well-formed statement may in cer-
tain circumstances be neither true nor false.   The circumstances in
question are those in which some singular term occurring in the
statement does not have a referent.   The present paper is not con-
cerned with Strawson's work on this subject (nor does it use his
terminology).   Rather, we wish to explore the consequences of this
view, as we have formulated it above, for logic and formal semantics.

In the above formulation, 'statement' is used to refer to formulas,
in simple artificial languages (such as are studied in elementary
logic), which do not contain free variables.   'Singular term' is used
to mean "name or definite description."   Strawson's own example
of a statement that is neither true nor false by the above criterion is

1. The king of France is wise.

Another such example, in which the singular term is a name, is

2. Pegasus has a white hind leg.

We assume that whatever reasons incline one not to assign a truth-
value to 1 are operative also in the case of 2.   Thus, the fact that
Pegasus does not exist may be taken to mean that the question
whether he has a white hind leg does not arise, and so forth.   (While
I do not intend to defend the position, I must admit that, although
I can think of many artificial ways to bestow a certain truth-value
on 2, I cannot find a single plausible reason to call it true or to
call it false.)   Since definite descriptions introduce certain com-
plexities, we shall from here on assume that all our singular terms
are names.

I

Among logicians, there have been two basic reactions to the
idea.   One is that the logic of a language for which this is so can
also be explored by the usual methods, though we may expect the
relevant logical system to be quite unordinary.   Thus Prior, in

[1] *Mind*, 59, 235 (July 1950): 320–344.

Time and Modality, explores many-valued logics for this reason. The other reaction is that, for ordinary purposes, the logician can restrict his attention to languages for which this is not so. Thus, if in a given language

3. The king of France is bald.

means what Russell said it meant, then it has a truth-value. And even if Strawson is correct concerning ordinary discourse, sentences that in his view are neither true nor false are "don't cares" for all ordinary purposes, and there is therefore no reason why we should not arbitrarily assign them some truth-value. This will be convenient, since it will make standard logical techniques applicable.

Both reactions appear to start from the tacit supposition that a language in which some statements are neither true nor false must have a very unordinary logical structure. (This supposition may have been motivated by the custom of logicians to treat 'is neither true nor false' on a par with 'has a third value which is neither *True* nor *False*'.) My thesis will be that this supposition is not quite correct. Before going on, I want to note that my conviction on this point is a matter of hindsight on the basis of the development of so-called *free logic* and that I have no quarrel with the reactions outlined above. I think they are correct (certainly not mutually exclusive) and well supported; only to accept them now would be to overlook something which has come to light since they were formulated.

II

The view we wish to consider is that a simple statement (an n-adic predicate followed by n proper names) has a truth-value if and only if all the names it contains have referents. As long as we deal with the logic of unanalyzed propositions, however, the only relevant distinction we are able to draw is that between those sentences which are true, those which are false, and those which are neither. (For example, we cannot distinguish there between those statements which contain names and those which do not.) This suggests that for our purposes it will be best to regard propositional logic as but a truncated part of the first-order predicate calculus. However, we shall begin with an intuitive discussion of the logic of propositions, with the purpose of introducing some relevant concepts.

III

For the sake of perspicuity, let us consider an argument with English sentences:

4. a. Mortimer is a man.
   b. If Mortimer is a man, then Mortimer is mortal.
   c. Mortimer is mortal.

Should our present view, that 4a is neither true nor false if 'Mortimer' does not refer, cause us to qualify our precritical reaction that 4 is a valid argument? I think not. This naive reaction is not based simply on the conviction inculcated by elementary logic courses that questions of validity can be decided on the basis of syntactic form. It can also be based quite soundly on the semantic characterization of validity found in many logic texts:

> 5. An argument is valid if and only if, were its premises true, its conclusion would be true also.[2]

The fact that Mortimer has to exist for the premises of 4 to be true is just as irrelevant to the validity of that argument as any other factual precondition for the truth of those premises. Were Mortimer a man and were it the case that if he is a man then he is mortal, then it would be the case that he is mortal—this is exactly why 4 is valid. Hence, acceptance of 5 leads to the very welcome conclusion that all the same arguments are still valid, as far as propositional logic is concerned.

The reader may have thought my conclusion about all arguments rather audacious, since my reasoning pertained to a single example. Let us therefore repeat this reasoning in a more abstract form. If a statement is neither true nor false, then it is not true. Hence the view we are presently considering has as consequence for truth that, under certain stated conditions, a given statement is not true. In the simple language of unanalyzed propositions which we are presently considering, these conditions cannot be made explicit in the premises of an argument. An argument is valid provided, for any conditions under which the premises are true, the conclusion is true also. On the view above, which entails that under certain conditions (failure of reference) a statement cannot be true, this is *redundantly equivalent* to: for any conditions under which the premises are true and the relevant names all refer, the conclusion is also true. But that means that this view has no consequences vis-à-vis validity (in the present context; the situation would be different if failure of reference could be made explicit in the premises or conclusion).

From this we conclude that, as long as this view is not accompanied by other aberrant views on the propositional connectives, classical propositional logic validates just the right arguments.

[2] Dr. Paul Benacerraf, of Princeton University, has pointed out to me that the above characterization of a valid argument, as a syntactic transformation preserving truth, has in the present context as alternative a characterization as "transformation preserving nonfalsehood." It would indeed be interesting to investigate the properties of those transformations, as well as the properties of those transformations which may take a statement that is neither true nor false into a falsehood but at least do not take a *truth* into a falsehood.

This property of propositional logic we may call its "argument-completeness" with respect to the present view.

However, validity of argument is not the only subject of interest in the logic of propositions. A second, related, subject is *logical truth* of statements. If we knew how to characterize logical truth, in accordance with the view we are presently investigating, then we could ask whether classical propositional logic is "statement-complete" with respect to this view—that is, whether all the logical truths are theorems. We could then also ask whether all the theorems of this logic are logical truths on this view (*soundness*).

It is not easy to see how we can characterize logical truth in the present context. The intuitive guide is the idea that logical truth is truth in all possible situations. But as long as we deal only with unanalyzed propositions, how can we reconstruct the relevant notion of situation? There is another possible approach: it seems plausible that the question of whether the theorems are exactly the logical truths cannot be independent of the question whether the arguments justified by the rules and theorems are exactly the valid arguments. It would seem that there is a certain connection between validity and logical truth, and if this connection were spelled out, we might be able to use the former notion to explicate the latter. We shall not look into this possibility further (though it suggests that the set of logical truths will turn out to be not too unfamiliar). Rather, we shall turn to the full first-order logic, with respect to which we *have* a plausible reconstruction of the notion of truth in all possible situations.

IV

Let us consider, then, a simple first-order language with predicates, names, variables, connectives, and quantifiers. We wish to investigate to what extent standard logic remains applicable to this language if we do not assume all its names to refer. It will be convenient to use the logical system—which we shall call *ML*—of Quine's *Mathematical Logic.*[3] ML was formulated for a language like ours, except that no names belong to its vocabulary (we disregard the part that concerns set theory). The logical system is essentially a specification to the effect that all statements in that language which have *a certain form* are theorems of ML. We shall analogously designate all the statements of our language which have that form as "theorems of ML." For example, Quine specifies that all statements of the form

$$(x)(A \supset B) \supset \cdot (x)A \supset (x)B$$

[3] Cambridge, Mass.: Harvard, rev. ed., 1951.

are theorems.   This means that, if F and G are monadic predicates, R a dyadic predicate, and b a name, of our language, then the following two statements are examples of theorems of ML (in our language):

$$(x)(Fx \supset Gx) \supset \cdot (x)Fx \supset (x)Gx$$
$$(x)(Rxb \supset Gx) \supset \cdot (x)Rxb \supset (x)Gx$$

Here our first example is a theorem of ML (in our language) that does not contain names, and the second is one that does.

Now truth-value gaps occur in the present context only in connection with names.   It follows that the standard logic will apply to all those statements in which no names occur.   So we conclude:

    6. If a statement is a theorem of ML and if no names occur
       in it, then it is logically true.

This much we know before we have even answered the question of how we can characterize logical truth here—provided, again, that the view regarding failure of reference which is presently under consideration is not accompanied by further exotic views on logic. I shall now try to show that furthermore we can drop the qualification about names from 6: *any* statement that is a theorem of ML in our language—even if it does contain names—can be regarded as logically true even on the present view.

To show this, we must first explicate what is meant by 'logically true', or 'true in all possible situations'.   The explication runs as follows.   To present a specific interpretation of a language, I must specify a *domain of discourse* and the *extensions* the predicates have in that domain.   Suppose L is the language, D—which may be any nonempty set of things—the domain of discourse, and f a function such that if F is a predicate of L then $f(F)$ is the extension of F in D. Then the couple $(f;D)$ is called "a model of L."   It is not difficult to see what counts as *truth* in such a model.   For example

$$(x)Fx$$

will be true in this model exactly if the extension $f(F)$ of F is the whole of the domain D.   These models are the reconstructed counterparts of Leibniz's possible worlds.   Thus, a sentence is *logically true* if and only if it is true in any model $(f;D)$, no matter how D and f are chosen.

In this account, no mention is made of singular terms.   Let us suppose that the language L has, in addition to predicates, variables, quantifiers and the usual propositional connectives, also a set of names.   Let us suppose that some of these names have referents and that some do not.   In terms of model, this means that, for

some names t, $f(t)$ is defined and is a member of the domain D, and that for some other names that function is not defined. For example, let e be a name, and let its referent $f(e)$ be a member of the extension $f(F)$ of the predicate F. Then

$$Fe$$

is true in this model. If, on the other hand, the name e does not have a referent, then, according to the position we are presently examining, the sentence

$$Fe$$

is neither true nor false in this model. We must now ask which sentences of *this* language are logically true. In answer, I shall characterize the set of logically true sentences of this language, proceeding in two steps.

First, recall the proposal that the troublesome truth-value gaps be eliminated by simply assigning truth-values to the offending statements in some arbitrary manner. We begin by taking up this suggestion.

7. A classical valuation over a model is a function v that assigns T or F to each statement, subject to:
   a. if A is an atomic statement containing no nonreferring names, then $v(A)$ is determined by the model, in the indicated manner, and
   b. if A is a complex statement, then $v(A)$ is determined by what v assigns to the simpler statements, in the usual manner.

We have not put any conditions on $v(A)$ when A is an atomic statement containing some nonreferring name—except that $v(A)$ is either T or F. This means that, if there is any name e that has no referent in the domain of a given model and if $A(e)$ is an atomic statement in which e occurs, then there are at least two classical valuations over this model: one which assigns T to $A(e)$ and one which assigns F to $A(e)$.

8. A statement is CL-*true* (*false*) if and only if it is assigned T (F) by all classical valuations over all models.

It will not surprise anyone that all theorems of ML are CL-true by the above definition. Some familiar theorems of other systems, however, are not considered:

9. $Fa \supset (\exists x)\, Fx$

This is not CL-true by the above definition. Nor can it be a theorem of ML, since the latter allows us to deduce only:

$$(y)(Fy \supset (\exists x)Fx)$$

If we added to ML some rule to replace variables by names, then 9 would become a theorem.   But since 9 is not logically true once we take into account the possibility that the name *a* does not refer, we shall of course not add such a rule.

The second point we wish to bring to the reader's attention is that the classical valuations go beyond the model to which they belong, just with respect to those terms which have no referent in the model.   But there, they go beyond the model *in all possible ways*.   The nature of the model determines in a given valuation not what is peculiar to it, but what it has in common with the other valuations over this model.   So what the classical valuations over a model have in common we can take as correctly reflecting truth and falsity in the model.

> 10. A *supervaluation over a model* is a function that assigns
> T (F) exactly to those statements assigned T (F) by
> all the classical valuations over that model.

Supervaluations have truth-value gaps.   The following diagram shows how the supervaluation compares with the classical valuations over a model $(f;D)$ when the language is supposed to contain exactly one predicate F (monadic) and exactly two names, *a* and *b*, such that $f(a)$ is defined and $f(b)$ is not defined.   There are here exactly two classical valuations, v_1 and v_2; the supervaluation we call *s*.   Let us assume that $f(a) \,\epsilon\, f(F)$ and that all of D is included in $f(F)$.

	v_1	v_2	*s*
Fa	T	T	T
$\sim Fa$	F	F	F
Fb	T	F	—
$\sim Fb$	F	T	—
$Fb \vee \sim Fb$	T	T	T
$(x)Fx$	T	T	T
$(x)Fx \supset Fb$	T	F	—

The dashes indicate truth-value gaps.

Now we can characterize logical truth in accordance with the view that is the subject of this paper.

> 11. A statement is *SL-true* if and only if it is assigned T by
> all supervaluations.

And the third point to make is that the set of CL-truths and the set of SL-truths are exactly the same!   For if a supervaluation assigns T to *A* only if all corresponding classical valuations do, then *all* supervaluations assign T to *A* only if all classical valuations do.   And conversely, since if all classical valuations over a model assign T to *A*, then so does the corresponding supervaluation, it

follows that if all classical valuations over all models assign T to A, then so do all supervaluations.

Hence, since all theorems of ML are CL-true, they are also all SL-true. We have now shown what we set out to show in this section, namely that the qualification about names may be dropped from principle 6 above. Along the way we have also answered a question left open in the preceding section: all tautologies may be regarded as logically true, in complete accordance with the present view. (Because of course, all theorems of the classical propositional calculus are also theorems of ML.)

We said above that the domain of discourse of a model must be a *nonempty* set. This was necessary because ML is not valid for the empty domain. However, Hailperin[4] and Quine[5] have shown how a slight revision in the axioms of ML—to yield what we shall call *revised ML*—makes it valid for empty domains also. The above argument comes through in its entirety when we allow domains to be empty, and substitute 'revised ML' everywhere for 'ML' (see footnote 11, below).

<div align="center">v</div>

Besides propositional inference and quantification, elementary logic comprises one further subject: *identity theory*. If we add the identity symbol to our language, we must ensure that at least the following hold:

12. $\vdash (x)(x = x)$

13. $\vdash (x)(y)(x = y \supset \cdot A(x) \supset A(y))$, no names occurring in $A(x)$.

(where '$\vdash$' is used as Quine uses H in ML: all instances of the schema given become theorems provided we supply initial universal quantifiers to bind any free variables in them.) This is simply to say that we must retain the usual identity theory at least for contexts in which truth-value gaps cannot occur. (From now on, the distinction between names and variables is very important, but we also want to be able to talk about both at once. Let us use x, y, z for variables; a, b, c for names; and t, t' for either.)

In addition, it seems to me that we cannot plausibly reject that

14. $t = t'$

is false when t has a referent and t' does not.[6] (We should point

[4] Theodore Hailperin, "Quantification and Empty Individual Domains," *Journal of Symbolic Logic*, 18, 3 (September 1953): 197–200.

[5] W. V. Quine, "Quantification and the Empty Domain," *ibid.*, 19, 3 (September 1954): 177–179.

[6] I was convinced of this by Dr. Karel Lambert, of West Virginia University (as the reader will see in section VI, this is only the least of my debts to him).

out that 14 may not be a statement, since either t or t' may be a free variable. A free variable is always to be regarded as having been assigned some referent.) For example, that Santa Claus does not exist is sufficient reason to conclude that the president of the U.S. is not Santa Claus; that the Scarlet Pimpernel is a fictional character, sufficient reason to conclude that the man in the iron mask was not the Scarlet Pimpernel.

Given only this, it already follows that

$$t \text{ exists}$$

is appropriately expressed as

$$(\exists y)(y = t)$$

and, furthermore, that the logical truth

15. $((x)A(x) \ \& \ (\exists y)(y = t)) \supset A(t)$

is the valid counterpart of "universal instantiation." In addition, however, I propose that we accept entirely without restriction:

13. $\vdash t = t$

14. $\vdash t = t' \supset \cdot A(t) \supset A(t')$

This extension appears to me to be reasonable because, first, it seems to me reasonable to adopt the principle that, if a sentence A is logically true, then any sentence obtained from A through a consistent substitution of singular terms for singular terms must also be logically true. Thus, if

$$\text{Cicero} = \text{Cicero}$$

is to be taken as *logically* true, then so must

$$\text{Pegasus} = \text{Pegasus}$$

Furthermore, the system obtained through this extension of revised ML is not only correct, but also *statement-complete* under the present interpretation (disregarding for the moment the complications of description theory).

VI

The logical system obtained through the extension discussed in the preceding paragraph is called "free logic" (the term is due to Karel Lambert). This variety of logic was developed by Leonard,[7]

Professor John Lemmon (Claremont Graduate College) does not agree to this. It seems plausible to him to say that such a statement is neither true nor false. He does seem to agree, however, when one of the terms is a definite description.

[7] Henry S. Leonard, "The Logic of Existence," *Philosophical Studies*, **7**, 4 (June 1956): 49–64.

Hailperin and Leblanc,[8] Hintikka,[9] and ⅟Lambert.[10] We should
mention here that the interpretation just given is not the only one
under which free logic is sound and statement-complete. That is,
its theorems are exactly the logical truths of the language by
definitions 8 and 11, but also by an infinity of other definitions.
These definitions yield a spectrum of interpretations. On the one
extreme, we find logical truth identified with CL-truth; on the
other extreme of the spectrum we find logical truth identified with
SL-truth. Equivalently, on the one extreme, all sentences in the
language are assigned a truth-value (classical valuations), and this
set of sentences that are either true or false diminishes as we move
to the other extreme, where it is at a minimum. It is not difficult
to see why some philosophers might prefer some intermediate
interpretation on this spectrum. For there certainly are sentences
in which there occur nonreferring singular terms and which we do
assign a truth-value. Examples are:

The ancient Greeks worshipped Zeus.

Pegasus is to be conceived of as a horse.

The wind prevented the greatest air disaster in history.

Of these examples, the first two are due to Leonard, and the third
to Chisholm. The proof of statement-completeness for the system,
for this whole spectrum of interpretations, is given in a forthcoming
article by this author.[11] The system has been extended to deal
with definite descriptions by Lambert[12] and by van Fraassen and
Lambert.[13]

VII

We may recall that in the section on propositional logic we
distinguished between *argument-completeness* and *statement-com-
pleteness*. Free logic is statement-complete under each of the
interpretations mentioned: the logical truths are exactly the
theorems of the system. But does it validate all the correct

  [8] Hugues Leblanc and Theodore Hailperin, "Nondesignating Singular
Terms," *Philosophical Review*, 68, 2 (April 1959): 239–244.

  [9] Jaakko Hintikka, "Existential Presuppositions and Existential Commit-
ments," this JOURNAL, 56, 3 (Jan. 29, 1959): 125–137.

  [10] Karel Lambert, "Existential Import Revisited," *Notre Dame Journal of
Formal Logic*, 4, 4 (October 1963): 288–292.

  [11] "The Completeness of Free Logic," *Zeitschrift für mathematische Logik
und Grundlagen der Mathematik*, 1966 (forthcoming).

  [12] "Notes on E! III: A Theory of Descriptions," *Philosophical Studies*, 13, 4
(June 1962): 51–59; and "Notes on 'E!' IV: A Reduction in Free Quantification
Theory with Identity and Descriptions," *ibid.*, 15, 6 (December 1964): 85–88.

  [13] "On Free Description Theory," *Zeitschrift für mathematische Logik und
Grundlagen der Mathematik* (forthcoming).

arguments? Its only rule is a rule for deriving theorems, not a rule for deriving true statements from other true statements. This rule, which is part of ML, reads:

15. If A and $A \supset B$ are both theorems, then B is also a theorem.

But the system can be used to show that arguments are valid, as follows:

16. Free logic validates an argument '$P_1, \ldots, P_n$; hence Q' if and only if '$P_1 \& \ldots \& P_n \cdot \supset Q$' is a theorem of free logic.[14]

What it means to say that an argument is valid depends of course on the interpretation chosen. If we consider only the two extreme interpretations, then we have the following two characterizations of validity.

17. An argument is C-valid if and only if every classical valuation that assigns T to all its premises also assigns T to its conclusion.

18. An argument is S-valid if and only if every supervaluation that assigns T to all its premises also assigns T to its conclusion.

It is not difficult to see that every argument that is C-valid is also S-valid. Furthermore, every argument validated by free logic is C-valid and, hence, S-valid. (If all classical valuations assign T to $A \supset B$, then there cannot be a classical valuation that assigns T to A but not to B.) This means that 16 is a correct definition.

But this does not yet answer the question whether free logic is argument-complete. Let us first ask whether all the C-valid arguments are validated by free logic. Suppose that A and B are such that every classical valuation that assigns T to A also assigns T to B. There are then two possibilities. Those classical valuations which assign T to A also assign T to $A \supset B$. There is only one other sort of classical valuation: those which assign F to A. These also assign T to $A \supset B$. Therefore, if 'A, hence B' is C-valid, then $A \supset B$ is CL-true, and a theorem of free logic. This establishes that free logic is *argument-complete* under the interpretation that uses classical valuations.

Unfortunately, this is not true for the interpretation that uses supervaluations.[15] The difference does not show up in arguments

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[14] We do not observe the use-mention distinction unless the context would not prevent confusion.

[15] I discovered this after a stimulating discussion at Wayne State University, and especially because of the comments of Mr. Lawrence Powers.

constructed from unanalyzed propositions. But it does show up in arguments involving some essential use of the quantifiers. This is because, using the quantifiers, *we can state in the language itself that a given name does not refer.* Hence,

    19. If $A(c)$ is atomic, then both '$A(c)$, hence $(\exists x)(x = c)$' and '$\sim A(c)$, hence $(\exists x)(x = c)$' are S-valid.

Of course, the corresponding conditionals '$A(c) \supset (\exists x)(x = c)$' and '$\sim A(c) \supset (\exists x)(x = c)$' are not SL-true. This is because those supervaluations which do not assign T to $A(c)$—respectively, $\sim A(c)$—comprise some which do not assign F to that statement either; and these assign neither T nor F to the corresponding conditional.

So we have found that, although free logic is statement-complete for the whole spectrum of interpretations considered, we can regard it as argument-complete only for the interpretation that uses classical valuations. This is not too surprising from a historical point of view, because several of the logicians who developed free logic have indicated repeatedly that they adhere firmly to the principle of bivalence (by name: Lambert and Leonard).

That not all the S-valid arguments are specifiable by means of free logic does not mean that we do not know which arguments are S-valid. Given any argument, we can in principle tell whether it is S-valid or not, because we have a complete semantic account of what this amounts to. (There is of course the interesting, but purely *technical* question, what sort of logical system validates exactly the S-valid arguments. That is exactly the question: What are the peculiar *syntactic* features of these arguments? From a philosophical point of view, this is not quite so important.)

This is also the situation for the interpretations intermediate between the two we have just discussed. All these interpretations take failure of reference seriously. For all of them, free logic gives us an exact syntactic description of what statements are logically true. All of them agree that all traditional *propositional* arguments are valid. All agree further that, if a given conditional statement is logically true, then the argument from its antecedent to its conclusion is valid. The differences are these. They differ on when failure of reference results in lack of truth-value; and they differ on which arguments, beyond the ones mentioned above, are valid. But the "upper limit" of these disagreements is reached in the interpretation using supervaluations: it withholds truth-values from the largest class of statements and recognizes as valid the largest class of arguments.[16]

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[16] By "largest" I do not mean "of greatest cardinality"; I use "K is larger than L" here to mean "L is included in K, but K also has some members that are not in L."

VIII

We have seen so far that the interpretations involving truth-value gaps do not lead to very unusual logical systems. Their unorthodoxy is mainly one of semantics. To throw a clearer light on this, we may distinguish between the logical law of the excluded middle and the semantic law of bivalence. The first says that any proposition of the form $P \vee {\sim}P$ is logically true. The second says that every proposition is either true or false, or, equivalently, that one of P and ${\sim}P$ is true, the other false.

Clearly the law of bivalence fails for supervaluations and, indeed, for all the interpretations other than that based on classical valuations. But all our interpretations agree that $P \vee {\sim}P$ is logically true. This shows that (contrary to usage) the two laws must be strictly distinguished. This distinction, just like the distinction between statement-completeness and argument-completeness, is a distinction without a difference in classical contexts. But the admission of truth-value gaps gives it content.

One interpretation of Aristotle's remarks on future contingencies is that he wished to deny the law of bivalence while retaining the law of the excluded middle. Whether this is historically accurate or not is not now to the point; what is important is that William and Martha Kneale[17] deny that this is a tenable position. Mrs. Kneale, who wrote this chapter, says on p. 48:

In other words Aristotle is trying to assert the Law of Excluded Middle while denying the Principle of Bivalence. We have already seen that this is a mistake.

The last sentence is a reference to her argument of pp. 46–47:

In chapter 9 of *De Interpretatione* Aristotle questions the assumption that every declarative sentence is true or false. It might seem that he is clearly committed to this thesis already, but this is not so; for when he says that to be true or false belongs to declarative sentences alone, this may be taken to mean that only these are capable of being true or false not that they necessarily are. . . . Given the definitions of truth which we have quoted, the principles [of Bivalence and of Excluded Middle] are, however, obviously equivalent; for if 'It is true that P' is equivalent to 'P', 'P or not-P' is plainly equivalent to 'It is true that P or it is false that P'.

It is clear that she is talking about a language in which, when P is a sentence, 'It is true that P' is also a sentence. Our language is not thus. But more important is the following observation. Her argument is: Aristotle may question the law of bivalence; but if he does reject this law, then his acceptance of the principle

20. P if and only if it is true that P

forces him also to reject the law of the excluded middle. Whether this is so depends on how 20 is understood. Mrs. Kneale apparently

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[17] *The Development of Logic*, New York: Oxford, 1962.

understands it in such a way that it justifies any argument of the form

21. —*P*—; hence: —It is true that *P*— (and conversely)

Thus understood, 20 validates her argument. But, since the language we constructed is one for which the law of excluded middle does and the law of bivalence does not hold, it follows that, thus understood, 20 is false. We may formulate this conclusion more strongly: this language provides a counterexample to her conclusion; hence any interpretation of 20 must be such that *either* it makes 20 false *or* it does not justify her reasoning.

Since 20 is such a plausible and widely accepted principle, there remains of course the question how it is to be understood. It seems to me that the answer to this is that 20 must be construed to mean simply that both the following kinds of argument are valid:

22. *P*; hence: It is true that *P*
It is true that *P*; hence: *P*

To say that these are valid simply means that they preserve truth: when the premise is true, so is the conclusion. This says nothing whatsoever about the truth-value of the conclusion when the premise is not true (that is, when the premise is false *or* when the premise neither true nor false[18]).

When 20 is understood in this manner, it does not lead to the incorrect conclusion that bivalence follows from the law of excluded middle. However, it may appear to do so, just because the use of 'if . . . then' to signify the validity of a certain argument may be misleading. For example, the following reasoning might at first sight appear valid: From 20 follow both:

a. If *P* then it is true that *P*

b. If not-*P* then it is true that not-*P*

but the excluded middle yields

c. *P* or not-*P*

hence, by the rule of Constructive Dilemma:

d. It is true that *P* or it is true that not-*P*

It is not difficult to spot the fallacy if we keep in mind that 20 is to be construed as 22. For then a and b amount to

a*. *P*; hence: It is true that *P*

b*. Not-*P*; hence: It is true that not-*P*

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[18] When *P* lacks a truth-value, we have a choice with respect to 'It is true that *P*': we may regard it as false or as neither true nor false. The above remarks apply regardless of which of these alternatives we adopt.

From the validity of these two arguments we can deduce the truth of *d* only if we are given not *c*, but

c*. Either *P* is true or not-*P* is true

which is not excluded middle but bivalence.

A similar argument is the following: from a we infer:

e. If not-(it is true that *P*) then not-*P*

by contraposition, and then from e and b by transitivity:

f. If not-(it is true that *P*) then it is true that not-*P*

But if e is really to follow from a in the sense of a*, then it can be understood to mean only:

e*. *P* is false; hence, not-*P*

in which case f amounts to the innocuous assertion that

f*. *P* is false; hence: not-*P* is true

is a valid argument.

The plausibility of these two fallacious arguments derives, it seems to me, from the fact that formulation 20 looks like a material biconditional. This suggests that certain familiar patterns of reasoning, which are in fact not applicable, do apply. For this reason, I should like to suggest that formulation 20 not be used. Instead, one may state explicitly that the arguments of form 22 are valid; or, if a biconditional formulation is more convenient, one might use

23. It is true that *P* if and only if it is true that (it is true that *P*)

which is less misleading.

<div align="right">BAS C. VAN FRAASSEN</div>

YALE UNIVERSITY

BOOK REVIEWS

Foundations of Historical Knowledge. MORTON WHITE. New York: Harper & Row, 229 p. $6.95.

Professor White has written a useful book in the best modern manner: clear, precise, systematic, honest in confessing difficulties. He begins from a careful discussion of the "covering-law" analysis of historical explanation, in which he attempts, first, to answer the principal objections that have been brought against that analysis and, secondly, admitting that it has certain defects, to remedy