Branching Space-Time
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BRANCHING SPACE-TIME

ABSTRACT. 'Branching space-time' is a simple blend of relativity and indeterminism. Postulates and definitions rigorously describe the 'causal order' relation between possible point events. The key postulate is a version of 'everything has a causal origin'; key defined terms include 'history' and 'choice point'. Some elementary but helpful facts are proved. Application is made to the status of causal contemporaries of indeterministic events, to how 'splitting' of histories happens, to indeterminism without choice, and to Einstein–Podolsky–Rosen distant correlations.

1. INTRODUCTION

Problem: How can we combine *relativity* and *indeterminism* in a rigorous theory?[1] The problem is difficult; indeed some have presented arguments that it is in principle insoluble – Stein (1991) combines a refutation of those arguments with an account of their apparent force. Here I directly confront the problem and offer a rigorously framed contribution to a solution.

The combinational question evidently presupposes both relativity and indeterminism, which is a lot to presuppose. I hope that those who reject one of these assumptions will nevertheless find helpful the present effort to devise a careful theory embodying them both. I further hope that those who reject one *because* of a belief that it is inconsistent with the other will come to see the reason for their rejection as flawed. Lastly, there are many who take the following as given: the *only* way to discuss relativity and determinism/indeterminism is by talking about psychology or epistemology or the history of science, or about theories or laws or models or other linguistic or quasi-linguistic phenomena.[2] I hope that some of these people will find it helpful to have an additional approach worked out in some detail.

The theory of this paper is simple in respect of vocabulary: although it involves several *defined* concepts intended as revelatory, its only *primitives* are (i) the set of 'possible point events' and (ii) the causal ordering relation on them. Disadvantage: The ideas developed will be remote from 'real' physics. Advantage: Such results as we obtain will be fundamental, rigorous, and clear.

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The underlying idea is that a true description of our world requires fusing Einstein–Minkowski *space-time* with Prior/Thomason *branching time*. For the resultant structure, *branching space-time* seems as good a name as any. Here is a historical reconstruction.

The 'old physicists', at least in my imagination, conceived of time as a linear ordering of spatially infinite but instantaneous Euclidean spaces. Ignoring all metrics, as I do throughout this paper, I call their structure *linear time* and I call its order *linear temporal order*. When I want a name for the individual instantaneous Euclidean spaces that are put in linear temporal order, I call them *moments*.

In articulating special relativity, the fundamental idea of Minkowski, after Einstein, was, from the present point of view, to revise the very terms of the relation. Rather than spatially infinite moments, it is infinitesimally small *point events* that are related, and what relates them is termed a *causal order*. By 'causal order' I mean what could be called 'time-like or light-like order', with the addition of a sense or 'direction'.[3] (Old physicists and relativity theorists agree that causal 'influences' pass (only) along the causal order, but they differ as to the nature of the terms of the relation.) The manifold of point events is called *space-time*. The idea of viewing space-time as a set of point events subject to a causal order seems to carry over from special to general relativity. For illustrative purposes, however, it is often useful to keep to special relativity, where a particular metric is available. I will use *Minkowski space-time*[4] to denote this case, leaving plain *space-time* for general relativistic use.

As for indeterminism, a history of physics might be able to obtain a nonrelativistic version from quantum mechanics, and then a relativistic version of indeterminism from quantum field theory. I do not have the background for essaying such a history. Instead I draw on the work of the logicians Prior and Thomason and McCall. To express some fundamental features of our world associated with indeterminism as a foundation for modal tense logic, Prior, and after him Thomason, started out as did the old physicists with *moments*. Then he generalized the linear temporal order to a *branching temporal order*. The manifold of moments ordered in this tree-like way is called *branching time*.[5]

Observe the contrast. On the one hand, the detailed physics of each of relativity and quantum mechanics is necessarily complicated. No wonder few persons claim to have much to say about their 'combination' in quantum field theory. But, on the other hand, neither the fundamental relativistic idea of Minkowski nor the fundamental indeterministic idea of Prior/Thomason is all that intricate. The hope arises that one can say something simple but useful about relativistic indeterminism by combining these ideas. The result is what I call 'branching space-time'. The idea of the combination has two parts: (1) the items related will be *point events*, as required by Minkowski but not by Prior/Thomason; and (2) the ordering will be a *branching* (*causal*) *order*, as required by Prior/Thomason but not by Minkowski. The following proportion thus describes what is wanted.

> linear time/space-time :: branching time/*branching space-time*.

Here is a table for the above 'historical' jargon.

Structure	*Relata*	*Relation*
linear time	moments	linear temporal order
space-time	point events	causal order
branching time	moments	branching temporal order
branching space-time	*point events*	*branching causal order*

The plan for the remainder of the paper is this. I develop the theory in Sections 2–7 through a mixture of (i) rigorous postulates, definitions and facts (each of which is numbered), and (ii) informal motivation. Then I apply the theory in Sections 8–11 to four problem areas, the last being the Einstein–Podolsky–Rosen 'paradox'. I summarize in Section 12. There is also an appendix suggesting a modest generalization.

2. OUR WORLD AND ITS CAUSAL ORDER

The rigorous theory commences with Postulate 1 below. Here I begin the informal gloss by introducing a suggestive name, explaining its meaning with the clearest language I know. Let **Our World** be the set of *point events* that are 'in suitable external relations'[6] to us. Accommodate indeterminism by including those point events that either are now future possibilities or were future possibilities.[7] Of the ones that were future possibilities, we might say that they 'could have been'. The following preliminary words will serve, if you keep in mind that there are opposed possibilities ahead of us in a causal direction: include any point events that are accessible from here-now by a possibly zigzagging combination of causal and reverse causal tracks. Let e (often marked) range over **Our World.**

I have put in plain indexical English that I do not mean to be speaking of point events that are mere creatures of belief or imagination or otherworldly recombinational possibility. In what follows I will try to avoid indexical language. In particular, I will not draw a distinction (inevitably indexical when not relational) between the actual and the possible – except in motivating or giving examples. 'Possible point events' are thus just 'point events'. These point events are to be taken not as mere spatiotemporal positions open for alternate concrete fillings, but as themselves concrete particulars.[8]

Some possible point events are incompatible with others. Here is an idealized illustration. There is an ideally small event, e_m, at which a certain electron is measured in a certain way. There are two possible outcomes: measured spin up or measured spin down. Take a possible point event, e_u, at which it is true to say, 'It has been measured spin up', and another, e_d, at which it is true to say 'It has been measured spin down'. The point events e_u and e_d are incompatible, though each is compatible with e_m.[9] Exactly how can two incompatible point events both fit into **Our World**? Answer: By means of the causal order.

Let $\leqslant$ be a relation on **Our World** having the significance that $e_1 \leqslant e_2$ just in case there is a causal order between e_1 and e_2, with the former earlier than the latter (in the weak sense that allows identity). Given $e_1 \leqslant e_2$, from the standpoint of e_2 we should say that e_1 *did* occur, and from the standpoint of e_1 we should say that e_2 *might* occur. Here are three paradigms. (If the indexical space-time annotation of the diagrams doesn't help, please pass on to the following text.)

Causal dispersion: Causal order can hold between a given point event, e_3, and two space-like separated future point events, e_1 and e_2, in a single (e.g.) Minkowski space-time, just as you might expect: $e_3 \leqslant e_1$ and $e_3 \leqslant e_2$. *Causal confluence:* Causal order also can hold between two given space-like separated point events, e_1 and e_2, in a single Minkowski space-time, and a single future point event, e_3, as you might equally expect: $e_1 \leqslant e_3$ and $e_2 \leqslant e_3$. *Causal branching:* Causal order also can hold between a given e_3 and two possible future point events e_1 and e_2 that might be said to be alternate possibilities for occupying the same 'spatiotemporal position': $e_3 \leqslant e_1$ and $e_3 \leqslant e_2$.[10] *No backward branching:* That a fourth diagram is missing from Figure 1 correctly suggests that I am denying that incompatible point events can lie in the

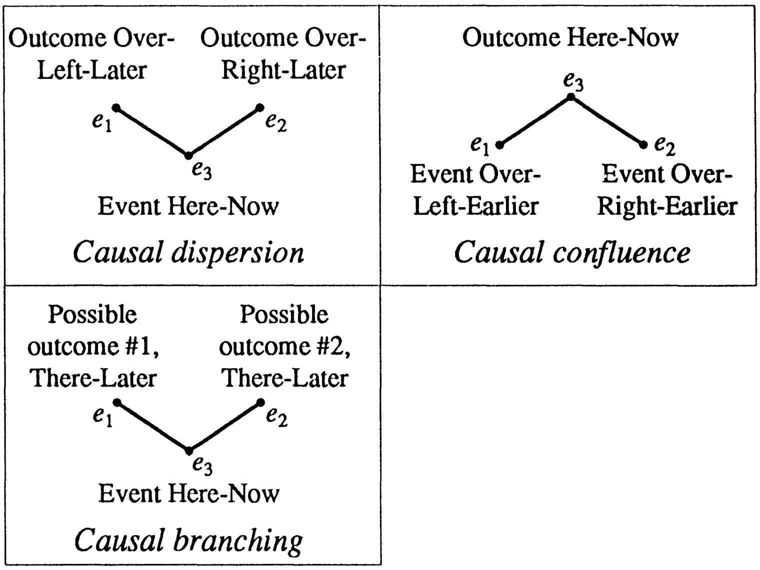

Fig. 1. Causal dispersion, confluence, and branching.

past, i.e., that some events could have incompatible 'incomes' in the same sense that some have incompatible outcomes. No backward branching is part of common sense, including that of scientists when speaking of experiments, measurements, probabilities, some irreversible phenomena, and the like. In many other contexts, however, scientists make a point of drawing no distinction between backward and forward. Because this paper lacks space for discussion of this controversial matter, I hope the following is noncontentious: the assumption of no backward branching is plausible enough to warrant making clear what it comes to. It will then be warranted to the extent that one finds helpful a theory of which it is a part.

What postulates hold for the causal order? For Minkowski space-time, Mundy (1986) describes the 1914–36 results of Robb and gives additional results for the light-like order. That research, however, does not immediately help here because a Minkowski space-time, as I understand it, never contains incompatible point events. We shall need to proceed more slowly. The first postulate is so natural and so vital that without it I would not know what to say next.

POSTULATE 1: **Partial Order.** The relation $\leqslant$ is a nontrivial partial ordering of **Our World**:

> Nontriviality: **Our World** is nonempty.
> Reflexivity: $e \leqslant e$.
> Transitivity: if $e_1 \leqslant e_2$ and $e_2 \leqslant e_3$, then $e_1 \leqslant e_3$.
> Antisymmetry: if $e_1 \leqslant e_2$ and $e_2 \leqslant e_1$, then $e_1 = e_2$.

I have not the slightest hope of making an instructive argument for this postulate. For example, some have questioned antisymmetry, asking us to consider 'causal chains' that double back upon themselves. I am unwilling to do so, but I am equally unwilling to argue the point. The following discussion would surely be unintelligible without antisymmetry – which is perhaps after all not a bad argument in its favor.[11]

The following simple definitions are for convenience.

DEFINITION 2: I use $<$ for the companion strict partial ordering: $e_1 < e_2$ if $e_1 \leqslant e_2$ but not $e_1 = e_2$.

I use 'causally earlier' and 'causally later,' etc., as English readings of the weak relation, $\leqslant$, but often drop the adjective 'causal'. I mark the stronger relation with 'proper', as in 'e_1 is properly earlier than e_2'. On the other hand, it is more convenient to use 'causal past' and 'causal future' for the strong relation, again dropping 'causal' more often than not. Thus, if $e_1 < e_2$, then the first is *properly earlier* than or in the *past* of the second. Also the second is *properly later* than or in the *future* of the first.

'The future' in the sense of the words used above contains incompatible possibilities. This should be borne in mind from the beginning, although it cannot yet be explained. You will remain forever confused (and think that I am confused, or perhaps that I mean to beguile you with amazing stories) if you identify *this* use of 'future' (which is of course jargon) with 'what *will* happen' instead of 'what *might* happen'. For example, in *this* use of 'future', to say that in the future of a chosen measurement event there is both a measured spin up and a measured spin down is to say with prosaic factuality that each *might* happen, not, incredibly, that each *will* happen. To help reduce confusion, I will sometimes speak of 'the future of possibilities' instead of just 'the future'. Below I will explain the Prior-inspired concept of futurity required for the future tense of English.

Keep in mind that none of these usages has anything to do with a 'frame of reference'. They all rely on the fundamental ordering of point events, and on nothing else. Also observe a purposeful omission: I have for good reason not yet defined 'space-like separation' or 'causal contemporaneity'.

DEFINITION 3: A *chain* is a subset of **Our World** all members of which are comparable by $\leqslant$: for e_1, e_2 in the chain, either $e_1 \leqslant e_2$ or $e_2 \leqslant e_1$.

A *causal track* or *interval* is a maximal chain of point events lying between two given point events. It is 'open' or 'closed' at one end or the other depending on whether one takes 'between' to exclude or include the given point events.

I extend the 'track' terminology to cases in which only one point is given, and the chain is maximized either upward or downward from the given point, or in which no point is given, and the chain is maximal in **Our World**. In the latter cases we may occasionally speak of causal tracks that are *upward maximal*, *downward maximal*, or (just) *maximal*.

Some people say that a causal track is a locus of a possible causal transmission.[12] On the present theory this is profoundly true but might be misleading if one neglects that a causal track is just the chain of (possible) point events that it is. The *spatiotemporal position* it occupies is, however, available for alternate possibilities. Thus, point events connected by a causal track are 'connect*ed*', not 'connect*ible*'.[13]

3. HISTORIES

How does one further describe the way that point events fit together in **Our World?** What will eventually emerge is a version of 'everything has a causal origin'. In order to state such a postulate rigorously, however, I shall need to devote three sections to the elaboration of some critical definitions that generalize from Prior's branching time. That theory arranges its moments into a tree: incompatible moments have a lower bounding moment in the tree ('historical connection' in Thomason's phrase), but never a common upper bound (no backward branching). The formal definition of a tree gives expression to the openness of the future in contrast to the settledness of the past. A key point to keep continually in mind is that in branching time, the entire tree is 'the world'. In addition there is the concept of a 'history', defined as a maximal chain of moments. Locate yourself at a moment in the tree, perhaps at the moment at which the spin measurement occurs. You will easily visualize that on this picture your 'world' is unique, whereas you belong to *many* 'histories'. Until and unless branching ceases, even long after your expiration, there is no such thing as 'your history'. Of course in branching time 'your history' makes sense when identified with 'your historical past'. Branching time takes uniqueness to fail only when histories are taken as stretching into the future. On this usage a 'world' contains incompatible possibilities, while a 'history' does not. A history represents a choice between incompatible possibilities, a resolution of all disjunctions unto the end that presumably never comes.

The present development keeps Prior's idea of **Our World** as involving many possible histories, each of which might be a Minkowski space-time.[14] It is obvious, however, that histories cannot be defined as maximal chains of point events; the latter are mere causal tracks without a spatial dimension. There is, however, something else on which to base a try: *for every two point events in a history, the history contains a later point event that has them both in its past*. For example, let two points in a Minkowski space-time be ever so far apart spatially (in some frame of reference). Eventually they will be in the past of some point sufficiently far in their respective futures. Contrariwise, suppose that for the measurement of an electron there are the incompatible possibilities of measured spin up and measured spin down. Then two later events each realizing one of these two possibilities cannot themselves be in the past of any single point event. Since this structural feature has a name, I will use it: a history must be a 'directed' set, defined as follows.

DEFINITION 4: A subset E of **Our World** is *directed* just in case for all e_1 and e_2 in E there is a point event e_3 in E that is their common upper bound: $e_3 \in E$ and $e_1 \leqslant e_3$ and $e_2 \leqslant e_3$. (See the picture of 'causal confluence' in Figure 1.)

There is a precedent for thinking of histories as directed sets in Whitehead (1929), though it may be hard to see it through his special vocabulary. For example, "the multiple nexus [between many actual entities] is how those actual entities are really together in all *subsequent* unifications of the universe . . ." (ibid., p. 351; my emphasis), or, "all real togetherness is togetherness in the formal constitution of an actuality" (ibid., p. 48). Here I am identifying point events in **Our World** with Whitehead's actual entities. In the course of a history, "the many become one, and are increased by one" (ibid., p. 32).

Not every directed set should be counted as a history; we expect a history to be maximal.

DEFINITION 5: A subset h of **Our World** is a *history* just in case h is a maximal directed subset of **Our World**: h itself is a directed subset of **Our World**, and no proper superset of h has this feature.

Histories are a key conceptual tool. I do not intend them to bear that name merely for mnemonic reasons: the proposal is that real histories are histories in this sense. They are analogous to the histories in branching time. Each history might be a Minkowski space-time; but (in the theory of branching space-time) **Our World** is no such thing, because a single Minkowski space-time, unlike **Our World**, fails to contain any incompatible possible point events. Here are some elementary facts about histories.

FACT 6: Every finite set of points contained in a history, h, has an upper bound in h.

> Infinite subsets of a history, for example a history itself, need have no common upper bound.
> Every directed set can be extended to a history.
> Zorn's lemma suffices to prove this.
> Every point event in **Our World** belongs to some history.
> Histories are closed downward: if $e_1 \leqslant e_2$ and $e_2 \in h$, then $e_1 \in h$.
> The complements of histories are closed upward: if $e_1 \leqslant e_2$ and $e_1 \notin h$, then $e_2 \notin h$.
> No history is a subset of a distinct history.
> Also, no history, h, is a subset of the union of a finite family, H, of histories of which it is not a member: if H is a finite set of histories, then that $h \subseteq \cup H$ implies that $h \in H$.

Argument: For each member of H find a point that is not in the member but is in h. Find a common upper bound in h for these points. Such a point will be in h but not in the union of H. (The argument rightly fails when H is infinite.)

Let $h_1 \oplus h_2$ be the disjoint union of h_1 and h_2: $(h_1 - h_2) \cup (h_2 - h_1)$. If it is nonempty (i.e., if the two histories are distinct), then *each* part $h_1 - h_2$ and $h_2 - h_1$ of the disjoint union is nonempty.

Otherwise one would be a proper subset of the other.

Evidently two point events (I will often say just '*point*') share some history just in case they have a common upper bound in **Our World**. Contrariwise, two points fail to have any history in common just in case they have no common upper bound. It is good to mark such a fundamental matter with a definition.

DEFINITION 7: Point events e_1 and e_2 are *compatible* if there is some history to which both belong, and otherwise are *incompatible*.

This definition relates to causal tense logic in the following way: e_1 and e_2 are compatible if and only if there (tenselessly) is a standpoint, e, at which one could truly say 'both e_1 occurred and e_2 occurred'.[15] 'Causal tense logic' here means: no 'frame of reference'. The causal past tense, for instance, never refers to causal contemporaries of the point event of utterance, as it might if a frame of reference were provided.

The definitions of 'history' and 'compatibility' involve at least three substantive commitments. I wish to make these clear without defending them piecemeal.

(1) If there is objective indeterminism toward the past (backward branching), then the fact that two point events have a common future is, contrary to the definition, no guarantee of their compatibility. As I said above, this study assumes that there is no backward branching – nor have I yet come across any clearly stated reason to assume other than epistemological backward indetermination. Some 'many worlds' theorists seem seriously to entertain the possibility of historical divergence followed by reconvergence. The present theory does not tolerate such entertainment, which I take as a mark in its favor. (See Belnap (forthcoming) for a little more discussion.)

(2) Perhaps some maximal directed sets 'can't happen', contrary to the idea of the definition of compatibility. One could certainly have a consistent theory on which this is so. I doubt the theory would be true, but it is so difficult to be sure that it seems best to make my theoretical commitments absolutely clear.

(3) If there are 'event horizons' such as are postulated near black holes, then it would appear that there can be compatible point events without a common causal future as required by the definitions. I ask that such difficult physical questions be tabled in the belief that the present theory can nevertheless serve as a useful approach.

It may have passed your mind that each Minkowski space-time looks the same upside down: each is not only directed, but also 'directed downward' in the following sense.

DEFINITION 8: A subset E of **Our World** is *directed downward* just in case for all e_1 and e_2 in E there is a point event e in E that is their common lower bound: $e \in E$ and $e \leqslant e_1$ and $e \leqslant e_2$.

That each Minkowski space-time is an upside-down image of itself is of course true, but this should not lead you to think that it makes no difference which way we define a 'history'. Consider, for instance, this. While a Minkowski space-time is indeed downward directed, it would be truly peculiar if it were *maximal* downward directed. For if it were maximal downward directed, it would be upward closed. And if it were upward closed, then if there were any incompatible possible point events in the future of any one of its members, it would have to contain both of them, which would, as advertised, be peculiar.

In this way, the concepts of branching space-time give a natural, unforced articulation of the 'direction of time' without complicated physics (e.g., the theory of entropy). They do so by looking beyond the properties of a single history so as to take account of how distinct histories fit together, something that becomes really clear only later in the context of further postulates. Here, however, is a definition and a fact that shift our attention from single to multiple histories.

DEFINITION 9: $H_{(e)}$ is the set of histories to which e belongs.

So:

FACT 10: $H_{(e)}$ is never empty. Also, if $e_1 \leqslant e_2$, then $H_{(e_2)} \subseteq H_{(e_1)}$.

The 'also' is just a baroque (but useful) repetition of the fact that histories are closed downward. It is another articulation of the 'direction of time'.

One should not necessarily expect the converse; for example, perhaps two compatible point events can belong to exactly the same histories.

With the concept of 'history' in hand (but not without it!), we can understand the future tense of English. Let us adapt the Prior/Thomason account to branching space-time. The key point is that the semantic value of tensed expressions depends not only on the point event of evaluation, but also on a specified history to which the point event belongs. For example, to evaluate 'the electron will be measured spin up', we need to be supplied both with an utterance event, e, and with a history, h, to which e belongs. In causal tense logic the statement is true if at some point event e_u that is both future to e and belongs to h, the electron is (tenselessly) measured spin up. Thus it makes sense to say that the electron might be measured spin up and might be measured spin down, but it is inconsistent to say that the electron will be measured spin up and will be measured spin down. This sounds obvious, but tends to be neglected in discussions of branching and of the 'many worlds' interpretation of quantum mechanics. These discussions could be improved by explicit use of Thomason's perfectly clear account of the future tense.[16] Still, everyone knows that this topic is addling; it is good that apart from some *obiter dicta*, I shall not have to fool around with tenses.

Another thing we can better understand is this: if branching space-time is right, then the phrase 'our history' or 'the actual history' is (if there are incompatible possibilities) senseless.[17] Scientists, for instance, no matter how hardheaded and downright empirical they wish to be, cannot confine their attention to 'our history' or to 'the actual history'. It is not just that they *ought* not. It is, rather, that (if branching space-time is true) they can no more do so than mathematicians can confine their attention to 'the odd prime number' and for exactly the same reason: there is more than one odd prime number, and there is more than one history to which we belong. On the other hand, just as a mathematician can deal with 'the odd prime numbers' (plural), so a scientist could manage to deal only with 'our histories' (plural), that is, with the set of all histories to which this indexically indicated context of utterance belongs. In fact such a policy is appropriate for astronomers; but physicists, in contrast, generally do not confine themselves in this way. Physics deals with what could have been as well as with what might be; it deals with all of **Our World**.[18] So physics is less tied to indexical language than is, say, astronomy.[19]

I can now define space-like separation.

DEFINITION 11: If e_1 and e_2 are (i) incomparable by $\leqslant$ but (ii) compatible, then they are *space-like separated*. We may also call them *causal contemporaries* (provided we bear in mind the failure of transitivity).

Observe that condition (ii) is essential. That is why it was not possible to become clear on space-like separation without the definitions of this section.

FACT 12: Incompatible points have neither a causal nor a space-like relation: they are with respect to each other neither causally future nor causally past nor causally contemporaneous.

This fact is a trivial, though I think helpful, consequence of definitions. It does not preclude a spatiotemporal notion of incompatible point events mediated by a concept of 'spatiotemporal position'. Even if such a concept becomes available, however, one cannot infer a spatiotemporal relation between the spatiotemporal positions of two point events from the mere fact that they are incompatible. In this sense, incompatibility, though defined from the causal order, is not itself a spatiotemporal relation.

4. HISTORICAL CONNECTION

This section adds a simple postulate and goes a little deeper into what we can do with the concepts of causal order, history, and compatibility.

In the theory of branching time, where histories are chains, one may postulate that every two histories overlap, following Thomason in labeling this property 'historical connection'. The same postulate holds in branching space-time, though 'history' now has a different meaning:

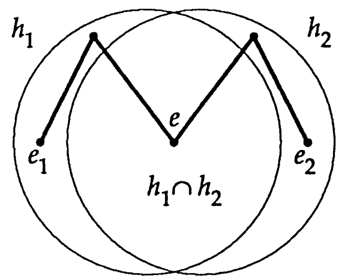

Fig. 2. The M property.

POSTULATE 13: **Historical Connection**. Every pair of histories has a nonempty intersection (*later we deduce this postulate from another*).

In the theory of branching time it would be equivalent to say that every two moments have a lower bound. Here, where the topic is point events instead of moments, the 'common lower bound' principle is not equivalent to historical connection, and is not postulated. For more detail, see Fact 17 below.

This postulate, unlike Postulate 1, does *not* imply the result of replacing $\leqslant$ by its converse, and is thus sensitive to the direction of time.

The following consequence of historical connection gives a good account of Lewis's notion of "suitable external relation": the trip from one point to another in **Our World** may be *long*, but it need not have a complicated shape.

FACT 14: **The M property.** Every pair of point events in **Our World** can at worst be connected by a $\leqslant/\geqslant$-path in the shape of an M.

See Figure 3. In causal tense logic we might say (here and now at e_1): for each point event, e_2, it might be true that it was true that it might be true that it was true that e_2 exists. (These tenses are just following the arms of the M. Also the formula neglects the possibility of a simpler path.)

It is true, but does not yet follow, that every finite set of histories has a nonempty intersection (Generalized Historical Connection).

FACT 15: Figure 3 is a little finite (six-point) example showing the

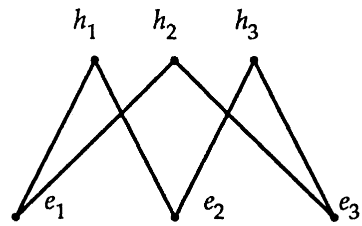

Fig. 3. Generalized Historical Connection not implied by Historical Connection.

independence of generalized historical connection from what has so far been postulated.

There are three-point events in each history. You see that each pair of histories overlaps (historical connection), but that no point event belongs to all three. A later postulate will rule this out as a possible model.

My general procedure has been and will be to make as few assumptions as possible about the spatiotemporal structure of individual histories. Instead I organize the distinctive concepts that *combine* indeterminism and relativity in such a way as to be as insensitive as possible to the texture of each individual history. For many purposes one can admit even finite models and the possibility in **Our World** of jumps and gaps. It is, however, even more important to make sense of 'Minkowski models' or 'special relativity models' of branching space-time:

DEFINITION 16: A *Minkowski branching space-time* is a model of **Our World** in which each history is a Minkowski space-time (in the standard sense found in the literature).

Here is a partial picture of a Minkowski branching space-time that has two histories, h_1 and h_2. The picture consists of two pictures: one of h_1 and one of h_2, with a stipulated point, e, of overlap, and a stipulated area of divergence (some fixed pair of 'triangles' of points properly greater than e). Since histories are closed downward, the shaded parts must be two pictures of the same points. Since the complements of histories are closed upward, the entire upper light cones (including their respective borders) have no overlap. (This and subsequent diagrams indicate where the borders go, which is sometimes important, as

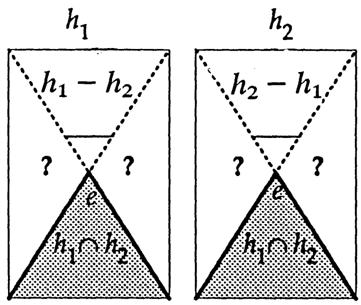

Fig. 4. A Minkowski branching space-time with two histories.

follows: solid borders must go with the area below, and dotted borders must go with the area above.)

The status of the 'wings' – the areas indicated by the question marks – appears not to be settled by stipulations to date. Some of the literature treated by Stein (1991) asks whether events in the wings are 'ontologically definite or indefinite', either absolutely or relatively. This terminology is suggestive but is used without the control of a rigorous theory. The present methodology permits the posing of a sharper question: Do point events in the wings belong to the intersection $h_1 \cap h_2$ or (in the picture of h_1) to the difference $h_1 - h_2$? One might suppose oneself to be entitled to ad-lib stipulations about how the wings separate by drawing a typical 'simultaneity slice' through e, putting points below the slice into the intersection and points above into the difference. That sounds as if it would be in the spirit of relativity. It turns out that this is profoundly wrong, but just how we cannot yet see. We shall have to wait for Section 8 for a definite solution to the easily mystifying 'problem of the wings'.

On the other hand, we can already see the truth of the following, which, although not later used in this paper, may be of some interest.

FACT 17: Suppose (*) that each individual history is downward directed. Then so is **Our World** as a whole.

The antecedent, (*), is true, for example, of a Minkowski branching space-time, so that in such a model even incompatible point events will be lower bounded.

Argument: Given e_1 and e_2 in **Our World,** the M property promises an e that shares a history with each. By (*), e_1 and e share a lower bound, e_3, which must, since histories are closed downward, also share a history with e_2. So e_3 and e_2 must by (*) share a lower bound, which will be a lower bound for e_1 and e_2, even if they are incompatible.

5. BRANCHING AND POSSIBILITIES AT POINT EVENTS

At a spin measurement exactly two outcomes are possible: measured spin up and measured spin down. The describable outcome that the electron should change its rest mass is not possible. What does this mean? It is standard to relegate the possible to the realm of mind or theory or laws or language or conversational practice.[20] In this section I look a little more closely at exactly how branching happens, and I offer a *thoroughly objective and fully rigorous account of possible outcomes*. I am first going to develop the ideas of 'branching' and 'possible outcomes' as they apply to point events. Later these ideas will need generalizing in a way requiring attention to certain *sets*.

The immediate order of development comes about like this. We are ultimately aiming at a postulational version of 'everything has a causal origin'. To state it, we need a concept of what is possible at a point event. For this concept of possibility, we need to be absolutely clear about branching at a point event. But it turns out to be technically easier to start with a definition of nonbranching, for which I introduce the term 'obviously undivided'.

DEFINITION 18: Two histories h_1 and h_2 in $H_{(e)}$ are *obviously undivided at e*, written $h_1 \approx_e h_2$, if they share some point that is properly later than e, if there are any.

> The final 'if' means that when e is a 'last point' of **Our World**, h_1 and h_2 are automatically defined as obviously undivided at e.

Otherwise, provided $e \in (h_1 \cap h_2)$, h_1 and h_2 *apparently split* or *divide at e*, written $h_i \not\approx_e h_2$.

> Thus, in this case, although there are points beyond e, none of them is shared by h_1 and h_2.

Note that both 'apparently divided at e' and 'obviously undivided at e' (both $\approx_e$ and $\not\approx_e$) presuppose that e is a member of each history involved.

The reason for the adverb 'apparently' is to match 'obviously', and the reason for 'obviously' is that we wish to save plain 'undivided' for the important relation that arises by taking the reflexive and transitive closure of 'obviously undivided'. Now it will eventually turn out that the latter is *already* reflexive (see Fact 20) and – except in what are very likely pathological cases – transitive (see Fact 46). The adverbs therefore do no permanently useful work. Keeping them temporarily, however, will simplify analysis.

I use the notion of 'obviously undivided' to help define an entirely objective concept of 'elementary possibility at e'. I will spend the next few paragraphs trying to make clear how the ideas fit together. (Here is perhaps the heart of the present essay.)

- An elementary possibility can be represented as a set of histories. This idea is copied from 'possible worlds' theories.

- To make sense of a possibility being *at* a particular point event e of **Our World**, however, more is needed. One might try to obtain that 'more' by considering sentences that mention e, but to do so is to lose hope of objectivity. An obviously objective (and obviously incomplete) constraint is that e should belong to each history in the set. In other words, any set representing an elementary possibility at e should be a subset of $H_{(e)}$.

- The entire set of elementary possibilities at e can be represented as a 'partition' of $H_{(e)}$; that is, as a pairwise disjoint and collectively exhaustive family of subsets of $H_{(e)}$. I will use 'π_e' for this partition once it is specified. This is just the familiar idea that given that e occurs, exactly one elementary possibility at e is bound to emerge.

  What English phrase shall we use for π_e? Here are some candidates, all of which seem to me clumsy: 'the (set of or pattern of) elementary possibilities at e (or open at e)'; 'the elementary e-possibilities'; 'the choice-partition for e'. More idiomatically, one might think of π_e as representing what might happen at e, or the way things might go immediately after e, or as the possible issues, outcomes, or results of e.

- Alternatively, we can represent the same information by an equivalence relation on $H_{(e)}$, where histories in $H_{(e)}$ are 'equivalent' at e if no elementary possibility open at e can distinguish the two histories. After it is defined, '$\equiv_e$' will serve for this equivalence relation.

- There remains the question of *which* partitions of $H_{(e)}$ to count as giving sense to 'elementary possibilities at e'. Here is a powerful constraint: the principle of *No Choice Between Obviously Undivided Histories*, suggested by P. Kremer in the context of the theory of agency. This principle says that no elementary possibility that is open at e can distinguish between histories that are obviously undivided at e. Suppose histories h_1 and h_2 do not appear to divide until properly after e, i.e., suppose that $h_1 \approx_e h_2$. Then nothing that can be realized (that can happen, be decided, be chosen, be settled, etc.) at e can distinguish between h_1 and h_2. It is too soon. In other words, let a point, e, be properly earlier than some point in the intersection of h_1 and h_2. Then e occurs too early for it to have a bearing on the split between h_1 and h_2. "No choice before its time". If the spin measurement will not occur until a few moments hence, then the possibilities 'measured spin up' and 'measured spin down' are not distinct possible outcomes for *now*. We shall have to wait for the measurement, which is a properly later point event that belongs to both histories and thus prevents them from being distinct possibilities *now*.[21]

- The principle of no choice between obviously undivided histories does not complete the analysis because there might be a variety of ways of partitioning $H_{(e)}$ each of which satisfies the condition. Perhaps there is even a unique such partition that is determined by a doctrine of universals or in another way. A fundamental hypothesis of the present theory is that nothing like this holds: the possible outcomes of a point event are entirely determined by (i.e., definable by) the causal ordering. The hypothesis is that *there is no other constraint on an elementary possibility than the constraint of no choice between obviously undivided histories*. Thus, a set of histories is an elementary possibility at e if it is a member of the *finest partition* of $H_{(e)}$ that does not separate any two histories that are obviously undivided at e. To repeat: The hypothesis is that there is nothing else except the no choice between obviously undivided histories condition that can limit the subtlety of the elementary possibilities open at e. The range of elementary possibilities open at e is therefore not an extra. It is an ingredient in (i.e., is definable from) the very structure of **Our World** given by $\leqslant$.

The following definitions encapsulate these considerations – but you will appreciate that I intend them as substantive.

DEFINITION 19: A partition of $H_{(e)}$ *respects the No Choice Between Obviously Undivided Histories Condition* if no two histories obviously undivided at e fall into different members of the partition: for $h_1, h_2 \in H_{(e)}$, if $h_1 \approx_e h_2$, then for each member H of the partition, $h_1 \in H$ iff $h_2 \in H$.

Let π_e be the finest partition of $H_{(e)}$ that respects the no choice between obviously undivided histories condition.

Let $\equiv_e$ be the reflexive and transitive closure on $H_{(e)}$ of $\approx_e$.

> Since π_e and $\equiv_e$ are mathematically equivalent, I will use them interchangeably.

By an *elementary possibility at e* I mean a member of π_e.

> Thus an elementary possibility at e is always a set of histories, all of which contain e.
> It may be typically or even always true in **Our World** that the unit set $\{h\}$ of a history is not an elementary possibility at any e. Thus, the competing definition of an elementary possibility as the unit set of a history would be too wide (though of course not too wide for every purpose).
>
> There are possibilities that are not elementary. At least any union of a set of elementary possibilities at e will need to be counted as itself a *possibility at e*; but this is beyond the scope of this paper. So are concepts of less immediate possibilities, important as they are.

I take the uniquely determined partition π_e as a proper locus for a ground-level theory of objective transition possibilities (or outcomes) in the single case. The significance is this: *the finest partition* is delivered by the causal structure of **Our World**, not by human interests, language, concepts, universals, other possible worlds, or evolutionary entrenchment. The possibility in question is conditional in form (the condition being that the point event occurs), but more than that, it has a concrete foothold in **Our World**.

How much does this have to do with *probabilities*? I suspect a great deal. It is not that the numbers themselves necessarily arise from the causal order (McCall suggests how they might). The point is that any serious theory of the nature of probabilities must start with an underlying *probability space* on which to fix the numbers. If this space comes from human interests, language, concepts, universals, other possible worlds, or evolutionary entrenchment, your finished theory will not be objective. So for objective transition probabilities in the single case, π_e recommends itself as a suitable underlying space.[22]

This scheme hides a threat that should be met before proceeding. Here are different ways of expressing the matter.

- From the surface form of the definitions, it might be that one of the elementary possibilities at e 'cannot happen' (is not really possible) because no way that **Our World** goes on realizes it. That is, the following could happen: **Our World** does not stop with e, but some history in $H_{(e)}$ stops with e (i.e., contains no point properly later than e).
- It would be bizarre if two histories h_1 and h_2 in $H_{(e)}$ appeared to 'split' at e in the defined sense although one of them contained no point beyond e; but this *seems* allowed by the definition of 'appeared to split'. If that could happen, it would be best not to speak of even apparent splitting.

Verdict: We are in a conceptual muddle unless every history in $H_{(e)}$ contains a proper upper bound for e (unless e is maximal in **Our World**). There is, however, no muddle; and as a corollary we have that $\approx_e$ is reflexive.

FACT 20: Provided e is not a maximal point in **Our World**, every history in $H_{(e)}$ contains a point properly later than e. Therefore obvious undividedness is reflexive.

Argument: Suppose $h \in H_{(e)}$, and that $e < e_1$. $\{e_2: e_2 \leqslant e\} \cup \{e_1\}$ is a directed proper superset of $\{e_2: e_2 \leqslant e\}$, so that the latter subset of h is not a history, so not identical with h, so a proper subset of h. Let $e_3 \in h - \{e_2: e_2 \leqslant e\}$. The upper bound in h that the directedness of h guarantees for e and e_3 must be properly later than e.

This result also guarantees that we may think of π_e as either a partition of $H_{(e)}$, or as a partition of $\{e_1 : e < e_1\}$, just as seems advantageous. Also we may extend our use of '$\equiv_e$' in the same way, including a convenient mixed use between point events and histories, as follows.

DEFINITION 21: For e_1 and e_2 both properly later than e, define that $e_1 \equiv_e e_2$ iff there are histories h_1 and h_2 such that $e_1 \in h_1$ and $e_2 \in h_2$ and $h_1 \equiv_e h_2$. In addition, define $e_1 \equiv_e h_2$ and $h_1 \equiv_e e_2$ in the same way. For all cases I use the *un*modified phrases *undivided at e* and *divided* (or *split* or *separated*) *at e* for $\equiv_e$ and $\not\equiv_e$, respectively.

Thus I use '$\equiv_e$' or 'undivided at' in multiple senses, between any pair each member of which is either a history containing e or a point event properly later than e. Since, as we have said, the ideas are equivalent, there should be no difficulty.

FACT 22: $\equiv_e$ is an equivalence relation on the point events properly later than e and in the mixed point event/history cases is symmetric and transitive.

We are finally in a position to be both relativistic and rigorous about indeterminism.

DEFINITION 23: A point event, e, is *indeterministic* if π_e has more than one member. Otherwise, it is *deterministic*.

As a rhetorical variant, we may say that **Our World** is indeterministic at e. Note that on this account it makes perfectly good sense to *locate* indeterminism not metaphorically in a theory, but literally in our world. It makes sense to say that **Our World** was indeterministic in Boston yesterday, but might not be so in Austin tomorrow.

There is one more logically trivial but psychologically critical definition before I state another postulate.

DEFINITION 24: A point event is a *choice point* if it is indeterministic.

For h_1, h_2 in $H_{(e)}$, if $h_1 \not\equiv_e h_2$, say that e is a *choice point for* h_1 *and* h_2. The same terminology extends to the cases when one or both arguments are point events instead of histories.

If a point event is not a choice point, it is *vacuous*.

The reason for introducing 'choice point' as a synonym is that although in the case of point events there is no difference between 'choice point' and 'indeterministic', the ideas will fall apart in a more general setting. The reason for the particular terminology is to anticipate a later postulate according to which choice points play a special role in **Our World** by being the places (literally) where choices (metaphorically) are made.

FACT 25: A choice point, e, for h_1 and h_2 is maximal in $h_1 \cap h_2$; that is, $e \in h_1 \cap h_2$, and no point event properly later than e has this feature.

The choice point, e, must be contained in their intersection since $h_1, h_2 \in H_{(e)}$, and it must be maximal therein because π_e never separates histories sharing a point properly later than e.

6. CHOICE PRINCIPLE

In the end I will suggest a postulate called 'the Prior Choice Principle' (Postulate 37). Stating this postulate in full generality will require concepts involving certain *sets* of point events, but its significance will be clearer if I first give two successively stronger versions involving only point events. The first version is called 'the Choice Principle'.

The choice principle is reminiscent of the ontological principle of Whitehead, who put the matter in various ways. Here is a sample from Whitehead (1929).[23]

> [A]ctual entities are the only *reasons*; so that to search for a *reason* is to search for one or more actual entities. (Ibid., p. 37)

> '[D]ecision' is the additional meaning imported by the word 'actual' into the phrase 'actual entity' . . . . The word 'decision' does not here imply conscious judgment . . . . The word is used in its root sense of a 'cutting off'. (Ibid., p. 68)

> [E]very decision expresses the relation of the actual thing, *for which* a decision is made, to an actual thing *by which* that decision is made. (Ibid., p. 68)

I am going to identify 'decision of some actual entity' with 'π_e for some point event, e'. This will make it easy for you to jettison the motivation if you wish; the substantive content will remain.

I also need to identify what sort of thing requires a reason in the present context, namely that at a certain point e_1, a point that belongs to perhaps many histories, we find ourselves in one history *rather than* another history that at the point e_1 is a might-have-been.[24] By the

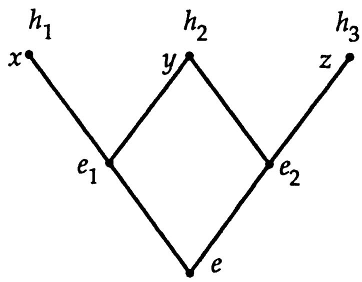

Fig. 5. Violation of choice principle.

considerations suggested above, a reason for the fact that we at e_1 are where we are *instead of* in some alternate history must be found in the definite choice made among the elementary possibilities π_e for some point event e.

POSTULATE 26: **Choice Principle**. For each two histories, there is at least one choice point (*this postulate is later strengthened*).

Figure 5 is a simple partial order that satisfies generalized historical connection (every finite set of histories has a nonempty intersection), but not the choice principle. We thus obtain a feel for the content of the latter. The picture is to be interpreted as a finite model (six-point events); the lines indicate the order, $\leqslant$. The three histories have to be $h_1 = \{x, e_1, e\}$, $h_2 = \{y, e_1, e_2, e\}$, and $h_3 = \{z, e_2, e\}$. The trouble is with h_1 and h_3, for which there is no choice point. The only candidate for such a choice point is e, for that is the only point in the intersection of h_1 and h_3. Since $h_1 \approx_e h_2$ and $h_2 \approx_e h_3$, it must be that $h_1 \equiv_e h_3$ by transitive closure (Definition 19), so that e, the only candidate, is not a choice point for h_1 and h_3 (Definition 24).

Evidently:

FACT 27: The choice principle implies historical connection (Postulate 13).

7. PRIOR CHOICE PRINCIPLE: POINT EVENT VERSION

There is a strengthening of the choice principle that answers to a deeply held conviction about causation: causes are prior to their effects. Thus, if I win ten dollars at the craps table, I look to the earlier roll of the dice for a reason that this happened *instead of* some contrary. I do not look to causal contemporaries, nor to the future. I look only in the causal past. Here is a statement of that conviction that is totally free of associations with habits of the mind. It is, as I see it, the crucial postulate of the present story about how indeterminism unites with relativity.

POSTULATE 28: **Prior Choice Principle, point event version.** If e belongs to $h_1 - h_2$, then there is a choice point for h_1 and h_2 lying in the past of e (*this postulate is later strengthened*).

The choice principle, Postulate 26, says that the divergence between two histories always requires at least one choice point. The prior choice principle trivially implies it, but says more: for each member of $h_1 \oplus h_2$, some choice point for h_1 and h_2 lies in its *past*.

The later strengthening of this principle will assert that chains of point events as well as individual point events require reasons.

Figure 3 satisfies the choice principle but not the stronger prior choice principle: e_1, for example, belongs to $h_1 - h_3$, but there is no (properly) prior choice point for h_1 and h_3.

Here are some elementary consequences of the prior choice principle.

FACT 29: Every pair of histories has a nonempty intersection (historical connection, Postulate 13).

Every finite set of histories has a nonempty intersection (generalized historical connection).

Argument: The inductive argument is easy. Suppose we have a set of histories, H, and that an inductive hypothesis promises that $e \in \cap H$. Choose a history, h, to which e does not belong (just to make it hard). Then by the prior choice principle, there is a point e_1 in the past of e that belongs to h, and *also* belongs to every member of H because histories are closed downward. Thus $e_1 \in \cap (H \cup \{h\})$.

Minimal points of **Our World** (if any) must belong to every history.

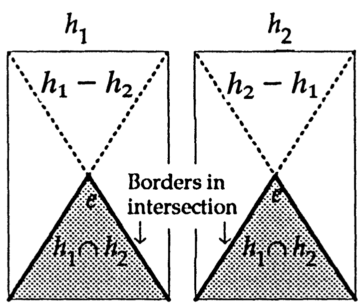

Fig. 6. 'Wings' in differences.

8. The problem of the 'wings'

A significant value of the theory as so far developed is that it settles in a principled way 'the problem of the wings' raised for Figure 4. By so doing it helps us to know our way around a relativistically indeterministic universe. You will recall that the problem was this. Suppose there is a measurement of spin with two possible outcomes (idealized as histories), measured spin up or measured spin down. How does this affect causal contemporaries of the measurement? Do they belong to the intersection of the two histories, or just to one or the other? Ontologically indefinite or ontologically definite (if that language helps), relatively or absolutely?

I show first that the choice principle alone does not settle the matter decisively. Then I show that the prior choice principle settles it definitely and (I should say) without ad hocery.

To see that the choice principle fails to settle the matter, assume that Figures 6 and 7 refer to a model of **Our World** satisfying the following *stipulations* (as I call them for later reference).

- There are exactly two histories.
- Each history is a Minkowski space-time.
- There is exactly one choice point.

FACT 30: Figures 6 and 7 are each consistent with both the stipulations and the choice principle.

Evidently e in Figure 6 is the only maximal point in the intersection and, therefore, the only choice point. Observe that we must put the

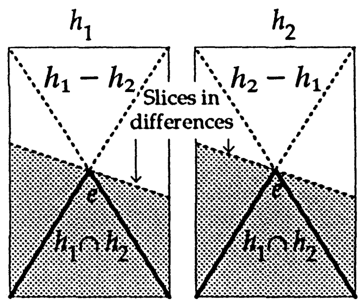

Fig. 7. 'Wings' divided by slice.

lower borders in the intersection because e, as a choice point, is stipulated to be in the intersection, and histories are closed downward.

In Figure 7, of those points on the 'simultaneity slice', only e is to be taken to be in the intersection. Thus e alone is a choice point.

Proof of Fact 30: The proof is by geometrical intuition: Figures 6 and 7 above clearly portray models that (i) satisfy the three stipulations, and (ii) satisfy the choice principle. Therefore the choice principle alone does not yield a definite answer to the problem of the wings.

The following gives the rest of the story.

FACT 31: In the presence of the prior choice principle, however, it must be that the 'wings' are in the *intersection* $h_1 \cap h_2$ of the two histories.

Argument: By the hypothesis that the model satisfies the stipulations, the points in the 'wings' have no choice point in their respective pasts: e is the only choice point, and it is not in the past of any point in the wings. Therefore, if any point in the wings failed to lie in the intersection $h_1 \cap h_2$, the prior choice principle would be violated.

Thus, given the prior choice principle, Figures 6 and 7 must be repudiated. The *true* picture of two Minkowski histories with exactly one choice point must be as in Figure 8. The intersection of the two histories

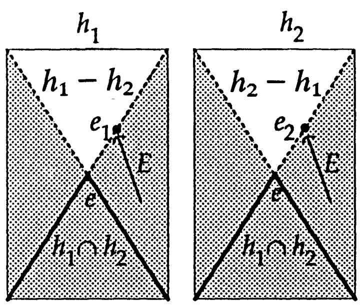

Fig. 8. The 'wings' must be in the intersection.

is shaded, and the upper borders belong 'on the light side' in the respective differences.

This formal but not *just* formal result deserves additional comment.

- Observe that the difference made by the choice at e pertains only to the future of possibilities of e. It does *not* pertain to the causal contemporaries of e.

- This 'not' is strong: whether the choice at e pertains to its causal contemporaries is not left undetermined – it definitely does not.

- One might imagine that whenever there is a tiny indeterministic situation such as spin up/spin down, the entire causally unrelated universe *simultaneously* splits in twain. Branching space-time gives a sharp explanation of how and why this picture is wrong. It also offers a competing rigorous and positive theory of what is right: *splitting in* **Our World** *occurs at point events, not at simultaneity slices, and affects only the causal future.*[25]

- Indeed, on the present theory it is *impossible* to draw a 'simultaneity slice' that exactly divides h_1 into $h_1 \cap h_2$ and $h_1 - h_2$. I do not know whether this should be taken to conflict with some form of special relativity. If it does, special relativity in that form should be abandoned. The true spirit of special relativity is maintained in the present context if each history is a Minkowski space-time. Part of what makes it possible to distance oneself from such issues is that in this study there is absolutely no reference to a concept of 'laws', much less the (linguistic?) 'form' they should take or what 'transformations' they should survive.

- Consider a chain, E, as marked in Figure 8, that approaches a 'spatiotemporal position' on the upper light cone from within $h_1 \cap h_2$. E will have *two* minimal upper bounds, say, e_1 on the upper light cone of h_1 and e_2 on the upper light cone of h_2. Thus e_1 and e_2 will in some sense be 'very close'.[26] No wonder it is hard to build this model of **Our World** with paper and cellophane tape.

- Of more substance, however, is the observation that this very situation permits us to begin to see just a little way into the following problem: What does it mean to say that two incompatible point events inhabit the same 'spatiotemporal position'?[27] The idea is that if each of two incompatible point events such as e_1 and e_2 is a minimal upper bound of the same directed set, E, then those two points should be taken to occupy 'the same spatiotemporal position'.[28] Observe that this scheme does not depend on a previously specified metric such as is available in a Minkowski space-time. On the other hand, although very many point events can by this means be identified across histories as 'occupying the same spatiotemporal position', one easily sees that vast regions are left untouched. I do not even know if a general doctrine of spatiotemporal position *should* be forthcoming. Does **Our World** contain, as Stein (1991) contemplates, histories that diverge into radically different topologies?

9. INDETERMINISM WITHOUT CHOICE

This section is in a way an insert, but its point is so important that I have chosen to state it as early as possible: *there can be indeterminism without choice*. For example, consider the paradigm Figure 8 above. Let E be the pictured chain approaching e_1 (and also e_2) from within the intersection, $h_1 \cap h_2$. If you are 'traveling along' this track, the situation as the track draws to a close is indeterministic: it is not determined whether you will wind up at e_1 or e_2. Still, there is no choice: the matter is entirely in the hands of your causal contemporary, e. The difference between the two cases seems to be this. The only reason that E underdetermines whether e_1 or e_2 is that it does not exhaust the entire past of either of these points: given the set of *all* proper predecessors of e_1, the outcome, e_1, is uniquely determined (and analogously for e_2). In contrast, the entire past culminating in e does not suffice to decide what happens next.

What is needed for a more general account of indeterminism? What I do is to extend the definitions of $\approx$, $\equiv$, and π beyond point events to chains.[29]

DEFINITION 32: Let E be a chain. $H_{[E]}$ is the set of histories extending E; that is, E is a subset of each member of $H_{[E]}$. For $h_1, h_2 \in H_{[E]}$, $h_1 \approx_E h_2$ iff either the two histories share a point properly later than each member of E or E is unbounded in **Our World**. π_E is the finest partition of $H_{[E]}$ respecting $\approx_E$. $\equiv_E$ is the companion equivalence relation on $H_{[E]}$, i.e., the reflexive and transitive closure of $\approx_E$. The language of 'divided/undivided', etc., is also extended to E.

A chain, E, is *indeterministic* if π_E is not vacuous (i.e., has more than one member), and is otherwise *vacuous*.

This language introduces rigor into our claim that E in Figure 8 is objectively indeterministic, since obviously $\pi_E = \pi_e = \{\{h_1\}, \{h_2\}\}$. We also need a rigorous account of why one should say what is intuitively obvious, that the 'choice' is at e and not E; but for this paper that need has to be left unmet.

10. SPLITTING ALONG A SIMULTANEITY SLICE?

Does branching space-time absolutely *forbid* that splitting between two histories occurs along a simultaneity slice? No, but branching space-time is so simple that it permits statement of at least one way in which such a situation appears weird. Figure 9 gives the diagram.

Let S name the simultaneity slice. Observe first that every point event in S must be in the intersection $h_1 \cap h_2$; or else prior choice would be violated. That means that every point in S is a choice point for h_1 and h_2, since each is maximal in their intersection. In other words, each point e_1 in S is a point of indeterminacy: π_{e_1} is nonvacuous. Also observe that the points in S are space-like related, without any being joined to any by the causal order. Here is what seems weird: the (metaphorical) choices made at each such e_1 are all perfectly correlated in spite of (i) each being objectively indeterministic, and (ii) the total absence of causal order between them. Could there really be such an uncanny synchronization of indeterministic events in the absence of

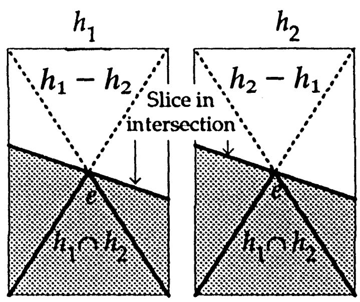

Fig. 9. Admissible split along a simultaneity slice.

causal order? Consider in particular that some of the correlation is between point events in S that are galaxies of galaxies apart.

In lieu of cranking up the rhetoric, let us go back and define 'perfect correlation', for what is distinctive here is that each concept is tightly defined on the basis of nothing but $\leqslant$. The idea of compatibility between sets of histories is first introduced as an auxiliary.

DEFINITION 33: Two sets of histories, e.g., two elementary possibilities, one from π_{e_1} and one from π_{e_2}, are *compatible* if they overlap, i.e., if some history belongs to both.

> Both elementary possibilities can be realized together, in a single history, if they are compatible; and otherwise not.
>
> This usage coheres with that of Definition 7, since point events e_1 and e_2 are compatible in the sense of that definition iff sets of histories $H_{(e_1)}$ and $H_{(e_2)}$ are compatible in the just-defined sense. (Furthermore, we occasionally speak of the compatibility of a point event e with a set of histories H, meaning of course the compatibility of $H_{(e)}$ with H.)

Point events e_1 and e_2 are *perfectly outcome-correlated* if each outcome in π_{e_1} is compatible with exactly one outcome in π_{e_2}, and vice versa.[30]

> In epistemic language, knowing which outcome of one point event is realized always suffices for deciding which outcome of the other is realized.

It is obvious that for every $e_1 \in S$, $\pi_{e_1} = \{\{h_1\}, \{h_2\}\}$, so that each is perfectly outcome-correlated with each, no matter their degree of separation. One may conjecture, however, that such a massive 'coincidence' never occurs in **Our World**:

CONJECTURE 34: Let E be a maximal set of pairwise space-like related choice points, all of which are included in some one history h (such as a simultaneity slice). Then it is false that all pairs of members of E are perfectly outcome-correlated.

The conjecture is evidently substantive, but I do not know of a relevant discussion. There is a related conjecture at the end of the next section.

11. DISTANT CORRELATIONS: EPR

In this section I apply the ideas of branching space-time to clarifying one of the famous puzzles of contemporary philosophy of science: what to make of the 'Einstein–Podolsky–Rosen paradox' in interpreting quantum mechanics.[31] The novel contribution here will be this: to state in absolutely rigorous terms a conjecture as to the exact nature of the puzzling phenomenon. An additional novelty will be to maintain rigor without using bewildering notation.

I propose that the essence of the EPR phenomenon is this: (i) space-like separated point events (ii) each of which is a genuine choice point but (iii) whose patterns of outcomes are perfectly correlated. I propose that the most deeply puzzling philosophical questions about the EPR phenomenon already arise for this stylized version, without any physics, probabilities, etc. For instance, it has often been observed, usually in the middle of intimidating notation, that the conjunction of (i)–(iii) surprises us because we have been taught to think that if there is no causal communication between genuinely random events,[32] then the patterns of possible outcomes should be radically independent.

Observe that (i)–(iii) are each sharply defined – and in terms of the causal order alone. Assuming branching space-time, if an EPR phenomenon actually occurs in our world, we can say what it is directly, without informal talk of theories or systems or states or the like. This capability might be useful even for principled anti-realists.

Branching space-time can clarify the nature of EPR phenomena, but it cannot settle whether they occur. Here is the positive conjecture.[33]

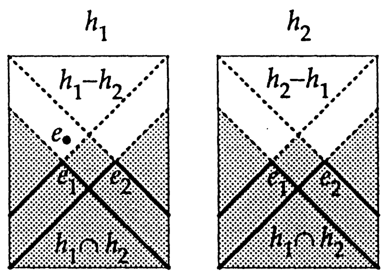

Fig. 10. Einstein–Podolsky–Rosen in miniature.

CONJECTURE 35: **Distant Correlations.** There exist at least two (i) space-like related (ii) choice points that are (iii) perfectly outcome-correlated.

The denial of this objective and rigorous statement is, I think, a (perhaps small) part of the content of the famous Reichenbach 'principle of the common cause', according to which each pair of correlated but distant outcomes must have a common cause.[34]

On the other hand, here is a simple model, given in terms of three 'stipulations', in which Conjecture 35 is true.

- There are exactly two histories, h_1 and h_2.
- Each history is a Minkowski space-time.
- There are exactly two choice points, e_1 and e_2.

The situation characterized by the three stipulations above is so simple that you can readily see that it exhibits the three crucial features of Einstein–Podolsky–Rosen phenomena. Here, for help in checking this, is a sketch.

(i) The points e_1 and e_2 are space-like separated. That is, although they share a history, there is no causal track from one to the other.

It needs verifying that the space-like separation of e_1 and e_2 is forced by no choice between obviously undivided histories. Of course it is so forced: the earlier of two points each belonging to two histories cannot be a choice point for those histories, since, by Fact 25, choice points are *maximal* in the intersection.

One may say that e_1 and e_2 are not causally connected. Note that no modal locution of connect*ability* is involved. It is just that each has "an existence independent of one another" since they are "situated in different parts of space" (Einstein, 1971, p. 170).

(ii) Each of e_1 and e_2 is a choice point. That is, at e_1, for instance, there is a real possibility of h_1 without h_2, and a real possibility of h_2 without h_1. One may suppose that in a particular case this amounts to the following joint outcomes: h_1 = up/down = the combination 'measured spin up after e_1' and 'measured spin down after e_2', and h_2 = down/up = the reverse combination. Before and at e_1 it is undecided – unsettled in Thomason's terminology – whether the historical continuation will be h_1 = up/down or h_2 = down/up, and the same is true of e_2. The two points are each indeterministic – which just means that at each point there is a pair of histories that split at that point.

These are cautious modes of speech; without probabilities we cannot say that the choice between h_1 and h_2 at e_1 is 'random'. Even so, the underlying idea is a *precisely defined* articulation of what others who have discussed EPR (and Bell) have aimed at in saying that at each of the two point events of measurement, it is random whether measured spin up or measured spin down results.[35]

(iii) Although each of e_1 and e_2 is a genuine choice point, there is perfect correlation between their outcomes.

It is obvious that in the little EPR model of Figure 10, $\pi_{e_1} = \{\{h_1\}, \{h_2\}\} = \pi_{e_2}$, and that therefore e_1 and e_2 are perfectly outcome-correlated. It is at bottom a matter of there being only two histories.

Of course all this shows mathematically is that the EPR phenomenon is consistent with what we have so far postulated, and is so in a simple way. But I think the branching space-time picture also helps us to be able to talk clearly about the phenomenon (if it exists) without tripping ourselves up quite so often.

(1) It clearly shows forth that at any point event e situated shortly after e_1 (as indicated in Figure 10), it is settled what happens immediately after e_2, even though e_2 is indeterministic and e lies

outside its causal future. In epistemic language, someone at e could know what happens immediately after e_2. They could know by putting together the information received from e_1 with the knowledge of the perfect outcome-correlation of e_1 and e_2. The very language of our discussion mentions neither particular types of 'signals' such as light nor even a postulate that there is a fastest signal – whatever that means. It thereby makes evident that it is irrelevant to consider signals that are faster than light or perhaps signals that are faster than the fastest signal. Either the point events e_1 and e_2 are space-like separated or not. If they are space-like separated, then it is plainly *inconsistent* to suppose that there is a causal order between them.

(2) As evidence that branching space-time helps us know our way around EPR, consider the following questions.

- Can a point event e_1 be (i) compatible with another point event e_2 but (ii) fail to be compatible with some outcome of e_2? Answer: Of course, if $e_2 < e_1$. In fact if e_1 lies in one outcome of e_2, it certainly will not be compatible with any other.
- Now ask the same question again, but suppose that e_1 and e_2 are space-like related. Can it happen? It sounds strange that some point event e_1 in Austin could be compatible with some causal contemporary e_2 in Boston, and yet fail to be compatible with some outcome (in Boston) of e_2. But that is exactly what happens in the case of an EPR phenomenon. Consider Figure 10. It is obvious that e in h_1 is compatible with e_2 (which is also in h_1) but not compatible with one of the outcomes of e_2, namely, $\{h_2\}$. So the answer is yes, provided an EPR phenomenon happens. You can *see* that this is so even in total ignorance of quantum mechanics.

(3) The picture shows that you will be permanently perplexed if you try to analyze EPR in terms of a simultaneity slice. Of course since e_1 and e_2 are space-like separated, you 'can' think of them as simultaneous. It is equally true that no good will come of it, for you also 'can' think of e_1 as simultaneous with some point event that occurs in the proper future of e_2. In this case you 'can' say that the possibilities at e_1 remain open at a time that is later than the time of e_2, which seems inconsistent with saying that

'after' e_2 it is settled what the outcome of e_1 'was' at some earlier time. In short, the picture shows that to talk sensibly about EPR, you should either refrain from tense/modal talk, or use causal tense constructions. This is so even though (or perhaps especially because) the EPR problem arose out of quantum mechanics, which is nonrelativistic.

(4) If you wish to help your understanding of the EPR phenomenon by means of counterfactuals, then you should rely only on their causal use. You will, I hope, find no room in Figure 10 for context-dependent 'similarity relations'. And permit me to add 'influence' to the list of words that, unless sharply defined, should not be relied upon for assisting us to understand the EPR phenomenon.

(5) There is the philosophical question of whether EPR-like outcome-correlations (if they exist) 'need' explanations (Fine 1989). There is much to say on both sides, and, I think, nothing in the branching space-time approach to settle the matter. Consider, for instance, the following pair of rhetorical questions: (i) How could there possibly exist a perfect correlation between the outcomes of an indeterministic point event in Austin and one in Boston?; (ii) What's the problem in awarding more than one maximum to the intersection of a couple of histories? These questions are rhetorically opposed, but although each is stated in the pure language of branching space-time, they sound equally persuasive.

(6) In addition to the above philosophical question, branching space-time permits consideration of the following strictly *scientific* (but not sharply posed) question: Does each such outcome-correlation *have* an explanation in the sense of some analog to a prior choice point? (Do not confuse this prospect with asking for a 'common cause'. To provide a common cause is to prove that the admittedly indeterministic space-like separated events are not really choice points.) Here is a relevant conjecture:

CONJECTURE 36: Let E be a set of pairwise space-like related choice points all of which belong to some one history. If every pair of members of E is perfectly outcome-correlated, then E has a lower bound in **Our World**.

> What makes the conjecture plausible is this: experiments creating EPR-like phenomena always seem to involve a careful preparation. In language that you should not trust because it is not sharp: although the outcomes of the correlated measurements do not seem to have a common cause in Reichenbach's sense, the fact of correlation itself seems caused.

(7) Branching space-time makes it easy to distinguish in structural terms the 'massive coincidences' that Conjecture 34 says never happen from the more modest distant correlations occurring in EPR phenomena (if they exist). On the one hand, the two choice points of Figure 10 evidently have a common lower bound at which to site the 'preparation' that gives rise to their modest correlation. On the other hand, a simultaneity slice in a Minkowski space-time paradigmatically has no lower bound at which to 'prepare' a massive correlation. Thus we might well take Conjecture 36 to speak for Conjecture 34 while permitting EPR phenomena to abound.

12. SUMMARY AND CHALLENGE

The aim was to contribute to the problem of uniting relativity and indeterminism in a fully rigorous theory. The grammar of the theory was based on just two primitives, **Our World** and $\leqslant$ (the causal order). The key postulate, namely 28, expressed in rigorously defined relativistic/indeterministic terms a version of a causal principle: if something contingent begins to be, then one can locate a definite choice point in its past. On the way several central concepts were defined in terms of 'causal order', each of which combined (I hope gracefully) the ideas of relativity and indeterminism: history, compatibility, space-like separation, undividedness/splitting of histories, elementary possibilities (transition possibilities) at a point event, (localized) indeterminism, and choice.

The entire apparatus provided a solid foundation for the notion that could be unreliably put by saying that indeterminism happens locally, and influences only the causal future. The fact that this view was expressed in rigorous terms made it possible to apply the theory with confidence to clarifying four problem areas for the combination of indeterminism and relativity, each of which is extremely difficult to talk about lucidly without the help of a constraining theory: (i) the status of the causal contemporaries of an indeterministic event; (ii) the existence of indeterministic events that are not themselves choice points; (iii) the question of whether histories might after all split along a simultaneity slice; and (iv) the problem of distant correlations brought to light by Einstein–Podolsky–Rosen. In the last case the theory was able to provide an absolutely clear candidate description of what counts as an EPR phenomenon. This account was so simple that no detailed knowledge of physics was needed to follow it.

Still, the theory is very abstract and very primitive and quite possibly very limited. My hope is that the approach has shown enough utility so that these features may be taken as a challenge. For example, the branching space-time treatment of EPR suggests the possibility of a more rigorous and objective approach to (i) the *Bell argument* (or its successors) and to *the principle of the common cause*, or to (ii) the *two slit experiment*, or even to (iii) the infamous *measurement problem*. These all seem to invoke both indeterminism and the causal order. The suggestion is not, however, that any of these can be approached with *only* the vocabulary of this paper; surely (i) involves probabilities and (ii) involves particles. Lastly, given the pioneering foundational account of (iv) *causation* in branching time due to von Kutschera (forthcoming), one should like to see similar ideas flourish in the context of branching space-time.

APPENDIX

The ideas of branching space-time can and should be extended beyond their simple application to single point events. The nearest beckoning target of generalization is given by *chains* of point events. I write here some pertinent definitions, suggested postulates, and elementary results. Definitions and postulates are given with minimal comment, and some results are given without proof, since the goal is only to forestall formation of the notion that the study could not possibly progress beyond its present stage.

A.1. *Prior Choice Principle, Extended to Chains*

Although Mother Nature can do just as she pleases, it seems plausible to postulate that if she has taken the trouble to provide a reason in the past of each point event for its being in one history rather than another, she would not withhold the same courtesy from a chain. In all empirical humility, I will therefore strengthen the prior choice principle as follows.

POSTULATE 37: **Prior Choice Principle.** Let E be a nonempty lower bounded chain of points in $h_1 - h_2$. Then there is a choice point for h_1 and h_2 lying in the past of E.

Evidently a downward maximal chain can have no reason, nor can the empty chain. (Perhaps, as in branching time, a downward maximal chain intersects every history.)

The theory of reasons for more complicated sorts of sets of point events goes beyond what I here present. I intend 'chains' here to be significant only for their lower ends; to be, so to speak, surrogate point events. Downward directed sets would have done as well.

FACT 38: Postulate 37 implies Postulate 28 and therefore also implies both Postulate 26 and Postulate 13.

Postulate 37 is *properly* stronger than Postulate 28.

Proof: For proof of the second part, stipulate **Our World** to consist of just two histories, h_1 and h_2, each of which is a two-dimensional Minkowski space-time. Distribute points between them as follows.

- There is a distinguished point, e. The upper light cone for e has two 'arms', the left and the right. There is a 'simultaneity slice'.
- The point, e, and all the points up the right arm are in the intersection, $h_1 \cap h_2$.
- Any point above the left arm of the simultaneity slice is in the appropriate difference. Any point on or below the simultaneity slice is in the intersection.

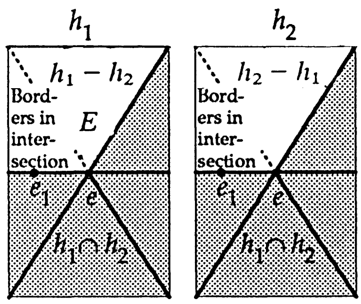

Fig. 11. Postulate 37 stronger than Postulate 28.

These stipulations are pictured in Figure 11.

You can see that if e_1 is in the left part of the simultaneity slice (excluding e), then e_1 is a choice point. For then e_1 is in the intersection but without any points properly above it that are in the intersection.

You also can see that e is *not* a choice point. Reason: All those points properly above it in the right arm of the upper light cone. (Nor is any point on the right arm of the upper light cone a choice point.)

Each point in $h_1 \oplus h_2$ is above some point on the left part of the simultaneity slice. Therefore, the prior choice principle in its point event formulation (Postulate 28) is satisfied.

Consider, however, any chain, E, of points in $h_1 - h_2$ descending toward e without limit. E does not overlap h_2. So what is its *raison d'être*, the ground of its beginning to be (instead of the continuance of h_2)? It cannot be any of the choice points in the left part of the simultaneity slice, since they do not lie in its past. It cannot be e, since that is not a choice point. Therefore, in this diagram there is a coming to be of the chain, instead of the continuance of h_2, without a reason in the past of the entire chain (though there is a reason in the past of each member of the chain). Figure 11 is thus allowed by Postulate 28 but forbidden by Postulate 37.

A.2. *Infima, Suprema, Density, and Transitivity*

This section considers some additional postulates relating to chains. Their role here is as objects of study insofar as they influence the *combination* of indeterminism and relativity – which is why I don't defend them much. First some (standard) terminology.

DEFINITION 39: A *lower bound* for E is a point e such that $e \leqslant e_1$ for every $e_1 \in E$. A *maximal* lower bound for E is a lower bound for E such that no lower bound for E is strictly above it. If there is a lower bound e for E such that $e_1 \leqslant e$ for every lower bound of E, it will be unique. One writes 'inf(E)', and calls inf(E) the *infimum* of E. Similarly for *upper bound*, for *minimal* upper bound, and for *supremum*, written 'sup(E)' when it exists.

In (for example) Minkowski space-time one expects that each nonempty lower bounded set of point events has (not of course a unique infimum but) a family of maximal lower bounds. The analog *should not* hold in branching space-time. For example, consult the paradigmatic Figure 8, where e_1 and e_2 are depicted as alternate 'fillings' of the same space-time 'position'. Consider the set $\{e_1, e_2\}$. You can plainly see that although this set is lower bounded, it has no maximal lower bound, and ought not to have one.

On the other hand, it is natural to expect infima for *chains*.

POSTULATE 40: **Existence of infima for chains.** Every nonempty lower bounded chain of point events has an infimum.

Attend now to suprema of nonempty upper bounded chains, which always exist in Minkowski space-time. One should *not* expect them to exist in branching space-time. The set, E, of Figure 8 is paradigmatic, having, as it does, two incomparable (and incompatible) minimal upper bounds. Guided by this example it is easy to see what is instead plausible:

POSTULATE 41: **Existence of historical suprema for chains**. Each nonempty upper bounded chain has a supremum *in each history* of which it is a subset.

Given this postulate, we may define a *relativized* notion of supremum, $\sup_h(E)$, with the following properties.

DEFINITION 42: Suppose that E is nonempty and upper bounded in **Our World**, and that $E \subseteq h$. Then $\sup_h(E)$ is characterized by the following.

$\sup_h(E) \in h$.
$e_1 \leqslant \sup_h(E)$ for every $e_1 \in E$.
$\sup_h(E)$ is the least such member of h: if $e_2 \in h$ and $e_1 \leqslant e_2$ for every $e_1 \in E$, then $\sup_h(E) \leqslant e_2$.

It is to be emphasized that even in Minkowski branching space-time, infima exist independently of histories, while suprema exist only relative to a history. These features are essential concomitants of branching space-time. Take a 'process' as represented by a bounded causal interval without a first or last point event, and interpret the following tenses from the standpoint of a point event within it. 'How this process will end' (i.e., the supremum of the process) is historically contingent, depending as it does on (perhaps metaphorical) choices made in the neighborhood of the process. 'How this process began' (i.e., the infimum of the process) is, in contrast, independent of histories.

A third key property in (for example) Minkowski branching space-time is density.

POSTULATE 43: **Density**. If $e_1 < e_2$, then there is a point event properly between them.

The burden of the remainder of this section is twofold: to confirm the technical difference between obvious undividedness as in Definition 18 and (plain) undividedness as in Definition 19; and (ii) to make clear that the distinction is nevertheless of interest only in finite or otherwise pathological circumstances, since the distinction collapses in the presence of infima, suprema, and density.

FACT 44: If we do not add the postulates for infima, suprema, and density, then none of the following is implied:

Transitivity of $\approx_e$ (i.e., obvious undividedness, the relation of sharing a point event properly later than e);
Transitivity of $\approx_E$ for E a chain (as defined in Definition 32); and
Reflexivity of $\approx_E$ for E a chain.

Proof omitted.

Here is what the added postulates imply for the combination of indeterminism and relativity.

FACT 45: Existence of historical suprema suffices for the reflexivity of $\approx_E$ for E an upper bounded chain with no last member. In fact it would be sufficient to have a plain upper bound in each history; minimality is not needed.

Existence of historical suprema also suffices for the transitivity of $\approx_E$ for E an upper bounded chain with no last member.

FACT 46: Density and existence of infima together imply that $\approx_e$ (i.e., obvious undividedness) is transitive and is thus the same as $\equiv_e$ (i.e., undividedness).

Argument: Suppose, where $e \in h_1 \cap h_2 \cap h_3$, that $h_1 \approx_e h_2$, $h_2 \approx_e h_3$, and $h_1 \not\approx_e h_3$, and that density holds and infima of nonempty lower bounded chains exist. I produce a contradiction.

Consider the portion of $h_1 \cap h_2$ properly above e. Since $h_1 \approx_e h_2$, and since $h_1 \not\approx_e h_3$ requires e not be maximal in **Our World**, this set is nonempty. So by Zorn's lemma, let E be a maximal chain of such points. Since e lower bounds E, $\inf(E)$ exists, and $e \leqslant \inf(E)$. Suppose $e < \inf(E)$. Since by maximality of E there are no points properly between e and E, this would contradict density; so $\inf(E) = e$.

That $h_1 \not\approx_e h_3$ says that no point later than e belongs to both histories, so $E \subseteq h_2 - h_3$. Thus by Postulate 37, there must be a choice point e_1 for h_2 and h_3 prior to E. Where is e_1? Since $e = \inf(E)$, by priority it must be that $e_1 \leqslant e$, and therefore contradiction: the assumption $h_2 \approx_e h_3$ rules out that either e or any point prior to it can be a choice point for h_2 and h_3.

COROLLARY 47: In the presence of the added postulates for infima, historical suprema, and density, there is no difference between $\approx_e$ and $\equiv_e$. Furthermore, where E is a nonempty upper bounded chain, there is no difference between $\approx_E$ and $\equiv_E$.

The following similar result helps the left brain by putting the transitivity of $\approx_e$ in formal perspective, and helps the right brain by sharpening our picture of branching space-time.

FACT 48: Without infima and density, it is not implied that if $h_1 \not\equiv_e h_3$

for $h_1, h_3 \in H_{(e)}$, then every point later than e in $h_1 - h_3$ is incompatible with every point later than e in $h_3 - h_1$.

FACT 49: In contrast, the transitivity of $\approx_e$, when it holds as for example in Minkowski branching space-time, suffices for this sort of fierce splitting.

A.3. *How Indeterminateness Becomes Determinateness*

Finally, let me explicitly note that on the present theory, and in the presence of the postulates of this section, a causal origin has always 'a last point of indeterminateness' (the choice point) and never 'a first point of determinateness'. I find the matter puzzling since it's *neither* clear to me how an alternate theory would work *nor* clear what difference it makes. In any event, the following corollary to density convincingly demonstrates how difficult it is to speak accurately about determinism/indeterminism. The question is, on the present theory, does the past determine the future? The answer is yes and no.

FACT 50: Yes: Given the entire past of any possible point event, there is no alternative to reaching that point event. That is, take any point event, e, and let E_1 be the set of point events lying in the proper past of e. Then given E_1, the event e is bound to happen: for each history, h, if $E_1 \subseteq h$ then $e \in h$.

Argument: We know that e belongs to some history, h_1. The 'hard' case is when e fails to belong to some history, h_2; we need to show that some member of E_1 also fails to belong to h_2. By the prior choice principle, some point event, e_1, is both prior to e and maximal in $h_1 \cap h_2$. By density, choose e_2 such that $e_1 < e_2 < e$. Then e_2 belongs to E_1 but not h_2, as wanted.

No: It is false that given the entire past of any lower bounded chain, there is no alternative to reaching that chain. That is, let E be a (perhaps open) lower bounded chain, and let E_1 be the set of point events lying in the proper past of E. What is false is that for each history, h, if $E_1 \subseteq h$ then $E \cap h \neq \emptyset$.

Argument: Just let E be any chain that is maximally lower bounded by any choice point, e_1. Let h_1 be any history such that $h_1 \cap E \neq \emptyset$. Now choose h_2 containing e_1 such that $h_1 \not\equiv_{e_1} h_2$. Evidently $E_1 \subseteq h_2$, but $E \cap h_2 = \emptyset$.

The first half of the above fact sounds downright deterministic. To put the matter in pseudo-epistemic terms, if you know the entire proper past of a point event, then you know what will happen next. The second half, however, tells us that in our naivete we were confused. Even if you know the entire past of an open lower bounded chain, you do not know what will happen next. It makes (on this theory) a difference!

NOTES

[1] I am indebted to many persons for constructive hearings, readings, and suggestions, and especially to the following: J. Haugeland for helping me to see what I could not see for myself; P. Bartha, A. Bressan, R. Brandom, M. Green, C. Hitchcock, H. Stein, M. Xu, B. Yi, and the referees supplied by this journal for finding errors or making significant suggestions; and L. Wessels for deeply valued encouragement.

[2] Should I add 'system' and 'state' to this list? I have also avoided these meta-scientific idioms because they seem not to help in the immediate enterprise.

[3] Some writers say 'non-space-like' for this type of order. That terminology, however, while acceptable in a deterministic cosmology, would severely hamper us later.

[4] I don't explain this terminology, which I will be using only in illustrations; see any treatment of relativity. The same is true for later uses of illustrations from quantum mechanics. Otherwise, this paper tries to be self-contained.

[5] See Prior (1967) and Thomason (1970 and 1984) for branching time. Just a little later, McCall (1976) began working on a combination of indeterministic and relativistic ideas expressed in his idea of a 'universe-tree'. McCall's line of thought has much influenced me. I am indebted to him for sharing some early versions of parts of a book that he is now preparing on these ideas and their applications. For some work on agency based on branching time, some of which concerns indeterminism, see joint and separate papers by Belnap and Perloff in the list of references.

[6] Lewis, 1986, p. 208.

[7] Since the sixties, Bressan has argued with appropriate logical sophistication for the need for a concept of real possibility in physics. This line of thought is fundamental to the present work. See Bressan (1972, 1972a, 1974, 1974a, and especially 1980). See also McCall (1976, sec. 7).

[8] Spatiotemporal positions or 'place-times', which are important, come in, I think, at a conceptually later stage. Branching space-time makes it easy to see that they should not be confused with point events.

[9] Different theories handle such an example in different ways. I do not offer the present articulation as persuasive, but only to help intuition grasp a key idea of branching space-time, be it right or wrong. (I trust it is evident that I invoke the quantum-mechanical measurement only as a putative example of an objectively indeterministic event, and that branching space-time does not pretend to be an 'interpretation' of quantum mechanics. I think there is no entirely noncontentious example available. If, however, coin-flipping or radioactive decay seems to you a more suitable illustration of indeterminism, please make an appropriate mental substitution.)

[10] These are inescapably heuristic remarks: I have not said what 'spatiotemporal position'

is to mean, and as I said before, I am not going to assume the availability of a Minkowski metric other than as an imagination-fixing illustration. Also, the words might suggest that there are several sorts of relations represented by $\leqslant$, or several sorts of point events; but this is not so.

[11] Hawking and Ellis (1973, p. 189) take free will as a premiss for antisymmetry. The theory of branching space-time can, I think, make sober sense out of their remarks. One may doubt that this is possible in their own cosmology, which has, I think, no *theory* of how incompatible real possibilities hang together. Here and below 'no *theory*' marks not a criticism but an important contrast. Everyone needs to use ideas uncontrolled or only partially controlled by rigorous theory. Still, as a counsel of perfection, everyone should recognize the difference! Incidentally, note that antisymmetry says that point events are identical if and only if they occupy the same place in the causal ordering of **Our World** by $\leqslant$. Without, however, a *theory* of causes and effects, which this paper does not offer, there is no deductive inference to the Davidsonian thesis that 'same causes and same effects' suffices for the identity of point events.

[12] Others think of the less jerky among such tracks as where a particle might be. This paper neither offers nor presupposes *any* theory of particles. This is one reason that I have avoided the customary language of 'world lines'. In addition, branching space-time would presumably need to think of there being *incompatible* possibilities for a given particle. Therefore each particle would at best have to be given a locus in **Our World** that looks more like a tree than a chain.

[13] The discussion in Reichenbach (1957), for example, may be marred by failure to appreciate this point; it is hard to be sure. In the attempt to elucidate the causal order between two *events*, Reichenbach speaks of "small variations" (ibid., p. 136) in them. He gives, however, no *theory* of small variations, so that one is entitled to wonder if small variations lead one to speak of two different events instead of the two one started with.

Suppose that instead of placing the causal order between events, one places it between *spatiotemporal positions*. One will still need a language and a *theory* that entitles one to speak of a given spatiotemporal position as occupied by alternate slightly varying possible events. A reason that it is easy to lose track of the point is this: a confessed determinist does not need to distinguish point events and spatiotemporal positions. Such a one can remain rigorous, however, only by abstaining from speaking of 'small variations', since he or she has, I think, no *theory* of them.

Another alternative keeps point events as the relata of the causal order. Instead of taking them as primitive, however, this alternative constructs point events from some combination of spatiotemporal positions and 'possibilities' for these spatiotemporal positions. Perhaps this alternative would equate possibilities with a certain range of properties. Development of such an alternative may be possible, but it will not be easy. One problem lies in elaborating a *theory* about the 'certain range of properties' that does not just leave it as an empty parameter. Another lies in identifying spatiotemporal positions as between distinct histories. As Bressan pointed out two decades ago, in general relativity the physical problem cannot be ignored. Perhaps at the end of the story one can justify such a picture, but, meanwhile, one should not just assume that it makes physical sense to use such phrases as "if the matter distribution around 'this spatio-temporal position' had differed in such and such a way from the way it actually was, then this is what would have happened at 'this spatio-temporal position'".

[14] I mean this as a heuristic remark; most people think that they know that histories are not really Minkowski space-times, and I will not postulate that they are.
[15] The past tense is critical. I don't care if you substitute 'existed' for 'occurred', provided you catch my meaning. Incidentally, one can see that precisely by considering the ontological need for such a standpoint, one might without warrant suddenly skip over to epistemology. This skip might then tempt one to introduce a *mind* placed at the standpoint, e, to *be aware of* influences from e_1 and e_2. Such a temptation is to be resisted in favor of reflecting on the ontological ideas themselves.
[16] Even the careful Earman (1986, p. 224) writes that "the different branches must represent simultaneously real situations and not merely unactualized possibilities", which is a tense/modal muddle – and nonrelativistic. This language presumably derives from the following tense/modal muddle of Everett (1957, p. 320): "All branches exist simultaneously in the superposition after any given sequence of observations". It may be, however, that there is nothing in Everett's own theory that requires this muddle. It would be good to know.
[17] This view is controversial, and I can explain it here only to the extent of a meager paragraph. Perhaps there is help in noting that I mean it in the same technical spirit in which one might say that the phrase 'the present time' is made senseless by relativistic considerations.
[18] Perhaps physics also considers worlds other than ours, such as those postulated by Lewis (1986); it is important to recognize this as an entirely different question.
[19] Bressan (1972, pp. 217–20, N53) makes the fundamental point about physics vs. astronomy adapted here.
[20] Even Lewis (1986, e.g., p. 8), the paradigmatic modal realist, seems to share the standard view that it is always all right to invoke 'the laws of our world'. He also writes that counterpart relations "are an inconstant and indeterminate affair" (ibid., p. 10). These features are desirable in giving an account of conversation, where "not anything goes, but a great deal does" (ibid., p. 8). The same features interfere, however, with the use of his constructions as a basis for rigorous theory.
[21] Does this help even a little in understanding 'superposition'? I think so, but I don't understand enough about quantum mechanics to warrant a settled opinion.
[22] "The only real probabilities in quantum mechanics, I maintain, are transition probabilities" (Cartwright, 1983, p. 179). I am suggesting π_e as the proper space of 'transition possibilities' underlying these probabilities.
[23] Since I am not endorsing much that Whitehead thought important about his ontological principle, I am quoting only selected phrases.
[24] See Belnap and Steel (1976) for a brief discussion of the doctrine that explanation-seeking why-questions always involve an 'instead of' clause (not just 'Why p?' but 'Why p instead of q?'). Bear in mind, however, that that was analysis of language, whereas this discussion is not. In particular, this use of 'instead of', is driven by contrasts existing deep in the structure of **Our World**. It has nothing to do with context or focus or emphasis or anything else mental or linguistic.
[25] Earman (1986, p. 224) balks "at trying to invent a causal mechanism by which a measurement of the spin of an electron causes a global bifurcation of space-time". Although his informal use of 'causal mechanism' is not in the spirit of the present line of inquiry, his instinct to reject his Figure XI.3 (p. 225) is squarely in line with our proposed solution to the problem of the wings.

[26] They are so close that this model of **Our World** is not Hausdorff: e_1 and e_2 cannot be separated by disjoint open sets. The model is, however, T_1: you can easily find an open set containing one but not the other. (These remarks are inspired by McCall (1990), which illuminates the topology of branching time. Though I am far from a topological understanding of branching space-time, here it seems enough to consider a set as 'open' if for every point event e in the set the following holds: for every interval E containing e of which e is not an end point, there are $e_1, e_2 \in E$ with $e_1 < e < e_2$ such that for every point event e_3 such that $e_1 \leq e_3 \leq e_2$, e_3 is in the set.) Please observe that in spite of page 224 of Earman (1986), none of this suggests that 'space-time' itself fails to be Hausdorff. For example, in this model each *history* is a Minkowski space-time and therefore Hausdorff in the usual way.

[27] This is, I take it, the same as the profound problem of identifying a natural absolute (rather than extensional) concept of point event as raised by Bressan in publications cited in Note 12, and worked on in Zampieri (1982 and 1982–83).

[28] I learned of this idea from A. Poteshman. Of course it won't work in the absence of additional assumptions. For starters, it makes little sense without Postulate 41 below.

[29] A merely formal generalization to arbitrary sets is easy, but pointless without an extended and controlled system of motivations.

[30] The relation expressed by saying that $H_{(e_1)} = H_{(e_2)}$ neither implies nor is implied by perfect outcome-correlation between e_1 and e_2. C. Hitchcock has observed, however, that if both relations obtain, then $\pi_{e_1} = \pi_{e_2}$, which we may call *absolute outcome-correlation*.

[31] There is a stupendously large literature on this topic. Any treatment of philosophical issues in quantum mechanics will give access to it.

[32] I regret to say that by 'genuinely random event' I mean just 'choice point'. The warning is needed because, as is spelled out in Section 9, there can be genuine indeterminism without choice. Given so much, it is easy to see that there can be a pair of space-like related perfectly outcome-correlated indeterministic events *without surprise* in the following sense: correlation and indeterminism alike are to be attributed to a single choice point that (take a breath) lies in the past of the future of possibilities of each given indeterministic event – a common cause. (Branching space-time *compels* accuracy in this matter.) Thus the 'surprise' arises only when the two space-like related perfectly outcome-correlated events are not just indeterministic but choice points, the transitions from which have no common cause.

[33] I should say explicitly that I have myself no doubt that quantum-mechanical theory-cum-experiment truly and conclusively proves the existence of EPR phenomena. No one should care about such undefended views, however, and I do not presuppose them in what follows.

[34] See Salmon (1984) for an enriching study. Incidentally, even without introducing probabilities, the denial of Conjecture 35 could be meaningfully strengthened to affirm the compatibility of any pair of outcomes of two space-like related choice points. This would be closer to saying that the two choice points are 'outcome independent' in the sense required equally for analysis of the common cause principle and analysis of the Bell argument. I think, however, the full meaning of 'outcome independence' requires probabilities of outcomes.

[35] Such discussions frequently give one an *epistemology* of randomness (repeated trials, etc.) without a *theory* of randomness.

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