# Robust Analysis via Simulation for a Merging-Conveyor Queueing Model

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Conveyor networks form critical components of many material-handling systems, so their performance is important in many areas of manufacturing and logistics. Often, conveyors merge and these are potential points of congestion, and thus bottlenecks and inefficiencies. In this paper we investigate the accuracy of a queueing-theoretic analysis of merging conveyors *vis á vis* direct simulation. We identify conditions under which the relatively simpler queueing-theoretic method can be used, and on the other hand when simulation would be necessary. The results have important implications for modeling, designing, and operating merging conveyor systems.

## 1. Introduction

Conveyor systems, as an essential component of material-handling systems, are widely used in transportation and manufacturing industries, such as mail hubs, airports, distribution centers, cargo carriers, warehouses, plants, and other sortation or delivery facilities. In many of these systems, the very first and most frequent situation they need to handle is a merging operation. After merging, cargo will be transported to downstream operations, such as sorting, splitting, or another merging, etc. The system, or portion of a system, that exclusively handles merging operations, is called a *conveyor network with merging configuration* (CNMC).

CNMCs play a key role in the performance of conveyor systems, since cargo conveyed on such induction lines may be delayed due to contention for space when they are inducted into the main line. In some systems where throughput is the primary concern, the space contention on the main line is very intense, which decreases the operational efficiency of the whole system. In this case, performance of CNMCs is critical to the performance of the whole system so obviously, it is very important to improve the performance of CNMCs.

The class of CNMCs discussed in this paper is shown in Fig. 1. In such a system, several induction conveyor lines connect into the main conveyor line at consecutive points. Cargo is loaded at the lower ends of the induction lines, transported into the main line and then downstream. There is an operator assigned to each induction line. Each operator attempts to load at a given rate. The operator could be a person, a machine or an upstream conveyor. If enough space is available on the induction conveyor, the operator places a package on the conveyor and then begins to unload the next package. Packages are random in size and require different amounts of space in the induction conveyor. Since the output of this CNMC could be the input of an induction line of another CNMC, several CNMCs can generate a complicated conveyor network.

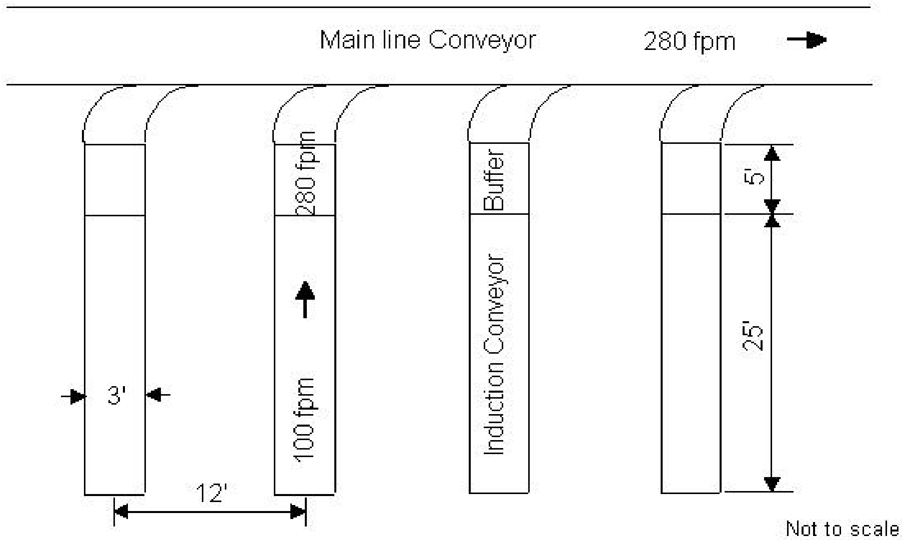


Figure 1 A Conveyor Network with Merging Configuration

The performance of a CNMC is primarily measured by its main-line throughput and utilization. Both high throughput and high utilization are desired. But a high utilization increases the contention for space, which causes unbalance of throughput among induction lines and decreases main-line throughput. Some parameters need to be carefully designed for a CNMC to reach high performance.

In the remainder of this section we discuss the balance of CNMCs and review a *queueing-theoretic method* (QTM) to analyze CNMCs. This is followed in Section 2 by the alternative simulation approach. Section 3 compares the two methods when the assumptions for the QTM apply, thus verifying its accuracy. Section 4 considers various violations of the QTM assumptions to measure its robustness *vis á vis* direct simulation, which serves as our benchmark. Section 5 draws some conclusions and suggests further research.

## 1.1 A Problem of CNMCs

A major problem in CNMCs is the unbalance of throughput among induction lines. Since the upstream lines have the advantage in seizing space, they are more likely to reach a higher throughput rate than those downstream, provided they have the same arrival rate. Balanced throughput rates among induction lines are desirable for even distribution of work load among the induction conveyors and corresponding staff, or for the balanced downstream demand, or for other reasons. For example, there might be several flights checking in at the same time. In this case, baggage might be checked in from different induction lines but transferred through the same main conveyor line. The carrier prefers that none of the flights or check-in stations be blocked. It would be better to give them a similar throughput rate.

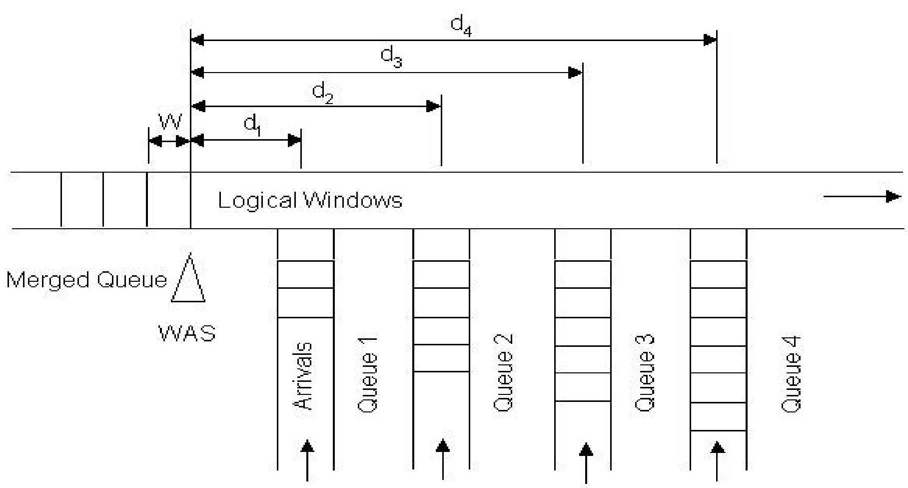


Figure 2 A Queueing-Theoretic Representation of CNMCs

The unbalance problem is caused by natural contention under no control. In advanced systems, the merging operations of induction lines are under control. The philosophy of the control is illustrated in Fig. 2. There is a section, called a *buffer* (queue 1, …, *N*), at the end of each induction line, which connects to the main line. A system that detects the size of each package is located at the entrance of each buffer. Based on the detected size, a control system allocates an appropriate space, called a *window*, for each package on the main line, at a location in front of the first merging point. This location is called the *window assignment station* (WAS). A package is held in the buffer until the window assigned for it arrives at its merging point, then it is released for merging. When the merging operation occurs, the package enters the main line and takes the space reserved for it. Technically, a gap is needed between two packages for detection systems to work. Due to specified limited capacity, a buffer might be full, in which case it blocks (stops) the induction conveyor and in this way cuts the throughput of the induction line. The blocked induction line will be resumed once a merging event occurs. By controlling some parameters such as the buffer sizes, one can influence the blocking rate of each induction line, allowing all lines to reach a balanced throughput.

### 1.2 A QTM to Design a Balanced CNMC

Queueing theory was introduced to conveyor-system analysis more than 30 years ago. Since Disney (1963), this application has been widely investigated by many researchers, such as Perros and Altiok (1981), Lee and Pollock (1989), Commault and Semery (1990), Yannopoulos (1994), and Karunaratne (1996). Those researchers applied queueing theory to various conveyor systems with various configurations, but none of them gives special attention to the relation between buffer sizes and balance control. Arantes and Deng (1998) developed a new algorithm for the design of a balanced conveyor network with merging configuration. The QTM discussed in this paper refers to this algorithm.

This QTM establishes relationships among parameters and reaches a balanced throughput design by controlling the buffer sizes of the induction conveyors. Fig. 2 shows the queueing representation of the CNMC in Fig. 1. The following assumptions and characteristics are used in the QTM:

- The system is in a stationary state;
- The inter-arrival times of packages are independent, following an exponential distribution (stationary Poisson distribution in terms of number of arrivals during a fixed interval of time);
- A buffer is measured by the number of packages it can hold;
- The package size is a random variable identified by the mean and variance of an empirical discrete distribution;
- The location of the WAS is fixed;
- The control logic is first-come first-serve;
- The induction lines are non-accumulative and the operation of operators and induction lines will be blocked (stopped) when the corresponding buffers are full;
- There are always enough spaces for operators to place new packages onto induction lines when the lines are not blocked, so the impact of the speeds and lengths of induction lines on system performance is ignored;

To solve the problem, the following parameters need to be specified:

$N$ = the number of induction lines,

$\boldsymbol{l}$ = the nominal arrival rate of packages to each of the induction lines,

$v$ = the velocity of the main line conveyor,

$E[w]$ = the expected value of a window size,

$Var[w]$ = the variance of a window size,

$d_i$ = the distance from buffer $i$ to WAS for $i = 1, \ldots, N$,

$\bar{U}$ = the desired utilization of induction lines,

$\bar{\lambda}$ = the minimum acceptable throughput rate of the main line.

The QTM uses a simplified queueing model when calculating $W_0$, the expected waiting time in the *merged queue* of the WAS. As a result the value of $W_0$, obtained by equation (1), overestimates the actual waiting time. This causes the calculated throughputs to be lower bounds of throughputs of the corresponding system. The following relations are developed.

$$W_0 = \frac{\rho_0^2 + \lambda_0^2 Var[w]/v^2}{2\lambda_0(1-\rho_0)} + \frac{1}{\mu_0} \tag{1}$$

where $\lambda_0$ is the arrival rate at the merged queue, $\mu_0$ is the service rate at the merged queue, and $\rho_0$ is the traffic intensity, as given below:

$$\lambda_0 = \lambda \sum_{i=1}^{N} [1 - p_i(K_i)] \tag{2}$$

$$\mu_0 = \frac{v}{E[w]} \tag{3}$$

$$\rho_0 = U_m = \frac{\lambda_0}{\mu_0} \tag{4}$$

where $U_m$ is the utilization of the main line. The utilization $U_i$ of induction line $i$ is

$$U_i = 1 - p_i(K_i) \qquad i = 1, \ldots, N \tag{5}$$

Where $K_i$ is the capacity of buffer $i$; $p_i(K_i)$ is the probability that buffer $i$ is full (has $K_i$ packages).

$$K_i \geq \frac{\lambda\, U_i(d_i + vW_0)}{v} \qquad i = 1, \ldots, N \tag{6}$$

$$\frac{\bar{\lambda}}{\lambda \bar{U}} \le N \le \frac{v}{\lambda \bar{U}\, E[W]} \tag{7}$$

$$p_i(K_i) = \frac{(\lambda / \mu_i)^{K_i} / K_i!}{\sum_{m=0}^{K_i} (\lambda / \mu_i)^m / m!} \qquad i = 1, \ldots, N \tag{8}$$

The algorithm to design a balanced CNMC is:

1. Verify that the previous specified data satisfy (7). If not, modify them until they fit;
2. Get $W_0$ from (5), (2), (1), then estimate the lower bound of $K_i$ by (6);
3. Compute $p_i(K_i)$ and $W_0$ for $i = 1, \ldots, N$ by solving the system of $N+1$ equations formed by (8), (1);
4. If $p_i(K_i) \le 1 - \bar{U}$ for all $i$, then stop. Otherwise, increment $K_i$ by 1 for all $i$ such that $p_i(K_i) > 1 - \bar{U}$. Go to step 3.

## 2. A Queueing-Theory-Based Simulation Model for CNMCs

Simulation has become more popular in conveyor-system analysis with the rapid improvement of simulation software and computer hardware. As some examples of those applications, Gourley (1973) simulated re-circulating conveyor systems; Woiret (1988) modeled and simulated CAD of conveyor systems; Bartlett and Harvey (1995) used SIMAN (Pegden, Shannon and Sadowski, 1995) to simulate a CIM cell in which two conveyors were considered. So far, an application that focuses on controlled conveyor networks with merging configuration described in this paper has not been studied systematically by simulation.

A general simulation model for CNMCs has been built and tested in Arena (Kelton, Sadowski and Sadowski, 1998). There are two sizing styles for assigning windows, fixed length (FWS) or variant length (VWS). For the fixed-length style, all windows have the same length. The length of the windows should be large enough to carry the longest package. Also, the capacity of buffers could be measured in one of two ways: number of packages and length of occupied space. There is no restriction on the distributions of arriving packages, but by default, a stationary Poisson distribution is applied. There is no restriction on the distribution of package size, but by default, an empirical distribution (Fig. 3) collected from the field is used in our analysis. The situation that QTM handles is for non-accumulative conveyors with buffers measured by the number of packages. Criteria used to evaluate the performance of CNMCs are as follows:

- Average *throughput rate* of the main line and induction lines
- *Utilization* of the main line and induction lines

The simulation experiments are designed as follows. CNMCs are primarily treated as a non-terminating system, assuming long runs without breaking. The batching technique (Kelton, 1994) is used for *confidence-interval* ( CI ) statistics. The length of a run is defined to cover no fewer than 10 batches while each batch covers no fewer than 10 correlation lags (Pegden, Shannon and Sadowski, 1995).

## 3. Comparing the QTM and Simulation Results

In this section, the QTM and the simulation model are both used to study a real-world CNMC introduced by Arantes and Deng (1998), in which the QTM was used to identify appropriate buffer sizes to balance the CNMC. Using the same configuration, we compare the utilization and throughput associated with each conveyor line obtained by both methods. This comparison allows us to evaluate the accuracy as well as the efficiency of each method.

The CNMC is as indicated in Fig. 1. It has four non-accumulative induction lines that merge at 5, 17, 29 and 41 feet from the WAS. The lengths of the induction lines are each 25 feet. The main line runs at 280 feet per minute (ft/m) while the induction lines run at 100 ft/m. Packages, arriving at the nominal rate of 16 packages per minute, have the size distribution (in inches) as shown in Fig. 3. This distribution has been empirically identified in a major American freight-delivery company. A twelve-inch gap is required between consecutive packages. Hence the window length equals ( the package size + 12 ) inches in the VWS case and 60 inches in the FWS case.

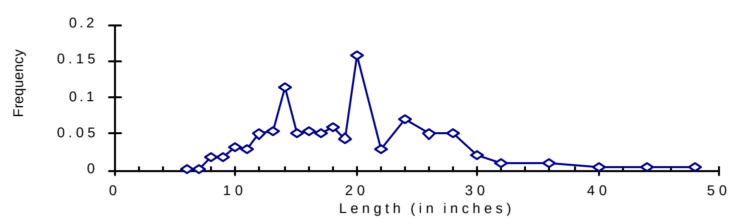


Figure 3 The Distribution of Package Size

As a non-terminating system, the length of a simulation run is selected to be 8 hours, which provides enough precision, and is close to the real situation. The simulation run is split into 24 batches of 1200 seconds. Identified by the trial simulation runs, the first 600 seconds corresponds to the warm-up period and is therefore excluded from data collection. In addition, the first batch is incomplete and is discarded. The design of the simulation experiments is the same for all cases referring to non-terminating systems. This facilitates the analysis while providing the necessary statistical precision.

### 3.1 Case 1 - Variant Window Size (VWS)

The buffer sizes used in the VWS case are (2, 3, 4, 5) corresponding respectively to the induction lines 1 through 4. The results are summarized in Table 1. The simulation results have a confidence level of 95%, which is kept through all the studies in this paper. "% difference" indicates the percent difference of the simulation results based on the QTM results, which is also provided through all the studies. Bonferroni multiple pairwise comparison is applied to check the balance among induction lines. The lines reach a balance in this case.

**Table 1. Analytical and Simulation Results of CNMC with Variant Window Size**

| Buffer Size (in No) (2, 3, 4, 5) | | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| Utilization | QTM | 0.527 | 0.902 | 0.899 | 0.909 | 0.919 |
| | Simu. | 0.533 $\pm$ 0.006 | 0.995 $\pm$ 0.001 | 0.977 $\pm$ 0.002 | 0.974 $\pm$ 0.002 | 0.974 $\pm$ 0.003 |
| | % Diff. | 1.1 % | 10.3 % | 10.9 % | 7.2 % | 6.0 % |
| Throughput (Pack. /sec.) | QTM | 0.968 | 0.240 | 0.240 | 0.242 | 0.245 |
| | Simu. | 0.980 $\pm$ 0.010 | 0.246 $\pm$ 0.005 | 0.246 $\pm$ 0.005 | 0.245 $\pm$ 0.006 | 0.242 $\pm$ 0.005 |
| | % Diff. | 1.2 % | 2.5 % | 2.5 % | 0.4 % | - 1.2 % |

### 3.2 Case 2 - Fixed Window Size (FWS)

The buffer sizes used in the FWS case are (3, 4, 5, 5). The results are summarized in Table 2. The induction lines reach a reasonable balance.

**Table 2. Analytical and Simulation Results of CNMC with Fixed Window Size**

| Buffer Size (in No) (3, 4, 5, 5) | | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| Utilization | QTM | 0.916 | 0.768 | 0.813 | 0.845 | 0.783 |
| | Simu. | 0.986 $\pm$ 0.005 | 0.809 $\pm$ 0.020 | 0.858 $\pm$ 0.016 | 0.893 $\pm$ 0.013 | 0.822 $\pm$ 0.013 |
| | % Diff. | 7.6 % | 5.3 % | 5.5 % | 5.7 % | 5.0 % |
| Throughput (Pack. /sec.) | QTM | 0.856 | 0.205 | 0.217 | 0.225 | 0.209 |
| | Simu. | 0.917 $\pm$ 0.009 | 0.229 $\pm$ 0.005 | 0.232 $\pm$ 0.004 | 0.235 $\pm$ 0.005 | 0.225 $\pm$ 0.004 |
| | % Diff. | 7.1 % | 11.7 % | 6.9 % | 4.4 % | 7.7 % |

### 3.3 Comments on the QTM and Simulation Results

1. The experiments confirm that the QTM yields conservative results. This is expected because the QTM uses approximations that tend to underestimate the utilization and throughput. Most of the QTM results fall outside, but close to, the lower bounds of the 95% CI of the simulation results that have a relative error ( CI half-width / mean) of less than 2.5%. The largest difference between simulation and QTM results is 11.7% in throughput in FWS. Thus, under a usual CI relative precision level 85% (Law and Kelton, 1991), QTM works well.
2. CNMCs reach their steady states quickly. Plots of time between outputs showed that CNMCs reach an apparently stationary state shortly after the first package exits the system. The apparent warm-up periods for all the investigated cases are less than 10 minutes. So, even though QTM is developed based on the assumption of studying non-terminating systems, it can also be used with reasonable precision for terminating systems;
3. Influence of the length and speed of the induction lines is ignored in the queueing model. Implicitly, it is assumed that the operator will always have enough space clear in the induction line to place his/her package. Yet, the simulation runs have indicated that a wait for a clear space may happen depending on the speeds and lengths of the induction lines. Hence, the interarrival-time distribution of packages at the loading operator and at the buffer may be different.
4. The correlation lags of simulation results are slightly larger than, but close to, the average time that a package stays in the system. This was confirmed by follow-up investigation. The physical meaning is that the packages that already exit the system

should have no impact on those that have not arrived, since the arrival of packages is memoryless. This feature could be used to design efficient simulation experiments.

In conclusion, both the QTM and the simulation model work well under the situations restricted by the QTM assumptions, with reasonable precision. QTM is a more efficient and conservative way to initiate a good design for CNMCs. To reach a design at the same level by simulation, many simulation runs have to be done.

## 4. Situations Violating the Assumptions of QTM

There are many situations in practice where the QTM assumptions are violated. Potentially most questionable is that QTM assumes the system is in a stationary state, which could be violated in several ways. One way is to terminate and restart the system frequently due to changes of unloaded vehicles, or scheduled breaks for workers. However, since the warm-up periods for CNMCs are short, QTM can usually be applied to such situations anyway. Another violation to the stationarity assumption is combined with the violation to the assumed form of the interarrival-time distribution. The time to reach the packages might increase with the evacuation of trucks. The workers might get fatigued, which increases the cycle time. In such situations, the assumed stationary Poisson arrival process becomes non-stationary and the arrival rate varies within a work shift (Law & Kelton, 1991). Moreover, some other distributions could also be applied to the interarrival times, while keeping the assumption of a stationary arrival process. The buffers could be controlled by the amount of space instead of the number of packages they can hold. The distribution of package sizes could also vary. Can the QTM results be applied to such situations? These situations, as the most important concerns in applying QTM, are investigated in this section. Except where specified, all configurations in the investigations are kept as described previously.

### 4.1 Terminating System with Non-stationary Poisson Arrival Process

It is quite possible that in a real system, the arrival of packages is memoryless but the arrival rate decreases during operation. If the system stops / restarts multiple times during one work shift, it should be treated as a terminating system. Considering the break time and unloading cycle time, the following situation is investigated: one hour-long operation with a linear arrival rate 50% higher than average at beginning and 50% lower than average at the end. The thinning method (Lewis and Shedler, 1979) is used to generate this nonstationary Poisson arrival process. For the convenience of comparison with the non-terminating situation, 23 replications are made, which corresponds to a relative error of 1.5%. The simulation results are summarized in Table 3. Number of packages per second is used as the unit of throughput in all tables.

**Table 3. Terminating System with Non-stationary Arrival Process**

| Case | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| 1 VWS | Utilization | 0.528 ± 0.004 | 0.992 ± 0.001 | 0.974 ± 0.001 | 0.969 ± 0.002 | 0.968 ± 0.002 |
| | % Difference | 0.2% | 10.0 % | 8.3 % | 6.6 % | 5.3 % |
| | Throughput | 0.971 ± 0.009 | 0.245 ± 0.003 | 0.242 ± 0.003 | 0.241 ± 0.003 | 0.242 ± 0.003 |
| | % Difference | 0.3 % | 2.1 % | 0.8 % | - 0.4 % | - 1.2 % |
| 2 FWS | Utilization | 0.902 ± 0.005 | 0.756 ± 0.006 | 0.802 ± 0.005 | 0.837 ± 0.007 | 0.774 ± 0.006 |
| | % Difference | - 1.5 % | - 1.6 % | - 1.4 % | - 1.0 % | - 5.8 % |
| | Throughput | 0.840 ± 0.007 | 0.205 ± 0.002 | 0.211 ± 0.002 | 0.218 ± 0.002 | 0.208 ± 0.003 |
| | % Difference | - 1.9 % | - 0.0 % | - 2.8 % | - 3.1 % | - 0.5 % |

Paired-*t* mean comparison is used to identify changes in simulation results. Significant changes are identified in FWS. The terminating system with non-stationary arrival process exhibits lower performance. The biggest throughput decrease in simulation is

10.5%, observed in line 1, FWS. The same trend of decrease for VWS, however, is not significant. The balance among induction lines becomes worse. Only VWS keeps the balance. As shown in the table, the results of simulation and QTM are getting closer, especially in FWS. The performance of simulation is even lower than that of QTM in FWS. In conclusion, FWS is sensitive to the variation of arrival rate and the violation of the stationarity assumption, while VWS is more robust to them; QTM design fits better in this situation.

### 4.2 Influence of Different Interarrival-Time Distributions

QTM is based on the assumption that the interarrival-time distribution of packages in each induction line is exponential. Even though they may be stationary, many practical situations will violate this distributional assumption. For instance, if an automated machine or procedure feeds the packages into the induction lines, then a deterministic or a uniformly distributed random variable may be a better representation of the interarrival times. Also, due to traffic congestion, even if the interarrival-time distribution of packages to the operator is exponential, the interarrival-time distribution of packages, from all the induction lines or from a specific one, exiting the mainline conveyor may look more like lognormal, as observed in our simulation experiments. Many distribution families such as lognormal, beta, erlang and gamma have been observed to fit interarrival-times of packages obtained as the output of the induction lines. The output of the mainline conveyor could be the input to another conveyor line downstream.

This section evaluates the validity of the queueing-theoretic results when the interarrival-time distribution is different from exponential. The following distributions are investigated: lognormal, beta, erlang-2, normal, triangular, uniform and deterministic. The same mean as the exponential distribution will be kept, as will the variance if possible. In some distributions, the variance cannot be kept the same with that of the exponential if the mean is same. In this case, the variance will be kept as close as possible. The distributions are listed in table 4. "% difference" here only indicates the maximum percent difference in throughput based on QTM results.

**Table 4. Variant Interarrival-time Distribution (in Seconds )**

| Distribution | Expression | Mean | STD | Feature | % Diff. |
|---|---|---|---|---|---|
| Lognormal | LOGN ( 3.75, 3.75 ) | 3.75 | 3.75 | Skewed | 9.3 % |
| Beta | 30 * BETA ( 0.75, 5.25 ) | 3.75 | 3.75 | Skewed | - 7.4 % |
| Erlang - 2 | ERLANG ( 1.875, 2 ) | 3.75 | 2.65 | Skewed | 10.2 % |
| Triangular | TRIANGULAR ( 0.01, 1.85, 9.39 ) | 3.75 | 2.46 | Skewed | 11.3 % |
| Normal | NORMAL ( 3.75, 1.25 ) | 3.75 | 1.25 | Symmetrical | 11.3 % |
| Uniform | UNIFORM (0.01, 7.49 ) | 3.75 | 2.74 | Symmetrical | 10.4 % |
| Deterministic | DETERM (3.75 ) | 3.75 | 0 | Symmetrical | 12.4 % |

The summarized simulation results are provided in the Appendix corresponding to a CI relative error of at most 3.5%. Bonferroni multiple pairwise comparison is applied to the main-line throughput and utilization. In VWS, lognormal, erlang and uniform fall into one group (group 1); triangular, normal and deterministic fall into another group (group 2). The performance of different distributions from low to high is respectively beta, exponential, group 1 and group 2. The exponential distribution serves as the midpoint among these distributions. In FWS, the main-line utilization reaches 100% for erlang, triangular, normal, uniform and deterministic (group 3). The same trends exist but there is no significant difference in group 3.

Comparisons are made with the QTM results. For the beta distribution, the throughputs in VWS are lower than those of QTM. So QTM is not always conservative for different interarrival-time distributions. Even though the differences between simulation and

QTM results are significant for different distributions, the largest difference in throughput does not exceed ± 15%, as is the case with the utilization in VWS. This means that QTM still works relatively well with the usual relative precision. Due to the fact that the main-line utilization of group 3 reaches 100% in FWS, the induction-line-blocking probability of group 3 significantly increases. Thus the difference in utilization, between the simulation and the QTM results, of many induction lines of group 3 in FWS largely exceed -15%. This situation reveals that QTM does not represent CNMCs very well when the main-line utilization is close to 100%. In conclusion, QTM can be applied to variant interarrival-time distributions provided the same mean is used in QTM.

### 4.3 Buffer Measured in Length of Space

Since a buffer is a segment of an induction conveyor, it is much easier to control its length than to control the number of packages it hold. Yet QTM is unable to do theoretical analysis if a buffer is controlled by length. Will there be much of an effect? If not, a number can be transformed into a corresponding length by multiplying the average length that packages take. As shown in Fig. 3, the mean package size is 18.5 inches in our studies. The simulation results of the corresponding buffer-length are summarized in Table 5 with a CI relative error of 3.5%.

The paired-*t* test identifies significant differences in simulation results. As a common trend, the throughput and utilization increase. The biggest difference between QTM and simulation results in this situation is 12.2%, slightly worse than the "number" situation but less than 15%. That means that even if the goal is to design a system with a buffer measured in length, QTM can still be used to find a conservative initial design. The induction lines reach a very good balance in this situation.

**Table 5. Buffer length**

| Case | Buffer Size | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|---|
| 1 VWS | 61,91,122,152 units in length | Utilization | 0.535 ± 0.006 | 0.998 ± 0.001 | 0.988 ± 0.002 | 0.986 ± 0.002 | 0.982 ± 0.003 |
| | | % Diff. | 1.5 % | 10.6 % | 9.9 % | 8.5 % | 6.9 % |
| | | Throughput | 0.980 ± 0.010 | 0.247 ± 0.005 | 0.246 ± 0.005 | 0.246 ± 0.006 | 0.243 ± 0.005 |
| | | % Diff. | 1.2 % | 2.9 % | 2.5 % | 1.7 % | - 0.8 % |
| 2 FWS | 91,122,152,152 units in length | Utilization | 0.990 ± 0.005 | 0.790 ± 0.022 | 0.848 ± 0.017 | 0.893 ± 0.013 | 0.811 ± 0.018 |
| | | % Diff. | 8.1 % | 2.9 % | 4.3 % | 5.7 % | 3.6 % |
| | | Throughput | 0.926 ± 0.009 | 0.230 ± 0.004 | 0.232 ± 0.004 | 0.236 ± 0.005 | 0.226 ± 0.004 |
| | | % Diff. | 8.2 % | 12.2 % | 6.9 % | 4.9 % | 8.1 % |

### 4.4 Different Distribution of Package Size

In QTM, the distribution of package size is summarized by its mean and variance. In our case studies, the empirical distribution from the real world is actually well represented by a Gamma distribution with parameters (*a*, *b*, *g*) = (3.5, 3.6, 6.0). As shown in Fig. 4(a) , it is skewed to the right. Is the system sensitive to different distributions with same mean and variance? To investigate the effect, a size distribution with the same mean and variance but skewed to the left is generated. As shown in Fig. 4(b), it is a beta distribution with parameters (*a*, *b*) = (2.5, 1.5). The simulation results corresponding to this beta distribution are summarized in Table 6 with a relative error of 3.0%. A paired-*t* test shows that the size distribution has no significant impact on CNMCs. The biggest difference between QTM and simulation results is 11.7%.

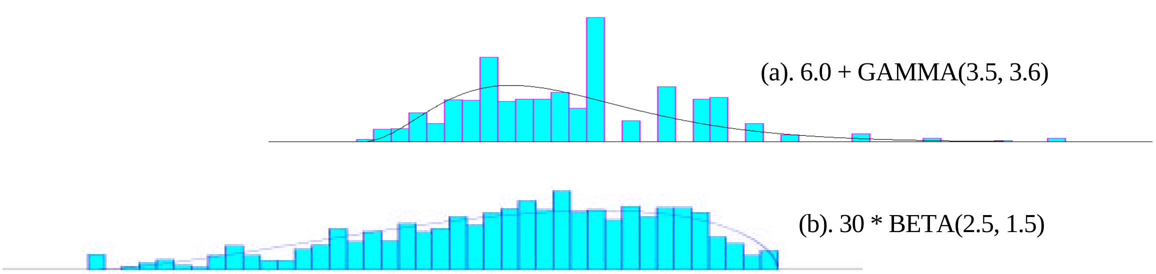


Figure 4 Various Distributions With Same Mean and STD (18.6, 6.45)

Table 6. Package Sizes follow Beta Distribution

| Case | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| 1 VWS | Utilization | 0.527 $\pm$ 0.007 | 0.995 $\pm$ 0.001 | 0.977 $\pm$ 0.003 | 0.974 $\pm$ 0.003 | 0.974 $\pm$ 0.003 |
| | % Difference | 0.0 % | 10.3 % | 8.7 % | 7.2 % | 6.0 % |
| | Throughput | 0.980 $\pm$ 0.010 | 0.248 $\pm$ 0.007 | 0.244 $\pm$ 0.005 | 0.247 $\pm$ 0.005 | 0.243 $\pm$ 0.006 |
| | % Difference | 1.2 % | 3.3 % | 1.7 % | 2.1 % | - 0.8 % |
| 2 FWS | Utilization | 0.986 $\pm$ 0.005 | 0.806 $\pm$ 0.020 | 0.853 $\pm$ 0.017 | 0.891 $\pm$ 0.013 | 0.816 $\pm$ 0.015 |
| | % Difference | 7.7 % | 5.0 % | 4.9 % | 5.4 % | 4.2.% |
| | Throughput | 0.917 $\pm$ 0.009 | 0.229 $\pm$ 0.004 | 0.233 $\pm$ 0.005 | 0.235 $\pm$ 0.004 | 0.224 $\pm$ 0.007 |
| | % Difference | 7.1 % | 11.7 % | 7.4 % | 4.4 % | 7.1 % |

## 5. Conclusions and Further Research

In this paper, we have developed a simulation model for CNMCs and, *vis á vis* simulation, investigated the robustness of an analytical queueing model for CNMCs, proposed by Arantes and Deng (1998). The investigation has lead to the following conclusions: CNMCs have short warm-up periods and reach steady state quickly, thus a terminating system can be treated as a non-terminating system with reasonable precision; QTM is a quick and conservative way to find a reasonable good initial design for CNMCs to reach a balanced throughput; QTM is not highly restricted to the assumption of a stationary arrival process, and is not sensitive to the interarrival-time distribution or the way to measure buffer length. Yet further simulation experiments reveal that the QTM results tend to deteriorate as the main line utilization approaches to 100%, because the approximation used in QTM causes a larger error when the main line utilization is high. As a result, under the same conditions, QTM fits VWS better than FWS since VWS has lower main-line utilization than FWS. Generally, under a relative precision of 85%, QTM works well. The results may need to be improved by other means such as simulation for high precision.

Further research in this topic is under development and shall include the study of: speed and length of induction lines; high utilization to throughput; accumulative vs. non-accumulative induction lines; different control logic, such as round robin; movable WAS vs. fixed WAS; simulation variance reduction and efficiency improvement with the help of QTM results as external control variates.

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## Appendix. Simulation results for Different Interarrival-time Distributions

**Table A1.** Lognormal Interarrival-time Distribution LOGN (3.75, 3.75)

| Case | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| 1 VWS | Utilization | 0.568 ± 0.009 | 0.995 ± 0.002 | 0.978 ± 0.002 | 0.969 ± 0.004 | 0.970 ± 0.003 |
| | % Difference | 7.8 % | 10.3 % | 8.8 % | 6.6 % | 5.6 % |
| | Throughput | 1.041 ± 0.014 | 0.261 ± 0.007 | 0.258 ± 0.005 | 0.264 ± 0.008 | 0.258 ± 0.007 |
| | % Difference | 7.5 % | 8.6 % | 7.5 % | 9.1 % | 5.3 % |
| 2 FWS | Utilization | 0.995 ± 0.002 | 0.699 ± 0.025 | 0.764 ± 0.020 | 0.798 ± 0.018 | 0.717 ± 0.019 |
| | % Difference | 8.6 % | - 9.0 % | - 6.0 % | - 5.6 % | - 8.4 % |
| | Throughput | 0.926 ± 0.000 | 0.224 ± 0.005 | 0.233 ± 0.004 | 0.245 ± 0.006 | 0.227 ± 0.003 |
| | % Difference | 8.2 % | 9.3 % | 7.4 % | 8.9 % | 8.6 % |

**Table A2.** Beta Interarrival-time Distribution 30 * beta (0.75, 5.25)

| Case | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| 1 VWS | Utilization | 0.498 ± 0.007 | 0.996 ± 0.001 | 0.982 ± 0.002 | 0.980 ± 0.006 | 0.979 ± 0.003 |
| | % Difference | - 14.3% | 10.4 % | 9.2 % | 7.8 % | 6.5 % |
| | Throughput | 0.917 ± 0.016 | 0.228 ± 0.006 | 0.227 ± 0.005 | 0.232 ± 0.007 | 0.227 ± 0.005 |
| | % Difference | - 5.3 % | - 5.0 % | - 5.4 % | - 4.1 % | - 7.4 % |
| 2 FWS | Utilization | 0.949 ± 0.007 | 0.903 ± 0.017 | 0.930 ± 0.012 | 0.942 ± 0.008 | 0.892 ± 0.014 |
| | % Difference | 3.6 % | 17.6 % | 14.4 % | 11.5 % | 13.9 % |
| | Throughput | 0.885 ± 0.008 | 0.220 ± 0.005 | 0.222 ± 0.004 | 0.228 ± 0.006 | 0.217 ± 0.003 |
| | % Difference | 3.4 % | 7.3 % | 2.3 % | 1.3 % | 3.8 % |

**Table A3.** Erlang Interarrival-time Distribution ERLANG (1.875, 2)

| Case | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| 1 VWS | Utilization | 0.570 ± 0.005 | 0.996 ± 0.001 | 0.984 ± 0.001 | 0.982 ± 0.002 | 0.983 ± 0.002 |
| | % Difference | 8.2 % | 10.4 % | 9.5 % | 8.0 % | 7.0 % |
| | Throughput | 1.046 ± 0.008 | 0.263 ± 0.004 | 0.264 ± 0.004 | 0.260 ± 0.004 | 0.260 ± 0.004 |
| | % Difference | 8.1 % | 9.6 % | 10.0 % | 7.4 % | 6.1 % |
| 2 FWS | Utilization | 1.000 ± 0.001 | 0.627 ± 0.018 | 0.725 ± 0.013 | 0.797 ± 0.013 | 0.683 ± 0.020 |
| | % Difference | 9.2 % | - 18.4 % | - 10.8 % | - 5.7 % | - 12.8 % |
| | Throughput | 0.935 ± 0.000 | 0.226 ± 0.002 | 0.237 ± 0.003 | 0.242 ± 0.003 | 0.228 ± 0.002 |
| | % Difference | 9.2 % | 10.2 % | 9.2 % | 7.6 % | 9.1 % |

**Table A4.** Triangular Interarrival-time Distribution TRIA (0.01,1.85,9.39)

| Case | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| 1 VWS | Utilization | 0.579 ± 0.005 | 0.997 ± 0.001 | 0.990 ± 0.001 | 0.989 ± 0.002 | 0.990 ± 0.002 |
| | % Difference | 9.9 % | 10.5 % | 10.1 % | 8.8 % | 7.7 % |
| | Throughput | 1.063 ± 0.006 | 0.267 ± 0.004 | 0.264 ± 0.003 | 0.267 ± 0.004 | 0.266 ± 0.003 |
| | % Difference | 9.8 % | 11.3 % | 10.0 % | 10.3 % | 8.6 % |
| 2 FWS | Utilization | 1.000 ± 0.000 | 0.512 ± 0.018 | 0.644 ± 0.015 | 0.727 ± 0.019 | 0.590 ± 0.013 |
| | % Difference | 9.2 % | - 33.3 % | - 20.8 % | - 14.0 % | - 24.7 % |
| | Throughput | 0.935 ± 0.000 | 0.221 ± 0.002 | 0.236 ± 0.002 | 0.247 ± 0.002 | 0.229 ± 0.001 |
| | % Difference | 9.2 % | 7.8 % | 8.8 % | 9.8 % | 9.6 % |

**Table A5.** Normal Interarrival-time Distribution NORMAL (3.75, 1.25)

| Case | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| 1 VWS | Utilization | 0.581 ± 0.003 | 0.999 ± 0.000 | 0.998 ± 0.001 | 0.999 ± 0.001 | 0.999 ± 0.001 |
| | % Difference | 10.3 % | 10.8 % | 11.0 % | 9.9 % | 8.7 % |
| | Throughput | 1.066 ± 0.004 | 0.267 ± 0.002 | 0.265 ± 0.002 | 0.267 ± 0.003 | 0.267 ± 0.002 |
| | % Difference | 10.1 % | 11.3 % | 10.4 % | 10.3 % | 9.0 % |
| 2 FWS | Utilization | 1.000 ± 0.000 | 0.392 ± 0.006 | 0.515 ± 0.010 | 0.628 ± 0.017 | 0.457 ± 0.009 |
| | % Difference | 9.2 % | - 49.0 % | - 36.7 % | - 25.7 % | - 41.6 % |
| | Throughput | 0.935 ± 0.000 | 0.217 ± 0.001 | 0.236 ± 0.002 | 0.249 ± 0.002 | 0.230 ± 0.001 |
| | % Difference | 9.2 % | 5.9 % | 8.8 % | 10.7 % | 10.1 % |

**Table A6.** Uniform Interarrival-time Distribution UNIFORM (0.01, 7.49)

| Case | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| 1 VWS | Utilization | 0.568 ± 0.005 | 0.997 ± 0.001 | 0.989 ± 0.002 | 0.988 ± 0.002 | 0.991 ± 0.002 |
| | % Difference | 7.8 % | 10.5 % | 10.0 % | 8.7 % | 7.8 % |
| | Throughput | 1.054 ± 0.008 | 0.265 ± 0.004 | 0.261 ± 0.004 | 0.265 ± 0.005 | 0.263 ± 0.003 |
| | % Difference | 8.9 % | 10.4 % | 8.8 % | 9.5 % | 7.3 % |
| 2 FWS | Utilization | 1.000 ± 0.000 | 0.567 ± 0.020 | 0.702 ± 0.016 | 0.773 ± 0.019 | 0.636 ± 0.019 |
| | % Difference | 9.2 % | - 26.2 % | - 13.7 % | - 8.5 % | - 18.8 % |
| | Throughput | 0.935 ± 0.000 | 0.222 ± 0.002 | 0.235 ± 0.002 | 0.246 ± 0.002 | 0.230 ± 0.002 |
| | % Difference | 9.2 % | 8.3 % | 8.3 % | 9.3 % | 10.1 % |

**Table A7.** Deterministic Interarrival-time Distribution (3.75)

| Case | Criteria | Main Line | Line 1 | Line 2 | Line 3 | Line 4 |
|---|---|---|---|---|---|---|
| 1 VWS | Utilization | 0.581 ± 0.002 | 1.000 ± 0.000 | 1.000 ± 0.000 | 1.000 ± 0.000 | 1.000 ± 0.000 |
| | % Difference | 10.3% | 10.9 % | 11.2 % | 10.0 % | 8.8 % |
| | Throughput | 1.067 ± 0.001 | 0.267 ± 0.000 | 0.267 ± 0.000 | 0.267 ± 0.000 | 0.267 ± 0.000 |
| | % Difference | 10.2 % | 11.3 % | 11.3 % | 10.3 % | 9.0 % |
| 2 FWS | Utilization | 1.000 ± 0.000 | 0.349 ± 0.004 | 0.442 ± 0.008 | 0.541 ± 0.010 | 0.406 ± 0.006 |
| | % Difference | 9.2 % | - 54.6 % | - 45.6 % | - 36.0 % | - 48.2 % |
| | Throughput | 0.935 ± 0.000 | 0.214 ± 0.001 | 0.236 ± 0.001 | 0.253 ± 0.001 | 0.229 ± 0.001 |
| | % Difference | 9.2 % | 4.4 % | 8.8 % | 12.4 % | 9.6 % |