MidTerm - Modern Physics for Engineers

October 31, 1997

Halloween

Name:____________________________

S.S. No.:__________________________

Do all work in this exam booklet. Use the backs of the sheets if necessary. The equation sheet given to you in class on Wednesday is the second sheet of this exam. No calculators or other aids are allowed. Leave all answers in a form that can be easily computed with a calculator if you had one. If the problem requires an integration, leave the answer as an integral. It is not necessary to evaluate the integral.

Good luck.

Remember, class on this Monday, Nov. 3, is cancelled.

Modern Physics for Engineers

Midterm - Fall '97

EQUATION SHEET

Special Relativity

$$x' = \gamma(x + \beta ct) \qquad p' = \gamma(p + \beta E/c)$$

$$t' = \gamma(t + \beta x/c) \qquad E' = \gamma(E + \beta cp)$$

$$E^2 = p^2c^2 + m^2c^4 \qquad KE = E - mc^2$$

$$p_i = p_f \qquad \nu' = \nu\sqrt{\frac{1-\beta}{1+\beta}}$$

$$E_i = E_f \qquad v' = \frac{v+u}{1+uv/c^2}$$

General Relativity

$$\frac{\Delta\nu}{\nu} = \frac{\phi_2 - \phi_1}{c^2} \qquad \theta = \frac{4GM}{Rc^2}$$

Quantum Mechanics

$$I = \sigma T^4 \qquad R(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

$$E = h\nu \qquad p = h/\lambda \qquad L = n\hbar$$

$$-i\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t)$$

$$E\psi(x) = -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)$$

$$p \rightarrow -i\hbar\frac{\partial}{\partial x} \qquad E \rightarrow i\hbar\frac{\partial}{\partial t} \qquad x \rightarrow x$$

$$\begin{aligned} E &= -\frac{mZ^2e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} \\ &= \frac{\hbar^2\pi^2n^2}{2mL^2} \\ &= (n + \frac{1}{2})\hbar\omega \end{aligned}$$

Relativity Short Answers - 5 points each

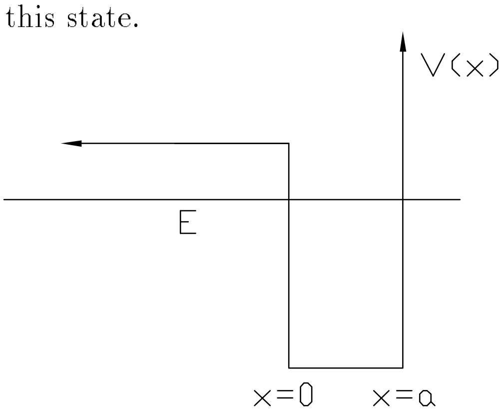
1. In a recent FAX News that I received, the editors asked me to think about "If you were in a vehicle going at the speed of light, what would happen when you turned on the headlights?" The proper question would be "If you were in a vehicle moving toward the earth at .999c and turned on your headlights, at what velocity would a person standing on earth measure the light coming at her?" How fast would she measure it?

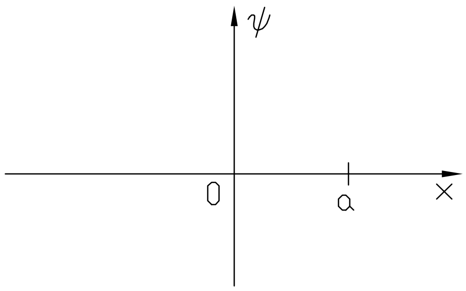
2. You notice that the clock in a space ship that is going past you is ticking at only half the rate your clock is (after you have corrected for the different distances the clock is away from you for each tick). Presuming that both clocks are accurate, how fast is the ship moving past you?

3. You and two of your friends are in an office building and each of you has a watch. You are standing on the third floor, your first friend is in an elevator accelerating upward at 1 g and your other friend is in another elevator accelerating downward at 1 g (both of these accelerations are in comparison to you). Which of your watches would most accurately match the watch of an observer far out in space?

Quantum Mechanics Short Answers - 5 points each

1. The energy for the second excited state is shown on the potential shown below. On the axis to the right, draw approximately the wave function for this state.





2. Give an example of an experiment or a phenomenon that illustrates the wave nature of an electron.

3. A particle of mass m is trapped in a hole of size a. Give a lower limit on the RMS (root mean square) of the momentum Δp assuming that the hole is stationary. Also give a lower limit on the average kinetic energy $\left\langle \frac{p^2}{2m} \right\rangle$.

Problem 1

The wave function for a particle in a certain potential is given by

$$\psi(x) = \begin{cases} 0; & \text{if } x < 0 \\ Axe^{-ax}; & \text{if } x \geq 0 \end{cases}$$

where a is given.

1. (**5 points**) Write the equation that you would use to evaluate the the constant A.

2. (**5 points**) Write the equation that you would use to evaluate the average position of this particle assuming A is known.

3. (**5 points**) Write the equation that you would use to evaluate the average of the square of the momentum for this particle again assuming A is known.

Bonus - 5 points extra credit! Do only if you have time! Sketch approximately the potential that could give such a wave function. Show approximately the level of the total energy on that potential.

Problem 2

A neutral kaon (K^0) is hadronic particle (a definition of which we will get by the end of the quarter) which has a rest mass of 494 MeV/c^2. It is moving in the lab with a velocity $v = .999c$. After a while the kaon decays into two pions, π^+ and π^-, each with mass 139 MeV/c^2. In the kaon's rest frame, the π^+ goes parallel to the kaon's direction of flight and the π^- goes backwards.

1. (**5 points**) Give formulae for the total energy and the momentum of the kaon in the laboratory.

2. (**10 points**) Write down the equations that you would need to solve to determine each pion's energy (E_{π^+} and E_{π^-}) and momentum (p_{π^+} and p_{π^-}) in the laboratory given the kaon's laboratory energy E_K and momentum p_K. Make sure you have as many equations as unknowns.