MidTerm II - Modern Physics for Engineers

November 1, 1999

Test No. ______________________

Name:_________________________

S.S. No.:_______________________

Do all work in this exam booklet. Use the backs of the sheets if necessary. Please compute all numerical answers using your calculator if necessary. For any equations that require integrals, simplify the integrals, but leave as integrals.

Good luck.

EQUATION SHEET

$$\int f\, dV = \int f\, dx\, dy\, dz = \int f\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$V = \frac{1}{2} m\omega^2 x^2 \qquad\qquad V = -\frac{kq^2}{r}$$

$$\begin{aligned} E\psi &= -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + V(x)\psi \\ &= -\frac{\hbar^2}{2mr^2}\left(\frac{\partial}{\partial r} r^2 \frac{\partial\psi}{\partial r} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\psi}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}\right) + V(x)\psi \end{aligned}$$

$$p = \frac{h}{\lambda}$$

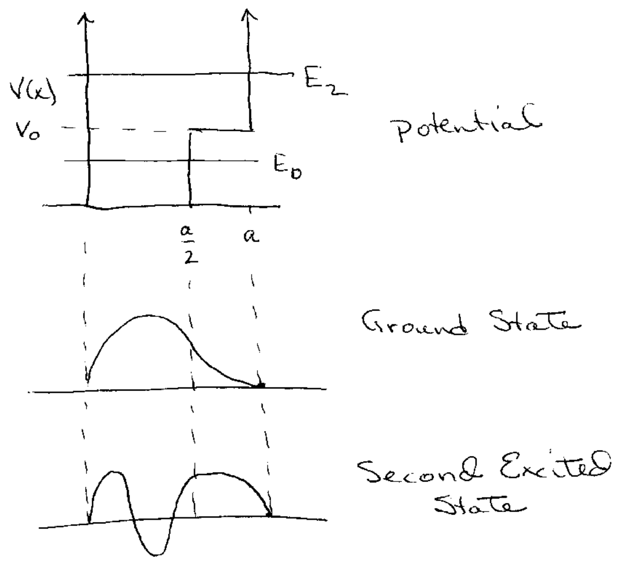
$$\hbar = 1.055 \times 10^{-34}\ \mathrm{J\cdot s} = 4.136 \times 10^{-15}\ \mathrm{eV\cdot sec}$$

Problem I Short Answers - 5 points each

1. What does "degeneracy" mean for quantum states?

If two or more different wave function have the same energy, then that energy is "degenerate". The "degeneracy" of an energy level is the number of different wave functions that have that energy.

2. An electron is captured in a potential of the form: $V = \infty$ for $x \leq 0$ and for $x \geq a$, $V = 0$ for $0 < x < a/2$ and $V = V_0$ for $a/2 < x < a$. Draw the potential, the wave function for the ground state where $E < V_0$, and the second excited state where $E > V_0$.



Problem I (continued)

3. Describe "Compton scattering". Tell whether it illustrates the wave or particle nature of light and why.

Compton scattering is the scattering of light off of electrons. It illustrates the particle nature of light (*i.e.*, that the quanta of light have a definite energy and momentum). As the scattering angle gets larger and larger, the wave length of the scattered light gets longer and longer indicating that it has lost momentum to the electron. If light were a classical wave, the electron would radiate light synchronously with the wave. Thus the wave length of the scattered light would be the same as the incoming light and would be independent of angle.

4. A laser can produce a light pulse that is 5 cm long. What is the minimum dispersion (error in wavelength) of such a pulse if the laser operates at 330 nm (a nitrogen laser)?

$$\begin{aligned}
\Delta p \Delta x &\geq \frac{\hbar}{2} \\
p &= \frac{h}{\lambda} \\
\frac{\Delta p}{\Delta \lambda} &\approx \left| \frac{dp}{d\lambda} \right| \\
&= \frac{h}{\lambda^2} \\
\Delta \lambda &\approx \frac{\lambda^2 \Delta p}{h} \geq \frac{\lambda^2}{4\pi \Delta x}
\end{aligned}$$

Problem II - 20 points

A particle is confined in the XY plane by the following potential:

$$V(x,y) = \begin{cases} 0 & \text{for } -a \le x \le a \text{ and } -a \le y \le a \\ \infty & \text{elsewhere.} \end{cases}$$

A solution to Schrödinger's equation for this potential is

$$\psi(x,y) = \begin{cases} A\cos\frac{\pi x}{2a}\sin\frac{\pi y}{a} & \text{where } V = 0 \text{ and} \\ 0 & \text{elsewhere.} \end{cases}$$

1. Give an equation which, when solved, will determine the value of A. **Do not solve the equation!**

$$A = \frac{1}{\sqrt{\int_{-a}^{a}\int_{-a}^{a}\cos^2\frac{\pi x}{2a}\sin^2\frac{\pi y}{a}\,dx\,dy}}$$

2. What is the energy of this state?

$$E\psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right)$$

$$E = \frac{5\hbar^2}{8\pi m a^2}$$

Problem II Continued

3. Is this the ground state or an excited state? If an excited state, which one? Why?

This is the first excited state because it crosses the axis once (on the line at $y = 0$).

4. Write out an equation that could be solved to determine the probability of finding the particle in the region $x > 0$. **Do not solve the equation!**

$$P = \int_{-a}^{a} \int_{0}^{a} A^2 \cos^2 \frac{\pi x}{2a} \sin^2 \frac{\pi y}{a} \, dx dy$$