MidTerm I - Modern Physics for Engineers

October 12, 1998

Test No. ______________________

Name: ______________________

S.S. No.: ______________________

Do all work in this exam booklet. Use the backs of the sheets if necessary. The equation sheet given to you in class on Friday is reproduced below. Please compute all numerical answers using your calculator if necessary. If the problem requires an integration, leave the answer as an integral. It is not necessary to evaluate the integral.

Good luck.

EQUATION SHEET

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c} \qquad f\lambda = c$$

$$x' = \gamma(x + \beta ct) \qquad p' = \gamma(p + \beta E/c)$$

$$t' = \gamma(t + \beta x/c) \qquad E' = \gamma(E + \beta cp)$$

$$E^2 = p^2c^2 + m^2c^4 \qquad KE = E - mc^2$$

$$\vec{p}_i = \vec{p}_f \qquad f' = f\sqrt{\frac{1-\beta}{1+\beta}}$$

$$E_i = E_f \qquad v' = \frac{v+u}{1+uv/c^2}$$

Problem I Short Answers - 5 points each

1. You observe a space ship going by at a speed of .995 c. You measure the space ship to be 100 m long. How long would it be if you measured it when it were stopped?

$$\beta = .995 \longrightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} = 10.01$$

   Moving objects measure shorter by a factor of $\frac{1}{\gamma}$

$$\begin{aligned} \longrightarrow \triangle x_{rest} &= \gamma \triangle x_{moving} \\ &= 10.01 \times 100 \text{ m} = 1001 \text{ m} \end{aligned}$$

2. On the same moving space ship, you observe a nuclear reactor that emits 10^3 neutrons/sec. How many neutrons/sec would be emitted if the space ship were stopped?

Moving clocks tick slower. The reactor is a clock. Thus in its rest frame it would give more "ticks" than measured in the moving frame by a factor of γ.

$$10^3 \gamma = 10^4 \text{ neutrons/sec}$$

3. As the space ship is coming toward you, you observe a light which you measure to have a wave length of 500 nm. What is the wave length that a person on the space ship would measure?

Coming toward you, the frequency increases which means the wave length decreases.

$$\lambda_{lab} = \sqrt{\frac{1-\beta}{1+\beta}}\lambda_{ship}$$

$$\longrightarrow \lambda_{ship} = 500\sqrt{\frac{1.995}{.005}} = 9987 \text{ nm} = 10\ \mu\text{m}$$

4. What is the wave length that you would measure after the space ship has passed you and is going away?

$$\lambda_{lab} = \sqrt{\frac{1+\beta}{1-\beta}}\lambda_{ship} \text{ for objects going away}$$
$$= \sqrt{\frac{1.995}{.005}} 10\ \mu\text{m} = 200\ \mu\text{m}$$

Problem II - 20 points

A 1.2 MeV photon (particle with 0 mass) that comes from Co^{60} decay hits an electron which is initially at rest. The photon scatters at an angle of 45° with respect to its original direction. Draw a picture of the reaction, write down the relevant conservation laws, and calculate the energy of the scattered photon. (Note: the electron does not come out at an angle of $-45°$ but at some other angle ϕ.) Remember:

$\sin^2\psi + \cos^2\psi = 1$

$\sin 45° = \cos 45° = \frac{\sqrt{2}}{2} \approx .707$

$m_e = .511$ MeV/c^2

Diagram:

Conservation of energy:

$$
\begin{aligned}
E_i &= E_f \\
p_0 c + m_e c^2 &= E_\gamma + E_e \\
1.2 \text{ MeV } + .511 \text{ MeV} &= E_\gamma + E_e \\
1.711 \text{ MeV} &= E_\gamma + E_e
\end{aligned}
$$

Y momentum conservation:

$$0 = p_\gamma^y + p_e^y \longrightarrow p_\gamma \sin 45 = p_e \sin \phi$$

X momentum conservation:

$$1.2 \text{ MeV/c} = p_\gamma^x + p_e^x = p_\gamma \cos 45 + p_e cos\phi$$

Kinematic relations:

$$
\begin{aligned}
E_\gamma &= p_\gamma c \\
E_e &= \sqrt{p_e^2 c^2 + m_e^2 c^4}
\end{aligned}
$$

Solution:

$$
\begin{aligned}
p_\gamma c &= \frac{m_e p_0 c^3}{p_0 c(1 - \cos 45) + m_e c^2} \\
&= .7109 \text{ MeV}
\end{aligned}
$$