Final - Modern Physics for Engineers

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Name:____________________________

S.S. No.:__________________________

Do all work in this exam booklet. Use the backs of the sheets if necessary. Two 8.5" x 11" sheets of notes is permissible. Please use your calculator. If integrals are needed that are not given on the next page, there is a CRC handbook available at the front of the classroom for your use. If you cannot find the integral in that, just leave it. On some questions, there are multiple parts. If you need the answer from some previous part to do a calculation, but were unable to obtain it, assume some reasonable value (informing me what you are doing) and use it.

Good luck!

Have a nice winter break.

Page of Useful Things

Useful Constants:

c = 3 x 10^8 m/sec

h = 4.14 x 10^{-15} eV sec

1 u = 931.5 MeV/c^2

Masses:

$^1n_0$ = Neutron = 1.008665 u

$^1H_1$ = Hydrogen = 1.007825 u

$^2H_1$ = Deuterium = 2.014102 u

$^3H_1$ = Tritium = 3.016049 u

$^4He_2$ = Helium-4 = 4.002603 u

Useful integrals:

$$\int_0^\pi \cos^{2n}\theta \sin\theta \; d\theta = \frac{2}{n+1}$$

$$\int_0^\pi \cos^{2n+1}\theta \sin\theta \; d\theta = 0$$

$$\int_0^\infty x^n e^{-ax} \; dx = \frac{n!}{a^{n+1}}$$

Short Answers – 10 points each

For each of the following, give a one to three sentence answer.

1. Systems with closed shell (*i.e.* multi-particle systems with all of the degenerate energy levels filled and where there is a sizeable gap between the degenerate levels and the next levels) seem to be particularly stable in nature. We have seen several examples of this this year. Name two such systems.

2. What are the quantum number of the seven electrons of a nitrogen atom?

3. $^{8}Be_4$ decays almost immediately into 2 $^{4}He_2$ nuclei. What does this tell you about the mass of $^{8}Be_4$ relative to $^{4}He_2$?

4. Name two predictions of the "Big Bang" model of the universe which can be tested.

5. State the "Pauli Exclusion Principle" and give an example of it.

6. The lowest three levels in nuclear systems are the 1s, 1p, and (2s,1d) levels. Give the quantum numbers of the three protons and the four neutrons in $^{7}Li_3$.

7. Our sun has and has had for the last few billion years quite a stable size. Yet it is giving off huge amounts of energy. Since the size is stable, the energy does not come from gravitational collapse. What is the source of energy?

8. How is the contact potential between two metals related to the work functions of the individual metals?

9. Neutrinos are created in the "Big Bang" model of the universe when the temperature was above 6 billion degrees. Are these neutrinos still around today? If not, where did they go? If so, what is their temperature today?

10. One of the mysteries of the pre-quantum mechanics era was the source of the lines seen in the X-Ray spectra of atoms. It was noted that the frequency of the shortest wave length line, know as the K_α line, increased like the square of the Z of the nucleus. What atomic transition(s) does the K_α line correspond to?

Problem 1:

In a fusion reactor, one possible combination of fuels is equal amounts of deuterium and tritium. The primary reaction would be:

$$^{2}H_1 \; + \; ^{3}H_1 \; \rightarrow \; ^{4}He_2 + ???$$

1. [**Points 5**] What must the unknown particle be?

2. [**Points 10**] How much energy is release for each $^{4}He_2$ made? (Mass tables can be found at beginning of exam.)

3. [**Points 10**] If both the tritium and deuterium are at rest when they fuse, what is the energy and the momentum of the helium nucleus? Write down all of the necessary equations to solve this problem, *but do not solve it!*

Problem 2

The 2p ($m_l = 0$) wave function for hydrogen is given by the product of three functions, $\psi = R(r)\Theta(\theta)\Phi(\phi)$, where:

$$R(r) = \frac{1}{\sqrt{24a_0^3}}\frac{r}{a_0}e^{-r/2a_0}$$
$$\Theta(\theta) = \sqrt{\frac{3}{2}}\cos\theta$$
$$\Phi(\phi) = \frac{1}{\sqrt{2\pi}}$$

[**Points 25**] Find the average of $\cos^2\theta$ for a particle with this wave function. [Note: the operator for $\cos^2\theta$ is $\cos^2\theta$.]

Problem 3

In any system that gives off photons, the most probable transitions are those that change the angular momentum quantum number by ± 1. For example, the 3s hydrogen state will de-excite by going to the 2p state and then to the 1s state.

1. [**Points 15**] For this sequence (3s $\rightarrow$ 2p $\rightarrow$ 1s), what are the energies of the photons given off for hydrogen?

2. [**Points 5**] What would be the sequence for de-exciting from the 3d state?

3. [**Points 5**] Sometimes, because of this de-excitation rule, atoms get stuck in meta-stable states. For example, since there is no 1p state, there is no place for the 2s state to go to. Show a sequence that brings the 6d state to the 2s state and not to the 1s state.

Problem 4

For the one dimensional potential:

$$V(x) = \begin{cases} -\frac{V_0}{x} & x > 0; \text{ and} \\ \infty & x \leq 0. \end{cases}$$

The ground state wave function is

$$\psi(x) = \begin{cases} 0 & x \leq 0; \text{ and} \\ 2\sqrt{\alpha^3} x e^{-\alpha x} & x > 0. \end{cases}$$

where $\alpha = mV_0/\hbar^2$. [Note that the zero in this wave function at zero does not indicate an excited state, but merely that the potential is infinite at this point.]

1. [**Points 5**] Sketch both the potential and the wave function for first excited state for this potential.

2. [**Points 10**] Find the average position $\langle x \rangle_{\text{avg}}$ of a particle in the ground state.

Problem 4 - Continued

3. [**Points 10**] Calculate the energy of a particle in the ground state.