Compton Scattering

Photon-Electron Scattering

An incoming photon of momentum p_i hits an electron at rest. The outgoing photon has a momentum p_f and deviates from the initial momentum by an angle θ. The outgoing electron has a momentum p_e. Momentum conservation gives:

$$\begin{aligned} \vec{p}_i =& \vec{p}_f + \vec{p}_e \\ \rightarrow \vec{p}_e =& \vec{p}_i - \vec{p}_f \\ \rightarrow p_e^2 =& p_i^2 + p_f^2 - 2p_i p_f \cos\theta \end{aligned}$$

Energy conservations gives:

$$\begin{aligned} m_e c^2 + E_i =& E_f + E_e \\ \rightarrow E_e =& E_i - E_f + m_e c^2 \end{aligned}$$

$$E_i = p_i c \qquad E_f = p_f c$$

$$\begin{aligned} E_e^2 =& p_e^2 c^2 + m_e^2 c^4 \\ =& E_i^2 + E_f^2 - 2E_i E_f + m_e^2 c^4 + 2E_i m_e c^2 - 2E_f m_e c^2 \\ \rightarrow p_e^2 c^2 =& p_i^2 c^2 + p_f^2 c^2 - 2p_i p_f c^2 + 2m_e c^3 (p_i - p_f) \\ p_i^2 + p_f^2 - 2p_i p_f \cos\theta =& p_i^2 + p_f^2 - 2p_i p_f + 2m_e c(p_i - p_f) \\ \rightarrow -p_i p_f \cos\theta =& - p_i p_f + m_e c(p_i - p_f) \end{aligned}$$

With the final result being:

$$\frac{1}{m_e c}(1 - \cos\theta) = \frac{1}{p_f} - \frac{1}{p_i}$$

Thus, the final momentum of the photon changes with outgoing angle. This is in contrast to the classical picture of electron/light scattering which would say that the incoming electromagnetic wave causes the electron to oscillate at the same frequency as the wave itself and thus radiate at that frequency to all angles. Thus classically there would be no change in energy (photon energy is proportional to its frequency). The classical result is correct as long as $p_i c \ll m_e c^2$.