Spin vectors

For a particle of momentum p^μ and spin s^μ:

$$s^2 = -1 \quad \text{and} \quad s \cdot p = 0$$

For the right-handed spin 1/2 particle travelling in the $+z$ direction ($m_z = +1/2$):

$$s_\mu = \left(\frac{p}{m}, 0, 0, \frac{E}{m}\right) \qquad p_\mu = (E, 0, 0, p)$$

For the left-handed spin 1/2 particle travelling in the $+z$ direction ($m_z = -1/2$):

$$s_\mu = \left(-\frac{p}{m}, 0, 0, -\frac{E}{m}\right) \qquad p_\mu = (E, 0, 0, p)$$

For the right-handed spin 1/2 particle travelling in the $-z$ direction ($m_z = -1/2$):

$$s_\mu = \left(\frac{p}{m}, 0, 0, -\frac{E}{m}\right) \qquad p_\mu = (E, 0, 0, -p)$$

For the left-handed spin 1/2 particle travelling in the $-z$ direction ($m_z = +1/2$):

$$s_\mu = \left(-\frac{p}{m}, 0, 0, \frac{E}{m}\right) \qquad p_\mu = (E, 0, 0, -p)$$

Useful γ^5 relationships

$$\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5 \qquad \gamma^5(\not{p} + m) == (-\not{p} + m)\gamma^5 \qquad \left(\gamma^5\right)^2 = \mathcal{I}$$

Spins and Colliding Beams

$$
\begin{array}{ccl}
\vec{P} & \vec{P} & \\
\Longrightarrow & \Longleftarrow & \\
\rightarrow & \leftarrow & \text{Both particles right-handed} \\
\vec{s} & \vec{s} & \text{Total spin} = 0
\end{array}
$$

$$
\begin{array}{ccl}
\vec{P} & \vec{P} & \\
\Longrightarrow & \Longleftarrow & \text{One right-handed and one left-handed} \\
\rightarrow & \rightarrow & \text{Total spin} = 1 \\
\vec{s} & \vec{s} &
\end{array}
$$